Mathematical Sciences Letters
An International Journal

http://dx.doi.org/10.18576/msl/050304

Multiobjective Capacitated Fractional Transportation Problem with Mixed Constraints

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Received: 28 Jul. 2015, Revised: 21 Dec. 2015, Accepted: 23 Jan. 2016
Published online: 1 Sep. 2016

Abstract: In this article, the problem of determining the optimal transportation schedule is formulated as multiobjective capacitated fractional transportation problem with mixed constraints, in which objectives are fractional functions and constraints are linear. The compromise solution of the problem is derived by using a fuzzy programming approach, in which we use three different forms of membership functions viz. linear, exponential and hyperbolic, and lexicographic goal programming with minimum distances techniques. The problem and solution procedures are demonstrated through a numerical example.

Keywords: Fractional transportation problem; Compromise solution; Fuzzy programming; Mixed constraints; Multiobjective programming.

1 Introduction

In mathematical optimization, linear fractional programming (LFP) is a generalization of linear programming (LP). As the objective functions in linear programs are linear functions, the objective function in a linear fractional program is a ratio of two linear functions. A linear program can be regarded as a special case of a linear fractional program in which the denominator is the constant function one.

The Transportation problem (TP) is a situation in which a product/products is/are to be transported from several sources (also called origin, supply or capacity centers) to several sinks (also called destination, demand or requirement centers). Hitchcock [7] developed the basic transportation problem. The TP in which the objective function is of fractional type are known as Fractional transportation problem (FTP). The FTP was originally proposed by Swarup [11]. The TP with fractional objective functions have been extensively used by several authors such as Verma and Puri [12] worked on paradox in LFTP, Gupta et al. [2] presented a paradox in linear fractional TPs with mixed constraints, etc. Recently, some authors who have discussed FTPs are Khurana and Arora [9], Gupta and Arora [3, 5, 4]. Joshi and Gupta [8] investigated the transportation problem with fractional objective function when the demand and supply quantities are varying.

Real life TPs are mostly multiobjective and in case of multiple conflicting objectives, it is not necessary that the optimum solution for one objective is also optimum for the others. So, in order to deal with such solutions, a compromise criterion is used in which a solution is obtained which is optimum for all the objectives in some sense. Also, real life TPs have mixed constraints but no systematic method for finding an optimal solution for TPs with mixed constraints are revealed in literature. Recently, some authors consider this situation such as Adlakha et al. [1], Mondal et al. [10],Gupta and Bari [6], etc.

In this article, multiobjective capacitated fractional transportation problem (MOCFTP) with mixed constraints is formulated in which the objective functions are fractional, that is, its a ratio of two linear functions. Fractional programs finds its application in a variety of real world problems such as stock cutting problem, resource allocation problems, routing problem for ships and planes, cargo loading problem, inventory problem and many other problems. The compromise solution is derived by two approaches viz, fuzzy programming and lexicographic goal programming with minimum distances. To demonstrate the problem and solution procedures a numerical example is provided.

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2 Formulation of the problem

Consider a fractional transportation problem with m origins having a_i $(i=1,2,\dots,m)$ units of supply to be transported among n destinations with b_j $(j=1,2,\dots,n)$ units of demand. Here we consider three fractional objective functions , which are

– Units transporting cost c_{ij} due to the traveled route and unit transporting cost due to preferring route r_{ij}, for transporting the product from i^{th} origin to j^{th} destination.
– Actual transportation time t^a_{ij} and a standard transportation time t^s_{ij}, for transporting the product from i^{th} origin to j^{th} destination.
– Unit transporting damage cost d_{ij} (lost of quality and quantity of transportation)due to the traveled route and unit transporting damage cost due to preferring route r_{ij}, for transporting the product from i^{th} origin to j^{th} destination.

The problem is to determine the transportation schedule of transporting the available quantity of products, to satisfy demand that minimizes the total transportation cost, time and damage charges. Let x_{ij} be the number of units transported from i^{th} origin to j^{th} destination. Then, the mathematical model for the MOCFTP with mixed constraints can be expressed as follows:

$$\left.\begin{aligned}
&\text{Minimize } C=\frac{\sum_{i=1}^m\sum_{j=1}^n c_{ij}x_{ij}}{\sum_{i=1}^m\sum_{j=1}^n r_{ij}x_{ij}}\\
&\text{Minimize } T=\max_{ij}\left\{\frac{t^a_{ij}|x_{ij}>0}{t^s_{ij}|x_{ij}>0}\right\}\\
&\text{Minimize } D=\frac{\sum_{i=1}^m\sum_{j=1}^n d_{ij}x_{ij}}{\sum_{i=1}^m\sum_{j=1}^n r_{ij}x_{ij}}\\
&\text{subject to } \sum_{i=1}^m x_{ij}\{\le/=/\ge\}a_i\\
&\qquad\qquad \sum_{j=1}^n x_{ij}\{\le/=/\ge\}b_j\\
&\qquad\qquad l_{ij}\le x_{ij}\le s_{ij};\;\; x_{ij}\ge 0.
\end{aligned}\right\}\tag{1}$$

where l_{ij} be the minimum and s_{ij} be the maximum amount of quantity transported from i^{th} origin to j^{th} destination i.e. $x_{ij}\le s_{ij}$ or the capacitated restriction on the route i to j.

3 Optimization Techniques

3.1 Fuzzy programming with different membership functions

To derive the compromise solution of MOCFTP we are using fuzzy programming (FP). To formulate the fuzzy model first we have to define the payoff matrix as:

$$\text{Payoff Matrix}=\begin{array}{c} \\ x_{ij}^{(1)}\\ x_{ij}^{(2)}\\ \vdots\\ x_{ij}^{(k)}\end{array}\begin{array}{c}\begin{array}{ccc}\;\;C\;\;&\;\;D\;\;&\;\;T\;\;\end{array}\\ \begin{bmatrix} C(x_{ij}^{(1)}) & D(x_{ij}^{(1)}) & T(x_{ij}^{(1)})\\ C(x_{ij}^{(2)}) & D(x_{ij}^{(2)}) & T(x_{ij}^{(2)})\\ \vdots&\vdots&\vdots\\ C(x_{ij}^{(k)}) & D(x_{ij}^{(k)}) & T(x_{ij}^{(k)})\end{bmatrix}\end{array}$$

where $x_{ij}^{(k)}$; $k=1,2,\dots,K$ are the individual optimum solutions.
Now, the membership functions for the problem are defined. Let the membership functions for the cost objective are:

3.1.1 Linear membership function

For cost objective function a linear membership function $\mu^L(C)$ is defined as:

$$\mu^L\{C\}=\begin{cases}1, & if\ C\le C_l\\ \frac{C_u-C}{C_u-C_l}, & if\ C_l<C<C_u\\ 0, & if\ C\ge C_u\end{cases}$$

where C_l and C_u are respectively the lower and upper tolerance limits of the objective functions such that the degrees of the membership function are 0 and 1, respectively.

3.1.2 Exponential membership function

For cost objective function an exponential membership function $\mu^E(C)$ is defined as:

$$\mu^E\{C\}=\begin{cases}1 & if\ C\le C_l\\ \dfrac{exp\left(\frac{-\alpha(C-C_l)}{C_u-C_l}\right)-exp(-\alpha)}{1-exp(-\alpha)} & if\ C_l<C<C_u\\ 0 & if\ C\ge C_u\ \ and\ \ \alpha\to\infty\end{cases}$$

where α is a non-zero parameter, prescribed by the decision maker and C_l, C_u have the usual meaning as described in section 3.1.

3.1.3 Hyperbolic membership function

For cost objective function a hyperbolic membership function $\mu^H(C)$ is defined as:

$$\mu^H\{C\}=\begin{cases}1 & if C\le C_l\\ \frac{1}{2}tanh\left(\left(\frac{C_u+C_l}{2}-C\right)\alpha_k\right)+\frac{1}{2} & if C_l<C<C_u\\ 0 & if C\ge C_u\end{cases}$$

where $\alpha_k=\frac{6}{(C_u-C_l)}$ and C_l, C_u have the usual meaning as described in section 3.1.
This membership function has the following formal properties given by Zimmermann [13]:

– $\mu^H(C)$ is strictly monotonously decreasing function with respect to C.
– $\mu^H(C) = \frac{1}{2} \Leftrightarrow C = \frac{1}{2}(C_u + C_l)$.
– $\mu^H(C)$ is strictly convex for $C \geq \frac{1}{2}(C_u + C_l)$ and strictly concave for $C \leq \frac{1}{2}(C_u + C_l)$.
– $\mu^H(C)$ satisfies $0 < \mu^H(C) < 1$ for $C_l < \mu^H(C) < C_u$ and approaches asymptotically $\mu^H(C) = 0$ and $\mu^H(C) = 1$ as $C \to \infty$ and $-\infty$ respectively.

Similarly, the membership functions for the other objectives i.e., time & damage charges, can be defined.
Now, the MOCFTP with mixed constraints given in eq. (1) can be written as an equivalent linear model, for linear membership function, as follows:

$$\left.\begin{aligned}
&\text{Minimize } \delta\\
&\text{subject to } \frac{C_u - C}{C_u - C_l} \leq \delta\\
&\frac{T_u - T}{T_u - T_l} \leq \delta; \quad \frac{D_u - D}{D_u - D_l} \leq \delta\\
&\sum_{i=1}^{m} x_{ij}\{\leq / = / \geq\} a_i; \quad \sum_{j=1}^{n} x_{ij}\{\leq / = / \geq\} b_j\\
&l_{ij} \leq x_{ij} \leq s_{ij}; \quad x_{ij} \geq 0; \quad \delta \geq 0.
\end{aligned}\right\} \tag{2}$$

Similarly, the MOCFTP with mixed constraints given in eq. (1) can be written as an equivalent nonlinear model, for exponential membership function, as follows:

$$\left.\begin{aligned}
&\text{Minimize } \delta\\
&\text{subject to } \frac{exp\left(\frac{-\alpha(C-C_l)}{C_u-C_l}\right) - exp(-\alpha)}{1 - exp(-\alpha)} \leq \delta\\
&\frac{exp\left(\frac{-\alpha(T-T_l)}{T_u-T_l}\right) - exp(-\alpha)}{1 - exp(-\alpha)} \leq \delta\\
&\frac{exp\left(\frac{-\alpha(D-D_l)}{D_u-D_l}\right) - exp(-\alpha)}{1 - exp(-\alpha)} \leq \delta\\
&\sum_{i=1}^{m} x_{ij}\{\leq / = / \geq\} a_i; \quad \sum_{j=1}^{n} x_{ij}\{\leq / = / \geq\} b_j\\
&l_{ij} \leq x_{ij} \leq s_{ij}; \quad x_{ij} \geq 0; \quad \delta \geq 0.
\end{aligned}\right\} \tag{3}$$

And, the MOCFTP with mixed constraints given in eq. (1) can be written as an equivalent nonlinear model, for hyperbolic membership function, as follows:

$$\left.\begin{aligned}
&\text{Minimize } \delta\\
&\text{subject to } \frac{1}{2}tanh\left(\left(\frac{C_u + C_l}{2} - C\right)\alpha_k\right) + \frac{1}{2} \leq \delta\\
&\frac{1}{2}tanh\left(\left(\frac{T_u + T_l}{2} - C\right)\alpha_k\right) + \frac{1}{2} \leq \delta\\
&\frac{1}{2}tanh\left(\left(\frac{D_u + D_l}{2} - C\right)\alpha_k\right) + \frac{1}{2} \leq \delta\\
&\sum_{i=1}^{m} x_{ij}\{\leq / = / \geq\} a_i; \quad \sum_{j=1}^{n} x_{ij}\{\leq / = / \geq\} b_j\\
&l_{ij} \leq x_{ij} \leq s_{ij}; \quad x_{ij} \geq 0; \quad \delta \geq 0.
\end{aligned}\right\} \tag{4}$$

where δ represents the deviations.

3.2 Lexicographic goal programming with minimum distance

Lexicographic goal programming (LGP), also termed as Preemptive goal programming, mainly features the existence of number of priority levels. Each priority level contains a number of unwanted deviations to be minimized or in other words unwanted deviations are placed into priority levels. LGP with minimum distance is an improved form of original LGP. For solving LGP with minimum distance, firstly the priorities are given to objectives one after the other and a set of solution is obtained, then an ideal solution is identified as follows:

Table 1: Calculations for ideal solutions

Priority Structure	x_{11}	x_{22}	$\cdots$	x_{pq}
$P^{(1)}$	$x_{11}^{(1)}$	$x_{22}^{(1)}$	$\cdots$	$x_{pq}^{(1)}$
$P^{(2)}$	$x_{11}^{(2)}$	$x_{22}^{(2)}$	$\cdots$	$x_{pq}^{(2)}$
$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$
$P^{(r)}$	$x_{11}^{(r)}$	$x_{22}^{(r)}$	$\cdots$	$x_{pq}^{(r)}$
Ideal Solution	x_{11}^*	x_{22}^*	$\cdots$	x_{pq}^*

Ideal Solution =

$$x_{ij}^* = \{min(x_{11}^{(1)},\ldots,x_{11}^{(r)}), min(x_{22}^{(1)},\ldots,x_{22}^{(r)},\ldots,min(x_{pq}^{(1)},\ldots,x_{pq}^{(r)})\}$$
$$= \{x_{11}^*, x_{22}^*, \ldots, x_{pq}^*\}.$$

A general procedure with K objectives is the following. As explained above, we will obtain K! (Factorial) different solutions by solving the K! problems arising for K! different priority structures.
Let $x_{ij}^{(r)} = \{x_{11}^{(r)}, x_{22}^{(r)}, \ldots, x_{pq}^{(r)}\}, 1 \leq r \leq K!$ be the K! number of solutions obtained by giving priorities to K objective functions. Let $(x_{11}^*, x_{22}^*, \ldots, x_{pq}^*)$ be the ideal solution. But in practice ideal solution can never be

achieved. The solution, which is closest to the ideal solution, is acceptable as the best compromise solution, and the corresponding priority structure is identified as most appropriate priority structure in the planning context. The D_1-distances of different solutions from the ideal solution defined below are then calculated. The solution corresponding to the minimum D_1-distance gives the best compromise solution.
Now,

$$(D_1)^r = \sum_{i=1}^{p}\sum_{j=1}^{q} |x_{ij}^* - x_{ij}^{(r)}|$$

is defined as the D_1-distance from the ideal solution $(x_{11}^*, x_{22}^*, \ldots, x_{pq}^*)$, to the r^{th} solution

$$\{x_{11}^{(r)}, x_{22}^{(r)}, \ldots, x_{pq}^{(r)}\}, 1 \leq r \leq P!$$

Therefore,

$$(D_1)_{opt} = min_{1\leq r\leq P!}(D_1)^r = min_{1\leq r\leq P!}\sum_{i=1}^{p}\sum_{j=1}^{q} |x_{ij}^* - x_{ij}^{(r)}|$$

D_1-distances are calculated from the ideal solution given below in table 2 as

Table 2: D_1-distances from the ideal solution

Priorities	x_{11}	$\cdots$	x_{pq}	$(D_1)^r$
$P^{(1)}$	$\lvert x_{11}^* - x_{11}^{(1)}\rvert$	$\cdots$	$\lvert x_{pq}^* - x_{pq}^{(1)}\rvert$	$\sum_{i=1}^{p}\sum_{j=1}^{q}\lvert x_{ij}^* - x_{ij}^{(r)}\rvert$
$P^{(2)}$	$\lvert x_{11}^* - x_{11}^{(2)}\rvert$	$\cdots$	$\lvert x_{pq}^* - x_{pq}^{(2)}\rvert$	$\sum_{i=1}^{p}\sum_{j=1}^{q}\lvert x_{ij}^* - x_{ij}^{(r)}\rvert$
$\vdots$	$\vdots$	$\cdots$	$\vdots$	$\vdots$
$P^{(r)}$	$\lvert x_{11}^* - x_{11}^{(r)}\rvert$	$\cdots$	$\lvert x_{pq}^* - x_{pq}^{(r)}\rvert$	$\sum_{i=1}^{p}\sum_{j=1}^{q}\lvert x_{ij}^* - x_{ij}^{(r)}\rvert$

Let the minimum be attained for r=t. Then

$$\{x_{11}^{(t)}, x_{22}^{(t)}, \ldots, x_{pq}^{(t)}\}$$

is the best compromise solution of the problem.

4 Numerical Illustration

In order to demonstrate the problem and the utility of the approaches discussed above, a numerical problem is presented. Here, we consider three origins and three destinations. The TP cost, time and the damage charges (both quality and quantity damage) during the transportation are represented in fractions.
Case I: When the $l_{ij} = 0$
Using the data given in matrices (3), (4) and (5), the multiobjective capacitated transportation problem with

Table 3: Cost Matrix

	b_1	b_2	b_3	Supply
a_1	5/3	7/4	15/13	≤ 12
a_2	8/12	17/14	12/7	=15
a_3	19/15	10/6	13/8	≥ 20
Demand	≥ 9	=13	≤ 21	

Table 4: Time Matrix

	b_1	b_2	b_3	Supply
a_1	17/9	5/2	10/3	≤ 12
a_2	1/2	11/4	6/5	=15
a_3	13/8	16/12	10/11	≥ 20
Demand	≥ 9	=13	≤ 21	

Table 5: Damage Charges Matrix

	b_1	b_2	b_3	Supply
a_1	13/8	15/9	8/11	≤ 12
a_2	11/15	14/6	19/7	=15
a_3	9/7	15/6	8/17	≥ 20
Demand	≥ 9	=13	≤ 21	

mixed constraints can be given as:

$$\left.\begin{array}{l}
\text{Minimize } C = \dfrac{5x_{11}+7x_{12}+15x_{13}+8x_{21}+17x_{22}+12x_{23}+19x_{31}+10x_{32}+13x_{33}}{3x_{11}+4x_{12}+13x_{13}+12x_{21}+14x_{22}+7x_{23}+15x_{31}+6x_{32}+8x_{33}} \\
\text{Minimize } D = \dfrac{13x_{11}+15x_{12}+8x_{13}+15x_{21}+14x_{22}+19x_{23}+9x_{31}+15x_{32}+8x_{33}}{8x_{11}+9x_{12}+11x_{13}+15x_{21}+6x_{22}+7x_{23}+7x_{31}+6x_{32}+17x_{33}} \\
\text{Minimize } T = \dfrac{17x_{11}+5x_{12}+10x_{13}+x_{21}+11x_{22}+6x_{23}+13x_{31}+16x_{32}+10x_{33}}{9x_{11}+2x_{12}+3x_{13}+2x_{21}+4x_{22}+5x_{23}+8x_{31}+12x_{32}+11x_{33}} \\
\text{Subject to } \sum_{j=1}^{3} x_{1j} \leq 12;\ \sum_{j=1}^{3} x_{2j} = 15;\ \sum_{j=1}^{3} x_{3j} \geq 20 \\
\qquad\qquad \sum_{i=1}^{3} x_{i1} \geq 9;\ \sum_{j=1}^{3} x_{i2} = 13;\ \sum_{j=1}^{3} x_{i3} \leq 21
\end{array}\right\}$$

The capacitated constraints are given below:
$0 \leq x_{11} \leq 6, 0 \leq x_{12} \leq 7, 0 \leq x_{13} \leq 13, 0 \leq x_{21} \leq 6, 0 \leq x_{22} \leq 2, 0 \leq x_{23} \leq 13, 0 \leq x_{31} \leq 4,$

$0 \leq x_{32} \leq 7, 0 \leq x_{33} \leq 14.$

4.1 Compromise solution using fuzzy programming with different membership functions

The payoff matrix for the case $[l_{ij} = 0]$obtained after solving the above problem separately for each problem using the optimizing software LINGO is as follows:

$$\text{Payoff Matrix} = \begin{array}{c} \\ x_{ij}^{(1)} \\ x_{ij}^{(2)} \\ x_{ij}^{(3)} \end{array} \begin{array}{c} \begin{array}{ccc} C & D & T \end{array} \\ \begin{bmatrix} 1.316832 & 1.16129 & 1.34472 \\ 1.37988 & 1.068410 & 1.79661 \\ 1.406433 & 1.170886 & 1.168285 \end{bmatrix} \end{array}$$

$C_u^1 = 1.406433, C_l^1 = 1.316832, D_u^2 = 1.170886, D_l^2 = 1.068410, T_u^3 = 1.79661$ and $T_l^3 = 1.168285$

Individual optimum solutions are obtained by solving the above problem separately for each objective using the optimizing software LINGO as follows:

Table 6: Individual optimum solution

Objectives	Objective values	Allocations								
		x_{11}	x_{12}	x_{13}	x_{21}	x_{22}	x_{23}	x_{31}	x_{32}	x_{33}
Cost	1.316832	0	4	5	2	6	7	4	7	9
Damage Charges	1.068410	0	7	0	6	2	7	3	4	14
Time	1.168285	0	6	0	6	0	9	3	7	12

An equivalent crisp problem for the linear membership function can be obtained as follows:-

$$\left.\begin{array}{l}
\text{Minimize } \delta \\
\text{Subject to} \\
\left(1.406433-\left(\dfrac{5x_{11}+7x_{12}+15x_{13}+8x_{21}+17x_{22}+12x_{23}+19x_{31}+10x_{32}+13x_{33}}{3x_{11}+4x_{12}+13x_{13}+12x_{21}+14x_{22}+7x_{23}+15x_{31}+6x_{32}+8x_{33}}\right)\right) \le 0.089601\delta \\
\left(1.79661-\left(\dfrac{17x_{11}+5x_{12}+10x_{13}+x_{21}+11x_{22}+6x_{23}+13x_{31}+6x_{32}+10x_{33}}{9x_{11}+2x_{12}+3x_{13}+2x_{21}+4x_{22}+5x_{23}+8x_{31}+12x_{32}+11x_{33}}\right)\right) \le 0.628325\delta \\
\left(1.170866-\left(\dfrac{13x_{11}+15x_{12}+8x_{13}+11x_{21}+14x_{22}+19x_{9}+9x_{31}+15x_{32}+8x_{33}}{8x_{11}+9x_{12}+11x_{13}+15x_{21}+6x_{22}+7x_{23}+7x_{31}+6x_{32}+17x_{33}}\right)\right) \le .102476\delta \\
\sum_{j=1}^{3} x_{1j} \le 12;\ \sum_{j=1}^{3} x_{2j} = 15;\ \sum_{j=1}^{3} x_{3j} \ge 20;\ \sum_{i=1}^{3} x_{i1} \ge 9;\ \sum_{i=1}^{3} x_{2i} = 13;\ \sum_{i=1}^{3} x_{i3} \le 21 \\
0 \le x_{11} \le 6, 0 \le x_{12} \le 7, 0 \le x_{13} \le 13, 0 \le x_{21} \le 6, 0 \le x_{22} \le 2, \\
0 \le x_{23} \le 13, 0 \le x_{31} \le 4, 0 \le x_{32} \le 7, 0 \le x_{33} \le 14
\end{array}\right\}$$

The compromise solution obtained by the LINGO software for linear membership function is as follows:-

$$x_{11}^{*}=4, x_{12}^{*}=4, x_{13}^{*}=4, x_{21}^{*}=5, x_{22}^{*}=2, x_{23}^{*}=8, x_{31}^{*}=4, x_{32}^{*}=7, x_{33}^{*}=9 \text{and} \delta=0$$

If we are using exponential membership function with parameter $\alpha = 1$, an equivalent crisp problem can be formulated as:

$$\left.\begin{array}{ll}
\text{Minimize } \delta & \\
\text{Subject to} & \dfrac{e^{\frac{-C-1.316832}{0.089601}}-e^{-1}}{1-e^{-1}} \le \delta \\
& \dfrac{e^{\frac{-T-1.168285}{0.628325}}-e^{-1}}{1-e^{-1}} \le \delta;\ \dfrac{e^{-\frac{D-1.068410}{.102476}}-e^{-1}}{1-e^{-1}} \le \delta \\
& \sum_{j=1}^{3} x_{1j} \le 12;\ \sum_{j=1}^{3} x_{2j} = 15;\ \sum_{j=1}^{3} x_{3j} \ge 20 \\
& \sum_{i=1}^{3} x_{i1} \ge 9;\ \sum_{i=1}^{3} x_{2i} = 13;\ \sum_{i=1}^{3} x_{i3} \le 21 \\
& 0 \le x_{11} \le 6, 0 \le x_{12} \le 7, 0 \le x_{13} \le 13, 0 \le x_{21} \le 6, 0 \le x_{22} \le 2, \\
& 0 \le x_{23} \le 13, 0 \le x_{31} \le 4, 0 \le x_{32} \le 7, 0 \le x_{33} \le 14
\end{array}\right\}$$

By optimizing software LINGO, the compromise solution is obtained as:

$$x_{11}^{*}=0, x_{12}^{*}=4, x_{13}^{*}=1, x_{21}^{*}=6, x_{22}^{*}=2, x_{23}^{*}=7, x_{31}^{*}=4, x_{32}^{*}=7, x_{33}^{*}=9 \text{and} \delta=0$$

If we are using hyperbolic membership function, an equivalent crisp problem (4) can be formulated as:

$$\left.\begin{array}{l}
\text{Minimize } \delta \\
\text{Subject to} \\
\frac{1}{2} tanh\left(\left(1.3616325-\left(\dfrac{17x_{11}+5x_{12}+10x_{13}+x_{21}+11x_{22}+6x_{23}+13x_{31}+6x_{32}+10x_{33}}{9x_{11}+2x_{12}+3x_{13}+2x_{21}+4x_{22}+5x_{23}+8x_{31}+12x_{32}+11x_{33}}\right)\right)66.9635\right)+\frac{1}{2} \le \delta \\
\frac{1}{2} tanh\left(\left(1.4824475-\left(\dfrac{17x_{11}+5x_{12}+10x_{13}+x_{21}+11x_{22}+6x_{23}+13x_{31}+6x_{32}+10x_{33}}{9x_{11}+2x_{12}+3x_{13}+2x_{21}+4x_{22}+5x_{23}+8x_{31}+12x_{32}+11x_{33}}\right)\right)9.5492\right)+\frac{1}{2} \le \delta \\
\frac{1}{2} tanh\left(\left(1.119648-\left(\dfrac{13x_{11}+15x_{12}+8x_{13}+11x_{21}+14x_{22}+19x_{9}+9x_{31}+15x_{32}+8x_{33}}{8x_{11}+9x_{12}+11x_{13}+15x_{21}+6x_{22}+7x_{23}+7x_{31}+6x_{32}+17x_{33}}\right)\right)58.5503\right)+\frac{1}{2} \le \delta \\
\sum_{j=1}^{3} x_{1j} \le 12;\ \sum_{j=1}^{3} x_{2j} = 15;\ \sum_{j=1}^{3} x_{3j} \ge 20;\ \sum_{i=1}^{3} x_{i1} \ge 9;\ \sum_{i=1}^{3} x_{2i} = 13;\ \sum_{i=1}^{3} x_{i3} \le 21 \\
0 \le x_{11} \le 6, 0 \le x_{12} \le 7, 0 \le x_{13} \le 13, 0 \le x_{21} \le 6, 0 \le x_{22} \le 2, \\
0 \le x_{23} \le 13, 0 \le x_{31} \le 4, 0 \le x_{32} \le 7, 0 \le x_{33} \le 14
\end{array}\right\}$$

By optimizing software LINGO, the compromise solution is obtained as:

$$x_{11}^{*}=4, x_{12}^{*}=4, x_{13}^{*}=4, x_{21}^{*}=5, x_{22}^{*}=2, x_{23}^{*}=8, x_{31}^{*}=4, x_{32}^{*}=7, x_{33}^{*}=9 \text{and} \delta=0$$

Case II: When the $l_{ij} \ge 0$
Using the data given in matrices (3), (4) and (5), the multiobjective capacitated transportation problem with mixed constraints can be given as:

$$\left.\begin{array}{ll}
\text{Minimize } C = & \dfrac{5x_{11}+7x_{12}+15x_{13}+8x_{21}+17x_{22}+12x_{23}+19x_{31}+10x_{32}+13x_{33}}{3x_{11}+4x_{12}+13x_{13}+12x_{21}+14x_{22}+7x_{23}+15x_{31}+6x_{32}+8x_{33}} \\
\text{Minimize } D = & \dfrac{13x_{11}+15x_{12}+8x_{13}+15x_{21}+14x_{22}+19x_{23}+9x_{31}+15x_{32}+8x_{33}}{8x_{11}+9x_{12}+11x_{13}+15x_{21}+6x_{22}+7x_{23}+7x_{31}+6x_{32}+17x_{33}} \\
\text{Minimize } T = & \dfrac{17x_{11}+5x_{12}+10x_{13}+x_{21}+11x_{22}+6x_{23}+13x_{31}+16x_{32}+10x_{33}}{9x_{11}+2x_{12}+3x_{13}+2x_{21}+4x_{22}+5x_{23}+8x_{31}+12x_{32}+11x_{33}} \\
\text{Subject to} & \sum_{j=1}^{3} x_{1j} \le 12;\ \sum_{j=1}^{3} x_{2j} = 15;\ \sum_{j=1}^{3} x_{3j} \ge 20 \\
& \sum_{i=1}^{3} x_{i1} \ge 9;\ \sum_{i=1}^{3} x_{i2} = 13;\ \sum_{i=1}^{3} x_{i3} \le 21
\end{array}\right\}$$

The capacitated constraints are given below:
$1 \le x_{11} \le 6, 2 \le x_{12} \le 7, 4 \le x_{13} \le 13, 2 \le x_{21} \le 6, 0 \le x_{22} \le 2, 5 \le x_{23} \le 13, 1 \le x_{31} \le 4,$

$2 \le x_{32} \le 7, \le x_{33} \le 14.$

4.2 *Compromise solution using fuzzy programming with different membership functions*

The payoff matrix for the case $[l_{ij} \ge 0]$ obtained after solving the above problem separately for each problem using the optimizing software LINGO is as follows:

$$\text{Payoff Matrix} = \begin{array}{c} \\ x_{ij}^{(1)} \\ x_{ij}^{(2)} \\ x_{ij}^{(3)} \end{array} \begin{array}{c} \begin{array}{ccc} C & D & T \end{array} \\ \begin{bmatrix} 1.31941 & 1.169133 & 1.360927 \\ 1.33 & 1.147303 & 1.333333 \\ 1.33333 & 1.152542 & 1.317881 \end{bmatrix} \end{array}$$

$$C_u^1 = 1.33333, C_l^1 = 1.31941, D_u^2 = 1.169133, D_l^2 = 1.147303, T_u^3 = 1.360927 \text{ and } T_l^3 = 1.317881$$

Individual optimum solutions are obtained by solving the above problem separately for each objective using the optimizing software LINGO as follows:

Table 7: Individual optimum solution

Objectives	Objective values	Allocations								
		x_{11}	x_{12}	x_{13}	x_{21}	x_{22}	x_{23}	x_{31}	x_{32}	x_{33}
Cost	1.31941	1	4	5	6	2	7	4	7	9
Damage Charges	1.147303	1	5	4	6	2	7	4	6	10
Time	1.168285	1	4	4	6	2	7	3	7	10

An equivalent crisp problem for the linear membership function can be obtained as follows:-

$$\left.\begin{array}{l}
\text{Minimize } \delta \\
\text{Subject to} \\
\left(1.33333-\left(\dfrac{5x_{11}+7x_{12}+15x_{13}+8x_{21}+17x_{22}+12x_{23}+19x_{31}+10x_{32}+13x_{33}}{3x_{11}+4x_{12}+13x_{13}+12x_{21}+14x_{22}+7x_{23}+15x_{31}+6x_{32}+8x_{33}}\right)\right)\le 0.01392\delta \\
\left(1.360927-\left(\dfrac{17x_{11}+5x_{12}+10x_{13}+x_{21}+11x_{22}+6x_{23}+13x_{31}+6x_{32}+10x_{33}}{9x_{11}+2x_{12}+3x_{13}+2x_{21}+4x_{22}+5x_{23}+8x_{31}+12x_{32}+11x_{33}}\right)\right)\le 0.043046\delta \\
\left(1.169133-\left(\dfrac{13x_{11}+15x_{12}+8x_{13}+11x_{21}+14x_{22}+19x_{9}+9x_{31}+15x_{32}+8x_{33}}{8x_{11}+9x_{12}+11x_{13}+15x_{21}+6x_{22}+7x_{23}+7x_{31}+6x_{32}+17x_{33}}\right)\right)\le .02183\delta \\
\sum_{j=1}^{3}x_{1j}\le 12;\ \sum_{j=1}^{3}x_{2j}=15;\ \sum_{j=1}^{3}x_{3j}\ge 20;\ \sum_{i=1}^{3}x_{i1}\ge 9;\ \sum_{i=1}^{3}x_{2i}=13;\ \sum_{i=1}^{3}x_{i3}\le 21 \\
1\le x_{11}\le 6, 2\le x_{12}\le 7, 4\le x_{13}\le 13, 2\le x_{21}\le 6, 0\le x_{22}\le 2, \\
5\le x_{23}\le 13, 1\le x_{31}\le 4, 2\le x_{32}\le 7, 5\le x_{33}\le 14
\end{array}\right\}$$

The compromise solution obtained by the LINGO software for linear membership function is as follows:-

$$x_{11}^{*}=4, x_{12}^{*}=4, x_{13}^{*}=4, x_{21}^{*}=5, x_{22}^{*}=2, x_{23}^{*}=8, x_{31}^{*}=4, x_{32}^{*}=7, x_{33}^{*}=9 \text{and} \delta=0$$

If we are using exponential membership function with parameter $\alpha = 1$, an equivalent crisp problem can be formulated as:

$$\left.\begin{array}{ll}
\text{Minimize } \delta & \\
\text{Subject to} & \dfrac{e^{\frac{-(C-1.31941)}{0.01392}}-e^{-1}}{1-e^{-1}}\le\delta \\
& \dfrac{e^{\frac{-(T-1.169133)}{0.021803}}-e^{-1}}{1-e^{-1}}\le\delta;\ \dfrac{e^{-\frac{-(D-1.360927)}{.043046}}-e^{-1}}{1-e^{-1}}\le\delta \\
& \sum_{j=1}^{3}x_{1j}\le 12;\ \sum_{j=1}^{3}x_{2j}=15;\ \sum_{j=1}^{3}x_{3j}\ge 20 \\
& \sum_{i=1}^{3}x_{i1}\ge 9;\ \sum_{i=1}^{3}x_{2i}=13;\ \sum_{i=1}^{3}x_{i3}\le 21 \\
& 1\le x_{11}\le 6, 2\le x_{12}\le 7, 4\le x_{13}\le 13, 2\le x_{21}\le 6, 0\le x_{22}\le 2, \\
& 5\le x_{23}\le 13, 1\le x_{31}\le 4, 2\le x_{32}\le 7, 5\le x_{33}\le 14
\end{array}\right\}$$

By optimizing software LINGO, the compromise solution is obtained as:

$$x_{11}^{*}=4, x_{12}^{*}=4, x_{13}^{*}=4, x_{21}^{*}=6, x_{22}^{*}=2, x_{23}^{*}=7, x_{31}^{*}=4, x_{32}^{*}=7, x_{33}^{*}=9 \text{and} \delta=0$$

If we are using hyperbolic membership function, an equivalent crisp problem (4) can be formulated as:

$$\left.\begin{array}{l}
\text{Minimize } \delta \\
\text{Subject to} \\
\frac{1}{2}tanh\left(\left(1.32637-\left(\dfrac{17x_{11}+5x_{12}+10x_{13}+x_{21}+11x_{22}+6x_{23}+13x_{31}+6x_{32}+10x_{33}}{9x_{11}+2x_{12}+3x_{13}+2x_{21}+4x_{22}+5x_{23}+8x_{31}+12x_{32}+11x_{33}}\right)\right)431.0344\right)+\frac{1}{2}\le\delta \\
\frac{1}{2}tanh\left(\left(1.15848-\left(\dfrac{13x_{11}+15x_{12}+8x_{13}+11x_{21}+14x_{22}+19x_{9}+9x_{31}+15x_{32}+8x_{33}}{8x_{11}+9x_{12}+11x_{13}+15x_{21}+6x_{22}+7x_{23}+7x_{31}+6x_{32}+17x_{33}}\right)\right)275.19151\right)+\frac{1}{2}\le\delta \\
\frac{1}{2}tanh\left(\left(1.339404-\left(\dfrac{17x_{11}+5x_{12}+10x_{13}+x_{21}+11x_{22}+6x_{23}+13x_{31}+6x_{32}+10x_{33}}{9x_{11}+2x_{12}+3x_{13}+2x_{21}+4x_{22}+5x_{23}+8x_{31}+12x_{32}+11x_{33}}\right)\right)139.38577\right)+\frac{1}{2}\le\delta \\
\sum_{j=1}^{3}x_{1j}\le 12;\ \sum_{j=1}^{3}x_{2j}=15;\ \sum_{j=1}^{3}x_{3j}\ge 20;\ \sum_{i=1}^{3}x_{i1}\ge 9;\ \sum_{i=1}^{3}x_{2i}=13;\ \sum_{i=1}^{3}x_{i3}\le 21 \\
1\le x_{11}\le 6, 2\le x_{12}\le 7, 4\le x_{13}\le 13, 2\le x_{21}\le 6, 0\le x_{22}\le 2, \\
5\le x_{23}\le 13, 1\le x_{31}\le 4, 2\le x_{32}\le 7, 5\le x_{33}\le 14
\end{array}\right\}$$

By optimizing software LINGO, the compromise solution is obtained as:

$$x_{11}^{*}=4, x_{12}^{*}=4, x_{13}^{*}=4, x_{21}^{*}=5, x_{22}^{*}=2, x_{23}^{*}=8, x_{31}^{*}=4, x_{32}^{*}=7, x_{33}^{*}=9 \text{and} \delta=0$$

4.3 Compromise solution using lexicographic goal programming with minimum distance

Since we have three objectives of minimizing transportation cost,time and damage charges in our problem, so we have to solve 3! = 6 problems according to the priority. The solutions obtained for the two cases I and II by giving priority to each of the objectives one by one are given below in the Tables (9) and (8)for the case I and II respectively (all the problems are solved by optimization software LINGO):

Table 8: Ideal solutions for case I

Priority structure	x_{11}	x_{12}	x_{13}	x_{21}	x_{22}	x_{23}	x_{31}	x_{32}	x_{33}
P[C,D,T]	0	4	1.8892	6	2	7	3	7	11.4361
P[C,T,D]	0	4	1.8892	6	2	7	3	7	11.4361
P[D,C,T]	0	7	0	6	2	7	3.1340	4	14
P[D,T,C]	0	7	0	6	2	7	3	4	14
P[T,C,D]	0	5.2547	0	6	2	7	3	5.7453	14
P[T,D,C]	0	5.2547	0	6	2	7	3	5.7453	14
Ideal solution (x_{ij}^{*})	0	4	0	6	2	7	3	4	11.4361

Table 9: Ideal solutions for case II

Priority structure	x_{11}	x_{12}	x_{13}	x_{21}	x_{22}	x_{23}	x_{31}	x_{32}	x_{33}
P[C,D,T]	1	4.002	4.323	6	2	7	3.325	6.998	9.677
P[C,T,D]	1	4	4.277	6	2	7	3.363	7	9.637
P[D,C,T]	1	4.556	4	6	2	7	3.556	6.444	10
P[D,T,C]	1	4.341	4	6	2	7	3.341	6.659	10
P[T,C,D]	1	4	4	6	2	7	3	7	10
P[T,D,C]	1	4	4	6	2	7	3	7	10
Ideal solution (x_{ij}^{*})	1	4	4	6	2	7	3	6.444	9.637

Using the ideal solution, D_1-distances are calculated as shown in Tables (10)and (11)for the case I and II respectively.

Table 10: Case I: The D_1-distance from the ideal solutions

Priority structure	x_{11}	x_{12}	x_{13}	x_{21}	x_{22}	x_{23}	x_{31}	x_{32}	x_{33}	$(D_1)^r$
P[C,D,T]	0	0	1.8892	0	0	0	0	3	0	4.8892
P[C,T,D]	0	0	1.8892	0	0	0	0	3	0	4.8892
P[D,C,T]	0	3	0	0	0	0	0.1340	0	2.5639	5.6979
P[D,T,C]	0	3	0	0	0	0	0	0	2.5639	5.6979
P[T,C,D]	0	1.2547	0	0	0	0	0	1.7453	2.5639	5.6979
P[T,D,C]	0	1.2547	0	0	0	0	0	1.7453	2.5639	5.6979

Table 11: Case II: The D_1-distance from the ideal solutions

Priority structure	x_{11}	x_{12}	x_{13}	x_{21}	x_{22}	x_{23}	x_{31}	x_{32}	x_{33}	$(D_1)^r$
P[C,D,T]	0	0.002	0.323	0	0	0	0.325	0.554	0.04	1.244
P[C,T,D]	0	0	0.277	0	0	0	0.363	0.556	0	1.196
P[D,C,T]	0	0.556	0	0	0	0	0	0.556	0.363	1.475
P[D,T,C]	0	0.341	0	0	0	0	0.341	0.215	0.363	1.26
P[T,C,D]	0	0	0	0	0	0	0	0.556	0.363	0.919
P[T,D,C]	0	0	0	0	0	0	0	0	0.556	0.363

5 Discussion

5.1 Case I

From the calculations done in section 4 & summarized results in Table (12), it can be seen that FP with exponential membership function derives the optimum compromise solution as compared to the other methods.

5.2 Case II

It can be seen that when $l_{ij} \geq 0$, the LGP gives the best result out of all the above methods used.

6 Conclusion and Summary

This article derives the compromise solution of MOCFTP with mixed constraints using FP approach, in which three different forms of membership functions viz. linear, exponential and hyperbolic are used along with LGP with minimum distances approach. also,two cases are studied for two values of The results are summarized in the Table (12)(13).

Table 12: Compromise optimum solution for case I

	Objective values		
Methods	Cost	Damage charges	Time
FP with Linear membership function	1.359296	1.238494	1.389058
FP with Exponential membership function	1.349030	1.229213	1.314935
FP with Hyperbolic membership function	1.359296	1.238494	1.389058
LGP with D_1-distance	1.353103	1.129854	1.237344

Table 13: Compromise optimum solution for case II

	Objective values		
Methods	Cost	Damage charges	Time
FP with Linear membership function	1.359296	1.238494	1.389058
FP with Exponential membership function	1.332506	1.201646	1.386503
FP with Hyperbolic membership function	1.359296	1.238494	1.389058
LGP with D_1-distance	1.3333	1.152542	1.317881

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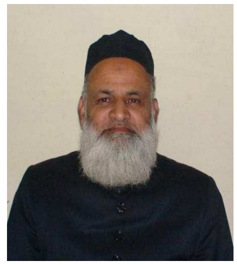

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