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Fuzzy Bi-Level Multi-Objective Fractional Integer Programming

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Abstract: We present an algorithm to solve a bi-level multi-objective fractional integer programming problem involving fuzzy numbers in the right-hand side of the constraints. The suggested algorithm combine the method of Taylor series together with the Kuhn Tucker conditions to solve fuzzy bi-level multi-objective fractional integer programming problem (FBLMOFIPP) then Gomory cuts are added till the integer solution is obtained. An illustrative example is discussed to demonstrate the correctness of the proposed solution method.

Keywords: Fuzzy programming; Bi-level programming; Multi-objective programming; Fractional programming; Integer programming.
MSC 2000: 90C70; 90C99; 90C29; 90C32; 90C10.

1 Introduction

The main idea behind a fuzzy set is that of gradual membership to a set without sharp boundary. This idea is in tune with human representation of reality that is more nuanced than clear cut. Some philosophical related issues ranging from ontological level to application level via epistemological level. In a fuzzy set, the membership degree of an element is expressed by any real number from 0 to 1 rather than the limiting extremes [3,7].

Bi-level programming (BLP) problem has a subset of their variables constrained to be an optimal solution of another problem parameterized by the remaining variables. They have been applied to decentralize planning problems involving a decision process with a hierarchical structure. The basic connect of the BLP technique is that a first level decision maker (FLDM) sets his goals and/or decisions and then asks each subordinate level of the organization for their optima which are calculated in isolation; the second level decision maker (SLDM) decision is then submitted and modified by the FLDM with consideration of the overall benefit for the organization; and the process continued until an optimal solution is reached.

In the literature, many researchers have focused to solve BLP problems. Some of them presented formulation and different version of problem. A number of studies have been done in the field of multi-level linear, nonlinear and integer fractional programming problems, some of them have multi-objective functions [13,15,16,20].

In [5], Emam presented a fuzzy approach for solving bi-level integer non-linear programming problem, then in [6] Emam proposed an interactive approach for solving bi-level integer multi-objective fractional programming problem. At the first phase of the solution approach, it begin by finding the convex hull of its original set of constraints using the cutting plane algorithm then the two level decision makers use the Charnes & Cooper transformation to convert the fractional objective functions to equivalent linear functions. At the second phase, the algorithm simplifies the equivalent problem by transforming it into separate multi-objective decision-making problem and solving it by using the ε-constraint method.

Multi-objective optimization problems are a class of difficult optimization problems in which several different

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objective functions have to be considered simultaneously. Usually, there is no solution optimizing simultaneously all the several objective functions. Therefore, we search the so-called efficient points [1,2].

Fractional programming problem is that in which the objective function is the ratio of numerator and denominator. These types of problems have attracted considerable research and interest. Fractional programming is useful in production planning, financial and corporate planning, health care and hospital planning etc. [7,8].

Integer programming problem is a common problem class where decision variables represent indivisibility or yes/no decisions. Integer Programming can also be interpreted as discrete optimizations which are extensively used in the areas of routing decisions, fleet assignment, and aircraft/aircrew scheduling [5,6].

Real world problems are often not deterministic ones but need to be described using fuzzy or stochastic goals and/or constraints. Fuzzy bi-level linear fractional programming (FBLLFP) problem is an application of fuzzy set theory in decision making problems and most of these problems are related to linear programming. Fuzzy mathematical programming was first formulated by Zimmermann [24].

Sharma and Bansa [18] proposed the mixed integer linear programming problems with fuzzy variables; a new method for solving integer linear programming problem with fuzzy variables was summarized by Pandian and Jayalakshmi [11]. Furthermore Nasseri [10] suggested simplex method for solving linear programming problems with fuzzy numbers.

Nachammai and Thangaraj [7] presented the solution of a fuzzy linear fractional programming problem using metric distance ranking, then they proposed the solution of a problem by fuzzy variables in [8].

Younes et al. [22] presented a simplex method for solving bi-level linear fractional integer programming problem with fuzzy numbers. Pramanik and Dey [12] proposed bi-level linear fractional programming problem based on fuzzy goal programming approach. The interactive fuzzy programming for stochastic two level linear programming problems through probability maximization was presented by Sakawa and Matsui [14]. A fuzzy interactive method for a class of bi-level multi-objective programming problem was suggested by Zheng [23].

In the present paper we consider a bi-level multi-objective fractional integer programming problem involving triangular fuzzy numbers. In this setting, we combine the different techniques which were presented by Nachammai [8] and Sarag [17] in addition we use the simplex method and cutting plane algorithm [19] to obtain the efficient integer solution for our problem.

This paper is organized as follows: In Section 2, the problem formulation and solution concept is introduced. An algorithm for solving fuzzy bi-level multi-objective fractional integer programming problem is suggested in Section 3. Section 4 presents a flowchart for solving FBLMOFIP problem. In addition, an example is provided to illustrate the developed results in Section 5 and in Section 6 the conclusion and some open points are stated for future research work in the field of interactive multi-level integer fractional optimization.

2 Problem Formulation and Solution Concept

The bi-level multi-objective fractional integer programming problem involving triangular fuzzy numbers can be formulated as the following:

$$\text{(FLDM)} \quad \max_{\tilde{x}_1} \tilde{F}_1(\tilde{x}) \max(f_{11}(\tilde{x}), f_{12}(\tilde{x}), \ldots, f_{1r_1}(\tilde{x})), \tag{1}$$

where $\tilde{x}_2$ solves

$$\text{(SLDM)} \quad \max_{\tilde{x}_2} \tilde{F}_2(\tilde{x}) \max(f_{21}(\tilde{x}), f_{22}(\tilde{x}), \ldots, f_{2r_2}(\tilde{x})), \tag{2}$$

Subject to

$$M(\tilde{x},\tilde{b}) = \{\tilde{x},\tilde{b} \in F(R); A\tilde{x} \leq \tilde{b}, \tilde{x} \geq 0 \text{ and integer}\} \tag{3}$$

The above problem (1)-(3) assumes that $\tilde{F}_1(\tilde{x}_1), \tilde{F}_2(\tilde{x}_1)$ are the first level and second level decision makers objective functions, respectively. $\tilde{x} = (\tilde{x}_J), \tilde{b} = (\tilde{b}_I), \tilde{x}_J, \tilde{b}_I \in F(R)$ for all $1 \leq J \leq n$ and $1 \leq I \leq m, F(R)$ be the set of all real triangular fuzzy numbers, A is $(m \times n)$ matrix of real numbers, $c_{ij}, d_{ij} \in R, d_{ij}\tilde{x} + \gamma_{ij} > \tilde{0}, \rho_{ij}, \gamma_{ij}$ are constant and each function $f_{ij}(\tilde{x})$ can be suggested in the form $f_{ij}(\tilde{x}) = \frac{c_{ij}\tilde{x}+\rho_{ij}}{d_{ij}\tilde{x}+\gamma_{ij}}; (j = 1,2,\ldots,r_i), (i = 1,2)$.

Definition.1 ([9]) A fuzzy set $\tilde{A}$ is defined by $\tilde{A} = \{(x, \mu_{\tilde{A}}(x)) : x \in R, \mu_{\tilde{A}}(x) \in [0,1]\}$. In the pair $(x, \mu_{\tilde{A}}(x))$, the first element x belong to the classical set A, the second element $\mu_{\tilde{A}}(x)$ belong to the interval $[0,1]$ and called membership function.

Definition.2 ([3]) A fuzzy number $\tilde{x}$ is a triangular fuzzy number denoted by (m, α, β) where m, α and β are real numbers and its membership function $\mu_{\tilde{b}}(x)$ is given below.

$$\mu_{\tilde{b}}(x) = \begin{cases} 0 & for \;\; x \geq \alpha - m, \\ 1 - \frac{\alpha - x}{m} & for \;\; \alpha - m < x < \alpha, \\ 1 & for \;\; x = \alpha, \\ 1 - \frac{x-\alpha}{\beta} & for \;\; \alpha < x < \alpha + \beta, \\ 0 & for \;\; x \geq \alpha + \beta. \end{cases}$$

If the membership function grade is 1 the point α will called the mean value and m, β are the left hand and right hand spreads of $\tilde{x}$ respectively.

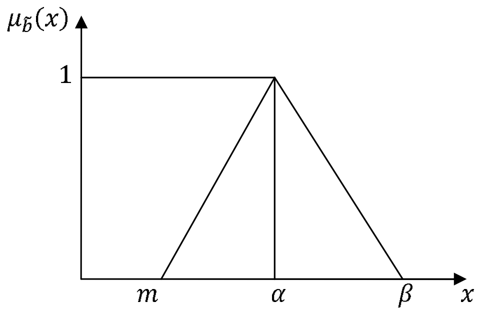

Figure .1 Membership function of triangular fuzzy number $\tilde{x}$

Definition.3 ([9]) A positive triangular fuzzy number $\tilde{x}$ is denoted as (m, α, β) where m, α and $\beta > 0$.
In the following some definitions are useful in solving the problem under consideration is stated as follows:
Definition.4. A fuzzy vector $(\tilde{x}, \tilde{b})$ is said to be a feasible solution of the problem (1)-(3) if $(\tilde{x}, \tilde{b}) \in M(\tilde{x}, \tilde{b})$.

Definition.5. A feasible solution $(\tilde{x}, \tilde{b})$ of the problem (1)-(3) is said to be an optimal solution if there exists no another feasible solution $(\tilde{u}, \tilde{b})$ of the problem such that $\tilde{F}(\tilde{u}, \tilde{b}) > \tilde{F}(\tilde{x}, \tilde{b})$.

Theorem.1. A fuzzy vector $\tilde{x} = (m^*, \alpha^*, \beta^*)$ is a solution of the fuzzy linear fractional programming problem:

$$(p') \qquad \max \tilde{F}(\tilde{x}) = \max(\tfrac{c\tilde{x}+\rho}{d\tilde{x}+\gamma}), \\ Subject\ to \\ A\tilde{x} \le \tilde{b}, \\ \tilde{x} \ge 0.$$

If and only if m^*, α^* and β^* are the solutions of the following crisp linear fractional integer programming problems (P_1'), (P_2') and (P_3') respectively:

$$(p_1') \qquad \max F(m) = \max(\tfrac{cm+\rho}{dm+\gamma}), \\ Subject\ to \\ Am \le b_1, \\ m \ge 0.$$

$$(p_2') \qquad \max G(\alpha) = \max(\tfrac{c\alpha+\rho}{d\alpha+\gamma}), \\ Subject\ to \\ A\alpha \le b_2, \\ \alpha \ge 0.$$

$$(p_3') \qquad \max D(\beta) = \max(\tfrac{c\beta+\rho}{d\beta+\gamma}), \\ Subject\ to \\ A\beta \le b_3, \\ \beta \ge 0.$$

where $\tilde{F} = (F, G, D)$ and $\tilde{b} = (b_1, b_2, b_3)$.

Proof. Let $\tilde{x} = (m^o, \alpha^o, \beta^o)$ is an optimal solution of the fuzzy linear fractional programming problem (P') this implies that:

$$A(m^o, \alpha^o, \beta^o) \le (b_1, b_2, b_3) \Rightarrow (Am^o, A\alpha^o, A\beta^o) \le (b_1, b_2, b_3)$$

That mean :

$$Am^o \le b_1, A\alpha^o \le b_2 \text{ and } A\beta^o \le b_3. \qquad (4)$$

Consider the optimal solution of the problems $(P_1'), (P_2')$ and (P_3') is given as m^*, α^* and β^* respectively, this implies that:

$$Am^* \le b1, A\alpha^* \le b_2 \text{ and } A\beta^* \le b_3 \qquad (5)$$

From (1) and (2):
$Am^o - Am^* \le b_1 - b_1$, $A\alpha^o - A\alpha^* \le b_2 - b_2$ and $A\beta^o - A\beta^* \le B_3 - b_3$.
$\Rightarrow Am^o \le Am^*$, $A\alpha^o \le A\alpha^*$ and $A\beta^o \le A\beta^*$
$\Rightarrow A(m^o, \alpha^o, \beta^o) \le A(m^*, \alpha^*, \beta^*)$
$\Rightarrow$ The vector $(m^*.\alpha^*.\beta^*)$ is the feasible solution of the problem (P').
In the solution of bi-level multi-objective fractional integer programming problem (BLMOFIPP) presented by Saraj and Safaei [17], the membership functions associated to each of the objectives in each level are firstly transformed by using Taylor series [21] and then a satisfactory value(s) for the variable(s) of the model is obtained by solving the fuzzy model, which has a single objective function. Here, the fractional linear membership function from each objective of each level is converted to a linear polynomial using Taylor series. Then, the bi-level multi-objective integer programming problem using Kuhn-Tucker conditions [4] can be reduced to a single objective. This model is representing as follows:

$$\max P(m), \qquad (6) \\ Subject\ to$$

$$A_1 m_1 + A_2 m_2 + s = b, \\ \omega A_2 - v = \frac{\partial \mu(f_{2j}(m_2^*))}{\partial m_2}, \\ \omega \le T\eta, s \le T(1-\eta), \\ m_2 \le T\xi, v \le T(1-\xi), \\ \eta, \xi \in \{0,1\}, \\ m_1, m_2 \ge 0, \text{and integers}, \\ \omega, s, v \ge 0.$$

where

$$\{P(m) = \sum_{j=1}^{m_1} \mu(f_{1j}(m_{1j}^*)) + \sum_{k=1}^{2} (m_k - m_{1j}^{k*}) \frac{\partial \mu(f_{1j}(m_{1j}^*))}{\partial m_k}\},$$

$\omega s = 0$ and T is a large positive number and the membership functions $\mu(f_{ij}(m_1, m_2))$ if $f_{ij}(m_1, m_2) > a_{ij}$

is given as the following:

$$\mu(f_{ij}(m_1,m_2)) = \begin{cases} 1 & if\ \ f_{ij}(m_1,m_2) \geq a_{ij}, \\ \frac{f_{ij}(m_1,m_2)-t_{ij}}{a_{ij}-t_{ij}} & if\ t_{ij} \leq f_{ij}(m_1,m_2) \leq a_{ij}, \\ 0 & if\ \ f_{ij}(m_1,m_2) \leq t_{ij} \end{cases} \quad (7)$$

where a_{ij} is the aspiration level of the FLDM and SLDM, and t_{ij} is the lower tolerance limits for each level.

3 A Solution Algorithm of FBLMOFIPP

A solution algorithm to solve fuzzy bi-level multi-objective fractional integer programming problem is described in a series of steps. Firstly the algorithm converts the (FBLMOFIPP) into three bi-level multi-objective fractional integer programming problems, then, an efficient solution for each formulated problem is obtained.
The suggested algorithm can be summarized in the following manner:

Step 1:

Convert a FBLMOFIPP into three separate problems $(P_1),(P_2)$ and (P_3).

Step 2:

Start with the problem (P_k), go to step3.

Step 3:

Set $k=1$.

Step 4:

Determine the integer solution $m^*_{ij} = (m^{1*}_{ij}, m^{2*}_{ij}); (i=1,2$ and $j=1,2,,r_i)$ of each the objectives $f_{ij}(m)$ in FLDM and SLDM.

Step 5:

Find the membership functions $\mu(f_{ij}(m))$ associated to the objectives $f_{ij}(m)$ of FLDM and SLDM functions, and then go to step 6.

Step 6:

Transform the linear fractional membership functions $\mu(f_{ij}(m))$ into linear membership functions by using first order Taylor series.

Step 7:

Form the mathematical model for the problem (P_k) as problem (4), and then solve it using dual simplex method, if this solution is integer, go to step 8, if not go to step 9.

Step 8:

Denote the integer solution for the problem (P_k) as (m^*_{1k}, m^*_{2k}), and then go to step11.

Step 9:

Select a row with a fractional right hand side: this is called the source row.

Step 10:

Augment the simplex tableau with a column for slack variables and add the new constraint row.

Step 11:

If $k=3$, go to step 12, otherwise set $k=k+1$, go to step 4.

Step 12:

Write the integer solution of (FBLMOFIP) problem on the form
$\tilde{x}_1 = (m^*_{11}, m^*_{12}, m^*_{13})$ and $\tilde{x}_2 = (m^*_{21}, m^*_{22}, m^*_{23})$

4 A flowchart for solving FBLMOFIP

In this section, a flowchart to explain the suggested algorithm for solving a fuzzy bi-level multi-objective fractional integer programming problem is shown in figure 1.

5 Numerical Example

To demonstrate the solution process for (FBLMOFIP), let us consider the following problem:

$$\text{(FLDM)} \quad \max_{\tilde{x}_1} \tilde{F}_1(\tilde{x}_1,\tilde{x}_2) = \max_{\tilde{x}_1}\left(\frac{6\tilde{x}_1+2\tilde{x}_2+1}{\tilde{x}_1+\tilde{x}_2+2}, \frac{3\tilde{x}_1-\tilde{x}_2}{\tilde{x}_2+1}\right),$$

$$\text{(SLDM)} \quad \max_{\tilde{x}_2} \tilde{F}_2(\tilde{x}_1,\tilde{x}_2) = \max_{\tilde{x}_2}\left(\frac{\tilde{x}_1+4\tilde{x}_2}{2\tilde{x}_1+1}, \frac{-\tilde{x}_1+2\tilde{x}_2+2}{\tilde{x}_1+\tilde{x}_2+2}\right),$$

Subject to

$$\tilde{x}_1+\tilde{x}_2 \leq (3,7,11),$$
$$\tilde{x}_1+\tilde{x}_2 \leq (12,17,27),$$
$$\tilde{x}_1+\tilde{x}_2 \geq 0, \text{ and integer.}$$

Let $\tilde{x}_1 = (m_1,\alpha_1,\beta_1)$, $\tilde{x}_2 = (m_2,\alpha_2,\beta_2)$, $\tilde{F}_1 = (F_1,G_1,D_1)$ and $\tilde{F}_2 = (F_2,G_2,D_2)$.
The bi-level linear multi-objective integer fractional programming problems $(P_1),(P_2)$ and (P_3) will be:

(P_1)

$$\text{(FLDM)} \quad \max_{m_1} F_1(m_1,m_2) = \max_{m_1}\left(\frac{6m_1+2m_2+1}{m_1+m_2+2}, \frac{3m_1-m_2}{m_2+1}\right),$$

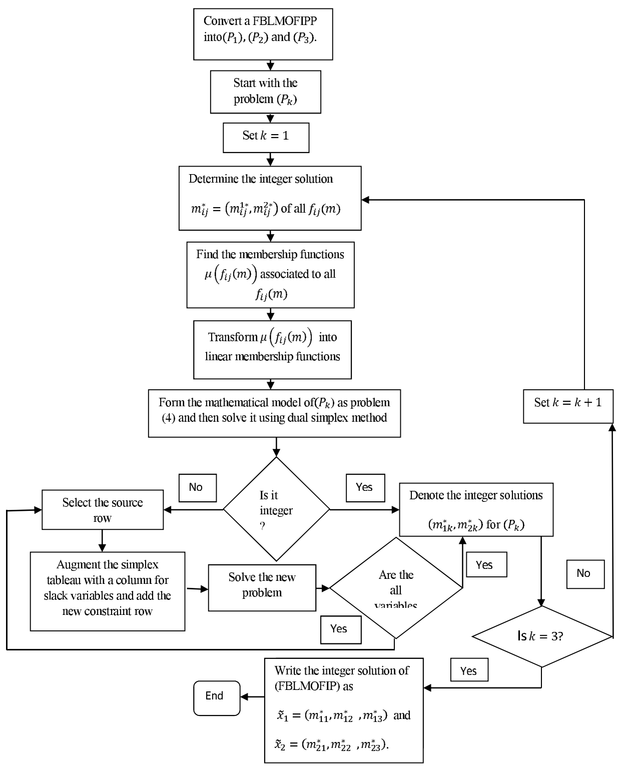

Fig. 1: Algorithm for solving a ta fuzzy bi-level multi-objective fractional integer programming problem

(SLDM) $\max_{m_2} F_2(m_1,m_2) = \max_{m_2}\left(\frac{m_1+4m_2}{2m_1+1}, \frac{-m_1+2m_2+1}{m_1+m_2+2}\right)$,

Subject to

$$m_1 + m_2 \le 3,$$
$$2m_1 + 3m_2 \le 7,$$
$$m_1, m_2 \ge 0, \text{ and integer.}$$

(P_2)

(FLDM) $\max_{\alpha_1} G_1(\alpha_1,\alpha_2) = \max_{\alpha_1}\left(\frac{6\alpha_1+2\alpha_2+1}{\alpha_1+\alpha_2+2}, \frac{3\alpha_1-\alpha_2}{\alpha_2+1}\right)$,

(SLDM) $\max_{\alpha_2} G_2(\alpha_1,\alpha_2) = \max_{\alpha_2}\left(\frac{\alpha_1+4\alpha_2}{2\alpha_1+1}, \frac{-\alpha_1+2\alpha_2+1}{\alpha_1+\alpha_2+2}\right)$,

Subject to

$$\alpha_1 + \alpha_2 \le 7,$$
$$2\alpha_1 + 3\alpha_2 \le 17,$$
$$\alpha_1 + \alpha_2 \ge 0. \text{ and integer.}$$

(P_3)

(FLDM) $\max_{\beta_1} D_1(\beta_1,\beta_2) = \max_{\beta_1}\left(\frac{6\beta_1+2\beta_2+1}{\beta_1+\beta_2+2}, \frac{3\beta_1-\beta_2}{\beta_2+1}\right)$,

Table 1: The integer solutions obtained will be in the following table:

	$f_{11}(m_1,m_2)$	$f_{12}(m_1,m_2)$	$f_{21}(m_1,m_2)$	$f_{22}(m_1,m_2)$
$m^*_{ij} = (m^{1*}_{ij}, m^{2*}_{ij})$	(3.0)	(3,0)	(0,2)	(0,2)
$f_{ij}(m_1,m_2)$	3.8	9	8	1.25

(SLDM) $\max_{\beta_2} D_2(\beta_1,\beta_2) = \max_{\beta_2}\left(\frac{\beta_1+4\beta_2}{2\beta_1+1}, \frac{-\beta_1+2\beta_2+1}{\beta_1+\beta_2+2}\right)$,

Subject to

$$\beta_1 + \beta_2 \le 11,$$
$$2\beta_1 + 3\beta_2 \le 27,$$
$$\beta_1 + \beta_2 \ge 0, \text{ and integer.}$$

For the problem (P_1), the integer solutions $m^*_{ij} = (m^{1*}_{ij}, m^{2*}_{ij})$; $(i = 1,2$ and $j = 1,2)$ will obtain by using dual simplex method and cutting-plane algorithm to solve the following problems:

$$max\, f_{ij}(m_1,m_2); (i = 1,2 \text{ and } j = 1,2),$$
$$subject\ to$$
$$m_1 + m_2 \le 3,$$
$$2m_1 + 3m_2 \le 7,$$
$$m_1, m_2 \ge 0, \text{and integer.}$$

Now, to find the membership functions $\mu f_{ij}(m_1,m_2)$, let the fuzzy aspiration levels of the objectives in the bi-level to be (3, 8, 7, 0) respectively and assume the tolerance limits of the objectives in the bi-level are (1, 6, 5, -2) respectively. The membership functions $\mu f_{ij}(m_1,m_2)$ will be:
$\mu(f_{11}) = \frac{5m_1+m_2-1}{2m_1+2m_2+4}$, $\mu(f_{12}) = \frac{3m_1-7m_2-6}{2m_2+2}$, $\mu(f_{21}) = \frac{-9m_1+4m_2-5}{4m_1+2}$ and $\mu(f_{22}) = \frac{m_1+4m_2+5}{2m_1+2m_2+4}$
By using first-order Taylor series the membership functions will be:
$\mu_{f_{11}} \cong 0.02m_1 - 0.2m_2 - 1.34$, $\mu_{f_{12}} \cong 1.5m_1 - 5m_2 - 3$, $\mu_{f_{21}} \cong -12.5m_1 + 2m_2 - 2.5$ and $\mu_{f_{22}} \cong -0.28m_1 + 0.09m_2 + 1.45$.
After applying the Kuhn-Tucker conditions of the membership functions for the SLDM, a new auxiliary problem by choosing $T = 100$ will be in the following form:

$$\max P(m_1,m_2) = \max(1.52m_1 - 5.26m_2 - 1.66),$$
$$Subject\ to$$
$$m_1 + m_2 + s_1 = 3,$$
$$2m_1 + 3m_2 + s_2 = 7,$$
$$\omega_1 + 3\omega_2 - v = 2.09,$$
$$\omega \le 100\eta, s \le 100(1-\eta),$$
$$m_2 \le 100\xi, v \le 100(1-\xi),$$
$$\eta, \xi \in \{0,1\},$$
$$m_1, m_2 \ge 0, \text{and integer},$$
$$\omega, s, v \ge 0.$$

By using the WINQSP package to solve this problem, the integer solution obtained for problem (P_1) is $m^*_1 = 2$ and $m^*_2 = 1$.
In the same way of solution of problem (P_1) the integer solution of the problems (P_2) and (P_3) will be

$(\alpha_1^* = 4, \alpha_2^* = 3)$ and $(\beta_1^* = 6, \beta_2^* = 5)$.
Now, the integer solution of (FBLMOFIP) problem is formed as: $\tilde{x}_1 = (2,4,6)$, $\tilde{x}_2 = (1,3,5)$, $\tilde{F}_1(\tilde{x}_1,\tilde{x}_2) = [(3,\frac{5}{2}),(\frac{37}{9},\frac{9}{4}),(\frac{47}{13},\frac{13}{4})]$ and $\tilde{F}_1(\tilde{x}_1,\tilde{x}_2) = [(\frac{7}{5},\frac{1}{7}),(\frac{16}{9},\frac{1}{3}),(2,\frac{5}{13})]$.

6 Conclusion

In the presented paper a solution algorithm has been proposed to solve fuzzy bi-level multi-objective fractional integer programming problem (FBLMOFIPP). The fuzzy numbers can be characterized by triangle fuzzy membership functions. The algorithm combines the techniques of Taylor series and Kuhn Tucker conditions; in addition cutting-plane method to obtain the integer solution of the problem is used.

Summarizing, many aspects and general questions remain to be studied and explored in the area of fuzzy linear fractional integer programming. Despite the limitations, we believe that this paper is an attempt to establish underlying results which hopefully will help others to answer of these questions. There are however several open points for future research in the area of (FBLMOFIPP), in our opinion, to be studied. Some of these points of interest are stated in the following:

(i) An algorithm for solving multi-level integer fractional multi-objective decision-making problems with random parameters in the objective functions; in the constraints and in both using Taylor series.

(ii) An algorithm for solving multi-level integer fractional multi-objective decision-making problems with rough parameters in the objective functions; in the constraints and in both using Taylor series.

(iii) An algorithm for solving fuzzy multi-level multi-objective linear fractional integer decision-making problems with general fuzzy parameters.

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