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The effects of thermal photons on Fisher information dynamics for a dispersive Jaynes-Cummings model

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Abstract: We give the exact analytical description for the dynamics of an atom coupled to field mode in dissipative cavity by using the master equation. The effects of thermal photon ($n\neq 0$) on entanglement and coherence loss are achievement. It is found that the field is inhibited from going into a pure state and coherence is lost faster than in the case of zero temperature ($n=0$).

Keywords: Entanglement; Quantum Fisher information; von Neumman entropy

1 Introduction

One of the simplest models that describe the interaction of quantized radiation field with matter is the Jaynes-Cumming (JC) model [1]. It has been successfully used to describe many physical systems [2,3] in cavity electrodynamics. When amplitude damping is taken, the exact solutions for the standard JC model [4] and for a dispersive JC model (in which, the far-off resonance limit for atom-field interaction is considered) [5,6], can be obtained under certain conditions.

In recent years attention has been focused on investigating open quantum systems because of their important role in building quantum computers and processing quantum information [7-9].

The entanglement between atomic and the field subsystems is one of the most interesting aspects of the dynamics of JC model. The quantum entanglement is emerged by the development of statistical correlations between the two subsystems, as a result of interaction. Many papers have focused on properties of entanglement [10-18] for the standard JC model without damping or with damping to reservoirs at zero temperature [5,19-21] or with phase damping cavity [22]. Also, entanglement generation and entropy growth due to essential decoherence in the JC model have been studied [23]. These studies used Shannon and von Neumman entropies and its variants to deal with these aspects. These entropies have been used successfully as quantifiers of entanglement, however, there are other information quantifiers. One of these quantifiers is the Fisher Information measure [24] that has been employed in many different aspects [25]. It has been recently used in the field of dynamics of a trapped ion in a laser field [26] as indicator to the quantum-classical limit.

In this article, we study the effect of thermal photon due to finite cavity temperature on an atom interacting with a single mode of the field. Specifically, we consider single-frequency photons of the cavity to be far from resonance with the atomic transition, i.e., the dispersive case. In section 2, the master equation is solved for any initial cavity field by using superoperator method. In section 3, we use partial entropies for the atom and the field, the total entropy and atomic quantum Fisher information to study the coherence properties and quantify entanglement. Concluding remarks are presented in section 4.

2 Dispersive regime in the presence of dissipation

The Hamiltonian of a two-level atom interacting with a single-mode quantized field under the rotating wave approximation (RWA) is given by

$$\hat{H} = \frac{\omega_0}{2}\hat{\sigma}_z + \omega\hat{a}_+\hat{a} + \kappa(\hat{a}_+\hat{\sigma}_- + \hat{a}\hat{\sigma}_+), \qquad (1)$$

where ω_0 is the transition frequency between the excited and ground states of the atom, $\hat{\sigma}_z = |e\rangle\langle e| - |g\rangle\langle g|$ is the population inversion operator, where the symbols $|e(g)\rangle$ refer to the excited (ground) state of the atom, $\hat{\sigma}_+$ and $\hat{\sigma}_-$ are the raising and lowering operators and ω is the frequency of the cavity field whose creation (annihilation) operator is $\hat{a}^+$ ($\hat{a}$) that satisfy the commutator relation $[\hat{a}, \hat{a}^+] = 1$, the parameter κ is the atom-field coupling constant, with the detuning parameter $\delta = \omega_0 - \omega.$ The dispersive limit of a one-photon process is obtained when the interaction Hamiltonian $\hat{H}_I$ can be considered as a small perturbation in the following regime $\frac{|\delta|}{\kappa} \geqslant \sqrt{n+1}$ for any "relevant" photon number n [5,27,28]. Under this condition, we get an effective interaction Hamiltonian in the form

$$\hat{H}_{eff} = \lambda(\hat{a}_+\hat{a}\,\sigma_z + |e\rangle\langle e|), \quad \lambda = \frac{\kappa^2}{\delta}. \qquad (2)$$

In the interaction picture and in the dispersive approximation, the master equation that governs the dynamics of a two-level atom interacting with an electromagnetic field in cavity and coupled to a reservoir at non-zero temperature [29] is given by

$$\frac{\partial\hat{\rho}}{\partial t} = -i[H_{eff}, \hat{\rho}] + \gamma(\bar{n}+1)(2\hat{a}\hat{\rho}\hat{a}^+ - \hat{a}^+\hat{a}\hat{\rho} - \hat{\rho}\hat{a}^+\hat{a}) + \gamma\bar{n}(2\hat{a}^+\hat{\rho}\hat{a} - \hat{a}\hat{a}^+\hat{\rho} - \hat{\rho}\hat{a}\hat{a}^+) \qquad (3)$$

Where γ is the decay rate, $\bar{n}$ the average number of thermal photons of the reservoir. This model has been solved for the case when $\bar{n} = 0$ in [5,6] and for the case $\lambda = 0$ in [6]. But here this model is studied for ($\lambda \neq 0, \ \bar{n} \neq 0$) in a generalization of earlier investigations. This is done to study the effects of black body photons on this system [30].

The density operator can be cast as follows:

$$\hat{\rho}(t) = \hat{\rho}_{++}(t)|e\rangle\langle e| + \hat{\rho}_{+-}(t)|e\rangle\langle g| + \hat{\rho}_{-+}(t)|g\rangle\langle e| + \hat{\rho}_{--}(t)|g\rangle\langle g| \qquad (4)$$

Equations of motion for each one of the matrix elements in (4) can be obtained from the master equation (3). As an example, the diagonal term $\hat{\rho}_{\pm\pm}(t)$ as done, for instance in [6] into the form:

$$\frac{\partial\hat{\rho}_{\pm\pm}(t)}{\partial t} = (-2\gamma\bar{n}\hat{I} + \hat{L}_1 + \hat{L}_2 + \pi^{\pm}\hat{M} + \pi^{\mp}\hat{N})\hat{\rho}_{\pm\pm}(0). \qquad (5)$$

with the superoperators

$$\hat{L}_1\hat{\rho} = 2\gamma(\bar{n}+1)\hat{a}\hat{\rho}\hat{a}^+, \qquad \hat{L}_2\hat{\rho} = 2\gamma\bar{n}\hat{a}^+\hat{\rho}\hat{a},$$
$$\hat{M}\hat{\rho} = \hat{a}^+\hat{a}\hat{\rho}, \qquad \hat{N}\hat{\rho} = \hat{\rho}\hat{a}^+\hat{a}, \quad \pi^{\pm} = -\gamma(2\bar{n}+1) \mp i\lambda,$$

and $\hat{I}$ is the identity operator. The superoperators $\hat{L}_1, \hat{L}_2, \hat{M}$ and $\hat{N}$ obeying the commutation relations displayed in ref [6]

$$[\hat{L}_2, \hat{M}]\hat{\rho} = -\hat{L}_2\hat{\rho}, \quad [\hat{L}_2, \hat{N}]\hat{\rho} = -\hat{L}_2\hat{\rho},$$
$$[\hat{L}_1, \hat{M}]\hat{\rho} = \hat{L}_1\hat{\rho}, \quad [\hat{L}_1, \hat{N}]\hat{\rho} = \hat{L}_1\hat{\rho}, \quad [\hat{M}, \hat{N}]\hat{\rho} = 0,$$

and

$$[\hat{L}_1, \hat{L}_2]\hat{\rho} = -4\gamma^2(\bar{n}^2+\bar{n})(\hat{M}+\hat{N}+1)\hat{\rho} \quad [\hat{L}_2, [\hat{L}_2, \hat{L}_1]]\hat{\rho} = 8\gamma^2(\bar{n}^2+\bar{n})\hat{L}_2\hat{\rho}$$

From the above equation, we can write the formal solution of (5) as

$$\hat{\rho}_{\pm\pm}(t) = \exp(-2\gamma\bar{n}t)\exp\left[\left(\hat{L}_1 + \hat{L}_2 + \pi^{\pm}\hat{M} + \pi^{\mp}\hat{N}\right)t\right]\hat{\rho}_{\pm\pm}(0). \qquad (6)$$

We can factorize the exponential in Eq (13) as

$$\hat{\rho}_{++}(t) = e^{2\gamma\bar{n}t + \theta_0(t)}e^{\theta_1(t)\hat{L}_2}e^{\theta_2(t)\pi^+\hat{M} + \theta_3(t)\pi^-\hat{N}}e^{\theta_4(t)\hat{L}_1}\hat{\rho}_{++}(0), \qquad (7)$$

$$\hat{\rho}_{--}(t) = e^{2\gamma\bar{n}t + \theta_0(t)}e^{\theta_1(t)\hat{L}_2}e^{\theta_3(t)\pi^-\hat{M} + \theta_2(t)\pi^+\hat{N}}e^{\theta_4(t)\hat{L}_1}\hat{\rho}_{--}(0). \qquad (8)$$

For the initial conditions $\theta_m(0) = 0$ for $m = 0..4,$ the solution for the θ' s given by [30]:

$$\theta_1(t) = \theta_4(t) = \frac{1}{2\gamma\bar{n}}\frac{N_t}{(1+N_t)},$$
$$\theta_2(t) = \frac{1}{\pi^+}[(-\gamma + i\lambda)t - \ln(1+N_t)] = \theta_3^*(t),$$
$$\theta_0(t) = 2\gamma\bar{n}t - \ln(1+N_t), \quad N_t = \bar{n}(1 - e^{-2\gamma t}). \qquad (9)$$

For the off diagonal operators $\hat{\rho}_{+-}(t)$ and $\hat{\rho}_{-+}(t)$, the master equations are given by

$$\frac{\partial\hat{\rho}_{\pm\mp}(t)}{\partial t} = [-(2\gamma\bar{n} \pm i\lambda)\hat{I} + \hat{L}_1 + \hat{L}_2 + \hat{J}^{\pm}]\hat{\rho}_{\pm\mp}(0), \qquad (10)$$

where

$\hat{J}_{\pm}\hat{\rho} = \pi_{\pm}(\hat{M} + \hat{N})\hat{\rho}.$

We get the solution of off diagonal elements in the form

$$\hat{\rho}_{\pm\mp}(t) = e^{(\mp i\lambda - 2\gamma\bar{n})t + \phi_0^{\pm}(t)} e^{\phi_1^{\pm}(t)\hat{L}_2} e^{\phi_2^{\pm}(t)\hat{J}^{\pm}} e^{\phi_3^{\pm}(t)\hat{L}_1} \hat{\rho}_{\pm\mp}(0). \tag{11}$$

The corresponding functions $\phi^{\pm}$ that satisfy $\phi_m^{\pm}(0) = 0$ are given by [30]:

$$\phi_0^{\pm}(t) = v_1^{\pm}t - \ln\left(\frac{v_1^{\pm}e^{2\xi^{\pm}t} - v_2^{\pm}}{2\xi^{\pm}}\right), \quad \phi_1^{\pm}(t) = \phi_3^{\pm}(t) = \frac{1 - e^{2\xi^{\pm}t}}{v_2^{\pm} - v_1^{\pm}e^{2\xi^{\pm}t}},$$
$$\phi_2^{\pm}(t) = \frac{\xi^{\pm}t}{\pi^{\pm}} - \frac{1}{\pi^{\pm}}\ln\left(\frac{v_1^{\pm}e^{2\xi^{\pm}t} - v_2^{\pm}}{2\xi^{\pm}}\right), \tag{12}$$

where

$$\xi^{\pm} = \sqrt{\pi^{\pm 2} - 4\gamma^2(\bar{n}^2 + \bar{n})}, \quad v_1^{\pm} = -\pi^{\pm} + \xi^{\pm}, \quad v_2^{\pm} = -\pi^{\pm} - \xi^{\pm}$$

eqs. (6) and (11) give the exact solutions for the density matrix elements.

In the following section we apply these time-dependent analytical solutions for the matrix elements for a specified initial cavity field to calculate the time evolution of some properties of this model especially coherence and entanglement through different quantifiers.

3 Temporal evolution of information quantifiers

In this section we study the effect of black body photons of the reservoir on entropies and atomic quantum Fisher information when the atom is prepared in a coherent superposition of its states and the cavity mode in coherent state. Thus, the initial state of the system may be expressed as follows:

$$|\psi_{AF}\rangle = (c_+|e\rangle + c_-|g\rangle)|\alpha\rangle, \quad |c_+|^2 + |c_-|^2 = 1, \quad |\alpha\rangle = \sum_{n=0}^{\infty} e^{-\frac{|\alpha|^2}{2}} \frac{\alpha^n}{\sqrt{n!}}|n\rangle \tag{13}$$

then the two subsystems start to interact at t=0. The elements (6) and (11) of the density operator are initially $\hat{\rho}_{++}(0) = |c_+|^2 |\alpha\rangle\langle\alpha|$, $\hat{\rho}_{--}(0) = |c_-|^2|\alpha\rangle\langle\alpha|$, $\hat{\rho}_{+-}(0) = c_+c_-^*|\alpha\rangle\langle\alpha|$ and $\hat{\rho}_{-+}(0) = c_-c_+^*|\alpha\rangle\langle\alpha|.$ Therefore, the matrix elements of the density operator are given as [30]:

$$\rho_{\pm\pm}(t) = \frac{e^{(z_{++}(t)-1)|\alpha^2|}|c_{\pm}|^2}{(1+N_t)} \sum_{m,n,\ell=0} \frac{\tilde{\alpha}_{\pm}(t)^m(\tilde{\alpha}_{\pm}^*(t))^n\sigma(t)^{\ell}}{\ell!\sqrt{m!n!}}(\hat{a}^+)^{\ell}|m\rangle\langle n|(\hat{a})^{\ell} \tag{14}$$

and

$$\rho_{\pm\mp}(t) = c_{\pm}c_{\mp}^* e^{(z_{+-}(t)-1)|\alpha^2| + \chi^{\pm}(t)} \sum_{m,n,\ell=0} \frac{(\alpha e^{\eta^{\pm}})(\alpha^* e^{\eta^{\pm}})^n\omega_{\pm}^{\ell}}{\ell!\sqrt{m!n!}}(\hat{a}^+)^{\ell}|m\rangle\langle n|(\hat{a})^{\ell} \tag{15}$$

where

$$\sigma(t) = \frac{N_t}{(1+N_t)}, \quad z_{++}(t) = \frac{(\bar{n}+1)}{\bar{n}}\sigma(t), \quad \tilde{\alpha}_{\pm} = \frac{\alpha e^{(-\gamma \pm i\lambda)t}}{(1+N_t)},$$
$$z_{+-}(t) = \frac{2\gamma(\bar{n}+1)(1-e^{2\xi^+t})}{v_2^+ - v_1^+e^{2\xi^+t}}, \quad \chi^{\pm}(t) = (\gamma - \xi^{\pm})t - \ln\left(\frac{v_1^{\pm} - v_2^{\pm}e^{-2\xi^{\pm}t}}{2\xi^{\pm}}\right),$$
$$\eta_{\pm} = \chi_{\pm} - \gamma t, \quad \omega_{\pm} = \frac{2\gamma\bar{n}(1-e^{2\xi_{\pm}t})}{v_2^{\pm} - v_1^{\pm}e^{2\xi_{\pm}t}}$$

The asymptotic values as $t \to \infty$ for the off diagonal elements $\rho_{+-}(t) = \rho_{-+}(t)$ vanish, but the asymptotic value for the diagonal elements are given by:

$$\rho_{\pm\pm}(t\to\infty) = \frac{|c_{\pm}|^2}{\bar{n}} \sum_{n=0}^{\infty}\left(\frac{\bar{n}}{(\bar{n}+1)}\right)^n |n\rangle\langle n| \tag{16}$$

which is the density operator for the thermal state as would be expected. In what follows we investigate the effects of damping on the system in which both the atom and cavity field mode start from pure state, we use the entropy and atomic quantum Fisher information to exhibit these effects.

3.1 Atomic quantum Fisher information

Fisher information has two basic roles to play in theory. First, it is a measure of the ability to estimate a parameter; this makes it a corner stone of the field of statistical studies called parameter estimation [31]. Second, it is a measure of the state of disorder of a system or phenomenon, that makes it a corner stone of physical theory [24,26]. Here we use quantum Fisher information to quantify entanglement. Suppose that θ is a parameter, it is often useful to express the log-likelihood derivative in terms of the symmetric logarithmic derivative or quantum score of ρ, denoted by $\rho_{//\theta}$ defined as

$$\rho_{/\theta} = \rho \circ \rho_{//\theta} \tag{17}$$

where $\circ$ denotes Jordan product, i.e.,

$$\rho \circ \rho_{//\theta} = \frac{1}{2}(\rho \circ \rho_{//\theta} + \rho_{//\theta} \circ \rho) \qquad (18)$$

$\rho_{/\theta}$ denoting the ordinary derivative of ρ with respect to θ.

Quantum Fisher information is given by [31]

$$I(\theta) = tr\{\rho(\theta)\rho^2_{//\theta}(\theta)\}. \qquad (19)$$

In this article , we replace the parameter θ with time t and use QFI to indicate entanglement for atomic state. Atomic QFI can be defined accordingly as

$$I(t) = tr\{\rho_A(t)\rho^2_{A//t}(t)\} \qquad (20)$$

We present in the following section the evolution of the atomic QFI $I(t)$ with the scaled time λt for different values of the dissipation constant γ and mean number of the thermal photons $\bar{n}$ to show how the atomic QFI gives results compatible with atomic entropy.

3.2 Total entropy and reduced entropies of bipartite systems

Total entropy $S(t)$ quantify the entanglement between the field-atom system and the environment. It can be expressed in terms of eigenvalues $\epsilon_i(t)$ of the density operator ρ as

$$S(t) = -\sum_{i=0}^{\infty} \epsilon_i(t) \ln \epsilon_i(t). \qquad (21)$$

We use the reduced quantum entropy to see what happen to coherence properties of the atom and field . The entropy of the subsystems can be defined through their respective reduced density matrix $\hat{\rho}_{A(F)}(t) = Tr_{F(A)}\{\hat{\rho}(t)\}$ as [10,11]

$$S_{A(F)} = -Tr_{A(F)}\{\hat{\rho}_{A(F)} \ln \hat{\rho}_{A(F)}\}. \qquad (22)$$

For the entropy of general two-component system, one has the Araki-Lieb theorem [32]: $|S_A - S_F| \le S \le S_A + S_F$. An immediate consequence of this inequality is that if the total system is in a pure state, then the component systems have equal entropies. Since we have assumed the atom and the field to be initially in a disentangled pure state , then the total entropy of the system is zero when the damping is ignored, but when damping is considered, entropy is non-zero, so the reduced entropies of the two subsystems are not identical. This means that to measure the correlation properties of the atom and field we use entropies of the subsystems. The eigenvalues of the reduced atom density operator $\hat{\rho}_A(t)$ are given by

$$\epsilon^A_{1,2} = \tfrac{1}{2}\{1 \pm \sqrt{(\rho^A_{++} - \rho^A_{--})^2 + 4 \mid \rho^A_{+-} \mid^2}\}. \qquad (23)$$

Then the atomic entropy is given by

$$S_A = -\sum_{i=1}^{2} \epsilon^A_i \ln \epsilon^A_i. \qquad (24)$$

The reduced field operator $\hat{\rho}_F(t)$ has infinite number of eigenvalues ϵ^F_i , so numerical computations are to be used to calculate the eigenvalues of the reduced field density operator. Therefore, the field entropy may be expressed in terms of the eigenvalues ϵ^F_i as,

$$S_F = -\sum_{i=0}^{\infty} \epsilon^F_i \ln \epsilon^F_i \qquad (25)$$

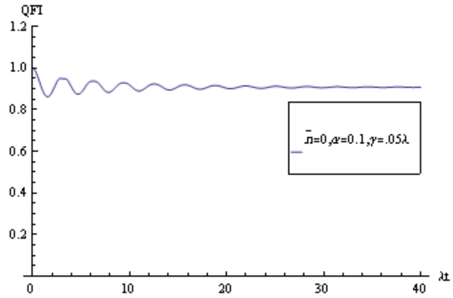
4 Numerical results and discussion

We take the initial atomic state $(|\alpha\rangle + |g\rangle)/\sqrt{2}, i.e., c_+ = c_- = \frac{1}{\sqrt{2}}$ and the field to be initially prepared in coherent state with $\alpha = 0.1$, 0.5 , 1. We present in Figs. 1,2,3 the time evolution of the atomic quantum Fisher information $I(t)$ (normalized to unity) and the atomic reduced entropy S_A with scaled time λt for the dissipation parameter $\gamma = 0.05$ and the mean number of thermal photons $\bar{n} = 0, 0.5, 1.$ Since it has been indicated [26] that the QFI increases strongly by increasing the coherent parameter α, therefore, we use the normalized QFI by relating $I(t)$ to $I(0)$. Thus we use the ratio $I(t)/I(0)$ in the following discussion.

4.1 Too-weak field ($\alpha = 0.1, 0.5$)

In Fig. 1 we display QFI and S_A for $\alpha = 0.1, \gamma = 0.05\lambda$ and $\bar{n} = 0, 0.5, 1$ respectively. At zero temperature (i.e., $\bar{n} = 0$) Fig. 1 (a,b) display QFI and atomic entropy, we observe that they oscillate near the values 1 and 0 respectively, after a long time both curves tend to stationary behavior. Increasing $\bar{n}$ to the value 0.5, we notice in Fig. 1(c,d) drops down and decay to an almost zero value after λt =40, while the atomic entropy increases monotonically until it reaches its stationary value of $\ln 2$ at almost the same time. When the thermal photons number increased to 1, it is observed that the QFI decays faster to almost zero, while S_A reaches the value $\ln 2$ faster than the earlier case (see Fig. 1e,f).

Comparing Fig. 2 ($\alpha = 0.5$) with Fig. 1, we find that at $\bar{n} = 0$ the amplitude of the oscillations in QFI curve (Fig. 2(a)) is increased, but about steady value less than the equivalent one in Fig. 1 due to higher $I(0)$ in this case due to increase of α and the atomic entropy oscillates about a value greater than that in Fig. 1due to the effect of the parameter α . The effect of thermal photons have made the number of oscillations decrease and QFI, S_A tend to steady values faster than the equivalent cases in Fig. 1 as indicated in Figs(2c-f) for the values $\bar{n} = 0.5$ and 1.



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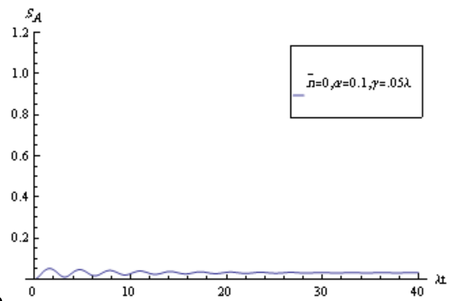

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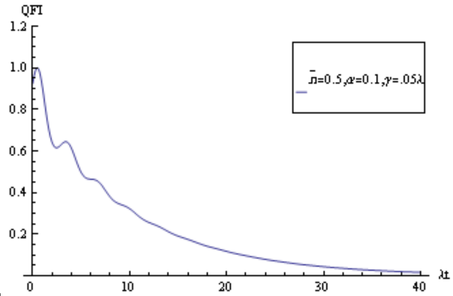

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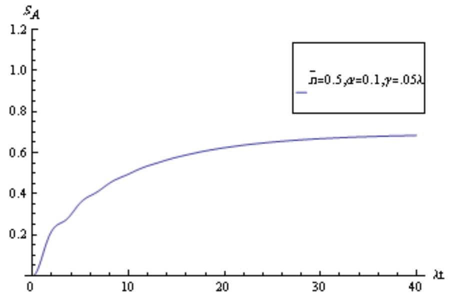

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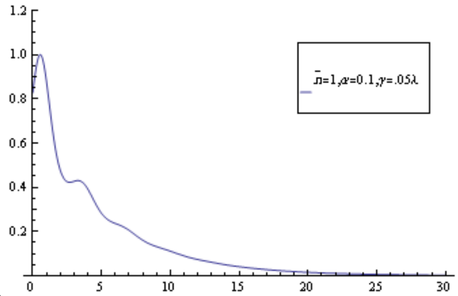

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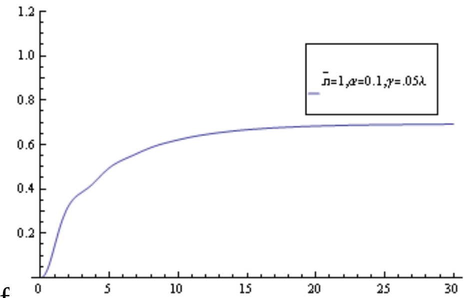
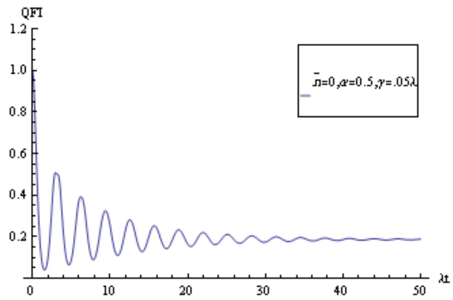


f

Figure 1: Time evolution of QFI and SA for α =0.1, γ =0.05 λ and $\bar{n}$ =0,0.5,1 respectively



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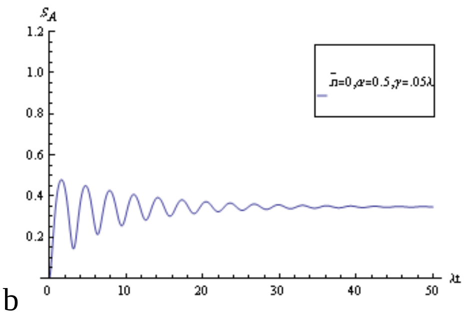

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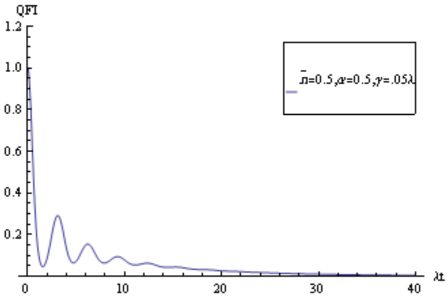

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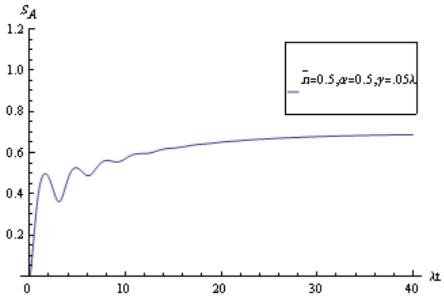

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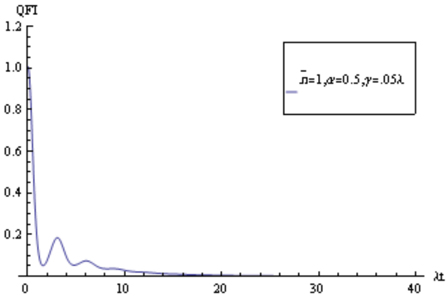

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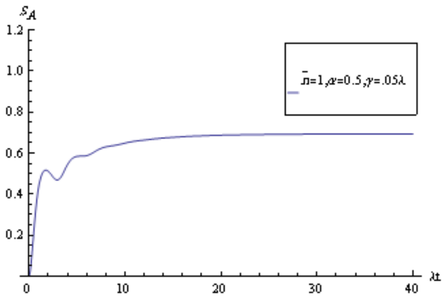
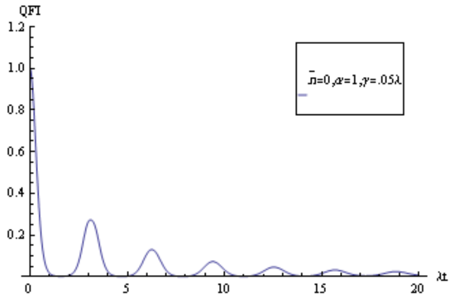


f

Figure 2: The same as Fig. 1 but for α =0.5

4.2 Not too-weak field

In this case we take $\alpha = 1$ and we keep the rest the parameters as before in the previous cases. What happen when we increase α to 1 ?. At $\bar{n} = 0$, we have the same number of oscillations but with small amplitudes. We note that the QFI has its minima near zero, while S_A has its maxima near $\ln 2$. Increasing $\bar{n}$ leads to decreasing in the number of oscillations, QFI and S_A tend to zero and $\ln 2$ respectively faster than in their equivalent cases Figs. 1,2. Therefore by increasing the number of the photons in the cavity field, it is found that information is lost faster as indicated by QFI, and decoherence becomes prominent as shown from the atomic entropy. These effects become more pronounced when this increase is accompanied with increase in the temperature by an increase in the thermal photons.



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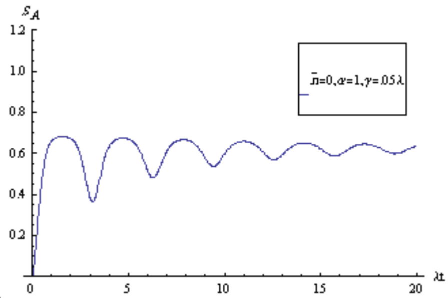

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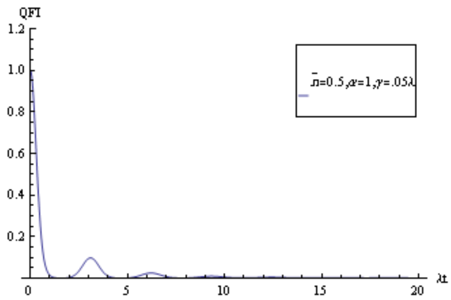

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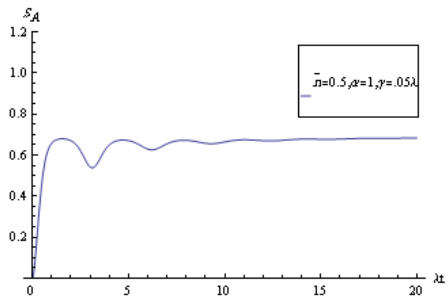

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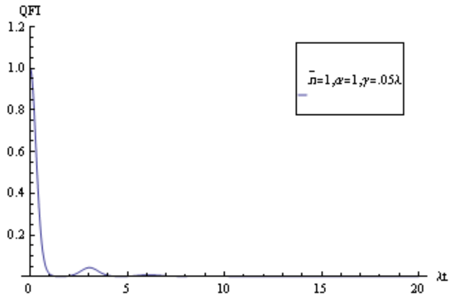

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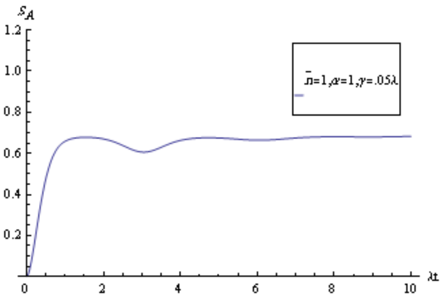

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Figure 3: The same as Fig. 1 but for α =1

5. Conclusions

In conclusion, we have solved the master equation for a quantized cavity mode for non-zero temperature by using superoperator techniques. The effects of thermal photons on Fisher information and atomic entropy dynamic for a dispersive JC model are investigated. It is found that, the thermal photons as well as the cavity field photons participate in the loss of information. The QFI finally tends to zero and information is lost completely as time develops. Further, the atomic coherence is lost faster than in the case of zero temperature on combining strong cavity field with higher temperature.

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