Progress in Fractional Differentiation and Applications
An International Journal

http://dx.doi.org/10.18576/pfda/030206

Fractional Order Controller Design and Analysis for Crane System

Abhaya Pal Singh[1,*], *Trisha Srivastava*[1], *Himanshu Agrawal*[1] *and Pallavi Srivastava*[2]

[1] Symbiosis International University, Symbiosis Institute of Technology, Pune, India
[2] SKN Sinhgad Institute of Technology and Science, Lonavala, India

Received: 18 May 2016, Revised: 17 Jul. 2016, Accepted: 20 Jul. 2016
Published online: 1 Apr. 2017

Abstract: This paper presents an example in which the fractional order calculus (FOC) provides better results than the traditional integer order system. The dynamical system considered here is under-actuated and has 2 degrees of freedom along with 1 control input which is 2-D Gantry crane system. Control of this crane system is essential, as its failure causes accident that can harm people nearby. This paper designs IOPID controller (integer order PID) and FOPID controller (fractional order PID) for the considered system and their performances are compared to validate that FOPID is better controller than IOPID.

Keywords: Mathematical Modeling, 2-D Gantry crane, FOC, PID controller, FOPID controller, Dynamical system, Euler-Lagrangian equation (EL), Lyapunov stability, Lagrangian, system modeling, launch vehicle (LV).

1 Introduction

The concept of fractional order calculus is as old as the conventional integer order calculus. In the last decade, a growing number of works by many researchers from various fields of science and engineering deal with dynamical systems described by fractional partial differential equations [1], [2], [3], [4], [5], [6], [7], [8], [9]. A FO system (fractional-order) characterized by fractional integral/differential operators/equations. Fractional controllers having 2 extra parameters as compared to conventional PID controllers; therefore, 2 more variables can be controlled and hence 2 more specifications can be achieved that may improves the system performance [10], [11], [12], [13], [14].

FOC emerges in $7^t h$ century when notions of operators $1^s t$ proposed by Leibniz's through a letter to L'Hopital. Since then the progress in the area of FDO were supported by many mathematicians/scientists [15], [16], [17], [18], [19]. FOC was not eligible for any applications in spite of its long history because of the lack of geometric interpretation, physical interpretation and its complexity [20]. Application of FOC into the real-world is only 4 decades old. Meanwhile, fractional calculus acquired much attraction of researchers due to its vast applications in engineering and technology, i.e. towards electrochemistry, towards porous media, towards control, towards viscoelasticity [21], [22] etc. The fractional order of the systems can give the information about the behavior of processes and materials [23]. Currently there are various concepts of FOC is being extended towards the development of control systems [24], [25], in signal filtering [26] and the observer discussed in [27] can also be extend with the help of FOC towards the fractional observer. [1] proposed a generalized PID controller as $PI^\alpha D^\beta$ which have integrator of order 'α' and differentiator of order 'β'.

1.1 Preliminaries

Fractional order calculus (FOC) gives the generalization of the Integer order calculus (IOC). The generalization to real order is as follows:

* Corresponding author e-mail: abhaya.aps@gmail.com

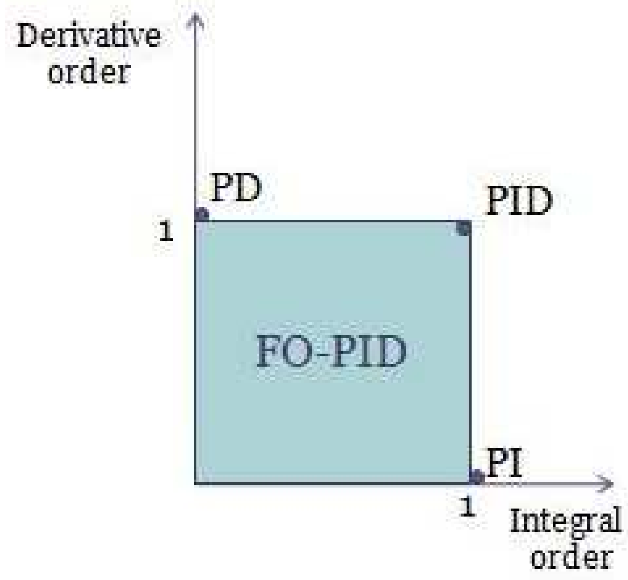

Fig. 1: Regions for FOC and IOC

$$aD_t^{\alpha} = \begin{cases} \frac{d^{\alpha}}{dt^{\alpha}} & \text{if } \alpha > 0 \\ 1 & \text{if } \alpha = 0 \\ \int_0^t d\tau^{\alpha} & \text{if } \alpha < 0, \end{cases}$$

where $\alpha \ \varepsilon \ R$.

Different notions of FOC has developed by various mathematicians [21], [22]:

–Riemann-Liouville (RL):
For $0 < \alpha < 1$ expression is:-

$$aD_t^{\alpha} = \frac{1}{\Gamma(n-\alpha)} \frac{d^n}{dt^n} \int_0^t \frac{f(\tau)}{(t-\tau)^{\alpha-n+1}} d\tau,$$

–Grünwald-Letnikov (GL):
Taking $n = \frac{(t-a)}{h}$ in which 'a' some constant then by GL:-

$$aD_t^{\alpha} = \lim_{h\to 0} \frac{1}{h^{\alpha}} \sum_{m=0}^{\frac{t-a}{h}} (-1)^m \frac{\Gamma(\alpha+1)}{m!\Gamma(\alpha-m+1)} f(t-mh),$$

–Caputo:
This is defined when '$n-1 < \alpha < n$':-

$$aD_t^{\alpha} = \frac{1}{\Gamma(m-\alpha)} \int_a^t \frac{f^m(\tau)}{(t-\tau)^{\alpha-m+1}} d\tau.$$

The Riemann-Liouville and Caputo formulation coincide with each other, when the initial condition taken to be zero.

This paper provides the FOPID controller design and analysis for the considered crane and compares it with the traditional PID controller to prove FOPID is better controller than PID controller.

2 Modeling of the System

Systems that can't be controlled to follow any trajectory are termed as under-actuated systems [28] because these systems have the actuators less than the degrees of freedom.

2-D Gantry crane is used to transport load from a point to another as shown in Fig. 2. These cranes able to handle heavy loads and are used in factories, depots and ships etc. So, the modeling and control of these systems are necessary which is discussed in this paper. For a basic 2-D Gantry crane, the equations of motion can be derived while assuming the following:

1. The system begins from equilibrium with zero initial conditions.
2. The load movement is limited for a few degrees to satisfy linear model.
3. Cable length is constant and weight of the cable is negligible.
4. A step input is applied.

Let us assume that $'m'$ is load mass and $'M'$ is trolley mass, the cable length to be 'l', force applied on this system is to be '$u(t)$' and '$(m*g)$' as gravity force. The position of the trolley is denoted by '$x(t)$' and '$\theta(t)$' is the tilt angle. Design Requirements:

– Settling time T_s less than 7 seconds.
– Peak overshoot value to be max $\leq 22.5\%$.

There are various methods in which one can get modeling equations of a dynamical system. This paper uses Euler-Lagrangian (EL) method. To model a system using EL it is required to find energies of the considered system.

$$\pounds = T - V \tag{1}$$

Where, £ = Lagrangian , V = Potential Energy, T = Kinetic Energy.

From Fig. 2, one can easily get the position coordinates of the load as $(x + lsin\theta, -lcos\theta)$.

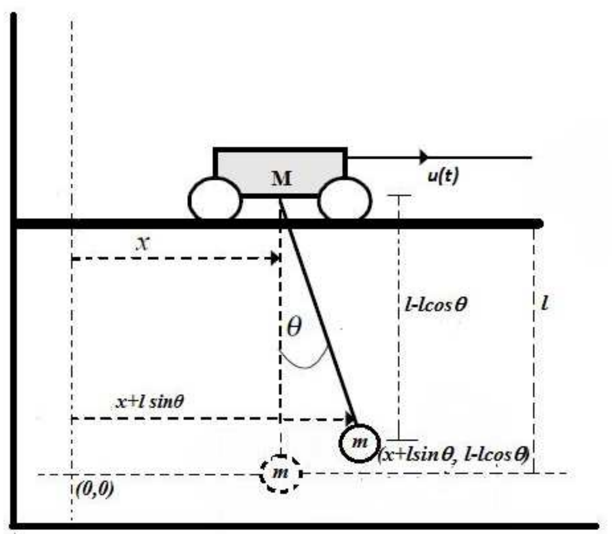

Fig. 2: Schematic diagram of 2-D Gantry crane

So, the total 'V' and the 'T' of the whole system can be obtained as

$$V_{total} = V_{trolly} + V_{load}$$
$$V_{total} = Mgl + mg(l - lcos\theta)$$
$$T_{total} = T_trolly + T_load$$
$$T_{total} = \frac{1}{2}M\dot{x}^2 + \frac{1}{2}m((\dot{x}^2) + l^2\dot{\theta}^2 + 2\dot{x}\dot{\theta}lcos\theta).$$

Therefore, the Lagrangian of the system can be represented as:

$$\pounds = T_{total} - V_{total}$$

$$\pounds = \frac{1}{2}M\dot{x}^2 + \frac{1}{2}m((\dot{x}^2) + l^2\dot{\theta}^2 + 2\dot{x}\dot{\theta}lcos\theta) - (Mgl + mg(l - lcos\theta)). \tag{2}$$

Putting Lagrangian '£' from (2) to EL equations shown below

$$\frac{d}{dt}\left(\frac{\partial \pounds}{\partial \dot{x}}\right) - \frac{\partial \pounds}{\partial x} = u$$

and

$$\frac{d}{dt}\left(\frac{\partial \pounds}{\partial \dot{\theta}}\right) - \frac{\partial \pounds}{\partial \theta} = 0.$$

We get,

$$(M+m)\ddot{x} + ml\ddot{\theta}cos\theta - ml\dot{\theta}^2 sin\theta = u \tag{3}$$

$$l\ddot{\theta} + \ddot{x}cos\theta + gsin\theta = 0. \tag{4}$$

Finding $\ddot{x}$ and $\ddot{\theta}$,

$$\ddot{x} = \frac{u + ml\dot{\theta}^2\sin\theta + gmsin\theta\cos\theta}{M + m - m\cos^2\theta} \tag{5}$$

$$\ddot{\theta} = \frac{u\cos\theta + ml\dot{\theta}^2\sin\theta\cos\theta + (M+m)gsin\theta}{mlcos^2\theta - (M+m)l}. \tag{6}$$

Casting eqn. (5) and eqn. (6) into standard non-linear state-space as

$$\frac{d}{dt}z = f(\underline{z}, u, t).$$

Let,$z_1 = x$, $z_2 = \dot{x} = \dot{z_1}$, $z_3 = \theta$, and $z_4 = \dot{\theta} = \dot{z_3}$.
The final nonlinear state-space matrices of 2-D Gantry crane system will be

$$\frac{d}{dt}\begin{bmatrix} z_1 \\ z_2 \\ z_3 \\ z_4 \end{bmatrix} = \begin{bmatrix} z_2 \\ \frac{u + mlz_4{}^2\sin z_3 + gm\sin z_3\cos z_3}{M + m - m\cos^2 z_3} \\ z_4 \\ \frac{u\cos z_3 + mlz_4{}^2\sin z_3\cos z_3 + g(M+m)\sin z_3}{ml\cos^2 z_3 - (M+m)l} \end{bmatrix}.$$

Linearizing the state-space obtained by taking Jacobian [28], [29], [30] at $(\underline{z}_0), u_0) = (0,0)$ as

$$\frac{d}{dt}\delta\underline{z} = J_{=\underline{z}}(\underline{z}_0, u_0)\delta\underline{z} + J_{=\underline{z}}(\underline{z}_0, u_0)\delta u.$$

After some mathematical manipulations the linearized matrix is,

$$\frac{d}{dt}\delta\underline{z} = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & \frac{mg}{M} & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & \frac{-(M+m)g}{Ml} & 0 \end{bmatrix}\delta\underline{z} + \begin{bmatrix} 0 \\ \frac{1}{M} \\ 0 \\ \frac{-1}{Ml} \end{bmatrix}\delta u.$$

And the output matrix as,

$$y = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}\begin{bmatrix} x \\ \dot{x} \\ \theta \\ \dot{\theta} \end{bmatrix}.$$

Table 1: System parameters

Parameter	Description	Value
m	Mass of the load	1kg
M	Mass of the trolley	0.25kg
g	Gravitational Constant	9.8m/s^2
l	Length of the cable	0.6m

Table 2: PID Controller parameters

Controller	Gains		
	k_p	k_i	k_d
PID Theta Controller	-5	4	-0.1
PID Position Controller	5	2	3

3 Controller Design

For comparing IOPID and FOPID controllers, we would design both type of controllers for the considered 2-D Gantry crane and then compare the responses.

3.1 Integer Order PID Controllers

Position controller controls the position of the movable trolley and angle (theta) controller is to control's the load position so that it moves to the desired position and suppress the swing angle of the cable as fast as possible. For the purpose of designing the controllers we need the values of the parameters of the crane system. Mahmud Iwan Solihin and Wahyudi proposed a lab-scale experimental set up in [31] for a gantry crane system. Referring the same, we take the parameters for controller design as:

In order to achieve fast motion of trolley with small overshoot, the PID position controller is optimized by considering the following desired specifications:

–Overshoot $\leq$ 22.5%,
–Settling time $\leq$ 7s,
–Steady state error $\leq \pm$1%.

After substituting the values of the system parameters into the linearized state space model of the system in equation, we get,

$$\frac{d}{dt}\delta \underline{z} = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 3.92 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & -81.67 & 0 \end{bmatrix} \delta \underline{z} + \begin{bmatrix} 0 \\ 4 \\ 0 \\ -6.67 \end{bmatrix} \delta u$$

And the output matrix as

$$y = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} x \\ \dot{x} \\ \theta \\ \dot{\theta} \end{bmatrix}$$

Using PID Tuner block of MATLAB SIMULINK, the PID controller parameters are.

Table 3: FOPID Controller parameters

Controller	Gains				
	k_p	k_i	k_d	α	β
FOPID Theta Controller	-5	4	-0.1	0.8	0.9
FOPID Position Controller	5	2	3	0.95	0.9

Table 4: Comparison of Controllers

Performance	Controller	
	IOPID	FOPID
Amplitude in radians (angle)	1	1
Settling Time in sec (angle)	6.4	2.5
Amplitude in m (trolley position)	1.3	1.4
Settling Time in sec (trolley position)	2.5	1.5

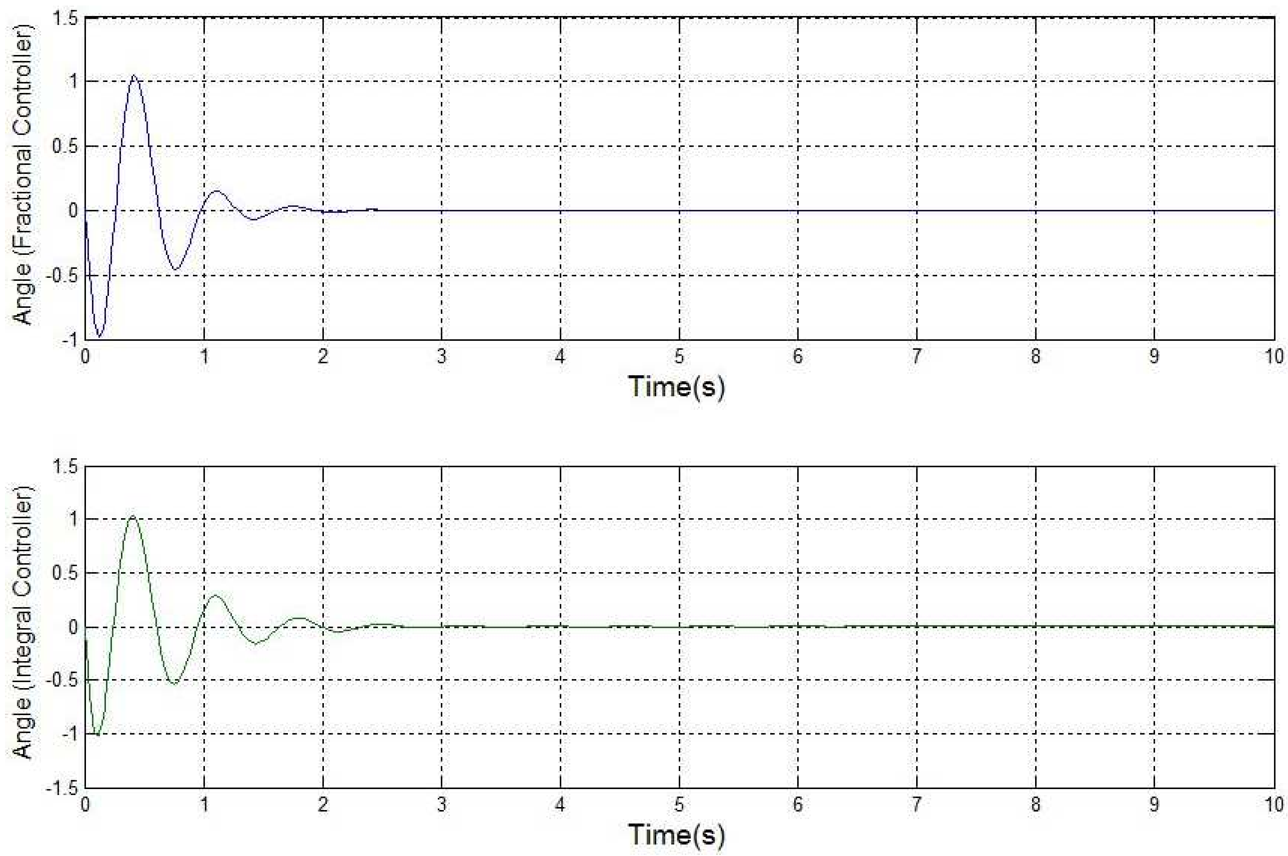

Fig. 3: Angle Comparison of the Controllers

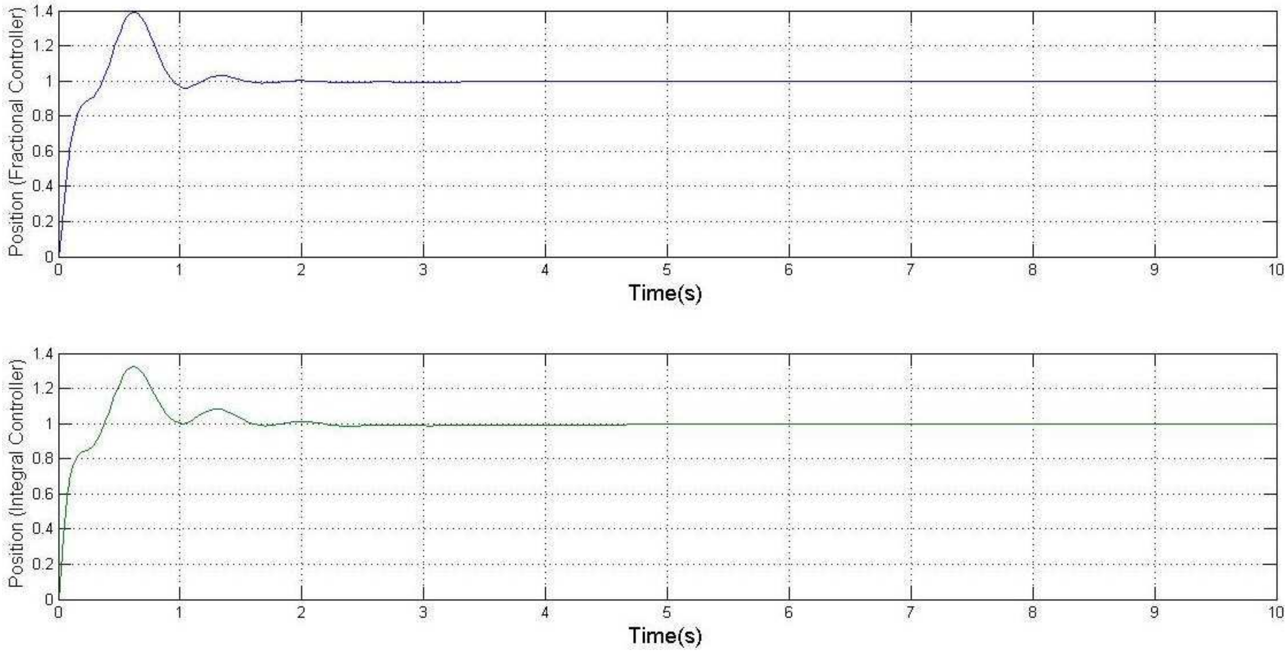

Fig. 4: Position Comparison of the controllers.

3.2 Fractional Order PID Controllers

FOPID controllers have five parameters that can be controlled while IOPID controllers have only three of them. Here α and β are the order of integration and differentiation respectively that can take any arbitrary value. The values of α and β are considered to be 1(only integral) in IOPID controllers. For the purpose of comparison we have kept the values of gains k_p, k_i and k_d in FOPID same as those in IOPID controllers and only change the values of α and β ranging between 0 and 1. Table 3 lists the obtained FOPID controller parameters.

3.3 Comparison

Comparing the results of IOPID and FOPID controllers, it can be observed that, the swing angle can be controlled in a very less time.

Observing Table 4, Fig. (4), and Fig. (5), we can conclude that settling time of FOPID controller is less than that of IOPID controller for theta controller as well as position controller. For the chosen values of α and β the amplitude of FOPID controller and IOPID controller is always under the specified range.

4 Conclusion

The main aim of controlling the swing angle as well as position is achieved in this paper for 2-D Gantry crane system. The FOPID proves better controller as compared on the basis of settling time which is summarizes in Table 4. The FOPID controllers utilize two more parameters to provide more flexible PID controllers. Even the small changes in these two parameters may provide better controller response. The result shows that the FOPID controllers perform better as compared to the IOPID controllers for controlling the swing angle as well as position of 2-D Gantry crane system.

References

[1] I. Podlubny, Fractional-order systems and PI$^\alpha$ D$^\beta$ controllers, *IEEE Trans. Aut. Contr.* **44**(1), 208-214 (1999).

[2] K. B. Oldhamand and J. Spanier, *The Fractional Calculus*, Academic Press, New York, 1974.

[3] C. H. Lubich, Discretized fractional calculus, *SIAM J. Math. Anal.*, 704-719 (1986).
[4] T. Srivastava, A. P. Singh and H. Agarwal, Modeling the under-actuated mechanical system with fractional order derivative, *Progr. Fract. Differ. Appl.* (**1**)(1) 57-64 (2015).
[5] A. Oustaloup, Fractional order sinusoidal oscillators: optimization and their use in highly linear FM modulators, *IEEE Trans. Circ. Syst.*, 1007-1009 (1981).
[6] M. Chengbin and Y. Hori, *The application of fractional order PID controller for robust two-inertia speed control*, Proceedings of the 4th International Power Electronics and Motion Control Conference, August, 2004.
[7] D. Baleanu and Om P. Agrawal, Fractional Hamilton formalism within Caputo's derivative, *Czech. J. Phys.* **56**(10-11), 1087-1092 (2006).
[8] D. Baleanu, S. I. Muslih and K. Tas, Fractional Hamiltonian analysis of higher order derivatives systems, *J. Math. Phys.* **47**(10), 103503 (2006).
[9] S. I. Muslih and D. Baleanu, Formulation of Hamiltonian equations for fractional variational problems, *Czech. J. Phys.* **55**(6), 633-642 (2005).
[10] C. A. Monje, B. M. Vinagre, Y. Chen, V. Feliu, P. Lanusse, J. Sabatier, Optimal tunings for fractional PI^{α} D^{β}, *Fractional Derivatives an Their Applications*, A. Le Mehaute, J. A. Tenreiro Machado, J. J. Trigeassou, J. Sabatier, J.(eds.) UBooks, Augsburg 3, 675-698 (2005).
[11] A. P. Singh, F.S. Kazi, N. M. Singh, P. Srivastava, $PI\alpha$ $D\beta$ controller design for under-actuated mechanical systems, *Control Automation Robotics & Vision (ICARCV), 2012*, 1654–1658, (2012).
[12] P. Shah, S. D. Agashe aand A. P. Singh, Fractional order modeling using state space theory, *Int. J. Engn. Tech.* **5**(3), 2891-2894 (2013).
[13] A. P. Singh, H. Agarwal and P. Srivastava, Fractional order controller design for inverted pendulum on a cart system (POAC), *WSEAS Trans. Syst. Contr.* (**10**), 172-178 (2015).
[14] A. P. Singh, F. Kazi, N. M. Singh and V. Vyawahare, *Fractional order controller design for under-actuated mechanical systems*, In The 5th IFAC Symposium on Fractional Differentiation and its Applications-FDA, 2012.
[15] I. Podlubny, *Fractional Differential Equations*, Academic Press, San Diego - New York - London, 1999.
[16] B. Ross, A brief history and exposition of the fundamental theory of fractional calculus, *Fractional Calculus and its Applications*, Springer Berlin Heidelberg, 1-36 (1975).
[17] J. T. Machado, V. Kiryakova and F. Mainardi, Recent history of fractional calculus, *Commun. Nonlin. Sci. Numer. Simul.* **16**(3), 1140-1153 (2011).
[18] J. T. Machado, V. Kiryakova and F. Mainardi, A poster about the recent history of fractional calculus, *Fract. Calc. Appl. Anal.* **13**(3), 329-334 (2010).
[19] J. T. Machado, V. Kiryakova and F. Mainardi, A poster about the old history of fractional calculus, *Fract. Calc. Appl. Anal.* **13**(4), 447-454 (2010).
[20] P. S. V. Nataraj, *Fractional Calculus and Fractional Differential Equations with SCILAB*, Systems and Control Engineering, Indian Institute of Technology Bombay, April 24, 2010.
[21] K. S. Miller and B. Ross, *An Introduction to the Fractional Calculus and Fractional Differential Equations*, 1993.
[22] K. S. Miller and R. Bertram, Fractional Green's functions, *Indian J. Pure Appl. Math.* **22**(9), 763-767 (1991).
[23] M. D. Ortigueira, An introduction to the fractional continuous-time linear systems: the 21st century systems, *Circ. Syst. Mag.IEEE* **8**(3), 19-26 (2008).
[24] G. W. Bohannan, Analog fractional order controller in a temperature control application, In *Fract. Differ. Appl.* **2**(1), 40-45 (2006).
[25] P. Shah, S. D. Agashe and A. P. Singh, Design of fractional order controller for undamped control system, In *Engineering (NUiCONE), 2013 Nirma University International Conference on*, 1-5 (2013).
[26] J. P. Clerc, A. M. S. Tremblay, G. Albinet and C. D. Mitescu, AC response of fractal networks *J. Phys. Lett.* **45**(19), 913-924 (1984).
[27] P. Srivastava and A. P. Singh, T. Srivastava and N. M. Singh, Observer design for non-linear systems, *Int. J. Appl. Eng. Res.* **8**(8), 957-967 (2013).
[28] R. O. Saber, *Nonlinear control of under-actuated mechanical systems with application to robotics and aerospace vehicles*, Diss. Massachusetts Institute of Technology, 2000.
[29] A. P. Singh and H. Agrawal, On modeling and analysis of launch vehicle system, in Proceedings of the International Conference on Signal, Networks, Computing, and Systems, *Lecture notes in Electrical Engineering*, Springer India, 273-279 (2016).
[30] J. Sidharth and A. P. Singh, Comparison of the performance of controllers for under-actuated systems, in *Power Electronics, Intelligent Control and Energy Systems (ICPEICES), IEEE International Conference on*, 1-5 (2016).
[31] M. I. Solihinand, Sensor less anti-swing control for automatic Gantry Crane system: model-based aapproach, *Int. J. Appl. Eng. Res.* **2**(1), 147-161 (2007).