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Application of Category Graph in Finding the Wiener Index of Rough Ideal based Rough Edge Cayley Graph

B. Praba[*] *and Benazir Obilia. X. A.*

Department of Mathematics, SSN College of Engineering, Chennai - 603110, India.

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Abstract: The Rough Ideal based Rough Edge Cayley Graph is defined on the Rough semiring (T,Δ,∇) along with its Rough Ideal J and this graph consists of $2^{n-m}3^m$ elements. The complexity in studying the Rough Ideal based Rough Edge Cayley Graph is made simpler by defining the category graph. Wiener index of the Rough Ideal based Rough Edge Cayley Graph is obtained through this category graph and the concepts are illustrated through examples.

Keywords: Rough set, Degree, Cardinality, Distance

1 Introduction

Informations found around the world are imprecise, incomplete, uncertain and vague. To give a conclusion to the information that we have obtained by various means, we have to deal with uncertainity. In early 1980's to deal with the uncertainities in the information sysytem Rough set theory was intoduced by Z.Pawlak [16]. Rough set is defined as a pair called lower and upper approximations, which are defined on any subset of the universal set. The concept of Rough lattice was discussed by B.Praba and R.Mohan [7]. In this, for any given information system the authors have proved the set of all rough sets T to be the lattice called rough lattice with respect to the two operations *praba*Δ and *praba*∇ on T. Manimaran, et.al [5], [8] studied the properties on ideals and homomorphism of this semiring in detain

Changzhang Wong, Degang Chen [3] in their paper pointed out that there are still some incomplete propositions in Rough sets and have discussed on Rough groups. B.Davvaz[4] has discussed about the roughness based on fuzzy ideals. Wei Cheng, et.al[14] have discussed on fuzzy group and rough ideals in semigroups. Edge rough graph was first introduced by Meilian Liang, et.al [6]. Roughness in Cayley graphs was discussed by M.H. Shahzamanian,et.al [12]. Ali Reza Naghipour and Meysam Rezagholibeig[1] discussed on the refinement of the unit and unitary Cayley graphs of the finite rings. Yingbin Ma and Zaiping Lu [15] have discussed on the Rainbow connection numbers of Cayley graphs.

Rough Ideal based Rough Edge Cayley graph was introduced by B.Praba, et al [9], [10]. In this paper, the authors have made an Algebraic graph theoretical study on an Rough semiring along with its Rough Ideal. In this work, the Wiener index of an Rough Ideal based Rough Edge Cayley graph is obtained through the category graph. This paper is as follows: In section 2, we give the preliminaries. In section 3, we introduce Category Graph corresponding to a Rough Ideal based Rough Edge Cayley Graph and obtain the degree and cardinalities of each category of vertices. In section 4, we obtain the Wiener index of Rough Ideal based Rough Edge Cayley Graph. In section 5, we give the conclusion.

2 Preliminaries

2.1 Rough Set Theory

In this section we consider an approximation space $I = (U,R)$ where U is a non empty finite set of objects, called universal set and R be an equivalence relation defined on U.

Definition 1.*[16] For any approximation space, the equivalence classes induced by R is defined by*

[*] Corresponding author e-mail: prabab@ssn.edu.in

$[x] = \{y \in U \mid (x,y) \in R\}$. *For any* $X \subseteq U$, *the lower approximation is defined as* $R_{-}(X) = \{x \in U \mid [x] \subseteq X\}$ *and the upper approximation is defined by* $R^{-}(X) = \{x \in U \mid [x] \cap X = \phi\}$. *The rough set corresponding to X is* $RS(X) = (R_{-}(X), R^{-}(X))$.

Definition 2.[7] *If* $X \subseteq U$, *then the number of equivalence classes(Induced by R) contained in X is called as the Ind.weight of X. It is denoted by* $I_W(X)$.

Definition 3.[7] *Let* $X,Y \subseteq U$. *The Praba* Δ *is defined as* $X\Delta Y = X \cup Y$, *if* $I_W(X\cup Y) = I_W(X)+I_W(Y)-I_W(X\cap Y)$. *If* $I_W(X\cup Y) > I_W(X)+I_W(Y)-I_W(X\cap Y)$, *then identify the equivalence class obtained by the union of X and Y. Then delete the elements of that class belonging to Y. Call the new set as Y. Now obtain* $X\Delta Y$. *Repeat the process until* $I_W(X\cup Y) = I_W(X)+I_W(Y)-I_W(X\cap Y)$.

Definition 4.[7] *Praba* ∇ *of X and Y is denoted by* $X\nabla Y$ *and it is defined as* $X\nabla Y = \{x \mid [x] \subseteq X\cap Y\} \cup P_{X\cap Y}$, *where* $X,Y \subseteq U$ *and* $P_{X\cap Y}$ *contains those elements of U whose corresponding equivalence classes are not a subset of* $X\cap Y$ *but will have non empty intersection with X and Y.*

Theorem 1.[8] *For any approximation space* $I = (U,R)$, (T,Δ,∇) *is a semiring called the Rough semiring.*

Remark.[8] Without loss of generality let us assume that there are n-equivalence classes $\{X_1, X_2, ..., X_n\}$ in U. Out of which there are m classes $\{X_1, X_2, ..., X_m\}$ (say) have cardinalities greater than one and the remaining n-m clsses have cardinality equal to one. Note that $|T| = 2^{n-m}3^m$. Let $B = \{x_i \mid x_i \in X_i, i = 1,2,...,m\}$ and $J = \{RS(X) \mid X \in P(B)\}$ then J is an ideal of the Rough semiring called the Rough ideal .

2.2 Rough Ideal based Rough Edge Cayley Graph

[9], [10] In this section, we consider an approximation space $I = (U,R)$ where U is the non empty finite set of objects and R is an equivalence relation on U. Let (T,Δ,∇) be the rough semiring induced by I. Let B be the set of representative elements of X_i, $i = 1,2,...,m$ and J be the rough ideal as in the previous section. We also assume that M is the union of none, one or more equivalence classes whose cardinality is equal to one and $M^{'}$ is the union of one or more equivalence classes whose cardinality is equal to one.

Definition 5.[9], [10] *Rough Ideal based Rough Edge Cayley Graph*
Rough Ideal based Rough Edge Cayley Graph denoted $G(T(J)) = (V,E)$ *where* $V \in T$ *and* $E = \left\{(RS(X),RS(Z)) \mid RS(X)\nabla RS(Y) = RS(Z), RS(Y) \in J\right\}$

Remark.[9], [10] Note that each element of T is connected to $RS(\phi)$. Hence to have a better understanding of the Rough Ideal based Rough Edge Cayley Graph, we are considering the edge $RS(X)\nabla RS(Y) = RS(\phi)$ for any $RS(Y) \neq RS(\phi)$ in J.

2.3 Category Graph

[10] Cardinality of T ($|T|$) is $2^{n-m}3^m$. We divide the vertices (the elements of T) into 13 categories in such a way that all elements belonging to a particular category will behave similarly. The vertices are grouped in categories by the following conditions:

(a)the degree of each vertex in a particular category will be same.
(b)the distances from any vertex of a particular category to elements of other ctegories will be same.

Definition 6.*Category Graph*[10]
The category graph corresponding to a given Rough Ideal based Rough Edge Cayley graph is defined as follows: The vertices of the category graph (CG) are $C_1, C_2, ..., C_{13}$. *Two vertices* C_i *and* C_j *are connected if elements in* C_i *are connected to elements in* C_j *by an edge in the Rough Ideal based Rough Edge Cayley graph.*

3 Degrees and Cardinalities of each category in a category graph

Now in the following theorems we are going to establish the degrees of all elements in a particular category and the cardinality of each category in a category graph corresponding to the Rough Ideal based Rough Edge Cayley Graph:

Theorem 2.*(a)The degree of* $RS(\phi)$ *is* $2^{n-m}(3^m - 2^m) - 1$.
(b)$|C_1| = 1$.

Proof.(a) Let

$$A = \left\{X_1, X_2, ..., X_m\right\}$$

$$B = \left\{x_1, x_2, ..., x_m\right\}$$

$$\text{and } C = \left\{X_{m+1}, X_{m+2}, ..., X_n\right\}$$

Note that $|P(B)| = 2^m$ and $2^m - 2$ elements in $P(B)$ are connected to $RS(\phi)$. Also $|P(C)| = 2^{n-m}$, hence $(2^m - 2)2^{n-m}$ elements will be connected to $RS(\phi)$. As $B = \left\{x_1, x_2, ..., x_m\right\}$, by a similar argument $(2^m - 2)2^{n-m}$ elements will be connected to $RS(\phi)$. As $C = \left\{X_{m+1}, X_{m+2}, ..., X_n\right\}$, $|P(C)| = 2^{n-m}$, $2^{n-m} - 1$

(excluding $RS(\phi)$) will be connected to $RS(\phi)$. Now considering the elements of $P(A)$ and $P(B)$ with $P(C)$, we have $\left[\sum_{i=1}^{m-2} mC_i \left(2^{m-i}-2\right)\right]2^{n-m}$ elements connected to $RS(\phi)$. Hence,
Degree of $RS(\phi)$

$$= 2^{n-m}\left[(2^m-2)+(2^m-2)+\left[\sum_{i=1}^{m-2} mC_i\left(2^{m-i}-2\right)\right]\right]-1$$

$$= 2^{n-m}\left[2\left(2^m-2\right)+\sum_{i=1}^{m-2} mC_i\left(2^{m-i}-2\right)\right]-1$$

$$= 2^{n-m}\left[3^m-2m\right]-1.$$

(b) It is trivial.

Theorem 3.*(a)The degree of $RS(\{x_i\})$ in $G(T(J))$ is $2\,(2)^{n-m}\,(3)^{m-1}$, for every $x_i \in B$.*
(b)$|C_2| = m$.

Proof.(a) Let $A_i = \{X_i, x_i, \phi\}$, for $i = 1,2,\ldots,m$ and $B_i = \{X_i, \phi\}$, for $i = m+1, m+2, \ldots, n$
An element $RS(X)$ will be connected to $RS(\{x_i\})$ iff

$$RS(X)\nabla RS(Y) = RS(\{x_i\}) \quad (1)$$

where $RS(Y) \in J$. The degree of $RS(\{x_i\})$ is obtained by considering those vertices $RS(X)$ which are connected to $RS(\{x_i\})$ and those vertices to which $RS(\{x_i\})$ is connected.
In the first case, we have to enumerate the vertex $RS(X)$ satisfying (1). This is possible only when X and Y contain $\{x_i\}$. Hence such X should be

$$\left\{\ \{x_i\} \cup \left(\bigcup_{i=2}^{m}\alpha_i\right) \cup \left(\bigcup_{i=m+1}^{n}\beta_i\right)\right\}\ (or)$$

$$\left\{\ X_i \cup \left(\bigcup_{i=2}^{m}\alpha_i\right) \cup \left(\bigcup_{i=m+1}^{n}\beta_i\right)\right\}$$

where $\alpha_i \in A_i$ and $\beta_i \in B_i$. The number of such $RS(X)$ will be $2\,(2)^{n-m}\,(3)^{m-1}-1$.
In the second case the vertices are obtained by $RS(\{x_i\})\nabla RS(Y)$ where $Y \in P(B)$. As we are not considering the self loop. the only vertex to which $RS(\{x_i\})$ is connected is $RS(\phi)$.
Hence the degree of $RS(\{x_i\})$ is $2\,(2)^{n-m}\,(3)^{m-1}-1+1 = 2\,(2)^{n-m}\,(3)^{m-1}$.

(b)It is trivial

Theorem 4.*For every X_i,where $|X_i| > 1$, $i = 1,2,\ldots,m$*

(a) the degree of $RS(\{X_i\}\cup M)$ is 2.
(b)$|C_3| = m(2^{n-m})$.

Proof.(a) Consider the vertex $RS(\{X_i\} \cup M)$ where $|X_i| > 1$, $i = 1,2,\ldots,m$ and M is the union of none, one or more equivalence classses whose cardinality is equal to one in $G(T(J))$.
The vertex $RS(\{X_i\} \cup M)$ will be connected to $RS(\{X_i\} \cup M)\nabla RS(Y)$ where $Y \in P(B)$. $RS(\{X_i\}\cup M)\nabla RS(Y)$ are $RS(\{x_i\})$ and $RS(\phi)$. Hence the degree of $RS(\{X_i\}\cup M)$ is 2.

(b) Since there are m-equivalence classes with cardinality greater than one, (i.e)$|X_i| > 1$ and $|P(M)| = 2^{n-m}$

$$|C_3| = m(|P(M)|).$$
$$= m(2^{n-m})$$

Theorem 5.*For every X_i,where $|X_i| = 1$,*

(a) the dergree of $RS(L), L \in M'$ is one.
(b)$|C_4| = 2^{n-m}-1$.

Proof.(a) Let X_i be the equivalence class such that $|X_i| = 1$. The vertices that are connected to $RS(\{X_i\})$ will be $RS(\{X_i\})\nabla RS(Y)$ where $RS(Y) \in J$. But as $RS(Y) \in J$ implies that Y doesnot contain X_i. Therefore, $RS(\{X_i\})\nabla RS(Y) = RS(\phi)$. Hence $RS(\{X_i\})$ is connected only to $RS(\phi)$. Hence the degree of $RS(L)$ is one.

(b)It is trivial.

Theorem 6.*(a)The degree of $RS(\{x_1 \cup x_2 \cup \ldots \cup x_r\})$,where $x_i \in B$ and $1 < r < m$, is $2^r\,(2)^{n-m}\,(3)^{m-r}+2^r-2$.*
(b)$|C_5| = 2^m-m-2$.

Proof. (a) Let $A_i = \{X_i, x_i, \phi\}$ for $i = 1,2,\ldots,m$ and $B_i = \{X_i, \phi\}$ for $i = m+1, m+2, \ldots, n$
Case:1 When $r = 2$
An element $RS(X)$ will be connected to $RS(\{x_1\}) \cup RS(\{x_2\})$ if and only if

$$RS(X)\nabla RS(Y) = RS(\{x_1\}) \cup RS(\{x_2\}) \quad (2)$$

where $RS(Y) \in J$.
The degree of $RS(\{x_1\}) \cup RS(\{x_2\})$ is obtained by considering those vertices $RS(X)$ are connected to $RS(\{x_1\}) \cup RS(\{x_2\})$ and those vertices to which $RS(\{x_1\}) \cup RS(\{x_2\})$ are connected.
In the first case we have to enumerate the vertex $RS(X)$ satisfying (2).
This is possible only when X and Y contain $\{x_1\} \cup \{x_2\}$. Hence we have to find $RS(X)$ in T satisfying (2). Hence such an X should be

$$\left\{\ \{x_1\} \cup \{x_2\} \cup \left(\bigcup_{i=3}^{m}\alpha_i\right) \cup \left(\bigcup_{i=m+1}^{n}\beta_i\right)\right\}\ \text{(or)}$$

$$\left\{\ \{x_1\} \cup X_2 \cup \left(\bigcup_{i=3}^{m}\alpha_i\right) \cup \left(\bigcup_{i=m+1}^{n}\beta_i\right)\right\}\ \text{(or)}$$

$$\left\{ X_1 \cup \{x_2\} \cup \left(\bigcup_{i=3}^{m}\alpha_i\right) \cup \left(\bigcup_{i=m+1}^{n}\beta_i\right) \right\} \text{ (or)}$$

$$\left\{ X_1 \cup X_2 \cup \left(\bigcup_{i=3}^{m}\alpha_i\right) \cup \left(\bigcup_{i=m+1}^{n}\beta_i\right) \right\}$$

Number of such vertices will be $2^2(3)^{m-2}(2)^{n-m}-1$ as we are avoiding the self loop at $RS(X)\nabla RS(\{x_1\})\cup RS(\{x_2\})$.
In the second case, the vertices to which $RS(\{x_1\}) \cup RS(\{x_2\})$ is connected are $RS(\{x_1\})$, $RS(\{x_2\})$ and $RS(\phi)$. Hence number of such vertices are 2^2-1.
Hence the degree of $RS(\{x_1\}) \cup RS(\{x_2\})$ is $2^2(3)^{m-2}(2)^{n-m} - 1 + 2^2 - 1 = 2^2(3)^{m-2}(2)^{n-m}+2^2-2$.
Case:2 When $r=3$
An element $RS(X)$ will be connected to $RS(\{x_1\}) \cup RS(\{x_2\}) \cup RS(\{x_3\})$ $\left(or\ RS\left(\{x_1,x_2,x_3\}\right)\right)$ if and only if

$$RS(X)\nabla RS(Y) = RS\left(\{x_1,x_2,x_3\}\right) \quad (3)$$

where $RS(Y)\in J$.

The degree of $RS\left(\{x_1,x_2,x_3\}\right)$ is obtained by considering those vertices $RS(X)$ are connected to $RS\left(\{x_1,x_2,x_3\}\right)$ and those vertices to which $RS\left(\{x_1,x_2,x_3\}\right)$ are connected. In the first case we have to enumerate the vertex $RS(X)$ satisfying (3).
This is possible only when X and Y contain $\{x_1\}\cup\{x_2\}\cup\{x_3\}$. Hence we have to find $RS(X)$ in T satisfying (3). Hence such an X should be

$$\left\{ \{x_1\} \cup \{x_2\} \cup \{x_3\} \cup \left(\bigcup_{i=4}^{m}\alpha_i\right) \cup \left(\bigcup_{i=m+1}^{n}\beta_i\right) \right\} \text{ (or)}$$

$$\left\{ \{x_1\} \cup \{x_2\} \cup X_3 \cup \left(\bigcup_{i=4}^{m}\alpha_i\right) \cup \left(\bigcup_{i=m+1}^{n}\beta_i\right) \right\} \text{ (or)}$$

$$\left\{ \{x_1\} \cup X_2 \cup \{x_3\} \cup \left(\bigcup_{i=4}^{m}\alpha_i\right) \cup \left(\bigcup_{i=m+1}^{n}\beta_i\right) \right\} \text{ (or)}$$

$$\left\{ X_1 \cup \{x_2\} \cup \{x_3\} \cup \left(\bigcup_{i=4}^{m}\alpha_i\right) \cup \left(\bigcup_{i=m+1}^{n}\beta_i\right) \right\} \text{ (or)}$$

$$\left\{ \{x_1\} \cup X_2 \cup X_3 \cup \left(\bigcup_{i=4}^{m}\alpha_i\right) \cup \left(\bigcup_{i=m+1}^{n}\beta_i\right) \right\} \text{ (or)}$$

$$\left\{ X_1 \cup \{x_2\} \cup X_3 \cup \left(\bigcup_{i=4}^{m}\alpha_i\right) \cup \left(\bigcup_{i=m+1}^{n}\beta_i\right) \right\} \text{ (or)}$$

$$\left\{ X_1 \cup X_2 \cup \{x_3\} \cup \left(\bigcup_{i=4}^{m}\alpha_i\right) \cup \left(\bigcup_{i=m+1}^{n}\beta_i\right) \right\} \text{ (or)}$$

$$\left\{ X_1 \cup X_2 \cup X_3 \cup \left(\bigcup_{i=4}^{m}\alpha_i\right) \cup \left(\bigcup_{i=m+1}^{n}\beta_i\right) \right\}$$

Number of such vertices will be $2^3(3)^{m-3}(2)^{n-m}-1$ as we are avoiding the self loop at $RS(X)\nabla RS\left(\{x_1,x_2,x_3\}\right)$.
In the second case, the vertices to which $RS\left(\{x_1,x_2,x_3\}\right)$ is connected are $P(S)$,where $S=\{\{x_1\},\{x_2\},\{x_3\}\}$. Hence number of such vertices are 2^3-1 (avoiding self loop).
Hence the degree of $RS(\{x_1\})\cup RS(\{x_2\})\cup RS(\{x_3\})$ is $2^3(3)^{m-3}(2)^{n-m} - 1 + 2^3 - 1 = 2^3(3)^{m-3}(2)^{n-m}+2^3-2$.
By continuing the similar argument, the degree of $RS(\{x_1\cup x_2\cup\ldots\cup x_r\})$ is $2^r(3)^{m-r}(2)^{n-m}+2^r-2$.

(b) Let $x_i\in B$

$$|C_5| = \sum_{r=2}^{m-1} mC_r. \qquad (since, 1<r<m)$$
$$= 2^m - mC_0 - mC_1 - mC_m.$$
$$= 2^m - m - 2.$$

Theorem 7. *(a)The degree of $RS(x_1\cup x_2\cup\ldots\cup x_r)\cup M'$, where $x_i\in B$ and $1<r<m$ is 2^r.*
(b)$|C_6| = (2^m-m-2)(2^{n-m}-1)$.

Proof.(a) Consider $RS(x_1\cup x_2\cup\ldots\cup x_r)\cup M'$ where $1<r<m$ and M' is the union of one or more equivalence classes whose cardinality equal to one in G(T(J)).
Note that $RS(x_1\cup x_2\cup\ldots\cup x_r)\cup M'$ will be connected to all elements of $P(B_1)$, where $B_1=\{x_1,x_2,\ldots,x_r\}$.
Also note that $(RS(x_1\cup x_2\cup\ldots\cup x_r)\cup M')\nabla RS(\{x_j\})$, for $r<j<m$ is $RS(\phi)$. Hence the degree of $RS(x_1\cup x_2\cup\ldots\cup x_r)\cup M'$ is 2^r.

(b) $|RS(x_1\cup x_2\cup\ldots\cup x_r)| = 2^m-m-2.$ and $|P(M')| = 2^{n-m}-1$

$$|C_6| = (|RS(x_1\cup x_2\cup\ldots\cup x_r)|)|P(M')|$$
$$= (2^m-m-2)(2^{n-m}-1).$$

Theorem 8. *(a)The degree of $RS((X_1\cup X_2\cup\ldots\cup X_r)\cup M)$ where X_i, $|X_i|>1$ and $1<r<m$ is 2^r.*
(b)$|C_7| = (2^m-m-2)(2^{n-m})$

Proof.(a) Consider $RS((X_1\cup X_2\cup\ldots\cup X_r)\cup M)$ where $1<r<m$,where $|X_i|>1$ and M is the union of none, one or more equivalence classes whose cardinality equal to one in G(T(J).
Note that $RS((X_1\cup X_2\cup\ldots\cup X_r)\cup M)$ will be connected to all elements of $P(B_1)$, where $B_1=\{x_1,x_2,\ldots,x_r\}$.

Also note that $RS((X_1 \cup X_2 \cup \ldots \cup X_r) \cup M) \nabla RS(\{x_j\})$, for $r < j < m$ is $RS(\phi)$.
Hence the degree of $RS((X_1 \cup X_2 \cup \ldots \cup X_r) \cup M)$ is 2^r.

(b)

$$|C_7| = (|RS(X_1 \cup X_2 \cup \ldots \cup X_r)|)\,|P(M)|.$$
$$= (2^m - mC_0 - mC_1 - mC_m)(2^{n-m})$$
$$= (2^m - m - 2)(2^{n-m}).$$

Theorem 9. *(a)The degree of $RS(x_1 \cup x_2 \cup \ldots \cup x_m)$, for every $x_i \in B$ is $2^n + 2^m - 3$.*
(b)$|C_8| = 1$.

Proof.(a) An element $RS(X)$ will be connected to $RS(\{x_i\})$ if and only if $RS(X)\nabla RS(Y) = RS(x_1 \cup x_2 \cup \ldots \cup x_m)$ where $RS(Y) \in J$.
The degree of $RS(x_1 \cup x_2 \cup \ldots \cup x_m)$ is obtained by considering those vertices to which $RS(X)$ which are connected to $RS(x_1 \cup x_2 \cup \ldots \cup x_m)$ and those vertices to which $RS(x_1 \cup x_2 \cup \ldots \cup x_m)$ is connected.
All those elements $RS(Z \cup M)$ where $Z = Z_1 \cup Z_2 \cup \ldots \cup Z_m$ such that $Z_i = x_i$ and all other $Z_j = x_j, i \neq j$ (or) $Z_i = x_i$, $Z_j = x_j$ and $Z_k = X_k$ $,k \neq i, j$, etc., (or) $Z_i = X_i, \forall i$ will be connected to $RS(x_1 \cup x_2 \cup \ldots \cup x_m)$.
This can be done in $(mC_0 + mC_1 + mC_2 + \ldots + mCm)\,2^{n-m} - 1$ $= (\sum_{i=0}^{m} mC_i)\,2^{n-m} - 1 = (2^m)(2^{n-m}) - 1$ ways as we avoid self loop.
Those vertices to which $RS(x_1 \cup x_2 \cup \ldots \cup x_m)$ is connected are $P(B)$ where $B = \{x_1, x_2, \ldots, x_m\}$.
$RS(x_1 \cup x_2 \cup \ldots \cup x_m)\nabla RS(\phi) = RS(\phi)$ and also by avoiding the self loop, $RS(x_1 \cup x_2 \cup \ldots \cup x_m)$ is connected to $2^m - 2$ elements.
Hence the degree of $RS(x_1 \cup x_2 \cup \ldots \cup x_m)$ is $(2^m)(2^{n-m}) - 1 + 2^m - 2 = 2^n + 2^m - 3$.

(b) Its trivial.

Theorem 10. *(a)The degree of $RS(x_1 \cup x_2 \cup \ldots \cup x_m) \cup M'$, for every $x_i \in B$ is $2^m - 1$.*
(b)$|C_9| = 2^{n-m} - 1$.

Proof.(a) Consider $RS(x_1 \cup x_2 \cup \ldots \cup x_m) \cup M'$, where $|X_i| > 1$ and M' is the union of one or more equivalence classes whose cardinality equal to one in G(T(J)).
Note that $RS(x_1 \cup x_2 \cup \ldots \cup x_m) \cup M'$ will be connected to all elements of $P(B)$ where $B = \{x_1, x_2, \ldots, x_m\}$.
But as $(RS(x_1 \cup x_2 \cup \ldots \cup x_m) \cup M')\nabla RS(\phi) = RS(\phi)$.
Hence we are not considering the edge $RS(x_1 \cup x_2 \cup \ldots \cup x_m) \cup M'$ to $RS(\phi)$.
Hence the degree of $RS(x_1 \cup x_2 \cup \ldots \cup x_m) \cup M'$ is $2^m - 1$.

(b) $|RS(x_1 \cup x_2 \cup \ldots \cup x_m)| = 1$ and Since, $|P(M')| = (2^{n-m} - 1)$.
$|C_9| = (|RS(x_1 \cup x_2 \cup \ldots \cup x_m)|)\,|P(M')| = 2^{n-m} - 1$.

Theorem 11. *(a)The degree of $RS((X_1 \cup X_2 \cup \ldots \cup X_m) \cup M)$, for every X_i, $|X_i| > 1$ is $2^m - 1$.*
(b)$|C_{10}| = 2^{n-m}$.

Proof.(a) Consider $RS((X_1 \cup X_2 \cup \ldots \cup X_m) \cup M)$, where $|X_i| > 1$ and M is the union of none, one or more equivalence classes whose cardinality equal to one in G(T(J)).
Note that $RS((X_1 \cup X_2 \cup \ldots \cup X_m) \cup M)$ will be connected to all elements of $P(B)$ where $B = \{x_1, x_2, \ldots, x_m\}$.
But as $RS((X_1 \cup X_2 \cup \ldots \cup X_m) \cup M)\,\nabla RS(\phi) = RS(\phi)$.
Hence we are not considering the edge $RS((X_1 \cup X_2 \cup \ldots \cup X_m) \cup M)$ to $RS(\phi)$. Hence the degree of $RS((X_1 \cup X_2 \cup \ldots \cup X_m) \cup M)$ is $2^m - 1$.

(b)Since there are m-equivalence classes with cardinality equal to one in G(T(J)), $|RS(X_1 \cup X_2 \cup \ldots \cup X_m)| = 1$ and $|M| = 2^{n-m}$.

$$|C_{10}| = (|RS(X_1 \cup X_2 \cup \ldots \cup X_m)|)\,|M| = 2^{n-m}.$$

Notation: For every $1 < r < m$, let $Q_r = \Big\{\{Z_1 \cup Z_2 \cup \ldots \cup Z_r\} \mid Z_i = \{x_i\}$ or X_i $, i = 1, 2, \ldots, r\Big\}$. In the following theorem we discuss the degree and cardinality of $RS(Q_r \cup M)$ but when all Z_i's are x_i or when all Z_i's are X_i , the degree and cardinality of such elements will fall under C_5, C_6 and C_7. Hence for the following discussion we assume that $\exists\ j,k \ni 1 \leq j,k \leq r, Z_j = \{x_j\}$ and $Z_k = X_k$.

Theorem 12. *(a)The degree of $RS(Q_r \cup M)$ is 2^r.*
(b)$|C_{11}| = 3(2^{n-m})(3^{m-1} - 2^m + 1)$.

Proof.(a) Consider the element $RS(Q_r \cup M)$. Note that $RS(Q_r \cup M)$ will be connected to all elements of $P(B_1)$, where $B_1 = \{x_1, x_2, \ldots, x_r\}$.
Also note that $RS(Q_r \cup M)\nabla RS(\{x_j\})$, for $r < j < m$ is $RS(\phi)$. Hence the degree of $RS(Q_r \cup M)$ is 2^r.

(b) Consider $A_1 = Z_1, Z_2, \ldots, Z_m$ where $Z_i = \{x_i\}$ or X_i.

*Case:*1 When $r = 2$
Two elements can be chosen from m elements in mC_2 ways and to satisfy the condition that atleast one should be $\{x_i\}$ and atleast on should be X_i, it can be done in $2C_1$ ways.
Hence when $r = 2$, the total number of ways is $mC_2\,(2C_1)$.

*Case:*2 When $r = 3$
We have mC_3 ways of choosing 3 elements from A_1 and to satisfy the condition that it can be done in $3C_1 + 3C_2$ ways.
Hence when $r = 3$,the toatal number of ways is $mC_3\,(3C_1 + 3C_2)$ ways.
Continuing this argument when $r = m - 1$, we have $mCm - 1$ waysof chosing $m - 1$ elements from A_1 and to

satisfy the condition it can be done in $(m-1)C_1+(m-1)C_2+\ldots+(m-1)C_{m-2}$ ways.
Hence when $r=m-1$ the total number of ways is $mCm-1\left[(m-1)C_1+(m-1)C_2+\ldots+(m-1)C_{m-2}\right]$.

$$|C_{11}| = 2^{n-m}\Big(mC_2(2C_1)+mC_3(3C_1+3C_2)+\ldots+mC_{m-1}$$
$$[(m-1)C_1+(m-1)C_2+\ldots+(m-1)C_{(m-2)}]\Big).$$
$$= 2^{n-m}\left[\sum_{r=2}^{m-1} mC_r\left(\sum_{i=1}^{r-1} rC_i\right)\right].$$
$$= 3(2^{n-m})\left[3^{m-1}-2^m+1\right].$$

Theorem 13.(a)*The degree of* $RS(Q_m\cup M)$ *is* 2^m-1.
(b)$|C_{12}|=2^{n-m}(2^m-2)$.

Proof.(a) Consider the element $RS(Q_m\cup M)$ where $Q_m=\{\{Z_1\cup Z_2\cup\ldots\cup Z_m\}\mid Z_i=\{x_i\}$ or $X_i\}$. Note that $RS(Q_m\cup M)$ will be connected to all elements of $P(B)$ where $B=\{x_1,x_2,\ldots,x_m\}$. But as $(RS(Q_m\cup M))\nabla RS(\phi)=RS(\phi)$. Hence we are not considering the edge $RS(Q_m\cup M)$ to $RS(\phi)$. Hence the degree of $RS(Q_m\cup M)$ is 2^m-1.

(b) Consider $A_1=Z_1,Z_2,\ldots,Z_m$ where $Z_i=\{x_i\}$ or X_i. The number of ways one element can be chosen from m elements is mC_1. The number of ways 2 elements can be chosen from m elements is mC_2. Continuing the process, $m-1$ elements can be chosen in mC_{m-1} ways.

$$|C_{12}| = [mC_1+mC_2+\ldots+mC_{m-1}]\,(|M|).$$
$$= \left[\sum_{r=1}^{m-1} mC_r\right]2^{n-m} = 2^{n-m}(2^m-2).$$

Theorem 14.(a)*The degree of* $RS(\{x_i\})\cup M'$*, for every* $x_i\in B$ *is* 2.
(b)$|C_{13}|=m(2^{n-m}-1)$.

Proof.(a) Consider the vertex $RS(\{x_i\})\cup M')$ where $x_i\in X_i,|X_i|>1$. The vertex $RS(\{x_i\})\cup M'$ will be connected to $RS(\{x_i\})\cup M')\nabla RS(Y)$ where $Y\in P(B)$. $RS(\{X_i\})\cup M')\nabla RS(Y)$ are $RS(\{x_i\})$ and $RS(\phi)$. Hence the degree of $RS(\{x_i\})\cup M'$ is 2.

(b) $|RS(\{x_i\})|=m$. and Since $|M'|=2^{n-m}-1$

$$|C_{13}| = (|RS(\{x_i)\}|)\,|M'| = m(2^{n-m}-1).$$

4 Application of Category Graph in finding the Wiener Index

We have assumed that there are n-equivalence classes with m classes whose cardinality greater than one and n-m classes whose cardinality equal to one. Since the cardinality of T ($|T|$) is $2^{n-m}3^m$, it is difficult to calculate the wiener index of a Rough Ideal based Rough Edge Cayley graphs $G(T(J))$ as it deals with the distance from each vertex to every other vertex. The categorization of the vertices will make the complexity in finding the weiner index simpler and hence we use the 13 categories of vertices. Now its enough to find the distances from each category to other. In Table:1 the vertices along with their degrees and cardinalities in each category are given:

Table 1: Degrees and cardinalities of the different category

Category (C_i)	Vertices	Cardinality	Degree of each vertex in the Category
C_1	$RS(\phi)$	1	$(2^{n-m})(3^m-2^m)-1$
C_2	$RS\{x_i\}$	m	$2(2^{n-m})(3^{m-1})$
C_3	$RS(X_i\cup M)$	$m(2^{n-m})$	2
C_4	M'	$2^{n-m}-1$	1
C_5	$RS(\{x_1x_2,\ldots,x_r\})$	$2^m-(m+2)$	$2^r(3^{m-r})(2^{n-m})+2^r-2$
C_6	$RS(\{x_1x_2,\ldots,x_r\}\cup M')$	$[2^m-(m+2)](2^{n-m}-1)$	2^r
C_7	$RS(\{X_1X_2,\ldots,X_r\}\cup M)$	$[2^m-(m+2)](2^{n-m})$	2^r
C_8	$RS(\{x_1x_2,\ldots,x_m\})$	1	2^m+2^n-3
C_9	$RS(\{x_1x_2,\ldots,x_m\}\cup M')$	$2^{n-m}-1$	2^m-1
C_{10}	$RS(\{X_1X_2,\ldots,X_m\}\cup M)$	2^{n-m}	2^m-1
C_{11}	$RS(Q_r\cup M)$	$3(2^{n-m})(3^{m-1}-2^m+1)$	2^r
C_{12}	$RS(Q_m\cup M)$	$2^{n-m}(2^m-2)$	2^m-1
C_{13}	$RS(\{x_i\}\cup M')$	$m(2^{n-m}-1)$	2

4.1 Distance between the vertices of each category to every other category:

In this section, we calculate the sum of the distances from one category to every other category. This means that we calculte the distance from the vertices of C_1 to the vertices of $C_2,C_3,\ldots,C_{13}$. Then we calculate the distance from the vertices of C_2 to the vertices of $C_3,C_4,\ldots,C_{13}$ and so on. And we calculate the distance from the vertices of C_{12} to the vertices of C_{13}. The sum of the distances from the vertices of one category to vertices of every other category are shown in tables 2 to 7.

Table 2: Sum of the distances from vertices of one category to the vertices of other categories

Categories	C_2	C_3	C_4	C_5
C_1	m	$m2^{n-m}$	$2^{n-m}-1$	$2^m-(m+2)$
C_2	$m(m-1)$	$m(2^{n-m})+2m(m-1)(2^{n-m})$	$2m(2^{n-m})$	$2m(2^m-(m+2))-m(2^{m-1}-2)$
C_3	-	$m(m-1)2^{n-m}+m^2 2^{n-m}(2^{n-m}-1)$	$2m(2^{n-m})(2^{n-m}-1)$	$2m2^{n-m}(2^m-(m+2))$
C_4	-	-	$(2^{n-m}-1)(2^{n-m}-2)$	$2(2^{n-m}-1)(2^m-(m+2))$
C_5	-	-	-	$\sum_{r=2}^{m-1} mC_r(mC_r-1)+mC_2\left\{2\sum_{r=2}^{m-1} mC_{r+1}-[(m-2)+(m-2)(m-3)+\ldots+(m-2)(m-3)\ldots(1)]\right\}+mC_3\left\{2\sum_{r=2}^{m-1} mC_{r+2}-[(m-3)+(m-3)(m-4)+\ldots+(m-3)(m-4)\ldots(1)]\right\}+\ldots+mC_{m-1}\left\{2\sum_{r=2}^{m-1} mC_{r+m-2}-(1)\right\}$

Table 3: Sum of the distances from vertices of one category to the vertices of other categories

Categories	C_6	C_7	C_8	C_9
C_1	$2^m-(m+2)(2^{n-m}-1)$	$2^m-(m+2)(2^{n-m})$	2	$2(2^{n-m}-1)$
C_2	$(2^{n-m}-1)\left[2m(2^m-(m+2))-m(2^{m-1}-2)\right]$	$(2^{n-m}-1)\left[2m(2^m-(m+2))-m(2^{m-1}-2)\right]$	m	$m(2^{n-m}-1)$
C_3	$2m2^{n-m}(2^m-(m+2))(2^{n-m}-1)$	$2m2^{n-m}(2^m-(m+2))(2^{n-m})$	$2m2^{n-m}$	$2m2^{n-m}(2^{n-m}-1)$
C_4	$2(2^{n-m}-1)(2^m-(m+2))(2^{n-m}-1)$	$2(2^{n-m}-1)(2^m-(m+2))(2^{n-m})$	$3(2^{n-m}-1)$	$3(2^{n-m}-1)(2^{n-m}-1)$
C_5	$(2^m-(m+2))(2^{n-m}-1)\left[2(2^m-(m+2))-1\right]$	$(2^m-(m+2))(2^{n-m})\left[2(2^m-(m+2))-1\right]$	$2^m-(m+2)$	$(2^m-(m+2))(2^{n-m}-1)$
C_6	$(2^m-(m+2))(2^{n-m}-1)\left[(2^m-(m+2))(2^{n-m}-1)-1\right]$	$2(2^m-(m+2))(2^{n-m}-1)(2^m-(m+2))(2^{n-m})$	$2(2^m-(m+2))(2^{n-m}-1)$	$2(2^m-(m+2))(2^{n-m}-1)(2^{n-m}-1)$

Table 4: Sum of the distances from vertices of one category to the vertices of other categories

Categories	C_{10}	C_{11}	C_{12}	C_{13}
C_1	$2(2^{n-m})$	$3(2^{n-m})(3^{m-1}-2^m+1)$	$2(2^{n-m})(2^m-2)$	$m(2^{n-m}-1)$
C_2	$m(2^{n-m})$	$m(2^{n-m})\left[5(3^{m-1})-9(2^{m-1})+3\right]$	$m(2^{n-m})(2^m-2)$	$m(2^{n-m})(2m-1)$
C_3	$2m2^{n-m}(2^{n-m})$	$2m2^{n-m}(3)(2^{n-m})(3^{m-1}-2^m+1)$	$2m2^{n-m}(2^{n-m})(2^m-2)$	$2m^2 2^{n-m}(2^{n-m}-1)$
C_4	$3(2^{n-m}-1)(2^{n-m})$	$6(2^{n-m}-1)(2^{n-m})(3^{m-1}-2^m+1)$	$3(2^{n-m}-1)(2^{n-m})(2^m-2)$	$2m(2^{n-m}-1)(2^{n-m}-1)$
C_5	$(2^m-(m+2))(2^{n-m})$	$(2^m-(m+2))(2^{n-m})$	$(2^m-(m+2))2^{n-m}(2^m-2)$	$2m(2^m-(m+2))(2^{n-m}-1)$
C_6	$2(2^m-(m+2))(2^{n-m}-1)(2^{n-m})$	$6(2^m-(m+2))(2^{n-m}-1)(2^{n-m})(3^{m-1}-2^m+1)$	$2(2^m-(m+2))(2^{n-m}-1)(2^m-2)$	$2(2^m-(m+2))(2^{n-m}-1)m(2^{n-m}-1)$

Table 5: Sum of the distances from vertices of one ctegory to the vertices of other categories

Categories	C_7	C_8	C_9	C_{10}
C_7	$(2^m-(m+2))(2^{n-m})\left[(2^m-(m+2))(2^{n-m})-1\right]$	$2(2^m-(m+2))(2^{n-m})$	$2(2^m-(m+2))(2^{n-m})(2^{n-m}-1)$	$2(2^m-(m+2))(2^{n-m})(2^{n-m})$
C_8	-	-	$2^{n-m}-1$	2^{n-m}
C_9	-	-	$(2^{n-m}-1)(2^{n-m}-2)$	$2(2^{n-m}-1)(2^{n-m})$
C_{10}	-	-	-	$(2^{n-m})(2^{n-m}-1)$

Table 6: Sum of the distances from vertices of one ctegory to the vertices of other categories

Categories	C_{11}	C_{12}	C_{13}
C_7	$2(2^m-(m+2))(2^{n-m})\left[(3)(2^{n-m})(3^{m-1}-2^m+1)\right]$	$2(2^m-(m+2))(2^{n-m})^2(2^m-2)$	$2m(2^m-(m+2))(2^{n-m})(2^{n-m}-1)$
C_8	$(6)2^{n-m}(3^{m-1}-2^m+1)$	$2^{n-m}(2^m-2)$	$2m(2^{n-m}-1)$
C_9	$6(2^{n-m}-1)(3^{m-1}-2^m+1)$	$(2)2^{n-m}(2^{n-m}-1)(2^m-2)$	$2m(2^{n-m}-1)(2^{n-m}-1)$
C_{10}	$6(2^{n-m})^2(3^{m-1}-2^m+1)$	$2(2^{n-m})^2(2^m-2)$	$2m2^{n-m}(2^{n-m}-1)$

Table 7: Sum of the distances from vertices of one category to the vertices of other categories

Categories	C_{11}	C_{12}	C_{13}
C_{11}	$(3)(2^{n-m})(3^{m-1}-2^m+1)\left[(3)(2^{n-m})(3^{m-1}-2^m+1)-1\right]$	$6(2^{n-m})(2^{n-m})(3^{m-1}-2^m+1)(2^m-2)$	$6m(2^{n-m})(3^{m-1}-2^m+1)(2^{n-m}-1)$
C_{12}	-	$2^{n-m}(2^m-2)\left[2^{n-m}(2^m-2)-1\right]$	$2m2^{n-m}(2^m-2)(2^{n-m}-1)$
C_{13}	-	-	$m(2^{n-m}-1)\left[m(2^{n-m}-1)-1\right]$

4.2 Wiener Index

The Weiner index $W(G(T(J)))$[2] of a graph G is defined as the sum half of the distance between every pair of vertices of G. In this section, we are going to obtain the Wiener index of a Rough Ideal Based Rough Edge Cayley Graph.

Wiener index of the Rough Ideal Based Rough Edge Cayley Graph $W(G(T(J)))$

= Sum of the distances from vertices of one category to the vertices of every other categories

$$= -3(k)3^{m-1}+5m(k)3^{m-1}-m(k)2^{m-1}-4m(k)-m^2 +2m(k)^2(3^m)-m^2(k)^2-2m^2(k)+(k)^2(2^m)-2m(k)^2 -(k)^2+6(k)^2(3^{m-1})-k+1+(2^m-(m+2)) \Big[-(2)(k)2^m-(4)k-(2)2^m+5+m+(6)(k)3^{m-1} -2^m(k)^2+2^m(k)+(6)3^{m-1}(k)^2-(6)k3^{m-1}-m(k) -2^m(k)+2^m-m(k)^2+m(k)+k-1+(6)3^{m-1}(k)^2 -2^{n-m}-3(2^m)(k)^2+m(k)^2-2m(k)+4(k)^2\Big] +\sum_{r=2}^{m-1} mC_r(mC_r-1)+mC_2\Big\{2\sum_{r=2}^{m-1} mC_{r+1}-\Big[(m-2) +(m-2)(m-3)+\ldots+(m-2)(m-3)\ldots(1)\Big]\Big\} +mC_3\Big\{2\sum_{r=2}^{m-1} mC_{r+2}-\Big[(m-3)+(m-3)(m-4) +\ldots+(m-3)(m-4)\ldots(1)\Big]\Big\}+\ldots+mC_{m-1} \Big\{2\sum_{r=2}^{m-1} mC_{r+m-2}-(1)\Big\}-2m-1+(6)(k)3^{m-1} -(5)(k)2^m+(6)(k)+2m(k)+(k-1)\Big[4(k) +6(k)3^{m-1}+2m(k)-4(k)2^m-2m+k-2\Big] +(k)\Big[6(k)3^{m-1}-4(k)2^m+2(k)-2m+(k)(1)+2m(k)\Big] +\Big[3(k)3^{m-1}-3(k)2^m+3(k)\Big]\Big[-(k)2^m-k+2m(k) +(3)(k)3^{m-1}-2m-1\Big]+\Big[(k)2^m-2(k)\Big]\Big[2m(k) -2m+(k)2^m-(2)(k)-1\Big]+(m(k)-m)(m(k)-m-1),$$

where $k=2^{n-m}$.

$$=(2^{n-m})(3^{m-1})\Big[9(2^{n-m})(3^{m-1})-6-m\Big]+(2^{n-m})(2^{m-1}) \Big[2(2^{n-m})-4(2^m)+7m+2\Big]+2^m\Big[2m-2^m+6\Big] +2^{n-m}\Big[4-3m-2(m^2)\Big]-m^2-5m-6 +\sum_{r=2}^{m-1} mC_r(mC_r-1)+mC_2\Big\{2\sum_{r=2}^{m-1} mC_{r+1}-\big[(m-2) +(m-2)(m-3)+\ldots+(m-2)(m-3)\ldots(1)\big]\Big\}+ mC_3\Big\{2\sum_{r=2}^{m-1} mC_{r+2}-\big[(m-3)+(m-3)(m-4) +\ldots+(m-3)(m-4)\ldots(1)\big]\Big\}+\ldots+mC_{m-1} \Big\{2\sum_{r=2}^{m-1} mC_{r+m-2}-(1)\Big\}.$$

Example 1.[7] Let $U=\{x_1,x_2,x_3,x_4\}$.
Let $X_1=[x_1]=\{x_1,x_3\}$ and $X_2=[x_2]=\{x_2,x_4\}$ induced by R.
If $B=\{x_1,x_2\}$ then
$J=\Big\{RS(\phi),RS(\{x_1\}),RS(\{x_2\}),RS(\{x_1,x_2\})\Big\}$
For example,
$RS(X_1)\nabla RS(\{x_1\}) = RS(\{x_1\})$ where $RS(X_1)\in T$ and $RS(\{x_1\})\in J$.
Hence there is an edge between $RS(X_1)$ and $RS(\{x_1\})$.
$RS(X_1)\nabla RS(\{x_2\}) = RS(\phi)$, where $RS(X_1)\in T$ and $RS(\{x_2\})\in J$.
Hence there is an edge between $RS(X_1)$ and $RS(\phi)$.
Continuing this process we obtain the Rough Ideal based Rough Edge Cayley Graph and is shown in Figure:1.
Consider the categories C_1 and C_3.
The only element in category C_1 is $RS(\phi)$.
The elements of the category C_3 are $RS(X_1)$ and $RS(X_2)$.
$RS(X_1)\nabla RS(\{x_2\}) = RS(\phi)$ where $RS(\{x_2\})\in J$.
$RS(X_2)\nabla RS(\{x_1\}) = RS(\phi)$ where $RS(\{x_1\})\in J$.
Hence, the elements of the category C_3 are connected to C_1.
Continuing this process we obtain the corresponding category graph when $n=m=2$ and is shown in figure:2 .

Wiener Index of the Rough Ideal based Rough Edge Cayley Graph, $W(G(T(J)))$

$=(1)(3)\big[9(1)(3)-6-2\big]+(1)(2)\big[2(1)-4(4)+7(2)+2\big]+4\big[2(2)-4+6\big]+(1)\big[4-3(2)-2(4)\big]-4-10-6=55.$

Example 2.[7] Let $U=\{x_1,x_2,x_3,x_4,x_5,x_6\}$. Let $X_1=\{x_1,x_3\}=[x_1]$, $X_2=\{x_2,x_4,x_6\}=[x_2]$ and $X_3=\{x_5\}=[x_5]$ induced by R.
$|X_1|,|X_2|>1$ and $\{x_1,x_2\}$ be the pivot elements of these classes.

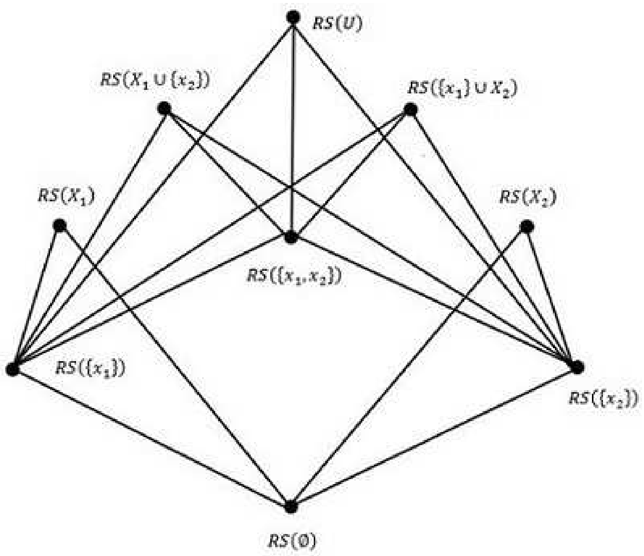

Fig. 1: Rough Ideal Based Rough edge Cayley graph when n=2 and m=2

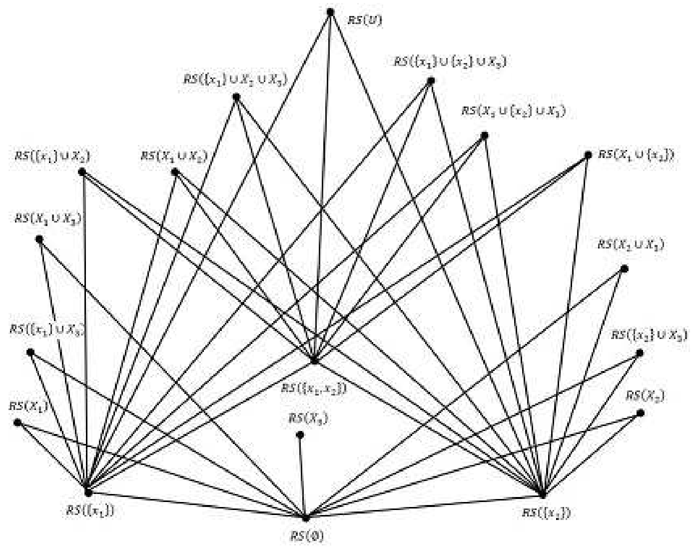

Fig. 3: Rough Ideal Based Rough edge Cayley graph for n=3 and m=2

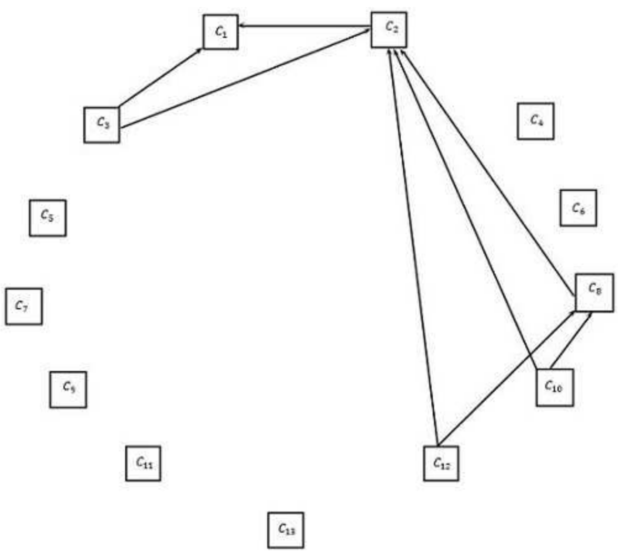

Fig. 2: Category graph corresponding to the Rough Ideal Based Rough edge Cayley graph when n=2 and m=2

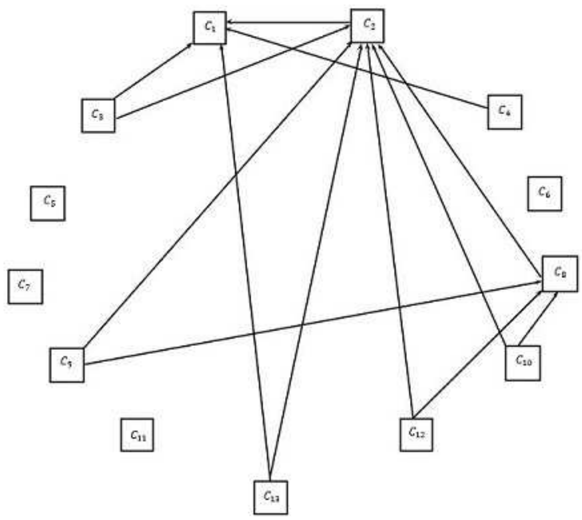

Fig. 4: Category graph corresponding to the Rough Ideal Based Rough edge Cayley graph for n=3 and m=2

$|X_3| = 1$.
$B = \{x_1, x_2\}$ then
$J = \left\{RS(\phi), RS(\{x_1\}), RS(\{x_2\}), RS(\{x_1, x_2\})\right\}$
The Rough Ideal based Rough Edge Cayley Graph when $n = 3$ and $m = 2$ is shown in Figure :3 and the corresponding Category Graph is shown in figure:4.

Wiener Index $W(G(T(J)))$

$= (2)(3)\left[9(2)(3) - 6 - 2\right] + (2)(2)\left[2(2) - 4(4) + 7(2) + 2\right] + 4\left[2(2) - 4 + 6\right] + (1)\left[4 - 3(2) - 2(4)\right] - 4 - 10 - 6.$
$= 276$

5 Conclusion

In this paper, degrees of all the vertices of a particular category and cardinalities of each category are obtained. Wiener Index of the Rough Ideal based Rough Edge Cayley Graph are obtained by using the categorization of vertices and cardinalities of each category. All these concepts are illustrated through examples. Our future work is to explore this category graph in the study of Rough Ideal based Rough Edge Cayley Graph.

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B. Praba is a professor in the Department of Mathematics, SSN College of Engineering, Chennai, India. Her area of research includes Rough set theory, Soft set theory and Algebraic Graph theory. She has more than 30 years of research and teaching experience and has published more than 100 papers in various indexed journals and conference proceedings. Five students have been awarded Ph.D. degree under her guidance. She is also a prolific writer and authored six books on various topics in Mathematics including Discrete Mathematics, Probability and Random Processes.

Benazir Obilia.X.A is a research scholar in the Department of Mathematics, SSN College of Engineering,Chennai, India. She has received her M.Sc., M.Phil degree in Mathematics from Stella Maris College, Chennai, India. Her area of research includes Rough set theory and Algebraic Graph theory.