Journal of Statistics Applications & Probability
An International Journal

http://dx.doi.org/10.18576/jsap/090209

Step Stress Partially Accelerated Life Tests and Estimating Costs of Maintenance Service Policy for the Power Function Distribution under Progressive Type-II Censoring

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Received: 1 Aug. 2019, Revised: 2 Sep. 2019, Accepted: 2 Oct. 2019
Published online: 1 Jul. 2020

Abstract: The present paper illustrates how to analyze and design the accelerated life testing (ALTg) plans for the improvement of the quality and reliability of the product. We focus on estimating the costs of maintenance service policy because it plays a very important role in manufacturing organization and providing cost-effective equipment and maintenance.. When the lifetime of units follows power function distribution, the partially step-stress accelerated life test is assumed. The maximum likelihood estimates (MLEs) are obtained under the progressive Type-II censoring. Using the Fisher Information matrix, the asymptotic variance and covariance matrix are obtained. The confidence intervals (CIs) of the estimators are also constructed. Furthermor, a simulation study is conducted to check the results accuracy.

Keywords: Partially Accelerated Life Testing, Maintenance Service Policy, Progressive Type-II censoring, Power Function distribution, Simulation Technique.

1 Introduction

To develop new and high-quality products/items, the manufacturer, faces more pressure and challenge while upgrading productivity, product field lifetime, and many other qualities of the current market situation in the present time. It is more challenging to identify failure information under normal situation under continuous improvement in the quality and reliability of items. Thus, the accelerated life test (ALT) is one of the best and most common ways that fulfill such demands. Accelerated life testing (ALTg) is attained through exposing the test units to conditions that are more severe than the normal ones, such as voltage, temperature, cycling rate, pressure, vibration, etc. Several methods can be adopted to apply stress under accelerated condition. The methods are progressive stress, constant stress, step stress, cyclic stress, random stress, or combinations of them. According to Nelson [1], stress can be applied in many ways. The Step-stress partially accelerated life test (SS-PALT) is widely used to get information on the lifetime of the product/item with high reliability, especially when the mathematical model related to testing conditions of mean lifetime of the product is unknown and cannot be assumed. The step stress arrangement applies stress to test units in the form that stress will be changed at a pre-specified time. Accordingly,, in SS-PALT, the test unit is first to run at normal use condition, and if it does not fail for a specified number of failures or time. Next, the test unit is run at accelerated use condition until failure occurs or the observation is censored. When the exact failure time of any item or product is unknown, observation is censored.

In this paper, we consider only partially accelerated life testing (PALTg) with the use of two stress levels. Many pieces of literature related to SS-PALT are available. For example, Ali Ismail [2] designed step-stress partially accelerated life tests for Weibull distribution with Type-II censored data. He estimated maximum likelihood estimates of model parameters under step-stress partially accelerated life tests using the Type-II censoring scheme. Rahman et al.[3] tackled step-stress partially accelerated life tests using the Type-I censoring for the Mukherjee-Islam failure model. Kamal et al.[4] presented

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step-stress accelerated life tests for two-parameter Pareto distribution. Ismail and Aly [5] proposed optimum plans for failure-step stress partially accelerated life tests under Type-II censoring for Weibull distribution. Amal and Abeer [6] handed the optimal design of failure step-stress partially accelerated life tests for Inverse Weibull distribution with Type-II censored data. They presented the optimum simple failure step stress partially accelerated life tests and statistical inferences for the distribution parameters and acceleration factor in which items are run at both use and accelerated conditions. Srivastava and Mittal [7] explored optimum step-stress partially accelerated life tests with censored data. They provided the optimal design of step-stress partially accelerated life tests in which items run at both accelerated and use conditions under censored data, and the lifetime of the items follows truncated logistic distribution. Abdallah et al. [8] covered estimation in step-stress partially accelerated life tests with Type-I censored data for Burr Type XII distribution. Amal [9] addressed the optimal design of failure step-stress accelerated life tests with Type-I censored data when the lifetime of units follows exponential inverted Weibull distribution.

The lifetime data is censored when the exact failure time of any item or product is unknown. There are many types of censoring, such as left, right, interval, Type-I, Type-II, hybrid, progressive, progressive Type-I, and progressive Type-II censoring, etc. Here we consider only the progressive Type-II censoring scheme. The Type-I and Type-II censoring schemes are the most common and popular schemes in reliability theory. The only difficulty in both Type-I and Type-II censoring schemes is that we cannot withdraw live items during the test. But in the progressive Type-II scheme, it is possible to withdraw live items during the experiment. It is a generalization of the classical Type-II censoring scheme. For literature, see the book by Balakrishnan and Aggrawalla [10], and an article by Balakrishnan [11]. The progressive Type-II censoring is described as follows

Let $X_1, X_2 \ldots, X_n$ be the lifetimes of units, which are put on the life testing experiment and also suppose that $X_i, i = 1, 2, \ldots, n$ are independent and identically distributed with cumulative distribution function *(cdf)*, $F(x)$ and probability distribution function *(pdf)*, $f(x)$. Before the life testing experiment, an integer $m(m < n)$ is resolved, and the progressive Type-II censoring scheme $(R_1, R_2, \ldots, R_n)$ and $n = m + \sum_{i=1}^{m} R_i$ is specified. Now, *ith* failure is observed, and after the failure, R_i functioning items are randomly removed from the test during the lifetime testing experiment. $X_{i:m:n}, i = 1, 2, \ldots, n$ and m are the totally observed lifetimes, which are observed samples for the progressively Type-II censoring scheme. $x_{1:m:n} < x_{2:m:n} < \ldots < x_{m:m:n}$ are the observed values of the progressively Type-II right censored samples.

Now, we present brief literature on SSPALT and progressive Type-II censoring related to our study. Ahmed A. Soliman [12] handled step-stress partially accelerated life tests for Inverse Weibull distribution with the progressive Type-II censoring scheme. M. M. Mohie El-Din et al. [13] considered a simple step-stress accelerated life test under the progressive Type-II censoring scheme for the extension of Exponential distribution. They analyzed a real data set to illustrate the proposed procedures. Rahman et al. [14] designed step-stress partially accelerated life tests for the Exponentiated Exponential distribution under the progressive Type-II censoring scheme. El-Din et al. [15] presented estimation in step-stress accelerated life tests using the progressive first failure censoring for Weibull distribution. Shi et al. [16] proposed a Bayesian inference for step-stress accelerated competing failure model. They estimated the maximum likelihood estimates of model parameters under step-stress partially accelerated life tests for Weibull distribution using the Type-II progressive censoring scheme. El-Din et al. [17] presented step-stress accelerated life testing using the progressive first-failure censoring scheme for Lindely distribution. They obtained point estimation and interval estimation for Lindely distribution parameter as well as the acceleration factor using progressive first failure samples under step-stress accelerated life test. In this paper, we present SS-PALT for the Power Function distribution based on Progressive Type-II censoring and estimating costs of maintenance service policy.

The rest of the paper is organized as follows: Section One involves an introduction of accelerated life testing, step-stress partially accelerated life testing, and progressive Type-II censoring. Section Two comprises the Model Description and Test Procedure. In this section, we describe the model and give the test procedure under step-stress partially accelerated life testing. Section Three is devoted to the Estimation Procedure of Model Parameter, the likelihood function, asymptotic Fisher Information matrix, and confidence intervals of the parameters. Section Four presents the estimation costs maintenance service policy for the model. The simulation study for verifying the theoretical results is considered in section Five. Section Six contains conclusion.

2 Model description and test procedure

2.1 Model description

In 1983, Mukherjee and Islam [18] presented an important and simple finite range lifetime failure distribution called the Power function failure model. This distribution includes the exponential and rectangular distribution as particular cases. The model is widely used as a simple lifetime distribution to assess system reliability. It exhibits a better fit for failure information and provides more appropriate information on hazard rate and other reliability measures. Hence it grabbed the attention of numerous the reliability practitioners in the world. This distribution is sometimes preferred to Weibull and lognormal distribution because of its easiness. Lai and Mukherjee [19] discussed some aging properties of this distribution, rectified the mistakes and revealed its further interesting properties. Lia et al. [20] stated it in the list of some important bathtub-shaped failure rate distributions. The applied stress and strength follows the Power function distribution. Saxena et al. [21] presented the reliability computation and Bayesian estimates of system reliability. Johnson et al. [22] reported the statistical properties of the distribution. M. Ahsanullah et al. [23] proposed a characterization of Power function distribution based on the lower record. Okorie et al. [24] proposed modified Power function distribution which is an extension of one parameter power function distribution. To measure the lifetime of the electrical component, Meniconi and Barry [25] compared the Power function distribution with Lognormal, Exponential, and Weibull distribution. Eun-Hyuk Lim and Min-Young Lee [26] provided a characterization of the power function distribution by lower record values. Tavanger [27] provided some characterization results on the power function distribution based on the properties of dual generalized order statistics.

The *pdf* is given as

$$f(t,p,\lambda) = \left(\frac{p}{\lambda^p}\right) t^{p-1} \quad p,\lambda > 0,\ 0 < t < \lambda \tag{1}$$

The cumulative distribution function *(cdf)* of the Power function distribution is given as

$$F(t,p,\lambda) = \left(\frac{t}{\lambda}\right)^p \quad p,\lambda > 0,\ 0 < t < \lambda \tag{2}$$

The reliability function of the Power function distribution is given as

$$R(t) = 1 - \left(\frac{t}{\lambda}\right)^p \tag{3}$$

The hazard function of the Power function distribution is given as

$$H(t) = \frac{\left(\frac{p}{\lambda^p}\right) t^{p-1}}{1-\left(\frac{t}{\lambda}\right)^p} \tag{4}$$

The shape parameter p plays an important role in reliability analysis. If the value of the shape parameter $p = 1$, the distribution becomes rectangular distribution. For the value $p > 1$, the distribution curve becomes steeper, and for $p < 1$, the density function diminishes monotonically. At least over the first quarter of its lifetime, the distribution has a decreasing failure rate. After that, the failure rate increases monotonically.

2.2 Test procedure

Under step stress partially accelerated life testing procedures, it is necessary to test the units under usual operating conditions. To induce failure from the remaining survived units, the test is switched to a higher stress level at a time y_{n_1}. This switch aims to divide the remaining lifetime of the unit(s) by the acceleration factor $\beta(>1)$. Hence it will shorten the life of the test item. Accordingly, the lifetime of the test unit in SS-PALT given by DeGroot and Goel [28] is indicated as follows

$$Y = \begin{cases} T & T \le y_{n_1} \\ y_{n_1} + \beta^{-1}(T - y_{n_1}) & T > y_{n_1} \end{cases} \tag{5}$$

T is the life of the unit at usual operating conditions and y_{n_1} is the time when stress is switched to a higher level. β is the tempering coefficient, which is the ratio of mean life of an experimental unit tested at the usual condition to accelerated condition, normally $\beta > 1$.

Under Progressive Type-II censoring, we got failures as the starting of the test. We remove the R_i units from the remaining units at the time *ith* failure's time. Finally, all the remaining $R_m = n - m - \sum_{i=1}^{m-1} R_i$ are removed from the test at the *mth* failure's, and the test is terminated.

Let's assume that the failure-times of the experimental units follow Power function failure distribution with scale parameter λ and a shape parameter p . Then, the *pdf* and *cdf* of the total lifetime Y of experimental units using transformations in equation (5), are given in the following equations.

The *pdf* takes the following forms

$$f(y) = \begin{cases} f_1(y) & 0 < T \le y_{n_1} \\ f_2(y) & T > y_{n_1} \end{cases} \tag{6}$$

Where the values of $f_1(y)$ and $f_2(y)$ take the following forms

$$f(y)=\begin{cases} f_1(y)=\frac{p}{\lambda^p}y^{p-1} & 0<y\le y_{n_1}\\ f_2(y)=\frac{p}{\lambda^p}(y_{n_1}+\beta^{-1}(y-y_{n_1}))^{p-1} & y>y_{n_1}\end{cases} \tag{7}$$

The *cdf* takes the following form

$$F(y)=\begin{cases} F_1(y) & 0<y\le y_{n_1}\\ F_2 & y>y_{n_1}\end{cases} \tag{8}$$

Where the values of $F_1(y)$ and $F_2(y)$ take the following forms

$$F(y)=\begin{cases} F_1(y)=\left(\frac{y}{\lambda}\right)^p & 0<y\le y_{n_1}\\ F_2(y)=\left(\frac{y_{n_1}+\beta^{-1}(y-y_{n_1})}{\lambda}\right)^p & y>y_{n_1}\end{cases}$$

Thus, the reliability function under SS-PALT takes the following form

$$S(y)=\begin{cases} S_1(y)=1-\left(\frac{y}{\lambda}\right)^p & 0<y\le y_{n_1}\\ S_2(y)=1-\left(\frac{y_{n_1}+\beta^{-1}(y-y_{n_1})}{\lambda}\right)^p & y>y_{n_1}\end{cases} \tag{9}$$

3 Estimation procedure of the model Parameters

In this section, we use the maximum likelihood method to estimate the model parameters based on progressively Type-II censoring under SS-PALT because it is simple and gives the estimate of parameters with accurate statistical properties.

Let n independent units are put on test with the corresponding lifetimes $Y_1,Y_2,...,Y_n$. These units are independently and identically distributed as Power function distribution with *pdf* , which is given in equation (1). The m completely ordered lifetimes are denoted by

$$y_{1:m:n}<y_{2:m:n}<...<y_{J:m:n}<y_{n_1}<y_{J+1:m:n}<...<y_{m:m:n}$$

Here, the number of failed units at use condition is denoted by J.

Hence, the likelihood function for progressively Type-II censored data under SS-PALT is given as

$$\begin{aligned} L(p,\lambda,\beta)&=\prod_{i=1}^{J}f_1(y_i)[1-F_1(y_i)]^{R_i}\times\prod_{i=J+1}^{m}f_2(y_i)[1-F_2(y_i)]^{R_i}\\ &=\prod_{i=1}^{J}\frac{p}{\lambda^p}y_i^{p-1}\left[1-\left(\frac{y_i}{\lambda}\right)^p\right]^{R_i}\times\prod_{i=J+1}^{m}\frac{p}{\lambda^p}\left[y_{n_1}+\beta^{-1}(y_i-y_{n_1})\right]^{p-1}\left[1-\left(\frac{y_{n_1}+\beta^{-1}(y_i-y_{n_1})}{\lambda}\right)^p\right]^{R_i} \end{aligned} \tag{10}$$

Where, $y_{1:m:n}<y_{2:m:n}<...<y_{n_1}<y_{J+1:m:n}<...<y_{m:m:n}$

The log-likelihood function is given as

$$\begin{aligned} lnL=&mln\left(\frac{p}{\lambda^p}\right)+(p-1)\sum_{i=1}^{J}lny_i+\sum_{i=1}^{J}R_iln\left(1-\frac{y_i}{\lambda}\right)^p+(p-1)\sum_{i=J+1}^{m}ln(y_{n_1}+\beta^{-1}(y_i-y_{n_1}))\\ &+\sum_{i=J+1}^{m}R_iln\left[1-\left(\frac{y_{n_1}+\beta^{-1}(y_i-y_{n_1}}{\lambda}\right)^p\right] \end{aligned}$$

The Maximum likelihood (ML) estimates of p,λ and β are estimated from the following equations.

$$\frac{\partial lnL}{\partial p}=m[p^{-1}-ln\lambda]+\sum_{i=1}^{J}lny_i+\sum_{i=1}^{J}R_p\left(1-\frac{y_i}{\lambda}\right)^{-1}+\sum_{i=J+1}^{m}(y_{n_1}+\beta^{-1}(y_i-y_{n_1}))+\sum_{i=J+1}^{m}R_ip\frac{\left(\frac{y_{n_1}+\beta^{-1}(y_i-y_{n_1})}{\lambda}\right)^{p-1}}{1-\left(\frac{y_{n_1}+\beta^{-1}(y_i-y_{n_1})}{\lambda}\right)^p}=0$$

$$\frac{\partial lnL}{\partial\lambda} = -m\left(\frac{p}{\lambda}\right) + \sum_{i=1}^{J} p\beta^{-2} + \sum_{i=J+1}^{m} pR_i\lambda^{-1}\frac{\left(\frac{y_{n_1}+\beta^{-1}(y_i-y_{n_1})}{\lambda}\right)^{p}}{1-\left(\frac{y_{n_1}+\beta^{-1}(y_i-y_{n_1})}{\lambda}\right)^{p}} = 0$$

$$\frac{\partial lnL}{\partial\beta} = (p-1)\sum_{i=J+1}^{m}\beta^{-2}\frac{(y_i-y_{n_1})}{y_{n_1}+\beta^{-1}(y_i-y_{n_1})} + \sum_{i=J+1}^{m} R_i p\beta^{-2}\frac{\left(\frac{y_i-y_{n_1}}{\lambda}\right)\left(\frac{y_{n_1}+\beta^{-1}(y_i-y_{n_1})}{\lambda}\right)^{p-1}}{1-\left(\frac{y_{n_1}+\beta^{-1}(y_i-y_{n_1})}{\lambda}\right)^{p}} = 0$$

It is impossible to get ML estimates from the above-mentioned three equations, so the Newton-Raphson technique is applied to obtained these estimetes.

The Fisher Information matrix is given as

$$I = \begin{bmatrix} -\frac{\partial^2 lnL}{\partial p^2} & -\frac{\partial^2 lnL}{\partial p\partial\lambda} & -\frac{\partial^2 lnL}{\partial p\partial\beta} \\ -\frac{\partial^2 lnL}{\partial\lambda\partial p} & -\frac{\partial^2 lnL}{\partial\lambda^2} & -\frac{\partial^2 lnL}{\partial\lambda\partial\beta} \\ -\frac{\partial^2 lnL}{\partial\beta\partial p} & -\frac{\partial^2 lnL}{\partial\beta\partial\lambda} & -\frac{\partial^2 lnL}{\partial\beta^2} \end{bmatrix} \tag{11}$$

The elements of the Fisher Information matrix are obtained by second partial derivatives of log-likelihood function with respect to parameters p, λ and β. Consequently, the elements are expressed by the following equations.

$$-\frac{\partial^2 lnL}{\partial p^2} = mp^{-2} + \sum_{i=1}^{J} R_i\left(1-\frac{y_i}{\lambda}\right)^{-1} + \sum_{i=J+1}^{m} R_i\left[p^{-1} + ln\left(\frac{y_{n_1}+(y_i-y_{n_1})}{\lambda}\right) + \frac{p\left(\frac{y_{n_1}+(y_i-y_{n_1})}{\lambda}\right)^{p-1}}{1-\left(\frac{y_{n_1}+(y_i-y_{n_1})}{\lambda}\right)^{p}}\right]$$

$$-\frac{\partial^2 lnL}{\partial p\partial\lambda} = m\lambda^{-1} + \sum_{i=1}^{J} R_i p\left(1-\frac{y_i}{\lambda}\right)^{-2} y_i\lambda^{-2} - \sum_{i=J+1}^{m} R_i p(y_{n_1}+(y_i-y_{n_1}))^{p-1}\left[\frac{(p+1)}{\lambda} + \frac{p\lambda^{-(p+1)}(y_{n_1}+(y_i-y_{n_1}))^{p}}{1-\left(\frac{y_{n_1}+(y_i-y_{n_1})}{\lambda}\right)^{p}}\right]$$

$$-\frac{\partial^2 lnL}{\partial p\partial\beta} = \sum_{i=J+1}^{m}\frac{R_i p}{\lambda^{p-1}}\left[\frac{(p-1)(y_{n_1}+\beta^{-1}(y_i-y_{n_1}))^{p-1}}{y_{n_1}+\beta^{-1}(y_i-y_{n_1})} - \frac{p\beta^{-2}\lambda^{-1}(y_i-y_{n_1})\left(\frac{y_{n_1}+\beta^{-1}(y_i-y_{n_1})}{\lambda}\right)^{p-1}}{1-\left(\frac{y_{n_1}+\beta^{-1}(y_i-y_{n_1})}{\lambda}\right)^{p}}\right] + \sum_{i=J+1}^{m}\frac{\beta^{-2}(y_i-y_{n_1})}{y_{n_1}+(y_i-y_{n_1})}$$

$$-\frac{\partial^2 lnL}{\partial\lambda\partial p} = \frac{m}{\lambda} - \sum_{i=1}^{J} y_i R_i\lambda^{-2}\left(1-\frac{y_i}{\lambda}\right)^{-1} + \sum_{i=J+1}^{m} p\lambda^{-1}R_i\frac{\left(\frac{y_{n_1}+\beta^{-1}(y_i-y_{n_1})}{\lambda}\right)^{p}}{1-\left(\frac{y_{n_1}+\beta^{-1}(y_i-y_{n_1})}{\lambda}\right)^{p}}\left[p^{-1} + ln\left(\frac{y_{n_1}+\beta^{-1}(y_i-y_{n_1})}{\lambda}\right) + p\frac{\left(\frac{y_{n_1}+\beta^{-1}(y_i-y_{n_1})}{\lambda}\right)^{p-1}}{\left(\frac{y_{n_1}+\beta^{-1}(y_i-y_{n_1})}{\lambda}\right)^{p}}\right]$$

$$-\frac{\partial^2 lnL}{\partial\lambda^2} = -\frac{mp}{\lambda^2} - \sum_{i=J+1}^{m}\lambda^{-2}R_i\left[\left(1-\frac{y_i}{\lambda}\right)^{-2} y_i\lambda^{-1}\left(1-\frac{y_i}{\lambda}\right)^{-1}\right] + \sum_{i=J+1}^{m} R_i(y_{n_1}+\beta^{-1}(y_i-y_{n_1})) \times\left[\frac{(y_{n_1}+\beta^{-1}(y_i-y_{n_1}))^{-1}}{\lambda^2} + \lambda^{-3}\left(1-\left(\frac{y_{n_1}+\beta^{-1}(y_i-y_{n_1})}{\lambda}\right)^{-1}\right)\right]$$

$$-\frac{\partial^2 lnL}{\partial\lambda\partial\beta}=\sum_{i=J+1}^{m}p\lambda^{-1}R_i\frac{\left(\frac{y_{n_1}+\beta^{-1}(y_i-y_{n_1})}{\lambda}\right)^p}{1-\left(\frac{y_{n_1}+\beta^{-1}(y_i-y_{n_1})}{\lambda}\right)^p}\left[\frac{p\beta^{-2}\left(\frac{y_i-y_{n_1}}{\lambda}\right)}{\frac{y_{n_1}+\beta^{-1}(y_i-y_{n_1})}{\lambda}}+\frac{p\beta^{-2}\left(\frac{y_{n_1}+\beta^{-1}(y_i-y_{n_1})}{\lambda}\right)^{p-2}}{1-\left(\frac{y_{n_1}+\beta^{-1}(y_i-y_{n_1})}{\lambda}\right)^p}\right]$$

$$-\frac{\partial^2 lnL}{\partial\beta\partial p}=-\sum_{i=J+1}^{m}\frac{\beta^{-2}(y_i-y_{n_1})}{y_{n_1}+\beta^{-1}(y_i-y_{n_1}}-\sum_{i=J+1}^{m}R_i\beta^{-2}\lambda^{-1}(y_i-y_{n_1})\frac{\left(\frac{y_{n_1}+\beta^{-1}(y_i-y_{n_1})}{\lambda}\right)^{p-1}}{1-\left(\frac{y_{n_1}+\beta^{-1}(y_i-y_{n_1})}{\lambda}\right)^p}$$
$$\times\left[ln\left(\frac{y_{n_1}+\beta^{-1}(y_i-y_{n_1})}{\lambda}\right)+\frac{p\left(\frac{y_{n_1}+\beta^{-1}(y_i-y_{n_1})}{\lambda}\right)^{p-1}}{p\left(\frac{y_{n_1}+\beta^{-1}(y_i-y_{n_1})}{\lambda}\right)^p}\right]$$

$$-\frac{\partial^2 lnL}{\partial\beta\partial\lambda}=-\sum_{i=J+1}^{m}pR_i\beta^{-2}(y_i-y_{n_1})\left(1-\frac{y_{n_1}+\beta^{-1}(y_i-y_{n_1})}{\lambda}\right)^{-2}\left(\frac{y_{n_1}+\beta^{-1}(y_i-y_{n_1})}{\lambda}\right)$$

$$-\frac{\partial^2 lnL}{\partial\beta^2}=-\sum_{i=J+1}^{m}(p-1)\beta^{-2}(y_i-y_{n_1})(y_{n_1}+\beta^{-1}(y_i-y_{n_1}))^{-1}\left[\frac{-2}{\beta}-\frac{\beta^{-2}(y_i-y_{n_1})}{y_{n_1}+\beta^{-1}(y_i-y_{n_1})}\right]$$
$$-\sum_{i=J+1}^{m}(y_i-y_{n_1})R_i\beta^{-2}\left(1-\frac{y_{n_1}+\beta^{-1}(y_i-y_{n_1})}{\lambda}\right)^{-1}\left[\frac{-2}{\beta}-\frac{\beta^{-2}\lambda^{-1}(y_i-y_{n_1})}{1-\frac{y_{n_1}+\beta^{-1}(y_i-y_{n_1})}{\lambda}}\right]$$

Now, the variance-covariance matrix is the inverse of the Fisher Information matrix and given as

$$\Sigma=I^{-1}$$

$$\Sigma=\begin{bmatrix}-\frac{\partial^2 lnL}{\partial p^2} & -\frac{\partial^2 lnL}{\partial p\partial\lambda} & -\frac{\partial^2 lnL}{\partial p\partial\beta}\\ -\frac{\partial^2 lnL}{\partial\lambda\partial p} & -\frac{\partial^2 lnL}{\partial\lambda^2} & -\frac{\partial^2 lnL}{\partial\lambda\partial\beta}\\ -\frac{\partial^2 lnL}{\partial\beta\partial p} & -\frac{\partial^2 lnL}{\partial\beta\partial\lambda} & -\frac{\partial^2 lnL}{\partial\beta^2}\end{bmatrix}^{-1}=\begin{bmatrix}AVar(\hat{p}) & ACov(\hat{p}\hat{\lambda}) & ACov(\hat{p}\hat{\beta})\\ ACov(\hat{\lambda}\hat{p}) & AVar(\hat{\lambda}) & ACov(\hat{\lambda}\hat{\beta})\\ ACov(\hat{\beta}\hat{p}) & ACov(\hat{\beta}\hat{\lambda}) & AVar(\hat{\beta})\end{bmatrix}$$

AVar is asymptotic variance and *ACov* is asymptotic covariance.

In large samples, the maximum likelihood estimates $\hat{p}$, $\hat{\lambda}$ and $\hat{\beta}$ are asymptotically normally distributed and consistent. So, the two-sided approximate $100(1-\gamma)\%$ confidence limits are obtained respectively in the following way:

$L_{\hat{p}}=\hat{p}-z_{\gamma/2}\sigma(\hat{p})$ and $U_{\hat{p}}=\hat{p}-z_{\gamma/2}\sigma(\hat{p})$

$L_{\hat{\lambda}}=\hat{\lambda}-z_{\gamma/2}\sigma(\hat{\lambda})$ and $U_{\hat{\lambda}}=\hat{\lambda}-z_{\gamma/2}\sigma(\hat{\lambda})$

$L_{\hat{\beta}}=\hat{\beta}-z_{\gamma/2}\sigma(\hat{\beta})$ and $U_{\hat{\beta}}=\hat{\beta}-z_{\gamma/2}\sigma(\hat{\beta})$

Where $Z_{\gamma/2}$ is the $[100(1-\gamma)/2]^{th}$ standard normal percentile and $\sigma(*)$ is the standard deviation for the maximum likelihood estimates $\hat{p}$, $\hat{\lambda}$ and $\hat{\beta}$. This standard deviation is calculated by taking the square root of the first diagonal element of the inverse of the Fisher Information matrix I.

4 Estimating costs of maintenance service policy

Several authors explored the problem of maintenance service policy. For instance, Yiwei et al. [29] studied a cost-driven predictive maintenance policy for structural airframe maintenance. Maintenance is formally derived based on the trade-off between probabilities of occurrence of unscheduled and scheduled maintenance. Yiwei et al. [30] proposed predictive airframe maintenance strategies using model-based prognostics. In this work, they proposed two predictive maintenance strategies based on the developed prognostic model and applied to fatigue damage propagation in fuselage panels. A preventive maintenance policy is also proposed by Lie et al. [31] for the single-unit system failures which have sudden socks and internal deterioration. The study aimed to minimize the minimization of the expected cost per unit time defining the help of determining the optimal preventive replacement interval, inspection interval, and the number of inspections. For designing and optimizing maintenance service policy, a study is presented by Sukhwa et al. [32]. Fabrian and Luis [33] presented a method to definite maintenance intervals to those of similar systems under development, and this method has been applied in an aircraft manufacturing company using the current operation database. Michail et al. [34] addressed the development of an aircraft maintenance planning optimization tool and its application to an aircraft component. Shey-Huei et al. [35] investigated the optimal preventive maintenance policy for multi-state systems.

The maintenance service policy ends when the arrangement period reaches time (usage level (L)). The system's renewal is not involved. The preventive and corrective maintenances are under this policy. At a constant interval of time (τ), the system should go for periodically preventive maintenance under this policy. At each failure between within successive preventive maintenances, the system should go for minimally repaired. A complicated repairable system with a long life is perfect for this type of service arrangement.

The main assumptions of the Maintenance Service Policy are

(i) The successive failures are mutually independent random actions.

(ii) The successive failures are known on the parameters of distributions.

(iii) Whether the repairs were completed in maintenance, only minimal repairs are conducted.

(iv) The Servicing activity restores life to a bit.

(v) The repairs times are minor to compared to the item's life.

(vi) After each preventive maintenance, the age renovation is stable.

(vii) The unit amount of minimal repairs between the unit amount of preventive maintenances and preventive maintenances have a constant average.

The expected cost of maintenance service policy is the sum of the total sum of expected costs, all minimal repairs, and the expected costs of all planned preventive maintenance over the policy's period. We can get the expected cost of maintenance service per unit time by dividing the expected total cost by the duration of service policy.

According to Rahman [36], the expected cost of maintenance service policy can be defined in the following steps:

(i) Taking the equal length of the preventive maintenance period (τ), the expected cost of minimal repairs between preventive maintenances is given as

$$E(C_{mr}) = C_{mr}\left[\sum_{k=0}^{N-1}\int_{k\tau}^{(k+1)\tau} h(t-k\tau)dt\right] \tag{12}$$

(ii) The expected cost of preventive maintenance is given as

$$E(C_{pm}) = NC_{pm} \tag{13}$$

Here, the system is periodically maintained at Nth preventive maintenance The total expected cost per unit time $C(\tau,N)$ is given as

$$E(C(\tau,N)) = \frac{E(C_{mr})+E(C_{pm})}{L} \tag{14}$$

Where $L = N\times\tau$

If the lifetime follows the Power Function distribution, the expected cost of maintenance service policy is

$$E(C_{mr}) = \frac{C_{mr}}{p}\left[\sum_{k=1}^{N-1} log\frac{(\lambda^p - k^p\tau^p)}{(\lambda^p-(k+1)^p\tau^p)}\right] \tag{15}$$

Finally, the total rate per unit time is given as

$$E(C(\tau,N)) = \frac{\frac{C_{mr}}{p}\left[\sum_{k=1}^{N-1} log\frac{(\lambda^p-k^p\tau^p)}{(\lambda^p-(k+1)^p\tau^p)}\right]+NC_pm}{L} \tag{16}$$

5 Simulation study and results

In this section, the simulation technique is implemented to check the performance of MLEs. It is hard to check the performance of the different censoring schemes or methods, it is a very tough task to check this theoretically. Here, a simulation technique is performed to compare different sampling schemes for different parameter values using the Monte-Carlo simulations method. The steps of the simulation technique are given follows (i) Here, we considered the following three progressive censoring schemes:

Scheme 1: $R_1 = \ldots R_{m-1} = 0$ and $R_m = n-m$, **Scheme** 2: $R_1 = n-m$ and $R_2 = \ldots R_m = 0$, **Scheme** 3: $R_1 = \ldots R_{m-1} = 1$ and $R_m = n-2m+1$. Where n is the sample size. The MLEs, biases, and confidence intervals are estimated for above-censoring schemes 1, 2, and 3 for 1000 replications.
(ii) Choose the values of n and m.
(iii) Also, choose the values for shape and scale parameters (p,λ) and the acceleration factor β. The chosen values of parameters are

$(p = 2.5, \lambda = 1.8, \beta = 1.6)$

$(p = 2, \lambda = 1.8, \beta = 1.8)$

$(p = 2, \lambda = 1.5, \beta = 2)$

(iv) All non-linear equations are solved by the Newton-Raphson method.

(v) Generate a random sample of size n $(n = 25, 50, 75)$ form Power function distribution with the specified values of p and λ.

(vi) The generation of the random sample is so easy. If $U(0,1)$ is Uniform distribution, by inverse transformation $y = \lambda U^{1/p}$ is Power function distribution from equation (2).

(vii) To compute the MLEs of unknown parameters, the progressive Type-II censoring scheme is used.

(viii) The average values of biases, MSEs of the parameter, and acceleration factor for all sample sizes are computed.

(ix) For different values of parameters n, m, p, λ and β, steps (v-viii) are replicated 1000 times.

(x) Using equations $(12-16)$, the expected cost of maintenance service policy is obtained for expected cost rate, minimal repairs, preventive maintenance, and total costs. The length of maintenance service policy (L) is three years, and the preventive maintenance every four months $\tau = 30$ at an average cost $(C_{pm} = 1000)$.

(xi) If there are failures between two successive preventive maintenance, the minimal repairs will be done at average cost $(C_{pm} = 700)$. Finally, the expected cost of preventive maintenance is 12000, $E(C_{pm} = 12000)$.

Table 1: The mean values of MLEs with bias and MSE of parameters $(p = 2.5, \lambda = 1.8, \beta = 1.6)$ for different size of samples for the progressive Type-II censoring scheme

(n,m)	Schemes	Estimates of p			Estimates of λ			Estimates of β		
		MLE	Bias	MSE	MLE	Bias	MSE	MLE	Bias	MSE
(25,10)	1	0.4521	0.7232	0.6828	0.6001	0.8123	0.5987	0.9087	0.9867	0.7865
	2	0.4763	0.7123	0.6809	0.6134	0.8001	0.6012	0.8977	0.9787	0.7765
	3	0.5112	0.7012	0.6799	0.6199	0.7981	0.5976	0.8723	0.9098	0.8776
(25,15)	1	0.4876	0.7001	0.6701	0.6245	0.7865	0.5876	0.9123	0.8876	0.6987
	2	0.4987	0.6912	0.6654	0.6366	0.7453	0.5342	0.9354	0.8009	0.5987
	3	0.5009	0.6876	0.6543	0.6543	0.8154	0.5009	0.9399	0.7432	0.5543
(50,25)	1	0.5134	0.7123	0.6433	0.6765	0.7432	0.4987	0.9412	0.8764	0.5321
	2	0.5087	0.6775	0.6512	0.6876	0.7223	0.5656	0.9435	0.8876	0.5476
	3	0.4887	0.6643	0.6387	0.6543	0.7008	0.5321	0.9967	0.7123	0.5112
(50,30)	1	0.5376	0.6423	0.5987	0.6987	0.6876	0.5112	0.8765	0.7009	0.5443
	2	0.5634	0.6565	0.5867	0.7154	0.6675	0.4876	0.8812	0.6898	0.4998
	3	0.6199	0.6123	0.5765	0.7009	0.7332	0.4454	0.9154	0.7654	0.4876
(75,35)	1	0.6009	0.6008	0.5662	0.7543	0.6432	0.4008	0.9645	0.6876	0.5665
	2	0.5876	0.6487	0.5223	0.7442	0.5765	0.3987	0.9465	0.5665	0.4554
	3	0.5798	0.5987	0.5009	0.7867	0.6543	0.3232	0.9986	0.6753	0.4112
(75,40)	1	0.6169	0.5632	0.4765	0.8007	0.6765	0.4232	0.9676	0.4987	0.3340
	2	0.6265	0.5432	0.4654	0.8154	0.5443	0.2987	0.9897	0.4232	0.2987
	3	0.5643	0.5321	0.4532	0.7979	0.5008	0.3212	0.9988	0.5432	0.2234

Table 2: The mean values of MLEs with bias and MSE of parameters $(p = 2, \lambda = 1.8, \beta = 1.8)$ for different size of samples for the progressive Type-II censoring scheme

(n,m)	Schemes	Estimates of p			Estimates of λ			Estimates of β		
		MLE	Bias	MSE	MLE	Bias	MSE	MLE	Bias	MSE
(25,10)	1	0.9234	0.7991	0.8265	0.9978	0.6882	0.8003	1.2154	0.8432	0.9006
	2	0.8786	0.7723	0.8165	1.1265	0.6667	0.7979	1.2987	0.8254	0.8948
	3	1.0092	0.7612	0.7987	1.0098	0.6554	0.7881	1.3154	0.8165	0.8876
(25,15)	1	0.8876	0.7443	0.7997	1.2354	0.5987	0.7453	1.4098	0.7987	0.8543
	2	0.7998	0.7865	0.8009	0.9001	0.5887	0.6986	1.4132	0.8009	0.8760
	3	0.8976	0.7009	0.7776	1.3241	0.5687	0.6886	1.3987	0.7665	0.8575
(50,25)	1	0.7543	0.6987	0.7443	1.2451	0.6198	0.7001	1.2987	0.7335	0.8432
	2	0.7987	0.6765	0.7254	1.1109	0.6254	0.6765	1.2765	0.7005	0.8283
	3	0.9154	0.7009	0.7654	0.9954	0.5543	0.6432	1.4321	0.6876	0.7709
(50,30)	1	0.6998	0.6543	0.7008	0.9065	0.5243	0.6116	1.4465	0.6776	0.7465
	2	0.8215	0.6341	0.6987	1.1098	0.5098	0.5876	1.0092	0.7114	0.7320
	3	1.0954	0.6009	0.6798	1.0098	0.4987	0.5776	1.3587	0.6543	0.7089
(75,35)	1	0.9154	0.6152	0.6543	1.1654	0.5165	0.5443	1.2967	0.6117	0.6765
	2	0.7243	0.5986	0.7132	1.2365	0.4876	0.5776	1.4325	0.5876	0.6543
	3	0.7878	0.5443	0.6443	1.2978	0.4687	0.5321	1.3934	0.6103	0.6987
(75,40)	1	0.9843	0.4987	0.6143	1.1180	0.4224	0.6009	1.2923	0.5438	0.5876
	2	0.8975	0.3987	0.57654	1.2009	0.4110	0.5087	1.1198	0.5003	0.5915
	3	1.1287	0.3465	0.5540	1.2298	0.3838	0.4987	1.4009	0.5174	0.4776

Table 3: At a confidence level 95%, the confidence intervals of the estimators $(p = 2, \lambda = 1.8, \beta = 1.6)$

(n,m)	Schemes	Estimates of p		Estimates of λ		Estimates of β	
		Lower Bound	Upper Bound	Lower Bound	Upper Bound	Lower Bound	Upper Bound
(25,10)	1	0.4238	1.4532	0.5622	1.5687	0.7765	1.7762
	2	0.4453	1.3459	0.5665	1.4887	0.7865	1.6998
	3	0.4628	1.2598	0.5880	1.5765	0.7965	1.7999
(25,15)	1	0.4692	1.3643	0.6009	1.4775	0.8009	1.7124
	2	0.4798	1.1542	0.6143	1.5987	0.8154	1.6765
	3	0.4766	1.0861	0.6223	1.6009	0.8143	1.8987
(50,25)	1	0.4987	1.1981	0.6343	1.4887	0.9154	1.6154
	2	0.5098	1.0665	0.6432	1.5243	0.9221	1.8654
	3	0.5001	1.1199	0.6545	1.3324	0.9354	1.5987
(50,30)	1	0.5122	1.0094	0.6678	1.3431	0.8976	1.2091
	2	0.5365	1.2154	0.6754	1.2776	0.9354	1.3165
	3	0.5412	1.1321	0.6832	1.1998	0.9465	1.4453
(75,35)	1	0.5632	1.0032	0.6987	1.2987	0.9587	1.4321
	2	0.5765	1.1165	0.7008	1.4231	0.8976	1.6542
	3	0.5668	1.0321	0.6987	1.0098	0.9765	1.5321
(75,40)	1	0.6876	1.3123	0.7132	1.4298	0.9987	1.7765
	2	0.7987	1.3330	0.7243	1.3154	0.8899	1.4876
	3	0.8001	1.3432	0.7454	1.4350	0.9991	1.3987

Table 4: At a confidence level 95%, the confidence intervals of the estimators $(p=2, \lambda=1.8, \beta=1.8)$

(n,m)	Schemes	Estimates of p		Estimates of λ		Estimates of β	
		Lower Bound	Upper Bound	Lower Bound	Upper Bound	Lower Bound	Upper Bound
(25,10)	1	0.4432	1.5122	0.6114	1.7117	0.7001	1.9232
	2	0.4465	1.4331	0.6176	1.7009	0.7132	1.8404
	3	0.4532	1.4509	0.6254	1.7132	0.7432	1.7875
(25,15)	1	0.4588	1.3032	1.6432	1.6453	0.7654	1.7123
	2	0.4593	1.2275	0.6588	1.8554	0.6987	1.6765
	3	0.4602	1.2994	0.6653	1.7997	0.7865	1.4098
(50,25)	1	0.4798	1.1091	0.7002	1.6187	0.7354	1.5154
	2	0.4654	1.2443	0.7112	1.5765	0.8164	1.7098
	3	0.4832	1.2996	0.8002	1.5932	0.8254	1.5254
(50,30)	1	0.4876	1.1112	0.8116	1.6998	0.9086	1.5543
	2	0.4988	1.1001	0.8276	1.4113	0.9543	1.7454
	3	0.5643	1.3223	0.9008	1.4908	0.8864	1.7143
(75,35)	1	0.5776	1.2886	0.8576	1.3256	1.0045	1.5845
	2	0.5887	1.2543	0.9409	1.3009	0.9870	1.4065
	3	0.5987	1.1776	0.9987	1.3976	1.1324	1.4765
(75,40)	1	0.6008	1.1776	0.9986	1.3976	1.1324	1.5650
	2	0.6112	1.0032	1.0101	1.4234	1.1123	1.4002
	3	0.6012	1.0887	1.1132	1.5176	0.9987	1.3540

Table 5: Estimation of expected cost rate, total cost,minimal repair time and its confidence level under the maintenence service policy

(n)	Minimal repair cost			Total rate			Cost rate		
	$E(C_{mr})$	Lower Bound	Upper Bound	$E(C_{total})$	Lower Bound	Upper Bound	$E(C(\tau,N))$	Lower Bound	Upper Bound
Case-I (p=2.5,λ=1.8,β=1.6)									
25	68429.4	22591.3	30432.4	72341.6	32458.0	42678.4	30981.3	6784.5	9981.5
50	64567.8	B26761.4	43154.4	71234.7	28761.4	37897.5	28761.7	7712.7	8786.5
75	62341.4	32761.7	48713.6	68145.8	26543.3	34578.5	27145.6	8903.5	9587.8
Case-II (p=2,λ=1.8,β=1.8)									
25	52341.7	15321.7	43781.6	41234.7	28870	32643.6	22654.2	4567.6	6671.6
50	48981.3	18342.6	26578.5	40987.8	28543.4	29986.6	21361.4	4789.9	6098.6
75	47892.1	22651.9	27453.3	39331.4	26543.6	28654.4	20153.5	5312.6	5754.5
Case-III (p=2,λ=1.5,β=2)									
25	40981.6	100000.4	15763.4	28453.6	29863.3	33870.6	15631.6	3876.7	4589.5
50	39124.3	12432.9	17880	26543.6	27893.1	31267.6	14321.6	3587.8	3912.6
75	37987.5	13981.5	21775.3	24786.2	24563.2	28967.5	14764.2	2908.6	3562.6

6 Conclusion

(i) This paper has presented the step-stress partially accelerated life tests (SS-PALT) for the Power function distribution using the progressive Type-II censoring scheme.

(ii) For the two-parameter of Power function distribution and acceleration factor, the maximum likelihood estimators are obtained using the Newton-Rapshon method.

(iii) Tables (1), (2), (3), and (4) indicate that as the sample size increases, the values of MSEs and bias reduce, and confidence intervals become narrower. Thus, the MLEs have favorable statistical properties. We can also observe that the numerical results and theoretical findings support each other, and our assumptions are also fulfilled. Accordingly, It is asserted that test design is stable and robust. Finally, we can say that the designing of the test is stable and robust.

(iv) Table (5) exhibits that values of the parameters and the cost of maintenance service policy have a direct relationship, while the cost of maintenance service policy and the sample size have an inverse relationship.

Acknowledgement

The authors are grateful to the anonymous referee whose beneficial comments improved this paper.

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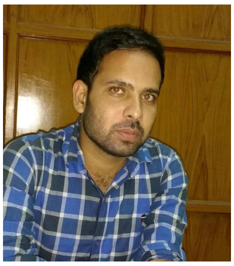

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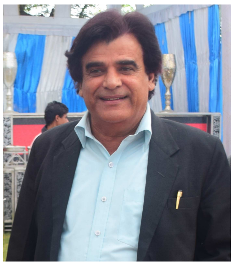

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