Several Ways to Solve the Jaynes-Cummings Model

R. Juárez-Amaro[1], *A. Zúñiga-Segundo*[2] *and H. M. Moya-Cessa*[3,*]

[1] Universidad Tecnológica de la Mixteca, Apdo. Postal 71, 69000 Huajuapan de León, Oax., Mexico
[2] Departamento de Física, Escuela Superior de Física y Matemáticas, IPN, Edificio 9, Unidad Profesional Adolfo López Mateos, 07738 México, DF, Mexico
[3] Instituto Nacional de Astrofísica, Óptica y Electrónica, Calle Luis Enrique Erro No. 1, 72840 Santa María Tonantzintla, Pue., Mexico

Abstract: We go through the several ways that the Jaynes-Cummings model, a cornerstone in the study of light-matter interactions, may be solved. We emphasize two not well known methods (one based on the London phase operator and the other one on the direct diagonalization of the Hamiltonian) considering that they may be of help for solving other systems like the interaction of light with a moving mirror, ion-laser interactions, etc.

Keywords: Atom-field interactions, Ion-laser interactions, JCM

1 Introduction

The Jaynes-Cummings (JC) model [1, 2] is one of the simplest representations of the interaction between light (a quantized field) and matter (a two-level atom). Because it may be solved exactly, it is an important tool in quantum optics. Its simplicity, however, does not affect the amount of phenomena arising from it. Among the many products we may think about, we may count: collapses and revivals in the atomic inversion [7], generation of Schrödinger cat states [3,4] of the quantized field, squeezing [5], transfer of atomic coherence to the quantized field [6], etc. Moreover, because of entanglement, the process of measuring atoms as they leave the cavity gives information about the field state, because while the atoms spend time in the cavity, they acquire knowledge of the field and as they leave the cavity and are measured, knowledge about the quantized field may be retrieved. This was noted by Satyanarayana *et al.* [7], when they studied the interaction between a field initially prepared in a squeezed state with a two-level atom and a feature of the field was imprinted in the atomic inversion producing ringing revivals [8]. It was possible later to obtain full information about the field via the Wigner function [9–11], one of the fundamental quasiprobability distribution functions, that together with the Husimi [12, 13] and Glauber-Sudarshan [14, 15]

functions have complete information of the quantum state of light.

The fact that some other problems, such as the ion-laser interaction [16] are similar to the atom-field interaction, has made possible to produce JC-type interaction in these systems [17–21], such as multiphonon and anti-JC interactions [22]. This has allowed the reconstruction of quasiprobability distributions also in such systems [23].

Generalizations of the JC model can either have more than one atom in the cavity [24], more than one field [25], multilevel atoms [26], and nonlinear media may also be considered [25,27]

2 Traditional approach

The JC Hamiltonian reads (we set $\hbar = 1$)

$$H = \omega\hat{n} + \frac{\omega_0}{2}\sigma_z + g(a^\dagger\sigma_- + \sigma_+ a), \tag{1}$$

where $a^\dagger$ and a are the creation and annihilation operators for the field mode, respectively, obeying $[a, a^\dagger] = 1$ and $\hat{n} = a^\dagger a$ is the number operator. The operators $\sigma_+ = |e\rangle\langle g|$, and $\sigma_- = |g\rangle\langle e|$ are the raising and lowering atomic operators, $|e\rangle$ being the excited state and $|g\rangle$ the ground state of the two-level atom. The atomic operators obey the commutation relations $[\sigma_+, \sigma_-] = \sigma_z$ and

* Corresponding author e-mail: hmmc@inaoep.mx

$[\sigma_z, \sigma_\pm] = \pm 2\sigma_\pm$. ω is the field frequency, ω_0 the atomic transition frequency and g is the interaction constant.

We can transform to a frame rotating at frequency ω via the transformation $H_I = RHR^\dagger$, with $R = e^{-i\omega t(\hat{n} + \sigma_z/2)}$ to produce the interaction Hamiltonian

$$H_I = \frac{\Delta}{2}\sigma_z + g(a^\dagger \sigma_- + \sigma_+ a), \tag{2}$$

where $\Delta = \omega_0 - \omega$ is the detuning. We may propose a solution of the form

$$|\psi(t)\rangle = \sum_{n=0}^{\infty} C_n(t)|n\rangle|e\rangle + D_n(t)|n+1\rangle|g\rangle, \tag{3}$$

which, if inserted in the Schrödinger equation

$$i\frac{\partial|\psi(t)\rangle}{\partial t} = H_I|\psi(t)\rangle, \tag{4}$$

yields the system of equations

$$i\dot{C}_n = \frac{\Delta}{2}C_n + g\sqrt{n+1}D_n, \tag{5}$$

$$i\dot{D}_n = g\sqrt{n+1}C_n - \frac{\Delta}{2}D_n. \tag{6}$$

We can rewrite the above system in the compact form

$$i\frac{d\mathbf{r}_n}{dt} = M\mathbf{r}_n, \tag{7}$$

with

$$\mathbf{r}_n = \begin{pmatrix} C_n \\ D_n \end{pmatrix}, \qquad M = \begin{pmatrix} \frac{\Delta}{2} & g\sqrt{n+1} \\ g\sqrt{n+1} & -\frac{\Delta}{2} \end{pmatrix}. \tag{8}$$

To solve the above system of differential equations, we can find the eigenvalues and eigenvectors of M in order to diagonalize the matrix. However we can also realize that

$$M^{2k} = \begin{pmatrix} \beta_n^{2k} & 0 \\ 0 & \beta_n^{2k} \end{pmatrix}, \tag{9}$$

and that

$$M^{2k+1} = \beta_n^{2k}M, \tag{10}$$

where we have defined

$$\beta_n^2 = \frac{\Delta^2}{4} + g^2(n+1). \tag{11}$$

The solution to (7) is given by

$$\mathbf{r}_n(t) = \exp(-iMt)\mathbf{r}_n(0), \tag{12}$$

and the exponential in the above equation may be developed in Taylor series, in particular we may split the series in even and odd powers

$$e^{-iMt} = 1_{2\times 2}\sum_{k=0}^{\infty}\frac{(-1)^k t^{2k}\beta_n^{2k}}{(2k)!}$$
$$- i\frac{M}{\beta_n}\sum_{k=0}^{\infty}\frac{(-1)^k t^{2k+1}\beta_n^{2k+1}}{(2k+1)!},$$

where $1_{2\times 2}$ is the 2×2 unity matrix. It is clear then that

$$e^{-iMt} = \begin{pmatrix} E_{11} & E_{12} \\ E_{21} & E_{22} \end{pmatrix}, \tag{13}$$

where

$$E_{11} = \cos(\beta_n t) - i\frac{\Delta}{2\beta_n}\sin(\beta_n t),$$

$$E_{12} = E_{21} = -i\frac{g\sqrt{n+1}}{\beta_n}\sin(\beta_n t),$$

and

$$E_{22} = \cos(\beta_n t) + i\frac{\Delta}{2\beta_n}\sin(\beta_n t).$$

We now may apply the solution (exponential) to any initial (condition) state.

3 Stenholm's method

Somehow a close relative to this way of solving the JC model is a method introduced by Stenholm [28], in which the Hamiltonian is written in terms of Pauli matrices

$$H_I = \begin{pmatrix} \frac{\Delta}{2} & ga \\ ga^\dagger & -\frac{\Delta}{2} \end{pmatrix}, \tag{14}$$

and realize that its powers are

$$H_I^{2k} = \begin{pmatrix} \beta_{\hat{n}}^{2k} & 0 \\ 0 & \beta_{\hat{n}-1}^{2k} \end{pmatrix},$$

and

$$H_I^{2k+1} = \begin{pmatrix} \frac{\Delta}{2}\beta_{\hat{n}}^{2k} & ga\beta_{\hat{n}-1}^{2k} \\ ga^\dagger\beta_{\hat{n}}^{2k} & -\frac{\Delta}{2}\beta_{\hat{n}-1}^{2k} \end{pmatrix},$$

with [see equation (11)] $\beta_{\hat{n}} = \sqrt{\frac{\Delta^2}{4} + g^2 aa^\dagger}$ and $\beta_{\hat{n}-1} = \sqrt{\frac{\Delta^2}{4} + g^2 a^\dagger a}$. Therefore, we can write a solution to equation (4) in the form

$$|\psi(t)\rangle = e^{-itH_I}|\psi(0)\rangle, \tag{15}$$

where $|\psi(0)\rangle$ is the initial (atom-field) wave function. As we have done in the previous Section, develop the exponential in Taylor series to finally recover an expresion in terms of trigonometric functions. We then write the evolution operator as

$$U(t) = e^{-iH_I t} = \begin{pmatrix} U_{11} & U_{12} \\ U_{21} & U_{22} \end{pmatrix}. \tag{16}$$

where

$$U_{11} = \cos(\beta_{\hat{n}}t) - i\frac{\Delta}{2\beta_{\hat{n}}}\sin(\beta_{\hat{n}}t),$$

$$U_{12} = -iga\frac{1}{\beta_{\hat{n}-1}}\sin(\beta_{\hat{n}-1}t),$$

$$U_{21} = -iga^\dagger \frac{1}{\beta_{\hat{n}}} \sin(\beta_{\hat{n}} t),$$

and

$$U_{22} = \cos(\beta_{\hat{n}-1} t)$$
$$+ i\frac{\Delta}{2\beta_{\hat{n}-1}} \sin(\beta_{\hat{n}-1} t).$$

We are now in a position to apply the evolution operator to any initial states in order to obtain the evolved wavefunction. Let us for simplicity take the detuning equal to zero, such that

$$U(t) = \begin{pmatrix} \cos(gt\sqrt{aa^\dagger}) & -i\sin(gt\sqrt{aa^\dagger})V \\ -iV^\dagger \sin(gt\sqrt{aa^\dagger}) & \cos(gt\sqrt{a^\dagger a}) \end{pmatrix}, \quad (17)$$

where V is the so-called London phase operator [29–31]

$$V = \frac{1}{\sqrt{\hat{n}+1}} a, \quad (18)$$

and we have used the property $af(\hat{n}) = f(\hat{n}+1)a$ [32]. The above equation gives us an introduction to next Section.

4 London phase operator method

Consider again the interaction Hamiltonian (14) and write it in terms of the London operator

$$H_I = \begin{pmatrix} \frac{\Delta}{2} & g\sqrt{\hat{n}+1}V \\ gV^\dagger\sqrt{\hat{n}+1} & -\frac{\Delta}{2} \end{pmatrix}, \quad (19)$$

we can rewrite it as [33]

$$H_I = \begin{pmatrix} V & 0 \\ 0 & 1 \end{pmatrix} \begin{pmatrix} \frac{\Delta}{2} & g\sqrt{\hat{n}} \\ g\sqrt{\hat{n}} & -\frac{\Delta}{2} \end{pmatrix} \begin{pmatrix} V^\dagger & 0 \\ 0 & 1 \end{pmatrix}. \quad (20)$$

Note that the matrix in the middle has only elements that commute with each other and therefore we can treat them as a c-numbers. Note that $V^\dagger V = 1 - |0\rangle\langle 0|$. Therefore (see Appendix A) we can obtain for the n-th power of the interaction Hamiltonian

$$H_I^n = \begin{pmatrix} V & 0 \\ 0 & 1 \end{pmatrix} \begin{pmatrix} \frac{\Delta}{2} & g\sqrt{\hat{n}} \\ g\sqrt{\hat{n}} & -\frac{\Delta}{2} \end{pmatrix}^n \begin{pmatrix} V^\dagger & 0 \\ 0 & 1 \end{pmatrix}, \quad (21)$$

and it is straightforward to calculate then the evolution operator (16) via Taylor series. The evolution operator then may be written as

$$U(t) = \begin{pmatrix} V & 0 \\ 0 & 1 \end{pmatrix} \exp\left\{ -it \begin{pmatrix} \frac{\Delta}{2} & g\sqrt{\hat{n}} \\ g\sqrt{\hat{n}} & -\frac{\Delta}{2} \end{pmatrix} \right\} \begin{pmatrix} V^\dagger & 0 \\ 0 & 1 \end{pmatrix}, \quad (22)$$

and the exponential may be obtained with the method outlined in Section 2.

5 Boson inverse operators method

We define the boson inverse operators as [34–36]

$$\frac{1}{a} = \sum_{k=0}^{\infty} \frac{1}{\sqrt{k+1}} |k+1\rangle\langle k|, \quad (23)$$

and

$$\frac{1}{a^\dagger} = \sum_{k=0}^{\infty} \frac{1}{\sqrt{k+1}} |k\rangle\langle k+1|. \quad (24)$$

Their actions on the Fock state $|k\rangle$ are

$$\frac{1}{a}|k\rangle = \frac{1}{\sqrt{k+1}} |k+1\rangle, \quad (25)$$

$$\frac{1}{a^\dagger}|k\rangle = \frac{1}{\sqrt{k}} |k-1\rangle, \quad k \neq 0 \quad (26)$$

and $\frac{1}{a^\dagger}|0\rangle = 0$.

We may note that, $\frac{1}{a}a \neq 1$, but $a\frac{1}{a} = 1$ just as the London operators also have a *preferred* order. In fact, we find that [34–36]

$$\left[\frac{1}{a^\dagger}, a^\dagger\right] = \left[a, \frac{1}{a}\right] = |0\rangle\langle 0|. \quad (27)$$

We mention in Section 2 that one of the approaches to solve the system of differential equations (7) was to diagonalize the matrix. Naively, we could think we can also diagonalize the interaction Hamiltnonian despite the fact that it contains non-commuting elements. We follow [37] in order to do this. However, we correct here the method proposed in there.

Consider the diagonal matrices

$$D = \begin{pmatrix} \beta_{\hat{n}-1} & 0 \\ 0 & -\beta_{\hat{n}-1} \end{pmatrix}, \quad (28)$$

$$T = \begin{pmatrix} a\frac{g}{2\beta_{\hat{n}-1}} & -a\frac{g}{2\beta_{\hat{n}-1}} \\ \frac{\beta_{\hat{n}-1}+\frac{\Delta}{2}}{2\beta_{\hat{n}-1}} & \frac{\beta_{\hat{n}-1}-\frac{\Delta}{2}}{2\beta_{\hat{n}-1}} \end{pmatrix}, \quad (29)$$

and

$$S = \begin{pmatrix} \frac{1}{ga}\left(\beta_{\hat{n}} - \frac{\Delta}{2}\right) & 1 \\ -\frac{1}{ga}\left(\beta_{\hat{n}} + \frac{\Delta}{2}\right) & 1 \end{pmatrix}. \quad (30)$$

It is not difficult to prove that $TS = 1$ but $ST \neq 1$. Moreover, we can also prove that $H_I = TDS$. Therefore it is straightforward to find the powers of H_I and then its exponential. So we write the evolution operator as (see Appendix B)

$$U(t) = T \begin{pmatrix} e^{-i\beta_{\hat{n}-1}t} & 0 \\ 0 & e^{i\beta_{\hat{n}-1}t} \end{pmatrix} S, \quad (31)$$

that is exactly the evolution operator given in equation (16).

6 Conclusions

We have made a small survey of the different forms the JC model may be solved. In particular, we have analyzed two not well known methods, one based on the London operator and the other one based on boson inverse operators.

Appendix A

We may find the square of H_I as

$$H_I^2 = \begin{pmatrix} V & 0 \\ 0 & 1 \end{pmatrix} \begin{pmatrix} \frac{\Delta}{2} & g\sqrt{\hat{n}} \\ g\sqrt{\hat{n}} & -\frac{\Delta}{2} \end{pmatrix} \begin{pmatrix} V^\dagger & 0 \\ 0 & 1 \end{pmatrix}$$
$$\times \begin{pmatrix} V & 0 \\ 0 & 1 \end{pmatrix} \begin{pmatrix} \frac{\Delta}{2} & g\sqrt{\hat{n}} \\ g\sqrt{\hat{n}} & -\frac{\Delta}{2} \end{pmatrix} \begin{pmatrix} V^\dagger & 0 \\ 0 & 1 \end{pmatrix}, \qquad (32)$$

so there is a term of the form

$$\begin{pmatrix} \frac{\Delta}{2} & g\sqrt{\hat{n}} \\ g\sqrt{\hat{n}} & -\frac{\Delta}{2} \end{pmatrix} \begin{pmatrix} |0\rangle\langle 0| & 0 \\ 0 & 0 \end{pmatrix} \begin{pmatrix} \frac{\Delta}{2} & g\sqrt{\hat{n}} \\ g\sqrt{\hat{n}} & -\frac{\Delta}{2} \end{pmatrix}$$
$$= \begin{pmatrix} \frac{\Delta^2}{4}|0\rangle\langle 0| & 0 \\ 0 & 0 \end{pmatrix}. \qquad (33)$$

that disappears when we apply the diagonal matrices with the London operators:

$$\begin{pmatrix} V & 0 \\ 0 & 1 \end{pmatrix} \begin{pmatrix} \frac{\Delta^2}{4}|0\rangle\langle 0| & 0 \\ 0 & 0 \end{pmatrix} \begin{pmatrix} V^\dagger & 0 \\ 0 & 1 \end{pmatrix} = \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix}. \qquad (34)$$

Appendix B

When doing the Taylor series for the evolution operator, we have terms of the form (for $k > 0$)

$$H_I^k = TDSTDSTDS\ldots TDS \qquad (35)$$
$$= T(DSTDSTDS\ldots TD)S,$$

that may be rewritten as

$$T\left[\begin{pmatrix} \beta_{\hat{n}-1}^k & 0 \\ 0 & (-1)^k\beta_{\hat{n}-1}^k \end{pmatrix} + \begin{pmatrix} 0 & 0 \\ |0\rangle\langle 0|h_k & |0\rangle\langle 0|g_k \end{pmatrix}\right]S$$

where the coefficients h_k and g_k may be calculated, however they are not important as the product of the matrices in the second term is identically zero, finally obtaining

$$H_I^k = T\begin{pmatrix} \beta_{\hat{n}-1}^k & 0 \\ 0 & (-1)^k\beta_{\hat{n}-1}^k \end{pmatrix} S.$$

References

[1] E.T. Jaynes and F.W. Cummings, Proc. IEEE 51, 89 (1963), *Comparison of Quantum and Semiclassical Radiation Theories with Application to the Beam Maser*.

[2] B.W. Shore and P.L. Knight, J. of Mod. Opt. **40**, 1195 (1993), *The Jaynes-Cummings model*.

[3] B. Yurke and D. Stoler, Phys. Rev. Lett. **57**, 13 (1986), *Generating Quantum Mechanical Superpositions of Macroscopically Distinguishable States via Amplitude Dispersion*.

[4] H. Moya-Cessa and A. Vidiella-Barranco J. of Mod. Optics **42**, 1547 (1995), *On the Interaction of Two-Level Atoms with Superpositions of Coherent States of Light*.

[5] J. R. Kuklinski and J.L. Madajczyk, Phys. Rev. A **37**, 3175 (1988), *Strong squeezing in the Jaynes-Cummings model*.

[6] A.F. Gomes and A. Vidiella-Barranco, App. Math. Inf. Sc. **8**, 727 (2014), *Enhancement of Atom-Field Transfer of Coherence in a Two-Photon Micromaser Assisted by a Classical Field*.

[7] M.V. Satyanarayana, P. Rice, R. Vyas, and H.J. Carmichael, J. Opt. Soc. Am. B **6**, 228 (1989), *Ringing revivals in the interaction of a 2-level atom with squeezed light*.

[8] H. Moya-Cessa and A. Vidiella-Barranco, J. of Mod. Optics **39**, 2481 (1992), *Interaction of squeezed states of light with two-level atoms*.

[9] E.P. Wigner, *Phys. Rev.* **40**, 749 (1932), *On the Quantum Correction For Thermodynamic Equilibrium*.

[10] P. Bertet, A. Auffeves, P. Maioli, S. Osnaghi, T. Meunier, M. Brune, J. M. Raimond, and S. Haroche, *Phys. Rev. Lett.* **89**, 200402 (2002), *Direct Measurement of the Wigner Function of a One-Photon Fock State in a Cavity*.

[11] L.G. Lutterbach, and L. Davidovich, Phys. Rev. Lett. **78**, 2547 (1997), *Method for Direct Measurement of the Wigner Function in Cavity QED and Ion Traps*.

[12] K. Husimi, *Proc. Phys. Math. Soc. Jpn.* **22**, 264 (1940), *Some Formal Properties of the Density Matrix*.

[13] J.R. Moya-Cessa, L.R. Berriel-Valdos and H. Moya-Cessa, App. Math. Inf. Sc. 2, 3, 309-316 (2008), *Optical production of the Husimi function of two Gaussian functions*.

[14] R.J. Glauber, *Phys. Rev.* **131**, 2766 (1963), *Coherent and Incoherent States of the Radiation Field* .

[15] E.C.G. Sudarshan, *Phys. Rev. Lett.* **10**, 277 (1963), *Equivalence of Semiclassical and Quantum Mechanical Descriptions of Statistical Light Beams*.

[16] H. Moya-Cessa, D. Jonathan and P.L. Knight, *J. of Mod. Optics* **50**, 265 (2003), *A family of exact eigenstates for a single trapped ion interacting with a laser field*.

[17] R.L. de Matos Filho and W. Vogel, Phys. Rev. Lett. 76, 608 (1996), *Even and Odd Coherent States of the Motion of a Trapped Ion*.

[18] R.L. de Matos Filho and W. Vogel, Phys. Rev. A 54, 4560 (1996), *Nonlinear coherent states*.

[19] H. Moya-Cessa and P. Tombesi, Phys. Rev. A **61**, 025401 (2000), *Filtering number states of the vibrational motion of an ion*.

[20] H. Moya-Cessa, F. Soto-Eguibar, J.M. Vargas-Martinez, R. Juarez-Amaro and A. Zúñiga-Segundo, Phys. Rep. 513, 229-261 (2012), *Ion-laser interactions: the most complete solution*.

[21] V. Penna, F. Raffa, arXiv:1403.2321, *Perturbative approach to the dynamics of a trapped ion interacting with a light field*.

[22] B.M. Rodriguez-Lara, H. Moya-Cessa and A.B. Klimov, Phys. Rev. A **71**, 023811 (2005), *Combining Jaynes-Cummings and anti-Jaynes-Cummings dynamics in a trapped ion system*.

[23] D. Leibfried, D. M. Meekhof, B. E. King, C. Monroe, W. M. Itano, and D. J. Wineland, *Phys. Rev. Lett.* **77**, 4281 (1996), *Experimental Determination of the Motional Quantum State of a Trapped Atom.*

[24] J.S. Zhang, A.X. Chen, and M. Abdel-Aty, Journal of Physics B: Atomic, Molecular and Optical Physics **43**, 025501 (2010), *Two atoms in dissipative cavities in dispersive limit: entanglement sudden death and long-lived entanglement.*

[25] P. Alsing, D.-S. Guo, and H.J. Carmichael, Phys. Rev. A **45**, 5135 (1992), *Dynamic Stark effect for the Jaynes-Cummings system.*

[26] M. Abdel-Aty, J. of Mod. Optics **50**, 161 (2003), *Quantum field entropy and entanglement of a three-level atom two-mode system with an arbitrary nonlinear medium.*

[27] M. Abdel-Aty, M.S. Abdalla, A.S.F. Obada, Journal of Physics A: Mathematical and General **34**, 9129 (2001), *Quantum information and entropy squeezing of a two-level atom with a non-linear medium.*

[28] S. Stenholm, Phys. Rep. **6**, 1 (1973), *Quantum theory of electromagnetic fields interacting with atoms and molecules.*

[29] F. London, Z. Phys. 37, 915 (1926), *Über die Jacobischen transformationen der quantenmechanik.*

[30] F. London, Z. Phys. 40, 193 (1927), *Winkervariable und kanonische transformationen in der undulationsmechanik.*

[31] L. Susskind and J. Glogower, Physics **1**, 49 (1964), *Quantum mechanical phase and time operator.*

[32] W.H. Louisell, Quantum Statistical Properties of Radiation (Wiley, NewYork, 1973).

[33] A.B. Klimov and S.M. Chumakov, A Group-Theoretical Approach to Quantum Optics (Wiley-VCH 2009).

[34] C. L. Mehta, Anil. K. Roy, and G. M. Saxena, Phys. Rev. A **46**, 1565 (1992), *Eigenstates of two-photon annihilation operators.*

[35] F. Hong-yi, Phys. Rev. A **47**, 4521 (1993), *Inverse operators in Fock space studied via a coherent-state approach.*

[36] A.K. Roy and C.L. Mehta, J. of Mod. Opt. **42**, 707 (1995), *Boson inverse operators and a new family of 2-photon annihilation operatos.*

[37] H. Moya-Cessa, S. Chavez-Cerda and J.J. Sanchez-Mondragon, Phys. Lett A. **205**, 51 (1995), *Interaction of quantized light with two-level atoms: comparison between the Stark and Kerr effects.*

Raúl Juárez-Amaro received his PhD from Instituto Nacional de Astrofísica, Óptica y Electrónica in Tonantzintla, Pue. He has authored over 15 papers in international peer reviewed journals.

Arturo Zúñiga-Segundo received his doctorate in physics from Centro de Investigacion y de Estudios Avanzados (CINVESTAV) in 1996. Since 1990 he is a physics teacher/researcher at Escuela Superior de Física y Matemáticas of Instituto Politécnico Nacional in Mexico City. He has published over 20 papers in international peer reviewed journals.

H. M. Moya-Cessa obtained his PhD at Imperial College in 1993 and since then he is a researcher/lecturer at Instituto Nacional de Astrofísica, Óptica y Electrónica in Puebla, Mexico where he works on Quantum Optics. He has published over 110 papers in international peer reviewed journals. He is fellow of the Alexander von Humboldt Foundation and a Regular Associate of the International Centre for Theoretical Physics (Trieste, Italy).