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Fractional Order Mathematical Modelling for Studying the Impact on the Emergence of Pollution and Biodiversity Pertaining to Incomplete Aleph Functions

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Abstract: Civilizations are always known for utilization, destruction and ignoring the environment. Different opinions have been put at the root of our environmental-disruptive tendencies, religion, social and economic structures and the adoption of technology. It is conventional to blame environmental conditions but we do not utilize practically our knowledge and understanding of nature. It seems that we continuously blame pollution and its destruction instead of our efforts. With the use of a fractional order mathematical model involving incomplete Aleph $(\aleph)$ functions (IAFs) and the Caputo fractional derivative, we explore the impact of the emergence of pollution and study the effect of industrial operations on the environment of the region. In this work, we evaluate the Caputo fractional derivative of incomplete Aleph $(\aleph)$ functions. Here, we define the concentration $\mathbb{C}(x,t)$ in terms of incomplete Aleph $(\aleph)$ functions and obtain some particular cases by giving specific values to the parameters of our primary results and expressing them in terms of special functions, notably H-functions, incomplete H-functions, and incomplete I-functions, and demonstrating their relationship with existing results.

Keywords: Caputo fractional derivative, incomplete Aleph functions, mathematical model, biodiversity.

1 Introduction, Motivation and Preliminaries

The ecological system is transformed due to the establishment of large numbers of industries and exponentially population growth. Spoilage of the ecosystem started due to the discharge of many types of solid materials, non-domestic pollutants and combustion of fuel and other industrial chemicals. In this 21^{st} century, the unbalanced ecological system is a subject of worried and scientists in many countries deal with this problem very seriously. Many visionary scientists in their research [1,2,3] already enumerated the unfavourable influence that arises due to unwanted industrial and biological pollutants.

Capitalism is the social hierarchy in this parable and capitalists earn and invest their wealth without thinking about the ecosystem. There are many ways to assess the level of pollutants in ecological and biological pollution where the venomous level depends on several factors. The recycling of waste is a major issue that will be of great importance in the fight against industrial pollution. There are many advanced waste treatments and recycling processes for the recycle industrial waste that have been started to minimize waste as much as possible. Moreover, it's important to measure regularly the pollution level of pollutant diffusion in the urban areas that happens due to the industries, sewage,

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wastewater system and vehicles having bad emissions. Many abominable activities within the ecosystem affect the environment badly [4,5].

If we are still ignoring the species destruction, the side effects of pesticides on biodiversity, slowly depletion of the stratospheric ozone layer and the effects on the life of water living organisms due to water pollution, it can destroy the entire ecosystem after an alarming situation. Gathering knowledge of nature and its implementation is the only way to protect us from a natural disaster. Unfortunately, many times fire or gas or oil or radioactive material leakage and harmful substances are released from the industries are also having a type of bad impact on the natural habits and breeding system of biodiversity. After a particular time, pollution can adversely affect the quality of the environment which may cause major damage to the system of biodiversity. Pollution can directly affect the immediate damage to vital chemical and physical components of living organisms. Many species of animals and plants are becoming extinct due to an unbalanced ecosystem and even many have become extinct thus the food chain affects the ecosystem directly or indirectly. The growth and development of species and their ability are controlled directly by the environment, but pollution and toxins increase the toxicity which directly affects the species. The entire world has a balanced approach to every aspect of beings and it always provides a support system to the system of nature but external pollutants and technological growth cause damage and destroyed the beauty of this ecology. Based on these factors, it is mandatory to take serious action against all kinds of pollution that is produced by rural and urban areas and make the public aware to create a healthy environment.

Several physical and mathematical models exist to execute the pollution diffusion. In the study of environmental pollution, mathematicians use deterministic and statistical models. In the environment, mathematical clarification of scientific processes is counted under the deterministic models. While analysis of the available environmental data which may be primary or secondary uses for statistical models. Both physically and scientific pieces of research [6,7,8,9] have been done by many investigators and investigated several effective and useful results. Undoubtedly, Industrial pollution is a huge problem for the entire biodiversity. Moreover, in this work, we observed the influence of natural pollutant growth and contiguity of two interconnecting biological inhabitants where the impurity causes harm to the significant structure of the biochemical and physiological ecosystem.

Through the investigation, we can understand the natural challenges of the current situation and highlight the importance of the mathematical models given by Patin [10] and modify it. Fractional calculus is an important tool to solve many practical life problems. Fractional calculus is used to solve civil engineering problems [11]. Recently, Several authors are used fractional calculus in the field of medical science [12,13]. By using the Incomplete Aleph functions and Caputo fractional derivative to get other results that might be more helpful to studying the environmental pollution. Thus we can choose the scientific models to get it as an effective medium that can help to evaluate the impacts on the ecosystem of resources-conservation. It also controls the involved pollution activities and helps decision-makers to express the arrangements of effective administration. We can be able to solve these issues with the help of advanced mathematical tools, suitable administration, the right arrangement of assets of resources and environmental structure. But it is not possible so far due to various reasons.

The paper is developed in the following way. In sections 2 and 3, we defined fractional calculus operators and incomplete Aleph functions respectively. And derived a new result Caputo derivative of incomplete Aleph functions in section 4. In section 5, We develop the mathematical model in terms of incomplete Aleph functions that can be applied to lessen the elements or issues to reduce the pollutants in the environment and this effort may keep the holiness of nature. In the last section, we conclude the summary of our main findings.

2 Fractional Derivatives

Fractional calculus is not a new topic, having lots of applications in Engineering, Medical Sciences and Mathematics. Few fractional derivatives are defined below:

Definition 2.1 Riemann-Liouville fractional derivative [14] of order α is defined as:

$$ {}_0D_t^{\alpha}(f(t)) = \frac{1}{\Gamma(n-\alpha)} \frac{d^n}{dt^n} \int_0^t f(\tau)(t-\tau)^{n-\alpha-1} d\tau, \quad n-1 < \alpha < n \in \mathbb{N}. \tag{1} $$

Definition 2.2 The Caputo fractional derivative (CFD) [14] is introduced by the Italian Mathematician Caputo in 1967 and for order α it's defined as:

$$ {}_0^C D_t^{\alpha}(f(t)) = \begin{cases} \frac{1}{\Gamma(n-\alpha)} \int_0^t \frac{f^{(n)}(\tau)}{(t-\tau)^{\alpha+1-n}} d\tau, & \text{if} \quad n-1 < \alpha < n \in \mathbb{N} \\ \frac{d^n f(t)}{dt^n}, & \text{if} \quad \alpha = n \in \mathbb{N}. \end{cases} \tag{2} $$

3 Incomplete Aleph ($\aleph$) Functions (IAFs)

The Incomplete Aleph ($\aleph$) functions (IAFs) ${}^{(\Gamma)}\aleph^{m,n}_{p_i,q_i,\rho_i;r}(z)$ and ${}^{(\gamma)}\aleph^{m,n}_{p_i,q_i,\rho_i;r}(z)$ was introduced and investigated by Bansal et al. [15] containing the incomplete Gamma functions $\Gamma(a,y)$ & $\gamma(a,y)$ [16] defined as:

$$ \begin{aligned} {}^{(\Gamma)}\aleph^{m,n}_{p_i,q_i,\rho_i;r}(z) &= {}^{(\Gamma)}\aleph^{m,n}_{p_i,q_i,\rho_i;r}\left[z \left| \begin{matrix} (\mathfrak{s}_1, \mathscr{S}_1, y), (\mathfrak{s}_j, \mathscr{S}_j)_{2,n}, [\rho_j(\mathfrak{s}_{ji}, \mathscr{S}_{ji})]_{n+1,p_i} \\ (\mathfrak{v}_j, \mathscr{V}_j)_{1,m}, [\rho_j(\mathfrak{v}_{ji}, \mathscr{V}_{ji})]_{m+1,q_i} \end{matrix} \right. \right] \\ &= \frac{1}{2\pi i} \int_L \varphi(\zeta, y) z^{\zeta} d\zeta, \end{aligned} \tag{3} $$

where $z \neq 0$, and

$$ \varphi(\zeta, y) = \frac{\Gamma(1-\mathfrak{s}_1+\mathscr{S}_1\zeta, y) \prod_{j=1}^{m} \Gamma(\mathfrak{v}_j - \mathscr{V}_j\zeta) \prod_{j=2}^{n} \Gamma(1-\mathfrak{s}_j+\mathscr{S}_j\zeta)}{\sum_{i=1}^{r} \rho_i \left[\prod_{j=m+1}^{q_i} \Gamma(1-\mathfrak{v}_{ji}+\mathscr{V}_{ji}\zeta) \prod_{j=n+1}^{p_i} \Gamma(\mathfrak{s}_{ji} - \mathscr{S}_{ji}\zeta) \right]}, \tag{4} $$

and

$$ \begin{aligned} {}^{(\gamma)}\aleph^{m,n}_{p_i,q_i,\rho_i;r}(z) &= {}^{(\gamma)}\aleph^{m,n}_{p_i,q_i,\rho_i;r}\left[z \left| \begin{matrix} (\mathfrak{s}_1, \mathscr{S}_1, y), (\mathfrak{s}_j, \mathscr{S}_j)_{2,n}, [\rho_j(\mathfrak{s}_{ji}, \mathscr{S}_{ji})]_{n+1,p_i} \\ (\mathfrak{v}_j, \mathscr{V}_j)_{1,m}, [\rho_j(\mathfrak{v}_{ji}, \mathscr{V}_{ji})]_{m+1,q_i} \end{matrix} \right. \right] \\ &= \frac{1}{2\pi i} \int_L \phi(\zeta, y) z^{\zeta} d\zeta, \end{aligned} \tag{5} $$

where $z \neq 0$, and

$$ \phi(\zeta, y) = \frac{\gamma(1-\mathfrak{s}_1+\mathscr{S}_1\zeta, y) \prod_{j=1}^{m} \Gamma(\mathfrak{v}_j - \mathscr{V}_j\zeta) \prod_{j=2}^{n} \Gamma(1-\mathfrak{s}_j+\mathscr{S}_j\zeta)}{\sum_{i=1}^{r} \rho_i \left[\prod_{j=m+1}^{q_i} \Gamma(1-\mathfrak{v}_{ji}+\mathscr{V}_{ji}\zeta) \prod_{j=n+1}^{p_i} \Gamma(\mathfrak{s}_{ji} - \mathscr{S}_{ji}\zeta) \right]}. \tag{6} $$

The IAFs ${}^{(\Gamma)}\aleph^{m,n}_{p_i,q_i,\rho_i;r}(z)$ and ${}^{(\gamma)}\aleph^{m,n}_{p_i,q_i,\rho_i;r}(z)$ are exist for all $y \geq 0$ and for more existing conditions (see [15,17]).

$$ k_i > 0, \quad |arg(z)| < \frac{1}{\pi} k_i \quad i = \overline{1,r}, \tag{7} $$

$$ k_i \geq 0, \quad |arg(z)| < \frac{1}{\pi} k_i \quad \text{and} \quad \mathbb{R}(L_i) + 1 < 0, \tag{8} $$

where

$$ k_i = \sum_{j=1}^{n} \mathscr{S}_j + \sum_{j=1}^{m} \mathscr{V}_j - \rho_i \left(\sum_{j=n+1}^{p_i} \mathscr{S}_{ji} + \sum_{j=m+1}^{q_i} \mathscr{V}_{ji} \right), \tag{9} $$

$$ L_i = \sum_{j=1}^{m} \mathfrak{v}_j - \sum_{j=1}^{n} \mathfrak{s}_j + \rho_i \left(\sum_{j=m+1}^{q_i} \mathscr{S}_{ji} - \sum_{j=n+1}^{p_i} \mathscr{V}_{ji} \right) + \frac{1}{2}(p_i - q_i), \quad i = \overline{1,r}. \tag{10} $$

The IAFs ${}^{(\Gamma)}\aleph^{m,n}_{p_i,q_i,\rho_i;r}(z)$ and ${}^{(\gamma)}\aleph^{m,n}_{p_i,q_i,\rho_i;r}(z)$ defined in (3) and (5) reduces to the several special functions as given below:
(i) Put $y = 0$ in equation (3), it converts IAF to the Aleph function [18]:

$$ {}^{(\Gamma)}\aleph^{m,n}_{p_i,q_i,\rho_i;r}\left[z \left| \begin{matrix} (\mathfrak{s}_1, \mathscr{S}_1, 0), (\mathfrak{s}_j, \mathscr{S}_j)_{2,n}, [\rho_j(\mathfrak{s}_{ji}, \mathscr{S}_{ji})]_{n+1,p_i} \\ (\mathfrak{v}_j, \mathscr{V}_j)_{1,m}, [\rho_j(\mathfrak{v}_{ji}, \mathscr{V}_{ji})]_{m+1,q_i} \end{matrix} \right. \right] = \aleph^{m,n}_{p_i,q_i,\rho_i;r}\left[z \left| \begin{matrix} (\mathfrak{s}_j, \mathscr{S}_j)_{1,n}, [\rho_j(\mathfrak{s}_{ji}, \mathscr{S}_{ji})]_{n+1,p_i} \\ (\mathfrak{v}_j, \mathscr{V}_j)_{1,m}, [\rho_j(\mathfrak{v}_{ji}, \mathscr{V}_{ji})]_{m+1,q_i} \end{matrix} \right. \right]. \tag{11} $$

(ii) Put $\rho_i = 1$ in equations (3) and (5), it converts IAFs to the incomplete I-functions [19,20]:

$$ {}^{(\Gamma)}\aleph^{m,n}_{p_i,q_i,1;r}\left[z \left| \begin{matrix} (\mathfrak{s}_1, \mathscr{S}_1, y), (\mathfrak{s}_j, \mathscr{S}_j)_{2,n}, [1(\mathfrak{s}_{ji}, \mathscr{S}_{ji})]_{n+1,p_i} \\ (\mathfrak{v}_j, \mathscr{V}_j)_{1,m}, [1(\mathfrak{v}_{ji}, \mathscr{V}_{ji})]_{m+1,q_i} \end{matrix} \right. \right] = {}^{(\Gamma)}I^{m,n}_{p_i,q_i;r}\left[z \left| \begin{matrix} (\mathfrak{s}_1, \mathscr{S}_1, y), (\mathfrak{s}_j, \mathscr{S}_j)_{2,n}, (\mathfrak{s}_{ji}, \mathscr{S}_{ji})_{n+1,p_i} \\ (\mathfrak{v}_j, \mathscr{V}_j)_{1,m}, (\mathfrak{v}_{ji}, \mathscr{V}_{ji})_{m+1,q_i} \end{matrix} \right. \right], \tag{12} $$

and

$$^{(\gamma)}\aleph^{m,n}_{p_i,q_i,1;r}\left[z\left|\begin{matrix}(\mathfrak{s}_1,\mathscr{S}_1,y),(\mathfrak{s}_j,\mathscr{S}_j)_{2,n},[1(\mathfrak{s}_{ji},\mathscr{S}_{ji})]_{n+1,p_i}\\(\mathfrak{v}_j,\mathscr{V}_j)_{1,m},[1(\mathfrak{v}_{ji},\mathscr{V}_{ji})]_{m+1,q_i}\end{matrix}\right.\right]={}^{(\gamma)}I^{m,n}_{p_i,q_i;r}\left[z\left|\begin{matrix}(\mathfrak{s}_1,\mathscr{S}_1,y),(\mathfrak{s}_j,\mathscr{S}_j)_{2,n},(\mathfrak{s}_{ji},\mathscr{S}_{ji})_{n+1,p_i}\\(\mathfrak{v}_j,\mathscr{V}_j)_{1,m},(\mathfrak{v}_{ji},\mathscr{V}_{ji})_{m+1,q_i}\end{matrix}\right.\right]. \tag{13}$$

(iii) Put $\rho_i=1$ and $y=0$ in equation (3), it converts IAF to the I-function [21]:

$$^{(\Gamma)}\aleph^{m,n}_{p_i,q_i,1;r}\left[z\left|\begin{matrix}(\mathfrak{s}_1,\mathscr{S}_1,0),(\mathfrak{s}_j,\mathscr{S}_j)_{2,n},[1(\mathfrak{s}_{ji},\mathscr{S}_{ji})]_{n+1,p_i}\\(\mathfrak{v}_j,\mathscr{V}_j)_{1,m},[1(\mathfrak{v}_{ji},\mathscr{V}_{ji})]_{m+1,q_i}\end{matrix}\right.\right]=I^{m,n}_{p_i,q_i;r}\left[z\left|\begin{matrix}(\mathfrak{s}_j,\mathscr{S}_j)_{1,n},(\mathfrak{s}_{ji},\mathscr{S}_{ji})_{n+1,p_i}\\(\mathfrak{v}_j,\mathscr{V}_j)_{1,m},(\mathfrak{v}_{ji},\mathscr{V}_{ji})_{m+1,q_i}\end{matrix}\right.\right]. \tag{14}$$

(iv) Put $\rho_i=1$ and $r=1$ in equations (3) and (5), it converts IAFs to the incomplete H-functions [22]:

$$^{(\Gamma)}\aleph^{m,n}_{p_i,q_i,1;1}\left[z\left|\begin{matrix}(\mathfrak{s}_1,\mathscr{S}_1,y),(\mathfrak{s}_j,\mathscr{S}_j)_{2,n},[1(\mathfrak{s}_{ji},\mathscr{S}_{ji})]_{n+1,p_i}\\(\mathfrak{v}_j,\mathscr{V}_j)_{1,m},[1(\mathfrak{v}_{ji},\mathscr{V}_{ji})]_{m+1,q_i}\end{matrix}\right.\right]=\Gamma^{m,n}_{p,q}\left[z\left|\begin{matrix}(\mathfrak{s}_1,\mathscr{S}_1,y),(\mathfrak{s}_j,\mathscr{S}_j)_{2,p}\\(\mathfrak{v}_j,\mathscr{V}_j)_{1,q}\end{matrix}\right.\right], \tag{15}$$

and

$$^{(\gamma)}\aleph^{m,n}_{p_i,q_i,1;1}\left[z\left|\begin{matrix}(\mathfrak{s}_1,\mathscr{S}_1,y),(\mathfrak{s}_j,\mathscr{S}_j)_{2,n},[1(\mathfrak{s}_{ji},\mathscr{S}_{ji})]_{n+1,p_i}\\(\mathfrak{v}_j,\mathscr{V}_j)_{1,m},[1(\mathfrak{v}_{ji},\mathscr{V}_{ji})]_{m+1,q_i}\end{matrix}\right.\right]=\gamma^{m,n}_{p,q}\left[z\left|\begin{matrix}(\mathfrak{s}_1,\mathscr{S}_1,y),(\mathfrak{s}_j,\mathscr{S}_j)_{2,p}\\(\mathfrak{v}_j,\mathscr{V}_j)_{1,q}\end{matrix}\right.\right]. \tag{16}$$

(v) Put $y=0,\rho_i=1$ and $r=1$ in equation (3), it converts IAF to the H-function [23]:

$$^{(\Gamma)}\aleph^{m,n}_{p_i,q_i,1;1}\left[z\left|\begin{matrix}(\mathfrak{s}_1,\mathscr{S}_1,0),(\mathfrak{s}_j,\mathscr{S}_j)_{2,n},[1(\mathfrak{s}_{ji},\mathscr{S}_{ji})]_{n+1,p_i}\\(\mathfrak{v}_j,\mathscr{V}_j)_{1,m},[1(\mathfrak{v}_{ji},\mathscr{V}_{ji})]_{m+1,q_i}\end{matrix}\right.\right]=H^{m,n}_{p,q}\left[z\left|\begin{matrix}(\mathfrak{s}_j,\mathscr{S}_j)_{1,p}\\(\mathfrak{v}_j,\mathscr{V}_j)_{1,q}\end{matrix}\right.\right]. \tag{17}$$

Several special functions can be generated from the IAFs where some interesting functions are given above.

We can write IAF $^{(\Gamma)}\aleph^{m,n}_{p_i,q_i,\rho_i;r}(zx^{\delta}t^{\theta})$ defined in (3) as below:

$$\begin{aligned}^{(\Gamma)}\aleph^{m,n}_{p_i,q_i,\rho_i;r}(zx^{\delta}t^{\theta})&={}^{(\Gamma)}\aleph^{m,n}_{p_i,q_i,\rho_i;r}\left[zx^{\delta}t^{\theta}\left|\begin{matrix}(\mathfrak{s}_1,\mathscr{S}_1,y),(\mathfrak{s}_j,\mathscr{S}_j)_{2,n},[\rho_j(\mathfrak{s}_{ji},\mathscr{S}_{ji})]_{n+1,p_i}\\(\mathfrak{v}_j,\mathscr{V}_j)_{1,m},[\rho_j(\mathfrak{v}_{ji},\mathscr{V}_{ji})]_{m+1,q_i}\end{matrix}\right.\right]\\&=\frac{1}{2\pi i}\int_L\varphi(\zeta,y)(zx^{\delta}t^{\theta})^{\zeta}d\zeta,\end{aligned} \tag{18}$$

where $z\neq 0$, and

$$\varphi(\zeta,y)=\frac{\Gamma(1-\mathfrak{s}_1+\mathscr{S}_1\zeta,y)\prod_{j=1}^{m}\Gamma(\mathfrak{v}_j-\mathscr{V}_j\zeta)\prod_{j=2}^{n}\Gamma(1-\mathfrak{s}_j+\mathscr{S}_j\zeta)}{\sum_{i=1}^{r}\rho_i\left[\prod_{j=m+1}^{q_i}\Gamma(1-\mathfrak{v}_{ji}+\mathscr{V}_{ji}\zeta)\prod_{j=n+1}^{p_i}\Gamma(\mathfrak{s}_{ji}-\mathscr{S}_{ji}\zeta)\right]}. \tag{19}$$

4 Caputo Fractional Derivative of Incomplete Aleph Functions

Now, we introduced the Caputo fractional derivative of IAF $^{(\Gamma)}\aleph^{m,n}_{p_i,q_i,\rho_i;r}(zx^{\delta}t^{\theta})$ defined in (18) for $n-1<\alpha<n\in\mathbb{N}$, as follows:

$${}^{C}_{0}D^{\alpha}_{t}{}^{(\Gamma)}\aleph^{m,n}_{p_i,q_i,\rho_i;r}(zx^{\delta}t^{\theta})=\frac{1}{t^{\alpha}}{}^{(\Gamma)}\aleph^{m,n+1}_{p_i+1,q_i+1,\rho_i;r}\left[zx^{\delta}t^{\theta}\left|\begin{matrix}(\mathfrak{s}_1,\mathscr{S}_1,y),(0,\theta),(\mathfrak{s}_j,\mathscr{S}_j)_{2,n},[\rho_j(\mathfrak{s}_{ji},\mathscr{S}_{ji})]_{n+1,p_i}\\(\mathfrak{v}_j,\mathscr{V}_j)_{1,m},[\rho_j(\mathfrak{v}_{ji},\mathscr{V}_{ji})]_{m+1,q_i},(\alpha,\theta)\end{matrix}\right.\right], \tag{20}$$

provide that $\theta>0,\ \zeta>0,\ 1+\zeta\theta>n,\ |arg(z)|<\frac{1}{\pi}k_i$ and $\mathbb{R}(L_i)+1<0$.
Similarly, we obtained CFD of IAF $^{(\gamma)}\aleph^{m,n}_{p_i,q_i,\rho_i;r}(zx^{\delta}t^{\theta})$ for $n-1<\alpha<n\in\mathbb{N}$, as follows:

$${}^{C}_{0}D^{\alpha}_{t}{}^{(\gamma)}\aleph^{m,n}_{p_i,q_i,\rho_i;r}(zx^{\delta}t^{\theta})=\frac{1}{t^{\alpha}}{}^{(\gamma)}\aleph^{m,n+1}_{p_i+1,q_i+1,\rho_i;r}\left[zx^{\delta}t^{\theta}\left|\begin{matrix}(\mathfrak{s}_1,\mathscr{S}_1,y),(0,\theta),(\mathfrak{s}_j,\mathscr{S}_j)_{2,n},[\rho_j(\mathfrak{s}_{ji},\mathscr{S}_{ji})]_{n+1,p_i}\\(\mathfrak{v}_j,\mathscr{V}_j)_{1,m},[\rho_j(\mathfrak{v}_{ji},\mathscr{V}_{ji})]_{m+1,q_i},(\alpha,\theta)\end{matrix}\right.\right], \tag{21}$$

provide that $\theta>0,\ \zeta>0,\ 1+\zeta\theta>n,\ |arg(z)|<\frac{1}{\pi}k_i$ and $\mathbb{R}(L_i)+1<0$.

Here, we examine the progressive collaboration and $\mathbb{N}_i(x,t),\ (i=1,2)$ is the density of one-dimensional dispersion of biological species in a straight environment $x\leq L/2=l\ (say)$, where the rate of growth and the environment carrying capacity is moving towards destruction because of the continued natural pollution happening in the territory. Well defined non-linear partial differential equations governing the growth and development of the species are supposed to be provided by the given system

$$\frac{\partial \mathbb{N}_i}{\partial t} = \mathbb{N}_i \mathbb{F}_i\left(\mathbb{N}_1, \mathbb{N}_2, \mathbb{R}_i(\mathbb{C}), \mathbb{K}_i(\mathbb{C})\right) + \mathbb{D}_i\left(\frac{\partial^2 \mathbb{N}_i}{\partial t^2}\right), \quad i = 1,2, \tag{22}$$

where $\mathbb{F}_i\left(\mathbb{N}_1, \mathbb{N}_2, \mathbb{R}_i(\mathbb{C}), \mathbb{K}_i(\mathbb{C})\right)$ determines the interactive function of species. $\mathbb{R}_i(\mathbb{C})$ and $\mathbb{K}_i(\mathbb{C})$ are the carrying capacity of the habitat and the intrinsic growth rate respectively, which are impacted by the concentration $\mathbb{C}(x,t)$. The positive constant $\mathbb{D}_i$ $(i = 1,2)$ is the dispersing coefficient of the species. The following equation considered for the flow of concentration $\mathbb{C}(x,t)$ is given by Patin [10] as:

$$\frac{\partial \mathbb{C}}{\partial t} = \mathbb{Q}_0 - \beta \mathbb{C} + \mathbb{D}_{\mathbb{C}}\left(\frac{\partial^2 \mathbb{C}}{\partial x^2}\right), \tag{23}$$

where $\mathbb{Q}_0 > 0$ is the constant that determines the exogenous input of impurities in the ecological system, $\beta > 0$ shows the first order decay constant due to a consequence of both the processes of biological as well as chemical. $\mathbb{D}_{\mathbb{C}} > 0$ represent the diffusion coefficient of the contaminant. While we formulate the model, as per the various researches, it has been observed that the organismal uptake of the impurities is directly proportional to the concentration of the impurities present in the habitant of the system. In nutshell, the approaches of advanced mathematical models can be helpful to analyse a cluster of issues of habitats effectively. Depending on the mathematical model (23) the result of the above equation will be obtained through the IAFs of one variable.
Hence, interestingly it brings attention that the exact solution of the given equation (23) that may arise in biomathematics, by applying the Adomians decomposition methods [24,25] in the given x and t axis. It is experienced through several types of research that this may minimise unrealistic assumptions and produce a solution with maximum accuracy.

5 Mathematical Model

Case 1: For $0 < \alpha < 1$, we modified the biological model (23) by applying well known Caputo fractional derivative as follows:

$${}_0^C D_t^{\alpha} \mathbb{C}(x,t) = \mathbb{Q}_0 - \beta \mathbb{C} + \mathbb{D}_{\mathbb{C}}\left(\frac{\partial^2 \mathbb{C}}{\partial x^2}\right), \quad 0 < \alpha < 1. \tag{24}$$

The concentration $\mathbb{C}(x,t)$ is expressed in term of incomplete Aleph function ${}^{(\gamma)}\aleph^{m,n}_{p_i,q_i,\rho_i;r}(zx^{\delta}t^{\theta})$ as given below:

$$\mathbb{C}(x,t) = {}^{(\gamma)}\aleph^{m,n}_{p_i,q_i,\rho_i;r}\left[zx^{\delta}t^{\theta}\,\middle|\,\begin{matrix}(\mathfrak{s}_1,\mathscr{S}_1,y),(\mathfrak{s}_j,\mathscr{S}_j)_{2,n},[\rho_j(\mathfrak{s}_{ji},\mathscr{S}_{ji})]_{n+1,p_i}\\(\mathfrak{v}_j,\mathscr{V}_j)_{1,m},[\rho_j(\mathfrak{v}_{ji},\mathscr{V}_{ji})]_{m+1,q_i}\end{matrix}\right]. \tag{25}$$

The Caputo fractional derivative of (25) for $0 < \alpha < 1$ is given as:

$${}_0^C D_t^{\alpha} \mathbb{C}(x,t) = \frac{1}{t^{\alpha}}{}^{(\gamma)}\aleph^{m,n+1}_{p_i+1,q_i+1,\rho_i;r}\left[zx^{\delta}t^{\theta}\,\middle|\,\begin{matrix}(\mathfrak{s}_1,\mathscr{S}_1,y),(0,\theta),(\mathfrak{s}_j,\mathscr{S}_j)_{2,n},[\rho_j(\mathfrak{s}_{ji},\mathscr{S}_{ji})]_{n+1,p_i}\\(\mathfrak{v}_j,\mathscr{V}_j)_{1,m},[\rho_j(\mathfrak{v}_{ji},\mathscr{V}_{ji})]_{m+1,q_i},(\alpha,\theta)\end{matrix}\right], \tag{26}$$

provide that $\theta > 0, \zeta > 0, 1 + \zeta\theta > n, |arg(z)| < \frac{1}{\pi}k_i$ and $\mathbb{R}(L_i) + 1 < 0$.
Now, first we write the IAF ${}^{(\gamma)}\aleph^{m,n}_{p_i,q_i,\rho_i;r}(zx^{\delta}t^{\theta})$ in the form of Mellin-Barnes integral as given in (18) and then partially differentiate (25) twice w.r.t. x and after simplification we arrive at:

$$\frac{\partial^2 \mathbb{C}}{\partial x^2} = \frac{1}{x^2}{}^{(\gamma)}\aleph^{m,n+1}_{p_i+1,q_i+1,\rho_i;r}\left[zx^{\delta}t^{\theta}\,\middle|\,\begin{matrix}(\mathfrak{s}_1,\mathscr{S}_1,y),(0,\delta),(\mathfrak{s}_j,\mathscr{S}_j)_{2,n},[\rho_j(\mathfrak{s}_{ji},\mathscr{S}_{ji})]_{n+1,p_i}\\(\mathfrak{v}_j,\mathscr{V}_j)_{1,m},[\rho_j(\mathfrak{v}_{ji},\mathscr{V}_{ji})]_{m+1,q_i},(1,\delta)\end{matrix}\right], \tag{27}$$

provide that $\theta > 0,\ \zeta > 0,\ 1 + \zeta\theta > n,\ |arg(z)| < \frac{1}{\pi}k_i$ and $\mathbb{R}(L_i) + 1 < 0$.
Using (25), (26), (27) in (24), we get the result as:

$$\begin{aligned}&\frac{1}{t^{\alpha}}{}^{(\gamma)}\aleph^{m,n+1}_{p_i+1,q_i+1,\rho_i;r}\left[zx^{\delta}t^{\theta}\,\middle|\,\begin{matrix}(\mathfrak{s}_1,\mathscr{S}_1,y),(0,\theta),(\mathfrak{s}_j,\mathscr{S}_j)_{2,n},[\rho_j(\mathfrak{s}_{ji},\mathscr{S}_{ji})]_{n+1,p_i}\\(\mathfrak{v}_j,\mathscr{V}_j)_{1,m},[\rho_j(\mathfrak{v}_{ji},\mathscr{V}_{ji})]_{m+1,q_i},(\alpha,\theta)\end{matrix}\right]\\
&= \mathbb{Q}_0 - \beta.{}^{(\gamma)}\aleph^{m,n}_{p_i,q_i,\rho_i;r}\left[zx^{\delta}t^{\theta}\,\middle|\,\begin{matrix}(\mathfrak{s}_1,\mathscr{S}_1,y),(\mathfrak{s}_j,\mathscr{S}_j)_{2,n},[\rho_j(\mathfrak{s}_{ji},\mathscr{S}_{ji})]_{n+1,p_i}\\(\mathfrak{v}_j,\mathscr{V}_j)_{1,m},[\rho_j(\mathfrak{v}_{ji},\mathscr{V}_{ji})]_{m+1,q_i}\end{matrix}\right]\\
&+\mathbb{D}_{\mathbb{C}}\frac{1}{x^2}{}^{(\gamma)}\aleph^{m,n+1}_{p_i+1,q_i+1,\rho_i;r}\left[zx^{\delta}t^{\theta}\,\middle|\,\begin{matrix}(\mathfrak{s}_1,\mathscr{S}_1,y),(0,\delta),(\mathfrak{s}_j,\mathscr{S}_j)_{2,n},[\rho_j(\mathfrak{s}_{ji},\mathscr{S}_{ji})]_{n+1,p_i}\\(\mathfrak{v}_j,\mathscr{V}_j)_{1,m},[\rho_j(\mathfrak{v}_{ji},\mathscr{V}_{ji})]_{m+1,q_i},(1,\delta)\end{matrix}\right],\end{aligned} \tag{28}$$

provide that $\theta > 0,\ \zeta > 0,\ 1+\zeta\theta > n,\ |arg(z)| < \frac{1}{\pi}k_i$ and $\mathbb{R}(L_i)+1<0$.
In similar manner, we can find another result in term of ${}^{(\Gamma)}\aleph^{m,n}_{p_i,q_i,\rho_i;r}(zx^{\delta}t^{\theta})$ as:

$$\frac{1}{t^{\alpha}}.{}^{(\Gamma)}\aleph^{m,n+1}_{p_i+1,q_i+1,\rho_i;r}\left[zx^{\delta}t^{\theta}\left|\begin{matrix}(\mathfrak{s}_1,\mathscr{S}_1,y),(0,\theta),(\mathfrak{s}_j,\mathscr{S}_j)_{2,n},[\rho_j(\mathfrak{s}_{ji},\mathscr{S}_{ji})]_{n+1,p_i}\\(\mathfrak{v}_j,\mathscr{V}_j)_{1,m},[\rho_j(\mathfrak{v}_{ji},\mathscr{V}_{ji})]_{m+1,q_i},(\alpha,\theta)\end{matrix}\right.\right]$$
$$=\mathbb{Q}_0-\beta.{}^{(\Gamma)}\aleph^{m,n}_{p_i,q_i,\rho_i;r}\left[zx^{\delta}t^{\theta}\left|\begin{matrix}(\mathfrak{s}_1,\mathscr{S}_1,y),(\mathfrak{s}_j,\mathscr{S}_j)_{2,n},[\rho_j(\mathfrak{s}_{ji},\mathscr{S}_{ji})]_{n+1,p_i}\\(\mathfrak{v}_j,\mathscr{V}_j)_{1,m},[\rho_j(\mathfrak{v}_{ji},\mathscr{V}_{ji})]_{m+1,q_i}\end{matrix}\right.\right]$$
$$+\mathbb{D}_{\mathbb{C}}\frac{1}{x^2}.{}^{(\Gamma)}\aleph^{m,n+1}_{p_i+1,q_i+1,\rho_i;r}\left[zx^{\delta}t^{\theta}\left|\begin{matrix}(\mathfrak{s}_1,\mathscr{S}_1,y),(0,\delta),(\mathfrak{s}_j,\mathscr{S}_j)_{2,n},[\rho_j(\mathfrak{s}_{ji},\mathscr{S}_{ji})]_{n+1,p_i}\\(\mathfrak{v}_j,\mathscr{V}_j)_{1,m},[\rho_j(\mathfrak{v}_{ji},\mathscr{V}_{ji})]_{m+1,q_i},(1,\delta)\end{matrix}\right.\right],\tag{29}$$

provide that $\theta > 0,\ \zeta > 0,\ 1+\zeta\theta > n,\ |arg(z)| < \frac{1}{\pi}k_i$ and $\mathbb{R}(L_i)+1<0$.

Case 2: For $\alpha = 1$, Our defined mathematical model (24) reduces to the model (23) obtained by Patin [10] and we take the concentration $\mathbb{C}(x,t)$ in term of IAF ${}^{(\gamma)}\aleph^{m,n}_{p_i,q_i,\rho_i;r}(zx^{\delta}t^{\theta})$ as follows:

$$\mathbb{C}(x,t)={}^{(\gamma)}\aleph^{m,n}_{p_i,q_i,\rho_i;r}\left[zx^{\delta}t^{\theta}\left|\begin{matrix}(\mathfrak{s}_1,\mathscr{S}_1,y),(\mathfrak{s}_j,\mathscr{S}_j)_{2,n},[\rho_j(\mathfrak{s}_{ji},\mathscr{S}_{ji})]_{n+1,p_i}\\(\mathfrak{v}_j,\mathscr{V}_j)_{1,m},[\rho_j(\mathfrak{v}_{ji},\mathscr{V}_{ji})]_{m+1,q_i}\end{matrix}\right.\right].\tag{30}$$

After partially differentiation (30) w.r.t. t, we get

$$\frac{\partial\mathbb{C}}{\partial t}=\frac{1}{t}{}^{(\gamma)}\aleph^{m,n+1}_{p_i+1,q_i+1,\rho_i;r}\left[zx^{\delta}t^{\theta}\left|\begin{matrix}(\mathfrak{s}_1,\mathscr{S}_1,y),(0,\theta),(\mathfrak{s}_j,\mathscr{S}_j)_{2,n},[\rho_j(\mathfrak{s}_{ji},\mathscr{S}_{ji})]_{n+1,p_i}\\(\mathfrak{v}_j,\mathscr{V}_j)_{1,m},[\rho_j(\mathfrak{v}_{ji},\mathscr{V}_{ji})]_{m+1,q_i},(1,\theta)\end{matrix}\right.\right],\tag{31}$$

provide that $\theta > 0,\ \zeta > 0,\ 1+\zeta\theta > n,\ |arg(z)| < \frac{1}{\pi}k_i$ and $\mathbb{R}(L_i)+1<0$.
Using (27), (30) and (31) in equation (23) and get the result for $\alpha = 1$, as follows:

$$\frac{1}{t}{}^{(\gamma)}\aleph^{m,n+1}_{p_i+1,q_i+1,\rho_i;r}\left[zx^{\delta}t^{\theta}\left|\begin{matrix}(\mathfrak{s}_1,\mathscr{S}_1,y),(0,\theta),(\mathfrak{s}_j,\mathscr{S}_j)_{2,n},[\rho_j(\mathfrak{s}_{ji},\mathscr{S}_{ji})]_{n+1,p_i}\\(\mathfrak{v}_j,\mathscr{V}_j)_{1,m},[\rho_j(\mathfrak{v}_{ji},\mathscr{V}_{ji})]_{m+1,q_i},(1,\theta)\end{matrix}\right.\right]$$
$$=\mathbb{Q}_0-\beta.{}^{(\gamma)}\aleph^{m,n}_{p_i,q_i,\rho_i;r}\left[zx^{\delta}t^{\theta}\left|\begin{matrix}(\mathfrak{s}_1,\mathscr{S}_1,y),(\mathfrak{s}_j,\mathscr{S}_j)_{2,n},[\rho_j(\mathfrak{s}_{ji},\mathscr{S}_{ji})]_{n+1,p_i}\\(\mathfrak{v}_j,\mathscr{V}_j)_{1,m},[\rho_j(\mathfrak{v}_{ji},\mathscr{V}_{ji})]_{m+1,q_i}\end{matrix}\right.\right]$$
$$+\mathbb{D}_{\mathbb{C}}\frac{1}{x^2}{}^{(\gamma)}\aleph^{m,n+1}_{p_i+1,q_i+1,\rho_i;r}\left[zx^{\delta}t^{\theta}\left|\begin{matrix}(\mathfrak{s}_1,\mathscr{S}_1,y),(0,\delta),(\mathfrak{s}_j,\mathscr{S}_j)_{2,n},[\rho_j(\mathfrak{s}_{ji},\mathscr{S}_{ji})]_{n+1,p_i}\\(\mathfrak{v}_j,\mathscr{V}_j)_{1,m},[\rho_j(\mathfrak{v}_{ji},\mathscr{V}_{ji})]_{m+1,q_i},(1,\delta)\end{matrix}\right.\right],\tag{32}$$

provide that $\theta > 0,\ \zeta > 0,\ 1+\zeta\theta > n,\ |arg(z)| < \frac{1}{\pi}k_i$ and $\mathbb{R}(L_i)+1<0$.
Similary, for $\alpha = 1$, we can derive the result for IAF ${}^{(\Gamma)}\aleph^{m,n}_{p_i,q_i,\rho_i;r}(zx^{\delta}t^{\theta})$ as we did in (32), then we have the following result:

$$\frac{1}{t}{}^{(\Gamma)}\aleph^{m,n+1}_{p_i+1,q_i+1,\rho_i;r}\left[zx^{\delta}t^{\theta}\left|\begin{matrix}(\mathfrak{s}_1,\mathscr{S}_1,y),(0,\theta),(\mathfrak{s}_j,\mathscr{S}_j)_{2,n},[\rho_j(\mathfrak{s}_{ji},\mathscr{S}_{ji})]_{n+1,p_i}\\(\mathfrak{v}_j,\mathscr{V}_j)_{1,m},[\rho_j(\mathfrak{v}_{ji},\mathscr{V}_{ji})]_{m+1,q_i},(1,\theta)\end{matrix}\right.\right]$$
$$=\mathbb{Q}_0-\beta.{}^{(\Gamma)}\aleph^{m,n}_{p_i,q_i,\rho_i;r}\left[zx^{\delta}t^{\theta}\left|\begin{matrix}(\mathfrak{s}_1,\mathscr{S}_1,y),(\mathfrak{s}_j,\mathscr{S}_j)_{2,n},[\rho_j(\mathfrak{s}_{ji},\mathscr{S}_{ji})]_{n+1,p_i}\\(\mathfrak{v}_j,\mathscr{V}_j)_{1,m},[\rho_j(\mathfrak{v}_{ji},\mathscr{V}_{ji})]_{m+1,q_i}\end{matrix}\right.\right]$$
$$+\mathbb{D}_{\mathbb{C}}\frac{1}{x^2}{}^{(\Gamma)}\aleph^{m,n+1}_{p_i+1,q_i+1,\rho_i;r}\left[zx^{\delta}t^{\theta}\left|\begin{matrix}(\mathfrak{s}_1,\mathscr{S}_1,y),(0,\delta),(\mathfrak{s}_j,\mathscr{S}_j)_{2,n},[\rho_j(\mathfrak{s}_{ji},\mathscr{S}_{ji})]_{n+1,p_i}\\(\mathfrak{v}_j,\mathscr{V}_j)_{1,m},[\rho_j(\mathfrak{v}_{ji},\mathscr{V}_{ji})]_{m+1,q_i},(1,\delta)\end{matrix}\right.\right].\tag{33}$$

Remark 1: For $y = 0, \rho_i = 1, r = 1$, we can reduce equations (32) and (33) to the known result investigated by Singh and Mehta [26].

Remark 2: For $\rho_i = 1$ and $r = 1$, we can reduce equations (32) and (33) to the known result investigated by Puothit et al. [27].

Remark 3: For $\rho_i = 0$, we can reduce equations (32) and (33) to the different form of result investigated by Suthar et al. [28].

6 Conclusion

It is observed that various observations and judgments of researchers find ways to control pollutants. Due to the abundant mass of pollutants, the extinction of species become an inevitable fact. Excessive use of resources, deforestation, accumulation of industrial effluences and change in technology reason for the depletion of natural resources and deterioration of the environment. Moreover, with these above specified reasons, the results become too vicious and the habitat of many species become in danger. The environment appears monotonically clean even at very low rates, reflecting that the species would not be available for years as the pollutant concentration continue to increase in the environment. Owing to the excessive concentration of waste materials, in any form, endanger the remaining species and will imbalance the ecosystem. The proposed mathematical model can be applied to lessen the elements or issues to reduce the pollutants in the environment and this endeavour may keep the holiness of nature. We derived the Caputo fractional derivative of IAFs could help the researchers to obtain new results. The concentration C(x, t) is defined in terms of incomplete Aleph functions and we also made some observations by assigning specific values to the parameters of our main findings and expressing them in terms of a number of special functions, namely H-functions, incomplete H-functions $\Gamma_{p,q}^{m,n}[z]$ and $\gamma_{p,q}^{m,n}[z]$, and incomplete I-functions ${}^{(\Gamma)}I_{p_i,q_i;r}^{m,n}$ and ${}^{(\gamma)}I_{p_i,q_i;r}^{m,n}$, as well as expressing their relationship with previously published results. With this effort, we can leave something for our future generation and thereby nature be a safer place.

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