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Ratio-Cum-Product Type Estimator of Finite Population Mean in Case of Post Stratification

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Abstract: In this paper, a ratio-cum-product type estimator in case of post-stratification has been proposed. The bias and mean squared error of the proposed estimator are obtained up to the first degree of approximation. The mean squared error for the proposed estimator has been obtained and it has been shown that the proposed estimator is more efficient than the usual unbiased estimator and the estimators suggested by Ige and Tripathi [7]. Theoretical conditions are obtained under which the proposed estimator is more efficient than considered estimators. Finally, in order to find the merits of the proposed estimator an empirical study has been carried out.

Keywords: Post-Stratification, Bias, Mean Squared Error

1 Introduction

Singh [14] used information on population mean of two auxiliary variates and proposed ratio-cum-product type estimator for population mean in simple random sampling. Motivated by Singh [14], Tailor et al. [16] studied the properties of Singh (1967) estimator in stratified random sampling. Ige and Tripathi [7] studied classical ratio and product estimators given by Cochran [3] and Robson [13] in case of post-stratification. Recently Lone and Tailor [10] and Lone and Tailor ([11], [12]) proposed ratio and product exponential type estimators in case of post-stratification. Initially, Hansen et al. [5] discussed the problem of post-stratification. Later many authors Fuller [4], Jagers [8], Holt and Smith [6], Jagers et al. [9], Ige and Tripathi [7] and Agrawal and Pandey [1] contributed significantly in this area of research.

Let $U=\left(U_1,U_2,...,U_N\right)$ be a finite population of size N. Using simple random sampling without replacement, a sample of size n is drawn from population U. After selecting the sample, it is observed that which units belong to h^{th} stratum. Let n_h be the size of the sample falling in h^{th} stratum such that $\sum_{h=1}^{L} n_h = n$. Here it is assumed that n is so large that possibility of n_h being zero is very small.

Let x_{hi} be the observation on i^{th} unit that fall in h^{th} stratum for auxiliary variate *x* and y_{hi} be the observation on i^{th} unit that fall in h^{th} stratum for study variate *y*, then

$\bar{X}=\sum_{h=1}^{L} W_h \bar{X}_h$: Population mean of auxiliary variate *x*,

$\bar{Y}=\sum_{h=1}^{L} W_h \bar{Y}_h$: Population mean of the study variate *y*

$\bar{X}_h=\frac{1}{N_h}\sum_{i=1}^{N_h} x_{hi}$: h^{th} stratum mean for auxiliary variate *x*,

$\bar{Y}_h=\frac{1}{N_h}\sum_{i=1}^{N_h} y_{hi}$: h^{th} stratum mean for study variate

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y.

2 Various Estimators

We considered the following estimators. The expressions $V(.)$, $M(.)$ and $B(.)$ denoted the variance, mean squared error and bias of the considered estimators.

(i) Usual unbiased estimator

$$\bar{y}_{PS} = \sum_{h=1}^{L} W_h \bar{y}_h \,, \tag{2.1}$$

Where $W_h = \dfrac{N_h}{N}$ is the weight of the h^{th} stratum and $\bar{y}_h = \dfrac{1}{n_h}\sum_{i=1}^{n_h} y_{hi}$ is sample mean of n_h sample units that fall in the h^{th} stratum.

Using the results from Stephen [15] the variance of $\bar{y}_{PS}$ to the first degree of approximation is obtained as

$$V(\bar{y}_{PS}) = \left(\frac{1}{n} - \frac{1}{N}\right)\sum_{h=1}^{L} W_h S_{yh}^2 + \frac{1}{n^2}\sum_{h=1}^{L}(1 - W_h) S_{yh}^2 \tag{2.2}$$

Where

$$S_{yh}^2 = \frac{1}{N_h - 1}\sum_{i=1}^{N_h}\left(y_{hi} - \bar{Y}_h\right)^2 .$$

(ii) Ige and Tripathi [7] estimators

Ige and Tripathi [7] introduced the following estimators

$$\hat{\bar{Y}}_{PS}^{R} = \bar{y}_{PS}\left(\frac{\bar{X}}{\bar{x}_{PS}}\right), \tag{2.3}$$

And

$$\hat{\bar{Y}}_{PS}^{P} = \bar{y}_{PS}\left(\frac{\bar{z}_{PS}}{\bar{Z}}\right). \tag{2.4}$$

Where $\bar{x}_{PS} = \sum_{h=1}^{L} W_h \bar{x}_h$ and $\bar{z}_{PS} = \sum_{h=1}^{L} W_h \bar{z}_h$.

The biases and mean squared errors of Ige and Tripathi [7] estimators $\hat{\bar{Y}}_{PS}^{R}$ and $\hat{\bar{Y}}_{PS}^{P}$ are obtained as

$$B\left(\hat{\bar{Y}}_{PS}^{R}\right) = \left(\frac{1}{n} - \frac{1}{N}\right)\frac{1}{\bar{X}}\sum_{h=1}^{L} W_h \left(R_1 S_{xh}^2 - S_{yxh}\right) \tag{2.5}$$

$$B\left(\hat{\bar{Y}}_{PS}^{P}\right) = \left(\frac{1}{n} - \frac{1}{N}\right)\frac{1}{\bar{Z}}\sum_{h=1}^{L} W_h S_{yzh} \,, \tag{2.6}$$

$$M\left(\hat{\bar{Y}}_{PS}^{R}\right) = \left(\frac{1}{n} - \frac{1}{N}\right)\sum_{h=1}^{L} W_h \left(S_{yh}^2 + R_1^2 S_{xh}^2 - 2R_1 S_{yxh}\right) \tag{2.7}$$

$$M\left(\hat{\bar{Y}}_{PS}^{P}\right) = \left(\frac{1}{n} - \frac{1}{N}\right)\sum_{h=1}^{L} W_h \left(S_{yh}^2 + R_2^2 S_{zh}^2 + 2R_2 S_{yzh}\right) \tag{2.8}$$

Where $R_1 = \dfrac{\bar{Y}}{\bar{X}}$ and $R_2 = \dfrac{\bar{Y}}{\bar{Z}}$.

3 Proposed Estimator

Singh [14] proposed a ratio-cum-product estimator in simple random sampling using two auxiliary variates as

$$\hat{\bar{Y}}_1 = \bar{y}\left(\frac{\bar{X}}{\bar{x}}\right)\left(\frac{\bar{z}}{\bar{Z}}\right). \tag{3.1}$$

Tailor et al. [16] defined Singh [14] estimator in stratified random sampling as

$$\hat{\bar{Y}}_{RPst} = \bar{y}_{st}\left(\frac{\bar{X}}{\bar{x}_{st}}\right)\left(\frac{\bar{z}_{st}}{\bar{Z}}\right). \tag{3.2}$$

Following Singh [14], we propose Tailor et al. [16] estimator in case of post- stratification as

$$\hat{\bar{Y}}_{Rp}^{Ps} = \bar{y}_{Ps}\left(\frac{\bar{X}}{\bar{x}_{Ps}}\right)\left(\frac{\bar{z}_{Ps}}{\bar{Z}}\right). \tag{3.3}$$

Where $\bar{y}_{Ps}$, $\bar{x}_{Ps}$ and $\bar{z}_{Ps}$ are unbiased estimators of population mean $\bar{Y}$, $\bar{X}$ and $\bar{Z}$ respectively.

To obtain the bias and mean squared error of the proposed estimator, we write

$\bar{y}_h = \bar{Y}_h(1 + e_{0h})$, $\bar{x}_h = \bar{X}_h(1 + e_{1h})$ and $\bar{z}_h = \bar{Z}_h(1 + e_{2h})$ such that

$$E(e_{0h}) = E(e_{1h}) = E(e_{2h}) = 0 ,$$

$$E(e_{0h}^2) = \left(\frac{1}{nW_h} - \frac{1}{N_h}\right)C_{yh}^2 ,$$

$$E(e_{1h}^2) = \left(\frac{1}{nW_h} - \frac{1}{N_h}\right)C_{xh}^2 ,$$

$$E(e_{2h}^2) = \left(\frac{1}{nW_h} - \frac{1}{N_h}\right)C_{zh}^2 ,$$

$$E(e_{0h}e_{1h}) = \left(\frac{1}{nW_h} - \frac{1}{N_h}\right)\rho_{yxh} C_{yh} C_{xh} ,$$

$$E(e_{0h}e_{2h})=\left(\frac{1}{nW_h}-\frac{1}{N_h}\right)\rho_{yzh}C_{yh}C_{zh} \quad \text{and}$$

$$E(e_{1h}e_{2h})=\left(\frac{1}{nW_h}-\frac{1}{N_h}\right)\rho_{xzh}C_{xh}C_{zh}.$$

Expressing (3.3) in terms of e_{ih}'s, we have

$$\left(\hat{\hat{Y}}_{RP}^{Ps}-\bar{Y}\right)=\bar{Y}\left[e_0-e_1+e_2-e_0e_1+e_1^2 + e_0e_2-e_1e_2\right] \quad (3.4)$$

Where $e_0=\frac{\sum_{h=1}^{L}W_h\bar{Y}_he_{0h}}{\bar{Y}}$, $e_1=\frac{\sum_{h=1}^{L}W_h\bar{X}_he_{1h}}{\bar{X}}$

And $e_2=\frac{\sum_{h=1}^{L}W_h\bar{Z}_he_{2h}}{\bar{Z}}$

Now taking expectation of both sides of (2.4), the bias of the suggested estimator to the first degree of approximation is obtained as

$$B\left(\hat{\hat{Y}}_{RP}^{Ps}\right)=\left(\frac{1}{n}-\frac{1}{N}\right)\sum_{h=1}^{L}W_h\left\{\frac{R_2}{\bar{Y}}S_{yzh}-\frac{R_1}{\bar{Z}}S_{xzh}. + \frac{R_1}{\bar{X}}S_{xh}^2-\frac{R_1}{\bar{Y}}S_{yxh}\right\} \quad (3.5)$$

Similarly the mean squared error of the suggested estimator up to the first degree of approximation are obtained as

$$M\left(\hat{\hat{Y}}_{RP}^{Ps}\right)=\left(\frac{1}{n}-\frac{1}{N}\right)\sum_{h=1}^{L}W_h\left[S_{yh}^2+R_1^2S_{xh}^2+R_2^2S_{zh}^2 -2\left(R_1S_{yxh}+R_1R_2S_{xzh}-R_2S_{yzh}\right)\right] \quad (3.6)$$

4 Efficiency Comparisons

From (2.2), (2.7), (2.8) and (3.6), it is observed that the proposed estimator $\hat{\hat{Y}}_{RP}^{Ps}$ would be more efficient than

(i) Usual unbiased estimator $\bar{y}_{PS}$ i.e

$$M\left(\hat{\hat{Y}}_{RP}^{Ps}\right)-V\left(\bar{y}_{PS}\right)<0 \text{ if}$$

$$\sum_{h=1}^{L}W_h\left[R_1^2S_{xh}^2+R_2^2S_{zh}^2-2R_1S_{yxh} -2\left(R_1R_2S_{xzh}-R_2S_{yzh}\right)\right]<0 \quad (4.1)$$

(ii) Ige and Tripathi [7] ratio estimator $\hat{\hat{Y}}_{PS}^{R}$ i.e

$$M\left(\hat{\hat{Y}}_{RP}^{Ps}\right)-M\left(\hat{\hat{Y}}_{PS}^{R}\right)<0 \text{ if}$$

$$\sum_{h=1}^{L}W_h\left[R_2^2S_{zh}^2-2R_1R_2S_{xzh}+R_2S_{yzh}\right]<0, \quad (4.2)$$

(iii) Ige and Tripathi [7] product estimator $\hat{\hat{Y}}_{PS}^{P}$ i.e

$$M\left(\hat{\hat{Y}}_{RP}^{Ps}\right)-M\left(\hat{\hat{Y}}_{PS}^{P}\right)<0 \text{ if}$$

$$\sum_{h=1}^{L}W_h\left[R_1^2S_{xh}^2-2R_1R_2S_{xzh}-2R_1S_{yxh}\right]<0 \quad (4.3)$$

5 Empirical Study

To show the performance of the proposed estimator in comparison to other considered estimators, we use a population data set. The description of population is given below.

Population I- [Source: Chouhan [2]]

y : Productivity (MT/Hectare)

x : Production in '000 Tons and

z : Area in '000 Hectare

N_1=10	N_2=10	n_1=4	n_1=4
$\bar{Y}_1$=1.70	$\bar{Y}_2$=3.67	$\bar{X}_1$ =10.41	$\bar{X}_2$ =289.14
$\bar{Z}_1$=6.32	$\bar{Z}_2$=80.67	S_{y1}=0.50	S_{y2}=1.41
S_{x1} =3.53	S_{x2} =111.61	S_{z1}=1.19	S_{z2}=10.82
S_{yx1} =1.60	S_{yx2} =144.87	S_{yz1}=-0.05	S_{yz2}=-7.04
S_{xz1} =1.38	S_{xz2}=-92.02		

Table 5.1 Percent relative Efficiencies of $\bar{y}_{PS}$, $\hat{\hat{Y}}_{PS}^{R}$, $\hat{\hat{Y}}_{PS}^{P}$ and $\hat{\hat{Y}}_{RP}^{Ps}$ with respect to $\bar{y}_{PS}$

Estimator	Percent Relative Efficiency
$\bar{y}_{PS}$	100.00
$\hat{\hat{Y}}_{PS}^{R}$	270.00
$\hat{\hat{Y}}_{PS}^{P}$	149.00
$\hat{\hat{Y}}_{RP}^{Ps}$	348.19

6 Conclusion

Section 4 provides the conditions under which the proposed estimators $\hat{\hat{Y}}_{RP}^{Ps}$ has less mean squared error in comparison to usual unbiased estimator $\bar{y}_{PS}$, Ige and Tripathi [7] estimators $\hat{\bar{Y}}_{PS}^{R}$ and $\hat{\bar{Y}}_{PS}^{P}$.Table **5.1** exhibits that the proposed estimators $\hat{\bar{Y}}_{RP}^{Ps}$ has highest percent relative efficiency in comparison to other considered estimators .Hence, it can be concluded that the proposed estimator is recommended for use in practice if the conditions defined in section 4 are satisfied .

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