Journal of Statistics Applications & Probability
An International Journal

http://dx.doi.org/10.18576/jsap/110114

Confidence Intervals for the Common Process Capability Index C_p of Normal Distributions

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Received: 29 Sep. 2020, Revised: 24 Dec. 2020, Accepted: 10 Feb. 2021
Published online: 1 Jan. 2022

Abstract: Herein, confidence intervals using the generalized confidence interval (GCI), large sample (LS), and the adjusted method of variance estimates recovery (Adjusted MOVER) approaches for the common process capability index C_p of normal distributions are proposed. Monte Carlo simulation was used to assess the performances of the proposed methods in terms of coverage probability and expected length. The results of the simulation indicate that Adjusted MOVER performed well in terms of the coverage probability and expected length under all conditions. The coverage probabilities of LS and GCI were differed greatly from the nominal level 0.95, except for some cases. The performances of the confidence intervals are also illustrated using real data from the area of industrial engineering.

Keywords: Confidence intervals, Process capability, Generalized confidence interval

1 Introduction

Statistical process control (SPC) is needed in product quality development planning that is used extensively in many industries. For example, James et al. [1] studied SPC to determine machine damage mitigation approach in the automotive industry. Mario de Araujo et al. [2] studied the waste problem in the weaving process to reduce the variation in the process by applying the SPC technique to reduce the amount of waste. Thomas et al. [3] used SPC as a development tool to create quality control chart in the wood industry to check average portion sizes and internal variation of part sizes for lumber in automatic sawmill. Adisak et al. [4] applied SPC in elevator assembly process when out of control action plan. The procedures for establishing an out of contral action plan was introduced. The process capability indices (PCIs) is a useful quality measurement tool that provides a numerical measure of whether a production process meets the quality requirements suggested by the manufacturer. Although there are many PCIs, we are only interested in the process capability index C_p that is a most commonly index (Kane [5], Zhang [6]).

The process capability index is a process performance measure commonly used to assess the ability of the production process, and there have been many studies published on its application. For example, Vannman [7] proposed a new class of the process capability index C_p(u,v), containing four indices. However, when the tolerance interval is asymmetric, these four indices may not be able to determine the process capability. Rado [8] showed that the process capability index can be used for program planning and growth to improve product development. Rezaie et al. [9] proposed a process capability index and a process parameter estimator and presented a method for determining the process capability. Zhang [6] proposed basic theories and computational methods for finding the conditional confidence intervals of two process capability indices (C_p, C_{pk}) and using programming code to calculate them.

In a situation such as selecting a supplier and assessing process improvement, testing the quality of two or more process capability index C_p based on independent samples collected from the different normal distributions can be accomplished with a common process capability index C_p using statistical inference, which is the premise of this study. We consider

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interval estimation, which is more accurate than hypothesis testing and point estimation, to approach the problem of making a statistical inference. In the literature, the problem when estimating the process capability index C_p has acquired attention in the literature. Putthipilun [10] studied the confidence intervals for the process capability index C_p after testing for normality using the Shapiro-Wilk test. The author found that the confidence intervals for the process capability index after normality testing gave a higher probability. Niwitpong and Kirdwichao [11] studied the confidence interval of the process capability index C_p via Monte Carlo simulation with the sample sizes of 10, 25, 50, and 100. Their findings show that for all of the non-normal distributions studied, the process capability index C_p provided higher coverage probabilities that the method of Kotz and Lovelace [12]. Panichkitkosolkul [13] proposed a new confidence interval for the process capability index C_p based on the bootstrap-t confidence interval for a standard deviation. Panichkitkosolkul [14] also constructed three confidence intervals (adjusted degrees of freedom, large sample, and augmented large sample) for the variance of the process capability index C_p under non-normality.

The construction of confidence intervals for the C_p in terms of a common PCI has not yet gained attention from other researchers. Therefore, to construct confidence intervals for the common C_p under the assumption of normality bring into a focus. An idea of constructing the confidence interval for a common C_p for normal distributions is extended. The concepts of the generalized confidence interval (GCI), large sample (LS), and the adjusted method of variance estimates recovery (Adjusted MOVER) approaches are used to construct confidence intervals for the common C_p of normal distributions.

According to the three approaches used to construct confidence intervals in this paper, there have been many published articles. For example, Smitpreecha and Niwitpong [15] constructed confidence intervals for the common mean of lognormal distributions. The results show that the Adjusted MOVER based on Angus's conservative method performs well in terms of coverage probability, close to the nominal level than other methods, but the average is wide. Also, Smitpreecha et al. [16] recommended the Adjusted MOVER approach to construct the confidence intervals for the common variance of normal distribution since the coverage probability of Adjusted MOVER approach provides the best confidence interval estimates than GCI and LS approach. Moreover, confidence intervals for common signal-to-noise ratio of several log-normal distributions based on GCI, Adjusted MOVER and computational approaches were constructed by Thangjai and Niwitpong [17]. Their simulation study results show that the GCI approach is better than the other approaches when sample case is 3 and is preferable when the sample sizes are large. Additionally, Thangjai et al. [18] established confidence intervals for the common coefficient of variation of several normal populations using the GCI and Adjusted MOVER approaches. The GCI approach performs better than the Adjusted MOVER approach in terms of coverage probabilities for all sample sizes.

The organization of this paper is as follows. In Section 2, the proposed confidence intervals for the common process capability index C_p of normal distributions are presented. In Section 3, the results of a simulation to evaluate the performance of the confidence intervals in terms of coverage probability and expected length are analyzed. In Section 4, real data is used to show the efficacy of the proposed approaches. In Section 5, conclusions on the study are presented.

2 Confidence Intervals for the common process capability index C_p

Suppose $X = (X_1, X_2, \ldots, X_n)$ are independently and identically distributed random variables with mean μ and variance σ^2. The process capability index C_p can be defined as (Kane, [5])

$$\theta = C_p = \frac{USL - LSL}{6\sigma}, \tag{1}$$

where USL is the upper specification limit, LSL is the lower specification limit, and σ is the process standard deviation. The estimator of μ is the sample mean $\bar{X}$, and the estimator of σ is the sample standard deviation S. The estimator of the process capability index C_p can be obtained by

$$\hat{\theta} = \hat{C}_p = \frac{USL - LSL}{6\hat{\sigma}} = \frac{d}{3S}, \tag{2}$$

where $\hat{\sigma} = S = \sqrt{\frac{1}{n-1}\sum_{i=1}^{n}(X_i - \bar{X})^2}$, $\hat{\mu} = \bar{X} = \frac{1}{n}\sum_{i=1}^{n} X_i$ and $d = \frac{USL-LSL}{2}$. According to Kotz and Lovelace [12], the respective mean and variance of $\hat{\theta}$ are

$$E(\hat{\theta}) = \left(\frac{n-1}{2}\right)^{\frac{1}{2}} \cdot \frac{\Gamma\left(\frac{n-2}{2}\right)}{\Gamma\left(\frac{n-1}{2}\right)} \cdot C_p = \frac{1}{b_f} \cdot C_p \tag{3}$$

$$Var(\hat{\theta}) = \left(\frac{n-1}{n-3} - b_f^{-2}\right) C_p, \tag{4}$$

where $b_f = \frac{\Gamma\left(\frac{n-1}{2}\right)}{\sqrt{\frac{n-1}{2}}\Gamma\left(\frac{n-2}{2}\right)}$. Consider k independent normal populations with a common process capability index C_p. Let $X_{i1}, X_{i2}, ..., X_{in_i}$ for $i = 1, 2, ..., k, j = 1, 2, ..., n_i$ be a random sample from the i-th normal distribution. The maximum likelihood estimator of θ_i is given by

$$\hat{\theta}_i = \hat{C}_{p_i} = \frac{d}{3S_i}, \tag{5}$$

and the variance of $\hat{\theta}_i$ can be written as

$$Var(\hat{\theta}_i) = \left(\frac{n_i-1}{n_i-3} - b_{f_i}^{-2}\right) C_{p_i}^2. \tag{6}$$

The estimator and variance of θ are used to construct confidence intervals for the common process capability index C_p in the next sections.

2.1 The Generalized Confidence Interval Approach

Weerahandi [19] proposed the concept of the generalized confidence interval (GCI) using the generalized pivotal quantity (GPQ) as a statistic that has a distribution free of unknown parameters and an observed value of generalized pivotal does not depend on nuisance parameters. In this proposed approach, we construct the generalized confidence interval (GCI), which performs well in terms of coverage probability and average length for various sample sizes [20,21]. Let $\bar{x}_i$ and s_i^2 be the observed values of the sample mean and sample variance $(\bar{X}_i, S_i^2)$, for the i-th sample.

Since

$$\frac{(n_i-1)S_i^2}{\sigma_i^2} \sim \chi^2_{n_i-1}, \tag{7}$$

we can get

$$\sigma_i \sim \frac{\sqrt{(n_i-1)}S_i}{\sqrt{\chi^2_{n_i-1}}}, \tag{8}$$

where $\chi^2_{n_i-1}$ is chi-squared distribution with $n_i - 1$ degree of freedom.

The GPQ for population standard deviation σ_i is defined as

$$R_{\sigma_i} = \frac{\sqrt{(n_i-1)}S_i}{\sqrt{\chi^2_{n_i-1}}}. \tag{9}$$

The GPQ for the process capability index C_p, θ based on the i-th sample is

$$R_{\theta_i} = \frac{d}{2R_{\sigma_i}}, \tag{10}$$

where R_{σ_i} is defined as in Equation (9).

Therefore, for the GPQ for the common process capability index C_p, θ, we propose the weighted average of the generalized pivotal quantity R_{θ_i} based on k individual sample as

$$R_\theta = \sum_{i=1}^{k} \frac{R_{\theta_i}}{R_{var(\hat{\theta}_i)}} \Bigg/ \sum_{i=1}^{k} \frac{1}{R_{var(\hat{\theta}_i)}}, \tag{11}$$

whrere

$$R_{var(\hat{\theta}_i)} = \left(\frac{n_i-1}{n_i-3} - b_{f_i}^{-2}\right) \left(\frac{d}{3R_{\sigma_i}}\right)^2. \tag{12}$$

Thus is, $R_{var(\hat{\theta}_i)}$ is $Var(\hat{\theta}_i)$ with σ_i replaced by s_i.

The generalized pivotal R_θ for interval estimation has the following two properties, which are in the line with the required properties of a generalized pivotal outlined above.

(i) The distribution of R_θ is independent of any unknown parameter,

(ii) The observed pivotal does not depend on the nuisance parameters.

Confidence intervals for θ based on the GPA can be constructed with the help of R_θ. If $R_\theta(1-\alpha)$ is the $100(1-\alpha)$th percentile of the R_θ distribution, then $R_\theta(1-\alpha)$ is the $(1-\alpha)100\%$ upper confidence limit for θ. Thus,

$$CI_1 = [R_\theta(\alpha/2), R_\theta(1-\alpha/2)] \tag{13}$$

is a$(1-\alpha)100\%$ two-sided GCI for the common process capability index C_p.

Algorithm 1. For given set X_{ij}, for $i = 1,2,...,k$ and $j = 1,2,...,n_i$, the GCI for θ can be computed by the following steps.

1. Compute $\bar{x}_i$ and s_i^2, for $i = 1,2,...,k$.
2. For g = 1 to m.
3. Generate $\chi^2_{n_i-1}$ from a Chi-squared distribution with $n_i - 1$ degrees of freedom.
4. Compute R_{σ_i} from Equation (9).
5. Compute R_{θ_i} from Equation (10).
6. Compute $R_{var(\hat{\theta}_i)}$ from Equation (12).
7. Compute R_θ from Equation (11).
8. End g loop.
9. Compute $R_\theta(\alpha/2)$ and $R_\theta(1-\alpha/2)$.

2.2 The Large Sample Approach

The large sample estimate of normal variance is a pooled estimate of the common process capability index C_p defined as

$$\hat{\theta} = \frac{\sum_{i=1}^{k} \frac{\hat{\theta}_i}{Var(\hat{\theta}_i)}}{\sum_{i=1}^{k} \frac{1}{Var(\hat{\theta}_i)}}, \tag{14}$$

where $\hat{\theta}_i$ is defined in Equation (5) and $Var(\hat{\theta}_i)$ is defined in Equation (6) with σ_i replaced by s_i, respectively.

Hence, the $100(1-\alpha)\%$ GCI for the common process capability index C_p, θ, based on this approach is given by

$$CI_2 = \left(\hat{\theta} - z_{1-\alpha/2} \sqrt{\frac{1}{\sum_{i=1}^{k} \frac{1}{Var(\hat{\theta}_i)}}}, \hat{\theta} + z_{1-\alpha/2} \sqrt{\frac{1}{\sum_{i=1}^{k} \frac{1}{Var(\hat{\theta}_i)}}} \right), \tag{15}$$

where $z_{1-\alpha/2}$ is the $(1-\alpha/2)$-th quantile of the standard normal distribution.

Algorithm 2. For given set X_{ij}, for $i = 1,2,...,k$ and $j = 1,2,...,n_i$, the GCI for θ can be computed by the following steps.

1. Compute $\bar{x}_i$ and s_i^2, for $i = 1,2,...,k$.
2. Calculate $Var(\hat{\theta}_i)$ from Equation (6), for $i = 1,2,...,k$.
3. Compute $\hat{\theta}$ following Equation (14).
4. Calculate the confidence interval estimation from Equation (15), for $i = 1,2,...,k$.

2.3 The Adjusted MOVER Approach

Donner and Zou [22] proposed the Adjusted MOVER approach as the confidence interval for two parameters of interest, θ_1 and θ_2. Using the central limit theorem, a general approach to set two-sided confidence interval for $\theta_1+\theta_2$ is given by

$$(\hat{\theta}_1+\hat{\theta}_2) \pm z_{\alpha/2}\sqrt{\widehat{Var}(\hat{\theta}_1)+\widehat{Var}(\hat{\theta}_2)}. \tag{16}$$

Now, consider confidence interval for θ_i as $(l_i,u_i), i=1,2$. Therefore, the variance estimates the lower $\widehat{Var}(\hat{\theta}_i)$ and upper $\widehat{Var}(\hat{\theta}_i)$ limits of θ_i, respectively, as

$$\widehat{Var}(\hat{\theta}_i)=\frac{(\hat{\theta}_i-l_i)^2}{z^2_{\alpha/2}}, \widehat{Var}(\hat{\theta}_i)=\frac{(u_i-\hat{\theta}_i)^2}{z^2_{\alpha/2}}. \tag{17}$$

For $i=1,2$, the two-sided $100(1-\alpha)\%$ confidence intervals for $\theta_1+\theta_2$ (L,U) are given by

$$L=(\hat{\theta}_1+\hat{\theta}_2)-\sqrt{(\hat{\theta}_1-l_1)^2+(\hat{\theta}_2-l_2)^2}$$

and

$$U=(\hat{\theta}_1+\hat{\theta}_2)+\sqrt{(u_1-\hat{\theta}_1)^2+(u_2-\hat{\theta}_2)^2}. \tag{18}$$

Let $\theta_1,\theta_2,...,\theta_k$, for k parameters of interest, where the estimates $\hat{\theta}_1,\hat{\theta}_2,...,\hat{\theta}_k$ are independent. The MOVER concept is used to construct the two-sided $100(1-\alpha)\%$ confidence intervals (L,U) for $\theta_1+\theta_2+...+\theta_k$ as

$$L=(\hat{\theta}_1+\hat{\theta}_2+...+\hat{\theta}_k)-z_{\alpha/2}\sqrt{\widehat{Var}(\hat{\theta}_1)+\widehat{Var}(\hat{\theta}_2)+...+\widehat{Var}(\hat{\theta}_k)}$$

and

$$U=(\hat{\theta}_1+\hat{\theta}_2+...+\hat{\theta}_k)+z_{\alpha/2}\sqrt{\widehat{Var}(\hat{\theta}_1)+\widehat{Var}(\hat{\theta}_2)+...+\widehat{Var}(\hat{\theta}_k)}. \tag{19}$$

The variance estimate for $\hat{\theta}_i$ at $\theta_i=l_i$ and $\theta_i=u_i$ can be written as

$$\widehat{Var}(\hat{\theta}_i)=\frac{(\hat{\theta}_i-l_i^2)}{z^2_{\alpha/2}}, \widehat{Var}(\hat{\theta}_i)=\frac{(u_i-\hat{\theta}_i)^2}{z^2_{\alpha/2}}. \tag{20}$$

Thus, the lower limit L and upper limit U for $\theta_1+\theta_2+...+\theta_k$ are given by

$$L=(\hat{\theta}_1+\hat{\theta}_2+...+\hat{\theta}_k)-\sqrt{(\hat{\theta}_1-l_1)^2+(\hat{\theta}_2-l_2)^2+...+(\hat{\theta}_k-l_k)^2}$$

and

$$U=(\hat{\theta}_1+\hat{\theta}_2+...+\hat{\theta}_k)+\sqrt{(u_1-\hat{\theta}_1)^2+(u_2-\hat{\theta}_2)^2+...+(u_k-\hat{\theta}_k)^2}. \tag{21}$$

According to Thangjai et al.[23], the Adjusted MOVER is based on the LS and MOVER approaches. The common process capability index C_p, θ, is the weighted average of the process capability index $\hat{\theta}_i$ based on k individual samples as follows:

$$\hat{\theta}=\frac{\sum_{i=1}^{k}\frac{\hat{\theta}_i}{V\hat{a}r(\hat{\theta}_i)}}{\sum_{i=1}^{k}\frac{1}{V\hat{a}r(\hat{\theta}_i)}}, \tag{22}$$

where

$$V\hat{a}r(\hat{\theta}_i)=\frac{1}{2}\left(\frac{(\hat{\theta}_i-l_i)^2}{z^2_{\alpha/2}}+\frac{(u_i-\hat{\theta}_i)^2}{z^2_{\alpha/2}}\right); i=1,2,...k, \tag{23}$$

where $z_{\alpha/2}$ denotes the $(\alpha/2)$-th quantile of the standard normal distribution.

Therefore, the lower limit L and upper limit U for the common process capability index C_p, θ, are respectively given by

$$L = \hat{\theta} - z_{1-\alpha/2}\sqrt{\frac{1}{\sum_{i=1}^{k}\frac{z_{\alpha/2}^2}{(\hat{\theta}_i - l_i)^2}}}, U = \hat{\theta} + z_{1-\alpha/2}\sqrt{\frac{1}{\sum_{i=1}^{k}\frac{z_{\alpha/2}^2}{(u_i - \hat{\theta}_i)^2}}}. \tag{24}$$

Chou et al. [24], and Pearn and Chang [25], proposed the confidence interval for the process capability index C_p. The lower limit l_i and upper limit u_i of the confidence interval are respectively given by

$$l_i = \hat{\theta}_i\sqrt{\frac{\chi^2_{\alpha/2(n_i-1)}}{n_i-1}} \text{ and } u_i = \hat{\theta}_i\sqrt{\frac{\chi^2_{1-\alpha/2(n_i-1)}}{n_i-1}}. \tag{25}$$

Therefore, the $100(1-\alpha)\%$ GCI for the common process capability index C_p, θ, based on the Adjusted MOVER approach can be written as

$$CI_3 = \left(\hat{\theta} - z_{1-\alpha/2}\sqrt{\frac{1}{\sum_{i=1}^{k}\frac{z_{\alpha/2}^2}{(\hat{\theta}_i - l_i)^2}}}, \hat{\theta} + z_{1-\alpha/2}\sqrt{\frac{1}{\sum_{i=1}^{k}\frac{z_{\alpha/2}^2}{(u_i - \hat{\theta}_i)^2}}}\right). \tag{26}$$

Algorithm 3. For given set X_{ij}, for $i = 1,2,...,k$ and $j = 1,2,...,n_i$, the GCI for θ can be computed by the following steps.
1. Compute $\bar{x}_i$ and s_i^2, for $i = 1,2,...,k$.
2. Compute l_i, u_i from Equation (25), for $i = 1,2,...,k$.
3. Compute $V\hat{a}r(\hat{\theta}_i)$ from Equation (23), for $i = 1,2,...,k$.
4. Compute $\hat{\theta}$ following Equation (22).
5. Calculate the confidence interval estimation from Equation (26), for $i = 1,2,...,k$.

3 Simulation Study

A simulation study was conducted to estimate the coverage probabilities (CP) and expected lengths (EL) of the three confidence intervals for the common process capability index C_p of normal distributions. The coverage probability and the expected length are given by [26]

$$CP = \frac{\sum_{i=1}^{10,000} I_i\,(L \le C_p \le U)}{10,000}$$

and

$$EL = \frac{\sum_{i=1}^{10,000} (U_i - L_i)}{10,000}$$

where $\sum_{i=1}^{10,000} I_i\,(L \le \alpha \le U)$ denotes the number of simulation runs. Various combinations of the number of sample cases $k = 2,4,6,10$, sample sizes $n_1 = ... = n_k = n$, (10, 25, 50, and 100), the population mean of the normal data for each sample, $\mu_1 = ... = \mu_k = \mu = 50$, and the population standard deviations $\sigma_1 = ... = \sigma_k = \sigma = 1$. In addition, the true values of the process capability index, $(C_{p_1} = ... = C_{p_k} = C_p)$, lower specification limit $(LSL_1 = ... = LSL_k = LSL)$, and upper specification limit $(USL_1 = ... = USL_k = USL)$were set as in Table 1 (Panichkitkosolkul [14]).

For each set of parameters, 10000 random samples were generated and 1000 R_0's were obtained for each of the random samples. The nominal level was fixed at 0.95. All simulation runs were performed using the R statistical package (Venbles et al. [27]). Figs. 1 to 4 provide simulation results for the estimated coverage probability of the confidence intervals for the 2, 4, 6, and 10 sample cases, respectively, and the expected lengths are presented in Figs. 5 to 8. The Adjusted MOVER approach had coverage probabilities close to the nominal level for all scenarios. However, both GCI and the LS provided coverage probabilities that were quite different from the nominal level: GCI underestimated the coverage probability for k = 4, 6, 10 but was close for the others, while LS overestimated the coverage probability, although this tended to decrease and come close to the nominal level when the sample size increased. Meanwhile, the Adjusted MOVER approach provided the shortest expected lengths compared with the other approaches for all situations.

Table 1: True values of C_p, LSL, and USL .

C_p	LSL	USL
1.00	47.00	53.00
1.33	46.01	53.00
1.50	45.50	54.50
1.67	44.90	55.01
2.00	44.00	56.00

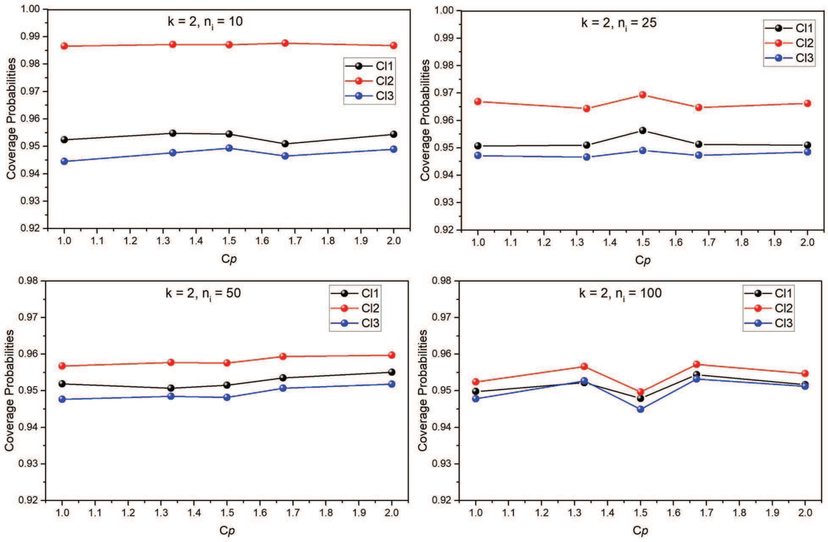

Fig. 1: Estimated coverage probabilities of 95% confidence intervals for common process capability index C_p in case of $k = 2$.

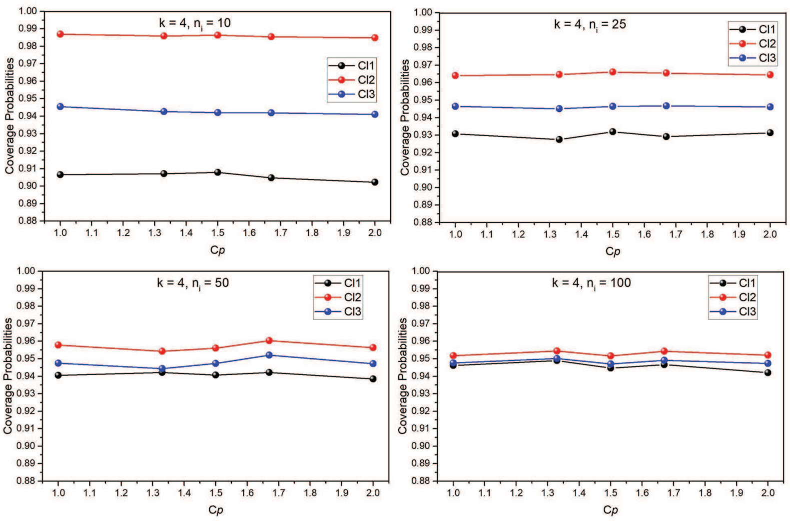

Fig. 2: Estimated coverage probabilities of 95% confidence intervals for common process capability index C_p in case of $k = 4$.

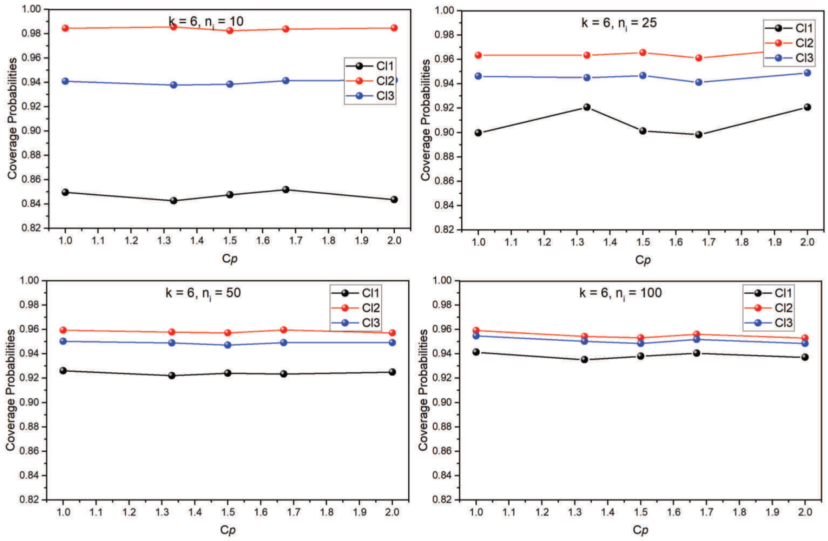

Fig. 3: Estimated coverage probabilities of 95% confidence intervals for common process capability index C_p in case of $k = 6$.

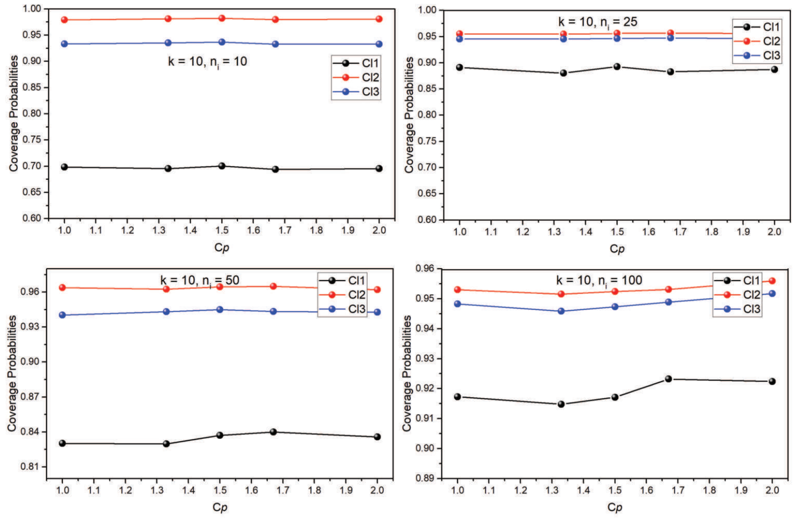

Fig. 4: Estimated coverage probabilities of 95% confidence intervals for common process capability index C_p in case of $k = 10$.

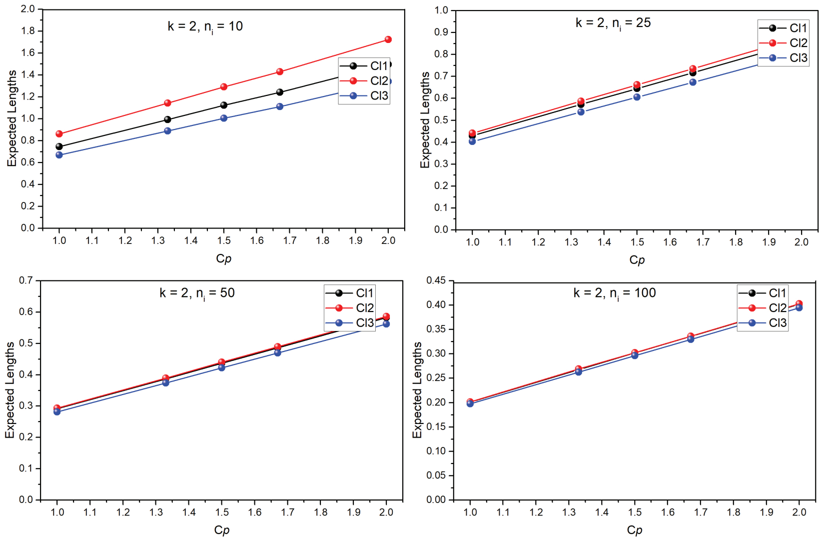

Fig. 5: Expected Lengths of 95% confidence intervals for common process capability index C_p in case of $k = 2$.

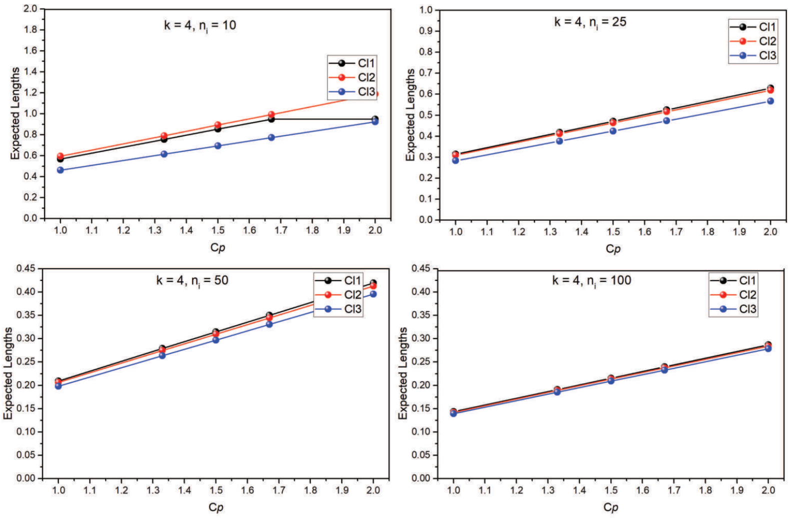

Fig. 6: Expected Lengths of 95% confidence intervals for common process capability index C_p in case of $k = 4$.

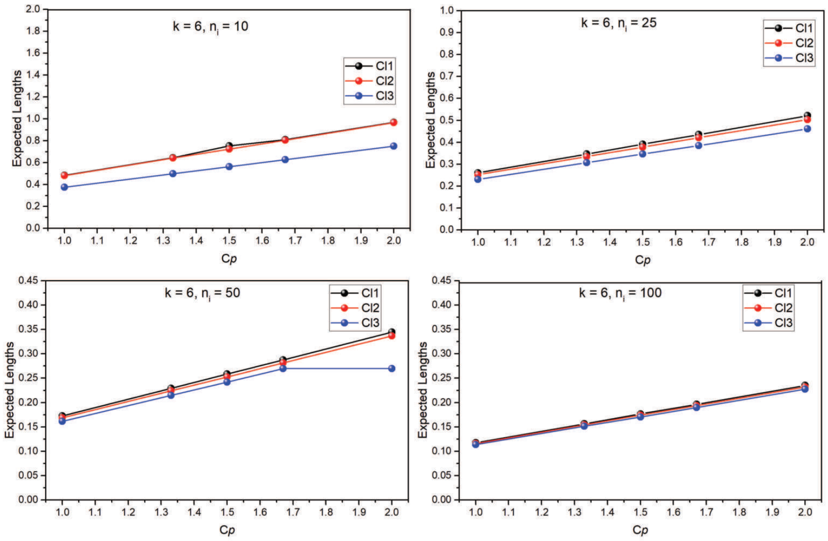

Fig. 7: Expected Lengths of 95% confidence intervals for common process capability index C_p in case of $k = 6$.

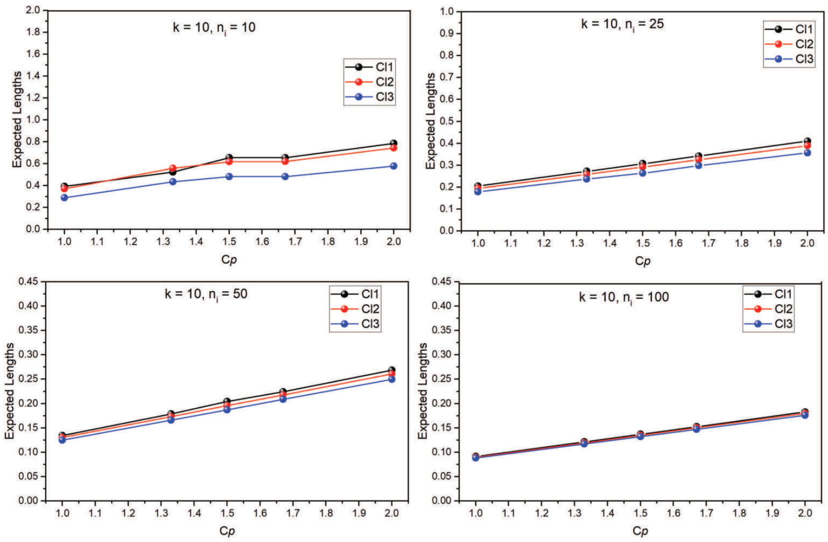

Fig. 8: Expected Lengths of 95% confidence intervals for common process capability index C_p in case of $k = 10$.

4 An Empirical Application

In this section, an example with real data is used to illustrate the given approaches. The data are from the PCIs of two hardness specimen cutting processes provided by Bangphan [28].

For process 1, the summary statistics are

$$n_1 = 25, \bar{x}_1 = 20.013, \text{ and } s_1 = 0.0014.$$

For process 2, the summary statistics are

$$n_2 = 25, \bar{x}_2 = 20.018, \text{ and } s_2 = 0.006.$$

The two processes have USL and LSL of 20.05 and 19.95 mm, respectively. For checking distributions of these two processes, the Akaike Information Criterion (AIC) values, the Bayesian Information Criterion (BIC) values and Normal Probability Plots are proposed. The AIC values and BIC values are demonstrated in Table 2 and Table 3, and Normal Probability Plots for two processes are given in Figure 9 and Figure 10.

Table 2: The values of AIC and BIC of process 1 .

	AIC	BIC
Normal	-137.9829	-135.5452
Exponential	200.8166	203.0355
Log-normal	-137.9901**	-135.5523**

Table 3: The values of AIC and BIC of process 2 .

	AIC	BIC
Normal	-184.6286	-182.1909
Exponential	200.8318	203.0507
Log-normal	-184.6348**	-182.1971**

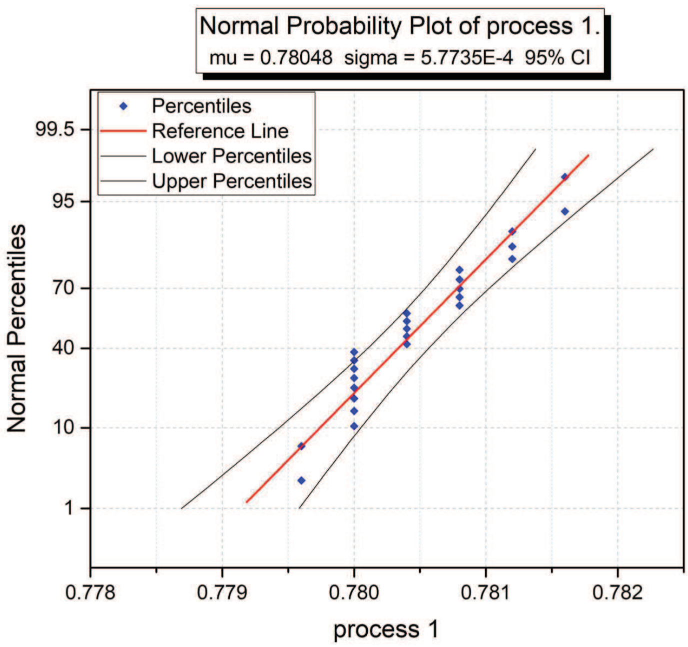

Fig. 9: Normal Probability Plot of process 1.

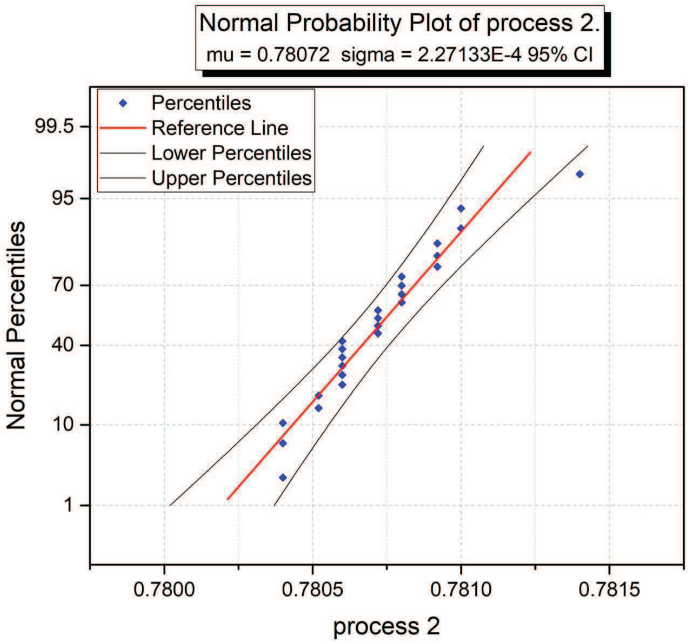

Fig. 10: Normal Probability Plot of process 2.

As Tables 2 and 3, both AIC and BIC values of the two processes from the lognormal population are minimum. However, the AIC values and BIC values of normal distribution are close to these of the lognormal distribution. Figures 9 and 10, Normal Probability Plot with the normal distribution curve that provides each process data based on normal population. In conclusion, from all statistic measure, the two processes come from normal populations.

The 95% confidence intervals for the common process capability index C_p, was CI_1=[0.9601, 1.7408] with an interval length of 0.7806 mm using the GCI approach, CI_2=[1.0477, 1.6990] with an interval length of 0.6513 mm using the LS approach, and CI_3=[1.0749, 1.6712] with an interval length of 0.5963 mm using the Adjusted MOVER approach. These results show that the Adjusted MOVER approach provided the shortest length, which clearly confirms the simulation study results in the previous section.

5 Conclusions

The aim of this paper is to propose three confidence intervals for the common process capability index C_p based on the k normal distributions. We compared the performances the GCI, LS, and Adjust MOVER approaches in terms of the coverage probability and the expected length via simulation studies. The results show that the coverage probability and the expected length of the Adjusted MOVER approach were far superior to GCI and LS in all cases. The Adjusted MOVER approach produced coverage probabilities close to the nominal level and the shortest expected lengths for all situations. The LS approach provided better coverage probabilities and expected lengths than the GCI approach when the sample size increased. Therefore, we recommend the Adjusted MOVER approach for constructing the confidence interval for the common process capability index C_p of normal distributions.

Conflict of interest

The authors declare that they have no conflict of interest.

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