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Common Fixed Point of Two R-Weakly Commuting Mappings in S_b-Metric Spaces

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Abstract: We prove some common fixed point results for two mappings satisfying generalized contractive condition in S_b-metric spaces. Note that the S_b-metrics of main results in this work are not necessarily continuous. So, our results extend and improve the several previous works. We also present one example that shows the applicability and usefulness of our results.

Keywords: Common fixed Point, R-weakly commuting, S-metric, S_b-metric.

1 Introduction

In 1922 the Banach contraction principle [4], is the most celebrated fixed point theorem and has been generalized in various directions. Fixed point problems for contractive mappings in metric spaces with a partially order have been studied by many authors (see [1], [3], [5], [7], [11], [12], [16], [17]).

The study of metric spaces has attracted, and continues to attract the interest of many authors. There are many generalized metric spaces such as 2-metric spaces [10], G-metric spaces [18], D^*-metric spaces [22], partial metric spaces [6], cone metric spaces [13], S-metric spaces [20], b-metric spaces [8] and G_b-metric spaces [2].

In 2012, Sedghi et al. [20] have introduced the notion of a S-metric space.

J. K. Kims, S. Sedghi and N. Shobkolaei [14], proved common fixed point theorems for the R-weakly commuting mappings in S-metric spaces.

On the other hand the concept of b-metric spaces was introduced by Czerwik in [8].

P. Kumam, W. Sintunavarat, S. Sedghi and N. Shobkolaei [15], proved common fixed point of two R-weakly commuting mappings in b-metric spaces.

The aim of this paper is to present some common fixed point results for two mappings under generalized contractive condition in a S_b-metric space, where the S_b-metric is not necessarily continuous.

First we recall some notions, lemmas and examples which will be useful later.

Throughout this paper $\mathbb{R}$, $\mathbb{R}^+$ and $\mathbb{N}$ denote the set of all real numbers, non-negative real numbers and positive integers respectively.First we refer the following preliminaries.

Definition 1.1. [20] Let X be a nonempty set. A S-metric on X is a function $S : X^3 \to [0,\infty)$ that satisfies the following conditions :

(S1)$0 < S(x,y,z)$ for all $x,y,z \in X$ with $x \neq y \neq z$,
(S2)$S(x,y,z) = 0$ if $x = y = z$,
(S3)$S(x,y,z) \leq S(x,x,a) + S(y,y,a) + S(z,z,a)$ for all $x,y,z,a \in X$.

The pair (X,S) is called a S-metric space.

Example 1.2. [20] Let $X = \mathbb{R}^2$ and d be an ordinary metric on X. Put $S(x,y,z) = d(x,y)+d(x,z)+d(y,z)$ for all $x,y,z \in \mathbb{R}^2$, that is, S is the perimeter of the triangle with vertices x,y,z. Then S is a S-metric on X.

Lemma 1.3. [19] In a S-metric space, we have $S(x,x,y) = S(y,y,x)$.

Definition 1.4. [21] Let (X,S) be a S-metric space and $A \subseteq X$.

(1)If for every $x \in X$ there exists $r > 0$ such that $B_s(x,r) \subseteq A$, then the subset A is called open subset of X.
(2)Subset A of X is said to be S-bounded if there exists $r > 0$ such that $S(x,x,y) < r$ for all $x,y \in A$.

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(3)A sequence $\{x_n\}$ in X is convergent to x if and only if $S(x_n,x_n,x)\to 0$ as $n\to\infty$. That is for each $\varepsilon>0$, there exists $n_0\in\mathbb{N}$ such that for each $n\geq n_0$, $S(x_n,x_n,x)<\varepsilon$ and we denote by $\lim_{n\to\infty} x_n = x$.

(4)Sequence $\{x_n\}$ in X is called a Cauchy sequence if for each $\varepsilon>0$, there exists $n_0\in\mathbb{N}$ such that for each $n,m\geq n_0$, $S(x_n,x_n,x_m)<\varepsilon$.

(5)The S-metric space (X,S) is said to be complete if every Cauchy sequence is convergent.

(6)Let τ be the of all $A\subseteq X$ with $x\in A$ if and only if there exists $r>0$ such that $B_s(x,r)\subseteq A$. Then τ is a topology on X.

Lemma 1.5. [21] Let (X,S) be a S-metric space. If there exist sequences $\{x_n\},\{y_n\}$ such that $\lim_{n\to\infty} x_n = x$ and $\lim_{n\to\infty} y_n = y$, then $\lim_{n\to\infty} S(x_n,x_n,y_n) = S(x,x,y)$.

Now we give our following definitions and examples of S_b-metric spaces.

Definition 1.6. Let X be a nonempty set and $b\geq 1$ be a given real number. Suppose that a mapping $S: X^3\to[0,\infty)$ satisfies :

(S_b1)$0<S(x,y,z)$ for all $x,y,z\in X$ with $x\neq y\neq z$,
(S_b2)$S(x,y,z)=0$ if $x=y=z$,
(S_b3)$S(x,y,z)\leq b(S(x,x,a)+S(y,y,a)+S(z,z,a))$ for all $x,y,z,a\in X$

Then S is called a S_b-metric and the pair (X,S) is called a S_b-metric space.

It should be noted that, the class of S_b-metric spaces is effectively larger than that of S-metric spaces. Indeed each S-metric space is a S_b-metric space with $b=1$.

Following example shows that a S_b-metric on X need not be a S-metric on X.

Example 1.7. Let (X,S) be a S-metric space, and $S_*(x,y,z)=S(x,y,z)^p$, where $p>1$ is a real number. Note that S_* is a S_b-metric with $b=2^{2(p-1)}$. Obviously, S_* satisfies condition $(S_b1),(S_b2)$ of Definition 1.6, so it suffice to show (S_b3) holds. If $1<p<\infty$, then the covexity of the function $f(x)=x^p$, $(x>0)$ implies that $(a+b)^p\leq 2^{p-1}(a^p+b^p)$.

Thus, for each $x,y,z,a\in X$, we obtain

$$\begin{aligned}
S_*(x,y,z) &= S(x,y,z)^p\\
&\leq ([S(x,x,a)+S(y,y,a)]+S(z,z,a))^p\\
&\leq 2^{p-1}([S(x,x,a)+S(y,y,a)]^p+S(z,z,a)^p)\\
&\leq 2^{p-1}(2^{p-1}(S(x,x,a)^p+S(y,y,a)^p)+S(z,z,a)^p)\\
&\leq 2^{2(p-1)}(S(x,x,a)^p+S(y,y,a)^p)+2^{p-1}S(z,z,a)^p\\
&\leq 2^{2(p-1)}(S(x,x,a)^p+S(y,y,a)^p+S(z,z,a)^p)\\
&\leq 2^{2(p-1)}(S_*(x,x,a)+S_*(y,y,a)+S_*(z,z,a))
\end{aligned}$$

so, S_* is a S_b-metric with $b=2^{2(p-1)}$.

Also in the above example, (X,S_*) is not necessarily a S-metric space. For example, let $X=\mathbb{R}$, $S_*(x,y,z)=(|y+z-2x|+|y-z|)^2$ is a S_b-metric on $\mathbb{R}$,with $p=2$, $b=2^{2(2-1)}=4$, for all $x,y,z\in\mathbb{R}$. But it is not a S-metric on $\mathbb{R}$.

To see this, let $x=3,y=5,z=7,a=\frac{7}{2}$. Hence, we get

$$\begin{aligned}
S_*(3,5,7) &= (|5+7-6|+|5-7|)^2 = 8^2 = 64\\
S_*(3,3,\tfrac{7}{2}) &= (\left|3+\tfrac{7}{2}-6\right|+\left|3-\tfrac{7}{2}\right|)^2 = 1^2 = 1\\
S_*(5,5,\tfrac{7}{2}) &= (\left|5+\tfrac{7}{2}-10\right|+\left|5-\tfrac{7}{2}\right|)^2 = 3^2 = 9\\
S_*(7,7,\tfrac{7}{2}) &= (\left|7+\tfrac{7}{2}-14\right|+\left|7-\tfrac{7}{2}\right|)^2 = 7^2 = 49.
\end{aligned}$$

Therefore, $S_*(3,5,7) = 64 \nleq 59 = S_*(3,3,\frac{7}{2})+S_*(5,5,\frac{7}{2})+S_*(7,7,\frac{7}{2})$.

Now we present some definitions and propositions in S_b-metric spaces.

Definition 1.8. Let (X,S) be a S_b-metric space. Then, for $x\in X, r>0$ we define the open ball $B_S(x,r)$ and closed ball $B_S[x,r]$ with center x and radius r as follows respectivly:

$$\begin{aligned}
B_S(x,r) &= \{y\in X : S(y,y,x)<r\},\\
B_S[x,r] &= \{y\in X : S(y,y,x)\leq r\}.
\end{aligned}$$

Example 1.9. Let $X=\mathbb{R}$.Denote $S(x,y,z)=(|y+z-2x|+|y-z|)^2$ is a S_b-metric on $\mathbb{R}$ with $b=2^{2(2-1)}=4$, for all $x,y,z\in\mathbb{R}$. Thus

$$\begin{aligned}
B_S(1,2) &= \{y\in\mathbb{R} : S(y,y,1)<2\}\\
&= \{y\in\mathbb{R} : |y-1|<\frac{\sqrt{2}}{2}\}\\
&= \{y\in\mathbb{R} : 1-\frac{\sqrt{2}}{2}<y<1+\frac{\sqrt{2}}{2}\}\\
&= (1-\frac{\sqrt{2}}{2}, 1+\frac{\sqrt{2}}{2}).
\end{aligned}$$

Lemma 1.10. In a S_b-metric space, we have $S(x,x,y)\leq b\,S(y,y,x)$ and $S(y,y,x)\leq b\,S(x,x,y)$.

Definition 1.11. Let (X,S) be a S_b-metric space. A sequence $\{x_n\}$ in X is said to be :

(1) S_b-Cauchy sequence if, for each $\varepsilon>0$, there exists $n_0\in\mathbb{N}$ such that $S(x_n,x_n,x_m)<\varepsilon$ for each $m,n\geq n_0$.

(2) S_b-convergent to a point $x\in X$ if, for each $\varepsilon>0$, there exists a positive integer n_0 such that for all $n\geq n_0$, $S(x_n,x_n,x)<\varepsilon$ or $S(x,x,x_n)<\varepsilon$ and we denote by $\lim_{n\to\infty} x_n = x$.

Definition 1.12. A S_b-metric space (X,S) is called complete if every S_b-Cauchy sequence is S_b-convergent in X.

Definition 1.13. Let f and g be mappings from a S_b-metric space (X,S) into itself. The mappings f and g are said to be weakly commuting if

$$S(fgx,fgx,gfx)\leq S(fx,fx,gx)$$

for each $x\in X$.

Definition 1.14. Let f and g be mappings from a S_b-metric space (X,S) into itself. The mappings f and g are said to be R-weakly commuting if there exists some positive real number R such that

$S(fgx, fgx, gfx) \leq R\, S(fx, fx, gx)$

for each $x \in X$.

Remark 1.15. Weak commutativity implies R-weak commutativity in S_b-metric space for every $R \geq 1$. However, R-weak commutativity implies weak commutativity only when $R \leq 1$.

Example 1.16. Let $X = \mathbb{R}$ and $S : X^3 \to \mathbb{R}^+$, defined as follow:

$S(x,y,z) = (|y+z-2x| + |y-z|)^2$

for all $x,y,z \in X$. Then (X,S) is a S_b-metric space. Define $fx = 2x-1$ and $gx = x^2$. Then

$S(fgx, fgx, gfx) = 16(x-1)^4 = 4\, S(fx, fx, gx) > S(fx, fx, gx)$.

Therefore, for $R = 4$, f and g are R-weakly commuting. But f and g are not weakly commuting.

2 Main Result

Let Φ denote the class of all functions $\phi : [0,\infty) \to [0,\infty)$ such that ϕ is non-decreasing, continuous, $\phi(t) < t$ for all $t > 0$ and $\phi(0) = 0$.

We start our work by proving the following one crucial lemma.

Lemma 2.1. Let (X,S) be a S_b-metric space with $b \geq 1$, and suppose that $\{x_n\}$ is a S_b-convergent to x, then we have

$$(i)\ \ \frac{1}{2b}S(y,y,x) \leq \liminf_{n\to\infty} S(y,y,x_n) \leq \limsup_{n\to\infty} S(y,y,x_n) \leq 2b\, S(y,y,x)$$

and

$$(ii)\ \ \frac{1}{b^2}S(x,x,y) \leq \liminf_{n\to\infty} S(x_n,x_n,y) \leq \limsup_{n\to\infty} S(x_n,x_n,y) \leq b^2\, S(x,x,y)$$

for all $y \in X$.

Proof. Using the condition (S_b3) of Definition 1.6, we have
$S(y,y,x_n) \leq 2b\, S(y,y,x) + b\, S(x_n,x_n,x)$ and
$S(y,y,x) \leq 2b\, S(y,y,x_n) + b\, S(x,x,x_n)$.
Taking the upper limit as $n \to \infty$ in the first inequality and the lower limit as $n \to \infty$ in the second inequality we obtain the desired result(i).
Similarly, from (S_b3) of Definition 1.6 and Lemma 1.10,we have
$S(x_n,x_n,y) \leq 2bS(x_n,x_n,x) + b^2S(x,x,y)$, and
$S(x,x,y) \leq 2bS(x,x,x_n) + b^2\, S(x_n,x_n,y)$. Taking the upper limit as $n \to \infty$ in the first inequality and the lower limit as $n \to \infty$ in the second inequality we obtain the desired result (ii).

The following is the main result of this section.

Theorem 2.2. Let (X,S) be a complete S_b-metric space. Let f and g be R-weakly commuting self-mappings on X satisfying the following conditions:

(2.2.1)$f(X) \subseteq g(X)$,
(2.2.2)f or g is continuous and
(2.2.3)$S(fx, fx, fy) \leq \frac{1}{4b^6}\phi(S(gx,gx,gy))$,
for all $x,y \in X$,where $\phi \in \Phi$.

Then there is a unique x in X such that $fx = gx = x$.

Proof. Let x_0 be an arbitrary point in X. By (2.2.1), there exist sequences $\{x_n\}$ and $\{y_n\}$ in X such that $y_n = fx_n = gx_{n+1}$ for all $n \in \mathbb{N} \cup \{0\}$.

Now we observe that for each $n \in \mathbb{N}$, we have

$$\begin{aligned} S(y_n,y_n,y_{n+1}) &= S(fx_n, fx_n, fx_{n+1}) \\ &\leq \frac{1}{4b^6}\phi(S(gx_n,gx_n,gx_{n+1})) \\ &= \frac{1}{4b^6}\phi(S(y_{n-1},y_{n-1},y_n)) \quad (1) \\ &\leq S(y_{n-1},y_{n-1},y_n). \end{aligned}$$

This implies that $\{c_n\} = \{S(y_n,y_n,y_{n+1})\}$ is a non-increasing sequence in $[0,\infty)$. Therefore, it tends to a limit $a \geq 0$. Suppose that $a > 0$. Making $n \to \infty$ in the inequality (1),we get

$$a \leq \frac{1}{4b^6}\phi(a) < \frac{1}{4b^6}a < a,$$

which is a contradiction. Hence $a = 0$. Thus
$\lim_{n\to\infty} c_n = \lim_{n\to\infty} S(y_n,y_n,y_{n+1}) = 0 \qquad (2)$

Suppose that $\{y_n\}$ is not a Cauchy sequence in X. Then there exist an $\varepsilon > 0$ and monotone increasing sequences of natural numbers $m(k)$ and $n(k)$ with $m(k) \geq n(k) \geq k$ such that $d_k = S(y_{n(k)},y_{n(k)},y_{m(k)}) \geq \varepsilon \qquad (3)$
and $S(y_{n(k)},y_{n(k)},y_{m(k)-1}) < \varepsilon \qquad (4)$
Using (3) and (4), we have

$$\begin{aligned} \varepsilon &\leq d_k = S(y_{n(k)},y_{n(k)},y_{m(k)}) \\ &\leq 2b\, S(y_{n(k)},y_{n(k)},y_{m(k)-1}) + b\, S(y_{m(k)},y_{m(k)},y_{m(k)-1}) \\ &< 2b\, \varepsilon + b^2S(y_{m(k)-1},y_{m(k)-1},y_{m(k)})\ from\ Lemma\ 1.10. \end{aligned}$$

Letting $k \to \infty$ and using (2), we get
$\varepsilon \leq \lim_{n\to\infty} d_k \leq 2b\varepsilon \qquad (5)$

Also notice that

$$\begin{aligned} d_k &= S(y_{n(k)},y_{n(k)},y_{m(k)}) \\ &\leq 2b\, S(y_{n(k)},y_{n(k)},y_{n(k)+1}) + b\, S(y_{m(k)},y_{m(k)},y_{n(k)+1}) \\ &\leq 2bc_{n(k)} + b^2\, S(y_{n(k)+1},y_{n(k)+1},y_{m(k)}) \\ &= 2bc_{n(k)}) + b^2[2b\, S(y_{n(k)+1},y_{n(k)+1},y_{m(k)+1}) + b\, S(y_{m(k)},y_{m(k)},y_{m(k)+1})] \\ &\leq 2bc_{n(k)} + 2b^3S(fx_{n(k)+1}, fx_{n(k)+1}, fx_{m(k)+1})) + b^3c_{m(k)} \\ &\leq 2bc_{n(k)} + \frac{2b^3}{4b^6}\phi(S(y_{n(k)},y_{n(k)},y_{m(k)})) + b^3c_{m(k)}. \end{aligned}$$

Letting $k \to \infty$ and using (2),(5) and properties of ϕ, we get
$\varepsilon \leq \frac{1}{2b^3}\phi(2b\varepsilon) < \varepsilon$
which is a contradiction.
Thus, $\{y_n\}$ is a Cauchy sequence in X and by the completeness of X, $\{y_n = fx_n\}$ converges to z in X. Also

$\{gx_n\}$ converges to z in X.
Let us suppose that the mapping f is continuous. Then $\lim_{n\to\infty} ffx_n = fz$ and $\lim_{n\to\infty} fgx_n = fz$.
Since f and g are R-weakly commuting,we have $S(fgx_n, fgx_n, gfx_n) \leq RS(fx_n, fx_n, gx_n)$ for all $n \in \mathbb{N}$.
Now consider

$$\begin{aligned}\frac{1}{b^2}S(fz, fz, gfx_m) &\leq \limsup_{n\to\infty} S(fgx_n, fgx_n, gfx_m)\\ &\leq R\limsup_{n\to\infty} S(fx_n, fx_n, gx_m)\\ &\leq Rb^2 S(z,z,gx_m).\end{aligned}$$

$$\begin{aligned}\frac{1}{b^2}\lim_{m\to\infty} S(fz, fz, gfx_m) &\leq Rb^2 \lim_{m\to\infty} S(z,z,gx_m)\\ &\leq Rb^3 S(z,z,z) = 0.\end{aligned}$$

Thus $\lim_{m\to\infty} gfx_m = fz$ (6)

Suppose that $z \neq fz$ .Then $S(z,z, fz) > 0$.
From (2.2.3)and by Lemma 2.1, we have using (6) and properties of ϕ that

$$\begin{aligned}\frac{1}{b^2}S(fz, fz, fx_m) &\leq \limsup_{n\to\infty} S(ffx_n, ffx_n, fx_m)\\ &\leq \frac{1}{4b^6}\limsup_{n\to\infty} \phi(S(gfx_n, gfx_n, gx_m)\\ &\leq \frac{1}{4b^6}\phi(b^2 S(fz, fz, gx_m)) \qquad (7)\end{aligned}$$

$$\begin{aligned}\frac{1}{2b}S(fz, fz, z) &\leq \limsup_{m\to\infty} S(fz, fz, fx_m)\\ &\leq \frac{1}{4b^4}\limsup_{m\to\infty} \phi(b^2\, S(fz, fz, gx_m)), from\ (7)\\ &\leq \frac{1}{4b^4}\phi(2b^3\, S(fz, fz, z)),\\ &< \frac{1}{2b}S(fz, fz, z),\end{aligned}$$

which is a contradiction. Therefore, $z = fz$.
Since $f(X) \subseteq g(X)$ we can find $z_1 \in X$ such that $z = fz = gz_1$.
From Lemma 2.1,(2.2.3) and properties of ϕ, we have

$$\begin{aligned}\frac{1}{b^2}S(fz, fz, fz_1) &\leq \limsup_{m\to\infty} S(ffx_n, ffx_n, fz_1)\\ &\leq \frac{1}{4b^6}\limsup_{m\to\infty} \phi(S(gfx_n, gfx_n, gz_1))\\ &\leq \frac{1}{4b^6}\phi(b^2\, S(fz, fz, gz_1)) = \frac{1}{4b^6}\phi(0) = 0.\end{aligned}$$

Thus $z = fz = fz_1 = gz_1$ (8)
Thus $S(fz, fz, gz) = S(fgz_1, fgz_1, gfz_1) \leq R\,S(fz_1, fz_1, gz_1) = 0$ which implies that $fz = gz$. Thus z is a common fixed point of f and g.

Now let $z' \neq z$ be another common fixed point of f and g.

Then $S(z,z,z') > 0$ and so

$$\begin{aligned}S(z,z,z') &= S(fz, fz, fz')\\ &\leq \frac{1}{4b^6}\phi(S(gz, gz, gz'))\\ &= \frac{1}{4b^6}\phi(S(z,z,z'))\\ &< \frac{1}{4b^6}S(z,z,z')\\ &< S(z,z,z'),\end{aligned}$$

which is a contradiction. Therefore, $z = z'$, that is, z is a unique common fixed point of f and g.Similarly the theorem follows when g is continuous.

Now we give an example to support Theorem 2.2.

Example 2.3. Let $X = \mathbb{R}$ and $S : X^3 \to \mathbb{R}^+$ defined by $S(x,y,z) = (|y+z-2x|+|y-z|)^2$ for all $x,y,z \in X$. Then (X,S) is a S_b-metric space for $b = 4$. Difine $f(x) = 1$ and $gx = 2x-1$ on X. It is evident that $f(X) \subseteq g(X)$ and f is continuous.

Now we observe that $S(fx, fx, fy) \leq \frac{1}{4^7}\phi(S(gx, gx, gy))$, for all $x,y \in X$ and for all $\phi \in \Phi$. Moreover, it is easy to see that f and g are R-weakly commuting. Thus all the conditions of Theorem 2.2 are satisfied and 1 is the unique common fixed point of f and g.

Corollary 2.4. Let (X,S) be a complete S_b-metric space and let f be a self mapping on X satisfying the following condition:
$S(fx, fx, fy) \leq \frac{1}{4b^6}\phi(S(x,x,y))$ for all $x,y \in X$ and $\phi \in \Phi$.
Then f has a unique fixed point.

Proof. If we take g as identity mapping on X in Theorem 2.2, then it follows that f has a unique fixed point.

Corollary 2.5. Let (X,S) be a complete S-metric space and let f and g be R-weakly commuting selfmapping of X satisfying the following condition:

1. $f(X) \subseteq g(X)$,
2. f or g is continuous,
3. $S(fx, fx, fy) \leq \frac{1}{4}\phi(S(gx, gx, gy))$,

for all $x,y \in X$ and $\phi \in \Phi$. Then f and g have a unique common fixed point.

Proof. If we take $b = 1$ in Theorem 2.2,then it follows that f and g have a unique common fixed point.

3 Conclusion

Our main result generalize and improve several previous results in fixed point theory like Banach fixed point theorem, Jungck theorem and Czerwik theorem etc. as our S_b-metric need not be continuous.

References

[1] R. P. Agarwal, M. A. El-Gebeily and D. Oregan, Generalized contractions in partially ordered metric spaces, Appl. Anal. 87(1) (2008), 109-116.

[2] A. Aghajani, M. Abbas, and J. R. Roshan, Common fixed point of generallized weak contractive mappings in partially ordered G_b-metric spaces, Filomat. 28(6),(2014),1087-1101.

[3] I. Altun and H. Simsek, Some fixed point theorems on ordered metric spaces and application, Fixed Point Theory App. (2010), Article ID 621492, 17 pages.

[4] S. Banach, Sur les operations dans les enseembles abstraits et leur application aux equations integrals. Fundam. Math. 3, 133-181 (1922).

[5] T. G. Bhaskar and V. Lakshmikantham, Fixed point theorems in partially ordered metric spaces and applications, Nonlinear Analysis 65 (2006) 1379-1393.

[6] M. Bukatin, R. Kopperman, S. Matthews, H. Pajoohesh, Partical metric spaces, Am. Math. Mon. 116, 708-718 (2009).

[7] Lj. Ciric, D. Mihet and R. Saadati, Monotone generalized contractions in partiality ordered probabilistic metric spaces, Topology and its Appl. (2009), doi:10.1016/j.na.2009.08.029.

[8] Czerwik, Nonlinear set-valued contraction mappings in b-metric spaces, Atti det Semmario Matematico e Fisico dell universita dr Modena, vol. 46, no. 2. pp. 263-276, 1998.

[9] N. V. Dung, On coupled common fixed points for mixed weakly monotone maps in partially ordered S-metric spaces. Fixed Point Theory Appl. 2013, Article ID 48 (2013).

[10] Gahler, 2-metrische Raume und ther topoloische Struktur, Math. Nachr. 26 (1963), 115-148.

[11] O. Hadzic, On common fixed point theorems in 2-metric spaces, Review of Research faculty of Science-University of Novi Sad, Vol. 12 (1982).

[12] J. Harjani and K. Sadarani, Fixed point theorems for weakly contractive mappings in partially ordered sets, Nonlinear Anal. 71(7-8) (2008), 3403-3410.

[13] L. G. Huang, X. Zhng, Cone metric spaces and fixed point theorems of contractive mappings, J. Math. Anal. App. 332, 1468-1476 (2007).

[14] J. K. Kim, S. Sedghi and N. Shobkolaei, Common fixed point theorems for the R-weakly commuting mappings in S-metric spaces, J. Computational Analysis and Applications. Vol. 19, No, 2015, pp. 751-759.

[15] P. Kuman, W. Sintunavarar, S. Sedghi, and N. Shobkolaei, Common fixed point of two R-wealky commuting mappings, Hindawi Publishing Corporation, Journal of Function Spaces, Article ID 350840.

[16] V. Lakshmikantham and Lj.B. Ćirić, Coupled fixed point theorems for nonlinear contractions in partially ordered metric spaces, Nonlinear Analysis 70 (2009) 4341-4349.

[17] N. V. Luong and N. X. Thuan, Coupled fixed points in partially ordered metric spaces and application, Nonlinear Anal. 74(3) (2011), 983-992.

[18] Z. Mustafa, B. Sims, A new approach to generalized metric spaces, J. Nonlinear Convex Anal. 7 (2006), 289-297.

[19] S. Sedghi, I. Alton, N. Shobe, M. Salahshour, Some properties of S-metric space and fixed point results, Kyung pook Math. J. 54 (2014), 113-122.

[20] S. Sedghi, N. Shobe, Aliouche, A generalization of fixed point theorem in S-metric spaces, Mat. Vesnik 64 (2012), 258-266.

[21] S. Sedghi, N. Shobe, T. Dosenovic, fixed point results in S-metric spaces, Nonlinear Functional Analysis and Applications, Vol. 20, 1 (2015), pp. 55-67.

[22] S. Sedghi, N. Shobe, H. Zhou, A common fixed point theorem in D^*-metric spaces, Fixed Point Theory Appl. Vol. 2007, Article ID 27906, 13 pages.

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