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Search of Some Effective Rotation Patterns in Estimation of Population Mean on Current Occasion in Two-Occasion Successive Sampling

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Abstract: In the present work, some efforts have been made to search of effective rotation patterns in estimation of precision of population mean of study variable on current occasion in two-occasion successive sampling. Information on an auxiliary character, which is readily available on previous and current occasion, has been used along with the information on study character from the first and second occasion. Exponential ratio and regression type estimators have been proposed, and their behaviors are examined. Optimum replacement strategy relevant to the proposed estimation procedure has been discussed. Theoretical and empirical results have been interpreted to justify the efficiency of the proposed estimators. Proposed estimators have been compared with natural estimators and suitable recommendations have been made.

Keywords: Successive sampling, auxiliary variable, bias, mean square error, optimum replacement strategy.

1 Introduction

If a survey is carried out on a certain point of time (occasion), then it provides information about the characteristics of the surveyed population for the given point of time (occasion). But, Information on the characteristics and nature of change the characteristics of the population are unable to observe over different point of time (occasions), when the character under study of a finite population changes over time. To detect and improve such situations, successive (rotation) sampling is very much helpful and useful to generate reliable estimate of population parameters such as mean, variance, etc. on different occasions.

Concept of optimal estimation for sampling units on successive occasions with partial replacement had been initiated by Jesson [7]. Further, the methods on successive (rotation) sampling was extended by Patterson [8], Rao and Graham [9], Gupta [6], Das [3], Chaturvedi and Tripathi [2] and among others. Sen [11] [12] applied this theory with success in designing the estimators of population mean on the current occasion using information on two or more auxiliary variables which were readily available on previous occasion. Singh et al.[23], Singh and Singh [21] made an efficient use of auxiliary variable on current occasion and subsequently Singh [13] extended this methodology for h-occasion successive sampling in estimation of current population mean in two-occasion successive sampling. Feng and Zou [5] and Biradar and Singh [1] used the auxiliary information on both the occasions for estimating the current population mean in two-occasion successive sampling.

In many situations, information on an auxiliary variable may be readily available on the first as well as on the second occasion, such as, tonnage (or seat capacity) of each vehicle or ship is known in survey sampling of transportation. Utilizing the auxiliary information on both the occasions, Singh [14], Singh and Priyanka [18] [19], Singh and Vishwakarma [22], Singh and Prasad [16] and Singh *et al.* [17], Singh *et al.* [15] and Singh and Sharma [20] have proposed several estimators of population mean on current (second) occasion in two-occasion successive sampling. Motivated with the above works, the objective of the present paper is to propose more effective and relevant estimators of current population mean in two-occasion successive (rotation) sampling. The behaviors of the proposed estimators are examined through empirical means of comparison. Consequently, suitable recommendations have been made.

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2 Desription of Notation

Let $U=(U_1,U_2,...,U_N)$ be the finite population of N units, which has been sampled over two occasions. The character under study is denoted by $x(y)$ on the first (second) occasion respectively. It is assumed that information on an auxiliary variable z whose population mean is known and stable over occasions is readily available on both the occasions. Also, z is positively correlated to x and y on first and second occasions respectively. Let a simple random sample (without replacement) s_n of size n be selected on the first occasion. A random sub-sample $s_m \subset s_n$ of $m=n\lambda$ units is retained (matched) for its use on the second occasion, while a fresh simple random sample (without replacement) s_u of size $u=(n-m)=n\mu$ are drawn on the second occasion from the entire population so that the sample size on the second occasion is n as well. Here λ and $\mu(\lambda+\mu=1)$ are the fractions of matched and fresh samples respectively on the current occasion. The values of λ and μ should be chosen optimally. The following notations have been considered for their further use.

$\bar{X}$, $\bar{Y}$: The population means of the study variables x and y respectively.

$\bar{Z}$: Population mean of the auxiliary variable z.

$\bar{y}_u$, $\bar{y}_m$, $\bar{x}_m$, $\bar{x}_n$, $\bar{z}_u$, $\bar{z}_m$, $\bar{z}_n$: The sample means of the respective variables based on the sample sizes shown in suffices.

ρ_{yx}, ρ_{yz}, ρ_{xz} : The correlation coefficients between the variables shown in subscripts.

S_x^2, S_y^2, S_z^2 : The population variances of the variables x, y and z respectively.

3 Formulation of Estimators

To estimate the population mean $\bar{Y}$ on the current occasion, we suggest two sets of estimators whose functional structures are exponential and regression types in their nature. First set of estimators (T_{1u}, T_{2u}) are based on the fresh sample s_u and the second set of estimators (T_{1m}, T_{2m}) are based on the matched sample s_m. The two sets of suggested estimators based on s_u and s_m are presented below:

$$T_{1u}=\bar{y}_u\left(\frac{\bar{Z}+k}{\bar{z}_u+k}\right)exp\left(\frac{\bar{Z}-\bar{z}_u}{\bar{Z}+\bar{z}_u}\right) \tag{1}$$

where k may be some known population parameters of auxiliary variable z such as coefficient of variation, standard deviation (σ), coefficient of skewness (β_1) and coefficient of kurtosis (β_2) etc.

$$T_{2u}=[\bar{y}_u+b_{yz}(u)(\bar{Z}-\bar{z}_u)] \tag{2}$$

$$T_{1m}=[\bar{y}_m+b_{yx}(m)(\bar{x}_n-\bar{x}_m)]\left(\frac{\bar{Z}+k}{\bar{z}_m+k}\right)exp\left(\frac{\bar{Z}-\bar{z}_m}{\bar{Z}+\bar{z}_m}\right) \tag{3}$$

and

$$T_{2m}=\bar{y}_m\left(\frac{\bar{Z}+k}{\bar{z}_m+k}\right)exp\left(\frac{\bar{x}_n-\bar{x}_m}{\bar{x}_n+\bar{x}_m}\right)exp\left(\frac{\bar{Z}-\bar{z}_m}{\bar{Z}+\bar{z}_m}\right) \tag{4}$$

where $b_{yz}(u)$ and $b_{yx}(m)$ are the sample regression coefficients between the variables shown in suffices and based on the sample sizes shown in braces.

Considering the convex linear combinations of the two sets of estimators $T_{iu}(i=1,2)$ and $T_{jm}(j=1,2)$, we have the final estimators of $\bar{Y}$ on the current occasion as

$$T_{ij}=\varphi_{ij}T_{iu}+(1-\varphi_{ij})T_{jm};(i,j=1,2) \tag{5}$$

where $\varphi_{ij}(0\leq\varphi_{ij}\leq 1)(i,j=1,2)$ are the unknown constants to be determined under certain criterion.

Remark 3.1: For estimating the mean on each occasion the estimators $T_{iu}(i=1,2)$ are suitable, which implies that more belief on T_{iu} could be shown by choosing $\varphi_{ij}(i=1,2)$ as 1 (or close to 1), while for estimating the change from one occasion to the next, the estimators $T_{jm}(j=1,2)$ could be more useful so $\varphi_{ij}(j=1,2)$ might be chosen as 0 (or close to 0). For asserting both the problems simultaneously, the suitable (optimum) choices of $\varphi_{ij}(i,j=1,2)$ are required.

4 Properties of the Proposed Estimators $T_{ij}(i,j=1,2)$

Since T_{iu} and $T_{jm}(i,j=1,2)$ are exponential and linear regression type estimators and they are biased for population mean $\bar{Y}$. Therefore, the final estimators $T_{ij}(i,j=1,2)$ defined in equation (5) are also biased estimators of $\bar{Y}$. The bias $B(.)$ and mean square errors $M(.)$ of the estimators $T_{ij}(i,j=1,2)$ are derived up to first order approximations under the large sample assumptions and using the following transformations:

$\bar{y}_u=\bar{Y}(1+e_1)$, $\bar{y}_m=\bar{Y}(1+e_2)$, $\bar{x}_m=\bar{X}(1+e_3)$, $\bar{x}_n=\bar{X}(1+e_4)$, $\bar{z}_u=\bar{Z}(1+e_5)$, $\bar{z}_m=\bar{Z}(1+e_6)$, $\bar{z}_n=\bar{Z}(1+e_7)$, $s_{yz}=S_{yz}(1+e_8)$, $s_z^2=S_z^2(1+e_9)$, $s_{yx}=S_{yx}(1+e_{10})$, $s_x^2=S_x^2(1+e_{11})$ such that $E(e_i)=0$ and $|e_i|\leq 1, \forall i=1,2,...,11$.

Under the above transformations the estimators T_{iu} and $T_{jm}(i,j=1,2)$ take the following forms:

$$T_{1u}=\bar{Y}(1+e_1)(1+ge_5)^{-1}exp\left[-\frac{1}{2}e_5\left(e_5+\frac{1}{2}e_5\right)^{-1}\right] \tag{6}$$

$$T_{2u}=\left[\bar{Y}(1+e_1)-\bar{Z}\beta_{yz}e_5(1+e_8)(1+e_9)^{-1}\right] \tag{7}$$

$$T_{1m}=\left[\bar{Y}(1+e_2)+\bar{X}\beta_{yx}(1+e_{10})(1+e_{11})^{-1}(e_4-e_3)\right](1+ge_6)^{-1}exp\left[-\frac{1}{2}e_6\left(1+\frac{1}{2}e_6\right)^{-1}\right] \tag{8}$$

and

$$T_{2m}=\bar{Y}(1+e_2)exp\left[\frac{1}{2}(e_4-e_3)\left(1+\frac{1}{2}(e_4+e_3)\right)^{-1}\right](1+ge_6)^{-1}exp\left[-\frac{1}{2}e_6\left(1+\frac{1}{2}e_6\right)^{-1}\right] \tag{9}$$

where $g=\frac{\bar{Z}}{\bar{Z}+k}$

Thus, we have the expressions of bias and mean square error of the estimators $T_{ij}(i,j=1,2)$ in the following theorems.

Theorem 4.1 Bias of the estimators $T_{ij}(i,j=1,2)$ up to the first order of approximations are derived as

$$B(T_{ij})=\varphi_{ij}B(T_{iu})+(1-\varphi_{ij})B(T_{jm});(i,j=1,2) \tag{10}$$

where

$$B(T_{1u})=\bar{Y}\left(\frac{1}{u}-\frac{1}{N}\right)\left[\left(g^2+\frac{1}{2}g+\frac{3}{8}\right)-\left(g+\frac{1}{2}\right)\rho_{yz}\right]C_y^2 \tag{11}$$

$$B(T_{2u})=\left(\frac{1}{u}-\frac{1}{N}\right)\beta_{yz}\left(\frac{\mu_{003}}{\mu_{002}}-\frac{\mu_{012}}{\mu_{011}}\right) \tag{12}$$

$$B(T_{1m})=\bar{Y}\left[\left(\frac{1}{m}-\frac{1}{N}\right)\left(\left(g^2+\frac{1}{2}g+\frac{3}{8}\right)-\left(g+\frac{1}{2}\right)\rho_{yz}\right)C_y^2+\left(\frac{1}{m}-\frac{1}{n}\right)\frac{\bar{X}}{\bar{Y}}\beta_{yx}\left(\left(g+\frac{1}{2}\right)C_y^2-\frac{1}{X}\left(\frac{\mu_{210}}{\mu_{110}}-\frac{\mu_{300}}{\mu_{200}}\right)\right)\right] \tag{13}$$

$$B(T_{2m})=\bar{Y}\left[\left(\frac{1}{m}-\frac{1}{N}\right)\left(\left(g^2+\frac{1}{2}g+\frac{3}{8}\right)-\left(g+\frac{1}{2}\right)\rho_{yz}\right)+\left(\frac{1}{m}-\frac{1}{n}\right)\left(\frac{3}{8}+\frac{1}{2}\left(g+\frac{1}{2}\right)\rho_{yz}-\frac{1}{2}\rho_{yx}\right)\right] \tag{14}$$

where $\mu_{rst}=E\left[(x_i-\bar{X})^r(y_j-\bar{Y})^s(z_i-\bar{Z})^t\right];(r,s,t)\geq 0$ are integers.

Proof : The bias of the estimators $T_{ij}(i,j=1,2)$ are given by

$$B(T_{ij})=E(T_{ij}-\bar{Y})=\varphi_{ij}E(T_{iu}-\bar{Y})+(1-\varphi_{ij})E(T_{jm}-\bar{Y}) \tag{15}$$

$$=\varphi_{ij}B(T_{iu})+(1-\varphi_{ij})B(T_{jm});(i,j=1,2). \tag{16}$$

where $B(T_{iu}) = E(T_{iu} - \overline{Y})$, $B(T_{jm}) = E(T_{jm} - \overline{Y})$; $(i, j = 1, 2)$.

Substituting the expressions of form equations (6) to (9) into the equation (16), expanding binomially and exponentially, taking expectations, and retaining the terms up to the first order approximations, we have the expression of the bias of the estimators $T_{ij}(i, j = 1, 2)$ as shown in equation (10).

Theorem 4.2. Mean square errors of the estimators $T_{ij}(i, j = 1, 2)$ up to the first order approximations are derived as

$$M(T_{ij}) = \varphi_{ij}^2 M(T_{iu}) + (1-\varphi_{ij}^2)^2 M(T_{jm}) + 2\varphi_{ij}(1-\varphi_{ij})C(T_{iu}, T_{jm}); (i, j = 1, 2) \tag{17}$$

where

$$M(T_{1u}) = \left(\frac{1}{u} - \frac{1}{N}\right)\left[1 + \left(g + \frac{1}{2}\right)^2 - 2\left(g + \frac{1}{2}\right)\rho_{yz}\right] S_y^2 \tag{18}$$

$$M(T_{2u}) = \left(\frac{1}{u} - \frac{1}{N}\right)(1 - \rho_{yz}^2) S_y^2 \tag{19}$$

$$M(T_{1m}) = \left[\left(\frac{1}{m} - \frac{1}{N}\right)\left(1 + \left(g + \frac{1}{2}\right)^2 - 2\left(g + \frac{1}{2}\right)\rho_{yz}\right) + \left(\frac{1}{m} - \frac{1}{n}\right)\left(2\left(g + \frac{1}{2}\right)\rho_{yx}\rho_{yz} - \rho_{yx}^2\right)\right] S_y^2 \tag{20}$$

$$M(T_{2m}) = \left[\left(\frac{1}{m} - \frac{1}{N}\right)\left(1 + \left(g + \frac{1}{2}\right)^2 - 2\left(g + \frac{1}{2}\right)\rho_{yz}\right) + \left(\frac{1}{m} - \frac{1}{n}\right)\left(\left(g + \frac{1}{2}\right)\rho_{yz} - \rho_{yx} + \frac{1}{4}\right)\right] S_y^2 \tag{21}$$

$$C(T_{1u}, T_{1m}) = C(T_{1u}, T_{2m}) = -\frac{1}{N}\left[1 + \left(g + \frac{1}{2}\right)^2 - 2\left(g + \frac{1}{2}\right)\rho_{yz}\right] S_y^2 \tag{22}$$

$$C(T_{2u}, T_{1m}) = C(T_{2u}, T_{2m}) = -\frac{1}{N}(1 - \rho_{yz}^2) S_y^2 \tag{23}$$

Proof: The mean square error of the estimators $T_{ij}(i, j = 1, 2)$ are given by

$$M(T_{ij}) = E\left[T_{ij} - \overline{Y}\right]^2 = E\left[\varphi_{ij}(T_{iu} - \overline{Y}) + (1-\varphi_{ij})(T_{jm} - \overline{Y})\right]^2 \tag{24}$$

$$= \varphi_{ij}^2 M(T_{iu}) + (1-\varphi_{ij})^2 M(T_{jm}) + 2\varphi_{ij}(1-\varphi_{ij})C(T_{iu}, T_{jm}); (i, j = 1, 2). \tag{25}$$

where $M(T_{iu}) = E[T_{iu} - \overline{Y}]^2$, $M(T_{jm}) = E[T_{jm} - \overline{Y}]^2$ and $C(T_{iu}, T_{jm}) = E[(T_{iu} - \overline{Y})(T_{jm} - \overline{Y})]$; $(i, j = 1, 2)$.

Substituting the expressions of T_{iu} and $T_{jm}(i, j = 1, 2)$ from equations (6) to (9) into the equation (25), expanding binomially and exponentially, taking expectations, and retaining the terms up to the first order approximations, we have the expression of the mean square error of the estimators $T_{ij}(i, j = 1, 2)$ as shown in equation (17).

Remark 4.1: The above results are derived under the assumption that the coefficient of variations of x, y and z are approximately equal, which has been considered by Feng and Zou [5].

5 Minimum Mean Square Errors of the Estimators $T_{ij}(i, j = 1, 2)$

Since the mean square errors of the estimators $T_{ij}(i, j = 1, 2)$ in equation (17) are functions of unknown constants $\varphi_{ij}(i, j = 1, 2)$, therefore, they are minimized with respect to φ_{ij} and subsequently the optimum values of φ_{ij} are obtained as

$$\varphi_{ij(opt)} = \frac{M(T_{jm}) - C(T_{iu}, T_{jm})}{M(T_{iu}) + M(T_{jm}) - 2C(T_{iu}, T_{jm})}; (i, j = 1, 2) \tag{26}$$

Now substituting the values of $\varphi_{ij(opt)}$ in equation (17), we have the optimum mean square errors of the estimators $T_{ij}(i, j = 1, 2)$as

$$M(T_{ij})_{(opt)} = \frac{M(T_{iu}).M(T_{jm}) - [C(T_{iu}, T_{jm})]^2}{M(T_{iu}) + M(T_{jm}) - 2C(T_{iu}, T_{jm})}; (i, j = 1, 2) \tag{27}$$

Further, substituting the values from equations (18) - (23) in equations (26) and (27), the simplified values of $\varphi_{ij(opt)}(i,j=1,2)$ and $M(T_{ij})_{(opt)}(i,j=1,2)$ are obtained as

$$\varphi_{11(opt)}=\frac{\mu_{11}[A_1+A_3\mu_{11}]}{[A_1+A_3\mu_{11}^2]} \tag{28}$$

$$\varphi_{12(opt)}=\frac{\mu_{12}[A_1+A_4\mu_{12}]}{[A_1+A_4\mu_{12}^2]} \tag{29}$$

$$\varphi_{21(opt)}=\frac{\mu_{21}[A_{10}-A_{11}\mu_{21}]}{[A_2+(A_{10}-A_2)\mu_{21}-A_{11}\mu_{21}^2]} \tag{30}$$

$$\varphi_{22(opt)}=\frac{\mu_{22}[A_{15}-A_{16}\mu_{22}]}{[A_1+(A_{15}-A_1)\mu_{22}-A_{10}\mu_{22}^2]} \tag{31}$$

$$M(T_{11})_{(opt)}=\frac{[A_5+A_6\mu_{11}-A_7\mu_{11}^2]}{[A_1+A_3\mu_{11}^2]}.\frac{S_y^2}{n} \tag{32}$$

$$M(T_{12})_{(opt)}=\frac{[A_5+A_8\mu_{12}-A_9\mu_{12}^2]}{[A_1+A_4\mu_{12}^2]}.\frac{S_y^2}{n} \tag{33}$$

$$M(T_{21})_{(opt)}=\frac{[A_{12}+A_{13}\mu_{21}-A_{14}\mu_{21}^2]}{[A_2+(A_{10}-A_2)\mu_{21}-A_{11}\mu_{21}^2]}.\frac{S_y^2}{n} \tag{34}$$

$$M(T_{22})_{(opt)}=\frac{[A_{16}+A_{17}\mu_{22}-A_{18}\mu_{22}^2]}{[A_2+(A_{10}-A_2)\mu_{22}-A_{15}\mu_{22}^2]}.\frac{S_y^2}{n} \tag{35}$$

where $A_1=1+\left(g+\frac{1}{2}\right)^2-2\left(g+\frac{1}{2}\right)\rho_{yz}$, $A_2=1-\rho_{yz}^2$, $A_3=2\left(g+\frac{1}{2}\right)\rho_{yx}\rho_{yz}-\rho_{yx}^2$, $A_4=\left(g+\frac{1}{2}\right)\rho_{yz}-\rho_{yx}+\frac{1}{4}$, $A_5=(1-f)A_1^2$, $A_6=A_1A_3$, $A_7=fA_6$, $A_8=A_1A_4$, $A_{10}=A_1+f(A_2-A_1)$, $A_{11}=(A_2-A_1)-A_3$, $A_{12}=(1-f)A_1A_2$, $A_{13}=A_2A_3+f^2(A_1A_2-A_2^2)$, $A_{14}=fA_2A_3+f^2(A_1A_2-A_2^2)$, $A_{15}=f(A_2-A_1)-A_4$, $A_{16}=f^2(A_1A_2-A_2^2)+A_2A_4$, $A_{17}=f^2(A_1A_2-A_2^2)+fA_2A_4$, $f=\frac{n}{N}$, and $\mu_{ij}=\frac{u}{n}(i,j=1,2)$ are the fractions of fresh sample drawn at the current (second) occasion.

6 Optimum Replacement Strategies of the Estimators $T_{ij}(i,j=1,2)$

To determine the optimum values of $\mu_{ij}(i,j=1,2)$ (fraction of samples to be drawn afresh at the second occasion) so that population mean $\bar{Y}$ may be estimated with maximum precision and minimum cost, we minimize mean square errors of $T_{ij}(i,j=1,2)$ given in equations (32)-(35) respectively with respect to μ_{ij}, which result in quadratic equations in μ_{ij} and respective solutions of μ_{ij} say $\hat{\mu}_{ij}(i,j=1,2)$ are given below:

$$Q_1\mu_{11}^2+2Q_2\mu_{11}-Q_3=0 \tag{36}$$

$$\hat{\mu}_{11}=\frac{-Q_2\pm\sqrt{Q_2^2+Q_1Q_3}}{Q_1} \tag{37}$$

$$P_1\mu_{12}^2+2P_2\mu_{12}-P_3=0 \tag{38}$$

$$\hat{\mu}_{12}=\frac{-P_2\pm\sqrt{P_2^2+P_1P_3}}{P_1} \tag{39}$$

$$R_1\mu_{21}^2+2R_2\mu_{21}+R_3=0 \tag{40}$$

$$\hat{\mu}_{21} = \frac{-R_2 \pm \sqrt{R_2^2 - R_1 R_3}}{R_1} \tag{41}$$

$$S_1 \mu_{22}^2 + 2 S_2 \mu_{22} + S_3 = 0 \tag{42}$$

$$\hat{\mu}_{22} = \frac{-S_2 \pm \sqrt{S_2^2 - S_1 S_3}}{S_1} \tag{43}$$

where $Q_1 = A_3 A_6$, $Q_2 = A_1 A_7 + A_3 A_5$, $Q_3 = A_1 A_6$, $P_1 = A_4 A_8$, $P_2 = A_1 A_9 + A_4 A_5$, $P_3 = A_1 A_8$, $R_1 = A_2 A_{14} - A_{10} A_{14} + A_{11} A_{13}$, $R_2 = A_{11} A_{12} - A_2 A_{14}$, $R_3 = A_2 A_{12} + A_2 A_{13} - A_{10} A_{12}$, $S_1 = A_2 A_{17} - A_{10} A_{17} + A_{15} A_{16}$, $S_2 = A_{15} A_{12} - A_2 A_{17}$, $S_3 = A_2 A_{12} + A_2 A_{16} - A_{10} A_{12}$.

From equations (37), (39), (41) and (43), it is clear that the real values of $\hat{\mu}_{ij}(i,j=1,2)$ exist, iff, the quantities under square roots are greater than or equal to zero. For any combinations of ρ_{xz} and ρ_{yz}, which satisfy the conditions of real solutions, two real values of $\hat{\mu}_{ij}$ are possible. Hence, while choosing the values of $\hat{\mu}_{ij}$, it should be remembered that $0 \leq \hat{\mu}_{ij} \leq 1$, all others values of $\hat{\mu}_{ij}$ are inadmissible. If both the values of $\hat{\mu}_{ij}$ are admissible, lowest one will be the best choice as it reduces the cost of the surveys. Substituting the admissible values of $\hat{\mu}_{ij}$ say $\mu_{ij}^*(i,j=1,2)$ from equations (37), (39), (41) and (43) into equations (32), (33), (34) and (35) respectively, we have the optimum values of mean square errors of the estimators $T_{ij}(i,j=1,2)$, which are shown below:

$$M(T_{11})^*_{(opt)} = \frac{[A_5 + A_6 \mu_{11}^* - A_7 \mu_{11}^{*2}]}{[A_1 + A_3 \mu_{11}^{*2}]} \cdot \frac{S_y^2}{n} \tag{44}$$

$$M(T_{12})^*_{(opt)} = \frac{[A_5 + A_8 \mu_{12}^* - A_9 \mu_{12}^{*2}]}{[A_1 + A_5 \mu_{12}^{*2}]} \cdot \frac{S_y^2}{n} \tag{45}$$

$$M(T_{21})^*_{(opt)} = \frac{[A_{12} + A_{13} \mu_{21}^* - A_{14} \mu_{21}^{*2}]}{[A_2 + (A_{10} - A_2)\mu_{21}^* - A_{11} \mu_{21}^{*2}]} \cdot \frac{S_y^2}{n} \tag{46}$$

$$M(T_{22})^*_{(opt)} = \frac{[A_{16} + A_{17} \mu_{22}^* - A_{18} \mu_{22}^{*2}]}{[A_2 + (A_{10} - A_2)\mu_{22}^* - A_{10} \mu_{22}^{*2}]} \cdot \frac{S_y^2}{n} \tag{47}$$

7 Efficiency Comparison

The percent relative efficiencies of the estimators $T_{ij}(i,j=1,2)$ with respect to (i) sample mean estimator $\overline{y}_n$, when there is no matching (ii) natural successive sampling estimator $\hat{\overline{Y}} = \varphi^* \overline{y}_u + (1-\varphi^*) \overline{y}'_m$, when there is no auxiliary information is used on any occasion, where $\overline{y}'_m = \overline{y}_m + b_{yx}(\overline{x}_n - \overline{x}_m)$, have been obtained for different choices of ρ_{yx} and ρ_{yz}. Since $\overline{y}_n$ and $\hat{\overline{Y}}$ are unbiased estimators of $\overline{Y}$, therefore, following Sukhatme *et al*. [24], the variance of $\overline{y}_n$ and optimum variance of $\hat{\overline{Y}}$ are respectively given as

$$V(\overline{y}_n) = \left(\frac{1}{n} - \frac{1}{N}\right) S_y^2 \tag{48}$$

$$V(\hat{\overline{Y}})_{opt} = \left[1 + \sqrt{1 - \rho_{yx}^2}\right] \frac{S_y^2}{2n} - \frac{S_y^2}{N} \tag{49}$$

For f = 0.1 and different choices of ρ_{yx} and ρ_{yz}, Tables 1 - 4 present the optimum values of $\mu_{ij}(i,j=1,2)$ and percent relative efficiencies E_{ij} and E_{ij}^* of $T_{ij}(i,j=1,2)$ (under their respective optimality conditions) with respect to $\overline{y}_n$ and $\hat{\overline{Y}}$, respectively, where

$$E_{ij} = \frac{V(\overline{y}_n)}{M(T_{ij})^*_{(opt)}} \times 100 \text{ and } E_{ij}^* = \frac{V(\hat{\overline{Y}})_{opt}}{M(T_{ij})^*_{(opt)}} \times 100 \; ; (i,j=1,2).$$

Table 1: Optimum values of μ_{11} and percent relative efficiencies of the estimator T_{11} with respect to $\bar{y}_n$ and $\hat{\bar{Y}}$.

$\rho_{yx}\downarrow$	$\rho_{yz}\downarrow$	$g\rightarrow$	0.2	0.4	0.6	0.8
0.1	0.7	μ_{11}^*	0.4801	0.4761	0.4757	0.4775
		E_{11}	187.45	172.21	141.23	109.21
		E_{11}^*	186.93	171.73	140.84	108.91
	0.8	μ_{11}^*	0.4696	0.4614	0.4608	0.4649
		E_{11}	252.14	247.31	203.05	151.25
		E_{11}^*	251.43	246.62	202.48	150.83
	0.9	μ_{11}^*	0.4491	0.4271	0.4259	0.4385
		E_{11}	386.18	442.36	364.32	247.18
		E_{11}^*	385.11	441.12	363.31	246.492
0.3	0.7	μ_{11}^*	0.4580	0.4476	0.4450	0.4475
		E_{11}	177.96	160.87	131.24	101.69
		E_{11}^*	173.41	156.76	127.88	100.00
	0.8	μ_{11}^*	0.4366	0.4189	0.4158	0.4220
		E_{11}	232.74	222.42	181.43	136.02
		E_{11}^*	226.78	216.73	176.78	132.53
	0.9	μ_{11}^*	0.3999	0.3628	0.3589	0.3762
		E_{11}	340.16	370.62	302.58	209.24
		E_{11}^*	331.45	361.13	294.84	203.88
0.5	0.7	μ_{11}^*	0.4519	0.4347	0.4287	0.4299
		E_{11}	175.36	155.82	125.97	**
		E_{11}^*	162.37	144.22	116.59	**
	0.8	μ_{11}^*	0.4245	0.3989	0.3923	0.3976
		E_{11}	225.68	210.90	170.27	127.47
		E_{11}^*	208.88	195.20	157.60	117.98
	0.9	μ_{11}^*	0.3804	0.3348	0.3275	0.3442
		E_{11}	322.25	339.95	274.25	190.13
		E_{11}^*	298.27	314.65	253.84	175.98
0.7	0.7	μ_{11}^*	0.4594	0.4331	0.4219	0.4204
		E_{11}	178.56	155.20	123.79	**
		E_{11}^*	150.20	130.55	104.13	**
	0.8	μ_{11}^*	0.4274	0.3923	0.3806	0.3835
		E_{11}	227.37	207.09	164.78	122.57
		E_{11}^*	191.26	174.20	138.61	103.11
	0.9	μ_{11}^*	0.3781	0.3231	0.3113	0.3259
		E_{11}	320.15	327.23	259.78	179.27
		E_{11}^*	269.31	275.26	218.52	150.80

Table 2: Optimum values of μ_{12} and percent relative efficiencies of the estimator T_{12} with respect to $\bar{y}_n$ and $\hat{\bar{Y}}$.

$\rho_{yx}\downarrow$	$\rho_{yz}\downarrow$	$g\rightarrow$	0.2	0.4	0.6	0.8
0.1	0.7	μ_{12}^*	0.3997	0.3914	0.3936	0.4017
		E_{12}	153.34	138.96	114.78	**
		E_{12}^*	152.92	138.58	114.47	**
	0.8	μ_{12}^*	0.3692	0.3533	0.3554	0.3679
		E_{12}	193.94	184.93	153.05	117.20
		E_{12}^*	193.40	184.41	152.62	116.87
	0.9	μ_{12}^*	0.3230	0.2890	0.2906	0.3140
		E_{12}	270.28	290.58	241.50	172.33
		E_{12}^*	269.53	289.78	240.83	171.85

continue....

$\rho_{yx}\downarrow$	$\rho_{yz}\downarrow$	$g\rightarrow$	0.2	0.4	0.6	0.8
0.3	0.7	μ_{12}^*	0.4229	0.4110	0.4098	0.4149
		E_{12}	163.03	146.54	119.92	**
		E_{12}^*	158.86	142.79	116.85	**
	0.8	μ_{12}^*	0.3934	0.3736	0.3722	0.3817
		E_{12}	207.70	196.44	160.86	121.96
		E_{12}^*	202.39	191.41	156.75	118.84
	0.9	μ_{12}^*	0.3476	0.3090	0.3072	0.3279
		E_{12}	292.39	312.04	256.14	180.49
		E_{12}^*	284.90	304.05	249.58	175.87
0.5	0.7	μ_{12}^*	0.4519	0.4347	0.4287	0.4299
		E_{12}	175.36	155.82	125.97	**
		E_{12}^*	162.31	144.22	116.60	**
	0.8	μ_{12}^*	0.4245	0.3989	0.3923	0.3976
		E_{12}	225.68	210.90	170.27	127.47
		E_{12}^*	208.88	195.20	157.60	117.98
	0.9	μ_{12}^*	0.3804	0.3348	0.3275	0.3442
		E_{12}	322.25	339.95	274.25	190.13
		E_{12}^*	298.27	314.65	253.84	175.98
0.7	0.7	μ_{12}^*	0.4906	0.4647	0.4514	0.4471
		E_{12}	191.97	167.65	167.65	101.60
		E_{12}^*	161.49	141.03	112.12	**
	0.8	μ_{12}^*	0.4675	0.4319	0.4169	0.4162
		E_{12}	250.90	230.00	181.95	133.97
		E_{12}^*	211.05	193.47	153.05	112.70
	0.9	μ_{12}^*	0.4282	0.3702	0.3534	0.3638
		E_{12}	366.54	378.74	297.60	201.80
		E_{12}^*	308.33	318.59	250.34	169.75

Table 3: Optimum values of μ_{21} and percent relative efficiencies of the estimator T_{21} with respect to $\bar{y}_n$ and $\hat{\bar{Y}}$.

$\rho_{yx}\downarrow$	$\rho_{yz}\downarrow$	$g\rightarrow$	0.2	0.4	0.6	0.8
0.1	0.7	μ_{21}^*	0.4801	0.3067	*	*
		E_{21}	187.45	177.39	**	**
		E_{21}^*	186.93	176.90	**	**
	0.8	μ_{21}^*	0.4195	0.4219	0.1709	*
		E_{21}	255.27	250.42	218.79	**
		E_{21}^*	254.56	249.72	218.18	**
	0.9	μ_{21}^*	0.2533	0.4271	0.2907	*
		E_{21}	415.62	442.36	394.56	**
		E_{21}^*	414.46	441.12	393.46	**
0.3	0.7	μ_{21}^*	0.4580	0.3690	0.1933	*
		E_{21}	177.96	166.31	144.66	**
		E_{21}^*	173.41	162.05	140.96	**
	0.8	μ_{21}^*	0.4126	0.4002	0.2783	0.1015
		E_{21}	235.72	225.30	199.15	161.14
		E_{21}^*	229.69	219.54	194.05	157.02
	0.9	μ_{21}^*	0.3004	0.3628	0.2902	0.1489
		E_{21}	369.77	370.62	330.03	263.60
		E_{21}^*	360.31	361.13	321.59	256.86
0.5	0.7	μ_{21}^*	0.4519	0.3716	0.2333	0.0709
		E_{21}	175.36	161.19	140.04	114.10
		E_{21}^*	162.31	149.19	129.62	105.61
	0.8	μ_{21}^*	0.4044	0.3841	0.2857	0.1533
		E_{21}	228.59	213.64	187.62	154.67
		E_{21}^*	211.58	197.74	173.65	143.16
	0.9	μ_{21}^*	0.2980	0.3348	0.2733	0.1680
		E_{21}	350.95	339.95	299.62	243.96
		E_{21}^*	324.83	314.65	277.32	225.80

continue....

$\rho_{yx}\downarrow$	$\rho_{yz}\downarrow$	$g\rightarrow$	0.2	0.4	0.6	0.8
0.7	0.7	μ_{21}^*	0.4594	0.3715	0.2434	0.1030
		E_{21}	178.56	160.56	137.97	112.86
		E_{21}^*	150.20	135.06	116.06	**
	0.8	μ_{121}^*	0.4065	0.3784	0.2850	0.1698
		E_{21}	230.30	209.79	181.82	150.15
		E_{21}^*	193.73	176.47	152.94	126.30
	0.9	μ_{21}^*	0.2974	0.3231	0.2628	0.1714
		E_{21}	348.73	327.23	284.00	231.78
		E_{21}^*	293.35	275.26	238.90	194.97

Table 4: Optimum values of μ_{22} and percent relative efficiencies of the estimator T_{22} with respect to $\bar{y}_n$ and $\hat{\bar{Y}}$.

$\rho_{yx}\downarrow$	$\rho_{yz}\downarrow$	$g\rightarrow$	0.2	0.4	0.6	0.8
0.1	0.7	μ_{22}^*	0.3997	0.3541	0.2634	0.1443
		E_{22}	153.34	143.92	128.87	109.48
		E_{22}^*	152.92	143.52	128.51	109.18
	0.8	μ_{22}^*	0.3580	0.3434	0.2781	0.1811
		E_{22}	196.48	187.36	169.27	144.75
		E_{22}^*	195.93	186.84	168.80	144.34
	0.9	μ_{22}^*	0.2705	0.2890	0.2484	0.1716
		E_{22}	295.34	290.58	264.21	223.74
		E_{22}^*	294.52	289.78	263.48	223.12
0.3	0.7	μ_{22}^*	0.4229	0.3650	0.2554	0.1175
		E_{22}	163.03	151.70	134.14	111.98
		E_{22}^*	158.86	147.82	130.70	109.11
	0.8	μ_{22}^*	0.3794	0.3620	0.2835	0.1714
		E_{22}	210.42	199.01	177.65	149.55
		E_{22}^*	205.03	193.92	173.11	145.72
	0.9	μ_{22}^*	0.2848	0.3090	0.2600	0.1712
		E_{22}	319.12	312.04	280.06	233.18
		E_{22}^*	310.95	304.05	272.89	227.21
0.5	0.7	μ_{22}^*	0.4519	0.3716	0.2333	0.0709
		E_{22}	175.36	161.18	140.04	114.10
		E_{22}^*	162.31	149.19	129.62	105.61
	0.8	μ_{22}^*	0.4044	0.3841	0.2857	0.1533
		E_{22}	228.59	213.64	187.62	154.67
		E_{22}^*	211.58	197.74	173.65	143.16
	0.9	μ_{22}^*	0.2980	0.3348	0.2733	0.1680
		E_{22}	350.95	339.95	299.62	243.96
		E_{22}^*	324.83	314.65	277.32	225.80
0.7	0.7	μ_{22}^*	0.4906	0.3487	0.1683	*
		E_{22}	191.97	173.06	146.24	**
		E_{22}^*	161.49	145.58	123.01	**
	0.8	μ_{22}^*	0.4207	0.4096	0.2776	0.1175
		E_{22}	254.03	232.96	199.68	159.82
		E_{22}^*	213.69	195.96	167.97	134.44
	0.9	μ_{22}^*	0.2886	0.3702	0.2877	0.1588
		E_{22}	396.77	378.74	324.70	256.29
		E_{22}^*	333.76	318.59	273.14	215.59

Note: * indicate do not exist and ** denote no gain.

8 Interpretations of Empirical Results

The following interpretations may be read out from Tables 1- 4:

(1) From Table-1, it is clear that

(a) For fixed values of ρ_{yx} and ρ_{yz}, the values of μ_{11}^* are first significantly decreased and little increased at the end while E_{11} and E_{11}^* are slightly increased at starting and significantly decreased with increased values of g. This behavior explains that the more the value of g, less the fraction of fresh sample is required at the current occasion.

(b) For fixed values of ρ_{yz} and g, the values of μ_{11}^*,E_{11} and E_{11}^* are decreased for lower values of ρ_{yx} and increased for higher values of ρ_{yx}. This behavior is an agreement with the Sukhatme *et al*. [24] results, which explains that the more the value of ρ_{yx}, more the fraction of fresh sample is required on the current occasion.

(c) For fixed values of ρ_{yx} and g, the values of E_{11}^* are decreasing with the increasing values of ρ_{yz} while E_{11} and E_{11}^* are increasing with the increasing values of ρ_{yz}. This behavior is highly desirable and indicates that higher the correlation coefficient between study variable and auxiliary variable at the first occasion, lower amount of fresh sample is required at the current occasion along with the increase in the precision of the estimates.

(d) Minimum value of μ_{11} is 0.3043, which indicates that the fraction to be replaced at the current occasion is as low as about 30 percent of the total sample size.

(2) From Table- 2, it is observed that

(a) For fixed values of ρ_{yx} and ρ_{yz}, the behavior of μ_{12}^*,E_{12} and E_{12}^* are similar to that of **1(a)**.

(b) For fixed values of ρ_{yz} and g, the values of μ_{12}^*,E_{12} and E_{12}^* are increased with increased values of ρ_{yx}. This behavior similar to **1(b)**.

(c) For fixed values of ρ_{yz} and g, the behavior of μ_{12}^*,E_{12} and E_{12}^* are same as it is discussed in **1(c)**.

(d) Minimum value of μ_{12}^* is 0.2857, which indicates that the fraction to be replaced on current occasion is as low as about 28 percent of the total sample size.

(3) From Table- 3, it can be seen that

(a) For fixed values of ρ_{yx} and ρ_{yz}, the values of μ_{21}^*, E_{21} and E_{21}^* are slightly increased at starting and then significantly decreased. But, for fixed values of ρ_{yx} and ρ_{yz}=0.6, the values of μ_{21}^*, E_{21} and E_{21}^* are uniformly decreased with increasing value of g.

(b) For fixed values of ρ_{yz} and g, the values of μ_{21}^*, E_{21} and E_{21}^* are increased with increased values of ρ_{yx}. This behavior is an agreement with the Sukhatme *et al*. [24] results.

(c) For fixed values of ρ_{yx} and g, the value of μ_{21}^* are slightly increased at starting and then significantly decreased, whileE_{21} and E_{21}^* are decreased with the increased value of ρ_{yx}. This behavior indicates that if the information on highly correlated auxiliary variable is available at the first occasion, it reduces the amount of fresh sample on the current occasion along with the increase in the precision of the estimates.

(d) Minimum value of μ_{21}^* is 0.0010, which indicates that the fraction of sample to be replaced at the current occasion is as low as about 1 percent of the total sample size, which leads to highly reduction in cost of the survey.

(4) From Table- 4, it is obvious that

(a) For fixed values of ρ_{yx} and ρ_{yz}, the behaviors of μ_{22}^*, E_{22} and E_{22}^* are similar to that of **3(a)**

(b) For fixed values of ρ_{yz} and g, the behaviors of μ_{22}^*, E_{22} and E_{22}^* are similar to that of **3(b)** with the increasing value of ρ_{yx}.

(c) For fixed value of ρ_{yx} and g, the behaviors of μ_{22}^*, E_{22} and E_{22}^* are same as it is discussed in **3(c)**.

(d) Minimum value of μ_{22}^* is 0.0171, which indicates that the fraction to be replaced at the current occasion is as low as about 10 percent of the total sample size, which is very helpful in reducing the cost of the survey.

9 Mutual Comparison of the Estimators $T_{ij}(i,j=1,2)$

Table 5: The values of $E_{ij}(i,j=1,2)$ for different choices of correlations.

$\rho_{yx}\downarrow$	$\rho_{yz}\downarrow$	$g\rightarrow$	0.2	0.3	0.4	0.5
0.2	0.7	E_{11}	181.67	176.29	165.54	151.27
		E_{12}	157.92	152.21	142.57	130.43
		E_{21}	**181.67**	**177.88**	**170.97**	**160.89**
		E_{22}	157.92	153.62	147.63	140.17
	0.8	E_{11}	240.40	242.14	232.50	213.89
		E_{12}	200.41	199.38	190.38	175.25
		E_{21}	**243.46**	**242.14**	**235.49**	**224.01**
		E_{22}	203.03	199.38	192.88	184.06
	0.9	E_{11}	357.97	391.87	398.14	373.06
		E_{12}	280.60	299.77	300.67	281.43
		E_{21}	**388.14**	**401.37**	**398.14**	**382.17**
		E_{22}	306.45	307.23	300.67	288.45
0.3	0.7	E_{11}	177.96	171.90	160.87	146.70
		E_{12}	163.03	156.82	146.54	133.73
		E_{21}	**177.96**	**173.46**	**166.31**	**156.65**
		E_{22}	163.03	158.26	151.70	143.59
	0.8	E_{11}	232.74	232.79	222.42	204.05
		E_{12}	207.70	206.23	196.44	180.32
		E_{21}	**235.72**	**232.79**	**225.30**	**213.93**
		E_{22}	210.42	206.23	199.01	189.34
	0.9	E_{11}	340.16	367.92	370.62	345.79
		E_{12}	292.39	311.89	312.04	291.14
		E_{21}	**369.77**	**376.91**	**370.62**	**354.29**
		E_{22}	319.12	319.62	312.04	298.39
0.4	0.7	E_{11}	175.93	169.14	157.73	143.47
		E_{12}	168.79	161.96	150.92	137.34
		E_{21}	**175.93**	**170.68**	**163.13**	**153.50**
		E_{22}	168.79	163.44	156.19	147.30
	0.8	E_{11}	228.02	226.62	215.50	197.09
		E_{12}	216.03	213.99	203.22	185.94
		E_{21}	**230.96**	**226.62**	**218.30**	**206.75**
		E_{22}	218.84	213.99	205.88	195.19
	0.9	E_{11}	328.86	352.25	352.31	327.35
		E_{12}	306.07	325.83	324.98	302.07
		E_{21}	**357.93**	**360.90**	**352.31**	**335.44**
		E_{22}	333.75	333.89	324.99	309.58

From Table-6, it is visible from the bold figures that the estimator T_{21} performs better in comparison to other three estimators.

10 Conclusions and Recommendations

From the preceding outcome, it may be concluded that the proposed estimators are more useful in estimation of the population mean of the study variable on current occasion in two-occasion successive sampling. Information on some suitable population parameters such as coefficient of variation, standard deviation (σ), coefficient of skewness (β_1) and coefficient of kurtosis (β_2) etc. of the auxiliary variable are enhanced the precision of the estimates. Consequently, the proposed estimators are more effective in reducing the cost of the survey in comparison to the sample mean estimator and the natural successive sampling estimator in the estimation of population mean on current occasion in two-occasion successive sampling. From the optimum replacement strategies, It is also vindicated that if a highly correlated auxiliary variable is used, relatively, only a smaller fraction of sample on the current (second) occasion is required to be replaced by a fresh sample which reduces the cost of the survey. Finally looking on the nice behaviors of the proposed estimators, they may be recommended to the survey practitioners for their practical applications.

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