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Characterization and Estimations of Weighted Generalized Beta Probability Distributions

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Abstract: In this research paper, a new class of weighted Generalized Beta Distribution of first kind and Size biased Generalized Beta Distribution second kind has been considered. The several structural properties of these probability models include measures of central tendency and dispersion are defined. Some important theorems have been derived to estimates the parameters of weighted and size-biased beta distributions and identify the relations with other related distributions. It was observed that the square of the sample coefficient of variation is asymptotically unbiased estimator of square of the population coefficient of variation of these weighted and size biased generalized beta distributions. Also, a likelihood ratio test of Weighted and size biased probability distributions are to be conducted.

Keywords: Size biased generalized beta distribution of second kind, weighted generalized beta distribution of first kind, Likelihood ratio test.

1 Introduction

The weighted distributions arise when the observations generated from a stochastic process are not given equal chance of being recorded; instead they are recorded according to some weighted function. First introduced by Fisher (1934) to model ascertainment bias, these were later formalized in a unifying theory by Rao (1965).When the weight function depends on the lengths of the units of interest, the resulting distribution is called length biased. More generally, when the sampling mechanism selects units with probability proportional to measure of the unit size, resulting distribution is called size-biased. Size biased distributions are a special case of the more general form known as weighted distributions. These distributions arise in practice when observations from a sample are recorded with unequal probability and provide unifying approach for the problems when the observations fall in the non –experimental, non –replicated and non –random categories.Other contributions have been made by Reshi et al (2012), Reshi et al (2013a, b), Reshi et al (2014a, b, c, d, e, f) and Ahmed et al (2013a, b, c).

The Weighted generalized beta distribution of first kind (WGBD1) is very versatile and a variety of uncertainties can be usefully modeled by them and weighted generalized beta distribution of first kind (WGBD1) was introduced by Reshi et.al (2013a). Many of the finite range distributions encountered in practice can be easily transformed into the standard distribution. It captures the characteristics of income distribution including skewness, peakedness in low-middle range, and long right hand tail. The Weighted Generalized Beta distribution of first kind includes several other distributions as special and particular cases, such as weighted beta distribution of first kind kind, Size-biased beta distribution of first kind (Mir et al (2013), area-biased beta distribution of first kind.

The probability density function (pdf) of the weighted generalized beta distribution of first kind (WGBD1) see (Reshi et al (2013a)) is given by:

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$$f_w^{*}(x;a,b,p,q)= \frac{a}{b^{ap+c}\ \beta\left(p+\frac{c}{a},q\right)} x^{ap+c-1}\left(1-\left(\frac{x}{b}\right)^{a}\right)^{q-1} \tag{1.1}$$

Where a, p, q are shape parameters, c is a weighted parameter and b is a scale parameter, $\beta(p+\frac{c}{a},q)=\frac{\Gamma\left(p+\frac{c}{a}\right)\Gamma q}{\Gamma\left(p+q+\frac{c}{a}\right)}$ is a beta function, a,b,p,q are positive real values.

The rth moment of weighted generalized beta distribution of first kind is given by:

$$\mu_r' = \frac{b^r}{\beta\left(p+\frac{c}{a},q\right)}\beta\left(p+\frac{r+c}{a},q\right) \tag{1.2}$$

Using the equation (1.2), the mean, variance and coefficient of variation of the WGBD1 is given by

$$\mu_1' = \frac{b\,\beta\left(p+\frac{c+1}{a},q\right)}{\beta\left(p+\frac{c}{a},q\right)}$$

$$\mu_2 = b^2\left[\frac{\beta\left(p+\frac{c+2}{a},q\right)}{\beta\left(p+\frac{c}{a},q\right)}-\left[\frac{\beta\left(p+\frac{c+1}{a},q\right)}{\beta\left(p+\frac{c}{a},q\right)}\right]^2\right]$$

$$CV = \sqrt{\frac{\beta\left(p+\frac{c+2}{a},q\right)\beta\left(p+\frac{c}{a},q\right)}{\beta^2\left(p+\frac{c+1}{a},q\right)}-1}$$

The mode and harmonic mean of weighted generalized beta distribution of first kind are given as:

$$x = b\left[\frac{ap+c-1}{a(p+q-1)+c-1}\right]^{\frac{1}{a}}$$

$$H = \frac{b\beta\left(p+\frac{c}{a},q\right)}{\beta(p+\frac{c-1}{a},q)}$$

2 Estimation of Parameters of the Weighted Generalized Beta Distribution of First Kind

In this section, we obtain estimates of the parameters for the weighted Generalized Beta distribution of first kind by employing the new method of moment (MOM) estimator.

2.1 New Method of Moment Estimators

Let $X_1, X_2, X_3 \ldots Xn$ be an independent sample from the WGBD1. The method of moment estimators are obtained by setting the raw moments equal to the sample moments, that is $E(X^r) = M_r$, where M_r is the sample moment corresponding to the $E(X^r)$. The following equations are obtained using the first and second sample moments.

$$\frac{1}{n}\sum_{j=1}^{n} X_j = \frac{b\,\beta\left(p+\frac{c+1}{a},q\right)}{\beta\left(p+\frac{c}{a},q\right)} \tag{2.1}$$

$$\frac{1}{n}\sum_{j=1}^{n} X_j^{\,2} = \frac{b^2\,\beta\left(p+\frac{c+2}{a},q\right)}{\beta\left(p+\frac{c}{a},q\right)} \tag{2.2}$$

Case 1: When p and q are fixed and a=1, then

$$\frac{\bar{X}}{M_2} = \frac{\Gamma(p+q+c+1)}{b\,\Gamma(p+c+1)}$$

$$\hat{b} = \frac{M_2}{\bar{X}}\left[1+\frac{q}{p+c+1}\right] \tag{2.3}$$

Case 2: When p and b are fixed and a=1, then dividing equation (2.1) by (2.2), we have:

$$\frac{\bar{X}}{M_2} = \frac{\Gamma(p+q+c+1)}{b\,\Gamma(p+c+1)}$$

$$\hat{q} = (p+c+1)\left[\frac{b\bar{X}}{M_2}-1\right] \tag{2.4}$$

Case 3: When b and q are fixed and a=1, then dividing equation (2.1) by (2.2), we have:

$$\frac{\bar{X}}{M_2} = \frac{\Gamma(p+q+c+1)}{b\,\Gamma(p+c+1)}$$

$$\hat{p} = \frac{qM_2}{b\,\bar{X}-M_2}-(c+1) \tag{2.5}$$

Case 4: When p and q are fixed, b=1 then we can calculate the value of $\hat{a}$ estimator by numerical methods.

3 Characterization of Weighted Generalized Beta Distribution of First Kind

Although Reshi et al (2014b) have presented a procedure to obtain the two parameters of the Size biased Gamma distribution, his procedure still quit complicated. In this research, we propose a simple procedure to obtained four estimators by using its characterization and moment estimation approach. Note that Ahmed et al (2013a) have obtained more general characterizations with the independence of sample coefficient of variation V_n with samplemean $\bar{X}_n$ as one of its special cases when random samples are drawn from the Size biased Generalized Gamma distribution. Their characterization is used to derive the expectation and the variance of $V_n^{\,2}$ and then the new estimators for the five parameters of weighted generalized Beta distribution of first kind are proposed. For deriving new moment estimators of three parameters of the weighted generalized Beta distribution of first kind, we need the following theorem obtained by using the similar approach of Reshi*et al* (2014b).

Theorem 3.1: Let $n \geq 3$ and let $x_1, x_2, x_3 \ldots x_n$ be a n positive identical independently distributed random samples drawn from a population having a weighted generalized Beta distribution of first kind

$$f_w^{\,*}(x;a,b,p,q) = \frac{a}{b^{ap+c}\ \beta\left(p+\frac{c}{a},q\right)} x^{a+c-1}\left(1-\left(\frac{x}{b}\right)^{a}\right)^{q-1}$$

Then

$$E(S_n^{\,2}) = \frac{b^2\left[\beta\left(p+\frac{c+2}{a},q\right)\beta\left(p+\frac{c}{a},q\right)-\beta^2\left(p+\frac{c+1}{a},q\right)\right]}{\beta^2\left(p+\frac{c}{a},q\right)}$$

Proof: Here,

$$E(X)^r = \frac{b^r}{\beta\left(p+\frac{c}{a},q\right)}\beta\left(p+\frac{r+c}{a},q\right)$$

$$E\left(\overline{X}_n\right) = \frac{b\,\beta\left(p+\frac{c+1}{a},q\right)}{\beta\left(p+\frac{c}{a},q\right)}$$

$$E(X_n^{\,2}) = \frac{b^2\left[\beta\left(p+\frac{c+2}{a},q\right)\beta\left(p+\frac{c}{a},q\right)+(n-1)\beta^2\left(p+\frac{c+1}{a},q\right)\right]}{n\beta^2\left(p+\frac{c}{a},q\right)}$$

And

$$E(S_n^{\,2}) = \frac{b^2\left[\beta\left(p+\frac{c+2}{a},q\right)\beta\left(p+\frac{c}{a},q\right)-\beta^2\left(p+\frac{c+1}{a},q\right)\right]}{\beta^2\left(p+\frac{c}{a},q\right)} \tag{3.1}$$

Where $\overline{X}_n$ and $S_n^{\,2}$ are respectively their sample mean and sample variance.

Theorem 3.2: Let $n \geq 3$ and let $x_1, x_2, x_3 \ldots x_n$ be a n positive identical independently distributed random samples drawn from a population having a weighted generalized Beta distribution of first kind

$$f_w^{\,*}(x;a,b,p,q) = \frac{a}{b^{ap+c}\ \beta\left(p+\frac{c}{a},q\right)} x^{a+c-1}\left(1-\left(\frac{x}{b}\right)^{a}\right)^{q-1}$$, Then

$$E\left(\frac{S_n^{\ 2}}{\overline{X}_n^{\ 2}}\right)=\frac{n\left[\beta\left(p+\frac{c+2}{a},q\right)\beta\left(p+\frac{c}{a},q\right)-\beta^2\left(p+\frac{c+1}{a},q\right)\right]}{\left[\beta\left(p+\frac{c+2}{a},q\right)\beta\left(p+\frac{c}{a},q\right)+(n-1)\beta^2\left(p+\frac{c+1}{a},q\right)\right]}$$

Where $\overline{X}_n$ and $S_n^{\ 2}$ are respectively their sample mean and sample variance.

Proof: By using theorem 3.1, we have

$$E(S_n^{\ 2})=E\left(\frac{S_n^{\ 2}}{\overline{X}_n^{\ 2}}.\overline{X}_n^{\ 2}\right)=E\left(\frac{S_n^{\ 2}}{\overline{X}_n^{\ 2}}\right)..E\left(\overline{X}_n^{\ 2}\right)$$

And hence
$$E\left(\frac{S_n^{\ 2}}{\overline{X}_n^{\ 2}}\right)=\frac{E\left(S_n^{\ 2}\right)}{E\left(\overline{X}_n^{\ 2}\right)}$$

Applying theorem 3.1 to the above identity yields that

$$E\left(\frac{S_n^{\ 2}}{\overline{X}_n^{\ 2}}\right)=\frac{n\left[\beta\left(p+\frac{c+2}{a},q\right)\beta\left(p+\frac{c}{a},q\right)-\beta^2\left(p+\frac{c+1}{a},q\right)\right]}{\left[\beta\left(p+\frac{c+2}{a},q\right)\beta\left(p+\frac{c}{a},q\right)+(n-1)\beta^2\left(p+\frac{c+1}{a},q\right)\right]} \tag{3.2}$$

Thus, theorem 3.2 is established.

Theorem 3.3: Let $n \geq 3$ and let $x_1, x_2, x_3 ... x_n$ be an n positive identical independently distributed random samples drawn from a population having a weighted generalized Beta distribution of first kind.

$$f_w^{\ *}(x;a,b,p,q)=\frac{a}{b^{ap+c}\ \beta\left(p+\frac{c}{a},q\right)}x^{a+c-1}\left(1-\left(\frac{x}{b}\right)^a\right)^{q-1}$$

$$E(S_n^{\ 2})=\frac{b^2\left[\beta\left(p+\frac{c+2}{a},q\right)\beta\left(p+\frac{c}{a},q\right)-\beta^2\left(p+\frac{c+1}{a},q\right)\right]}{\beta^2\left(p+\frac{c}{a},q\right)}$$

$$E\left(\frac{S_n^{\ 2}}{\overline{X}_n^{\ 2}}\right)=\frac{n\left[\beta\left(p+\frac{c+2}{a},q\right)\beta\left(p+\frac{c}{a},q\right)-\beta^2\left(p+\frac{c+1}{a},q\right)\right]}{\left[\beta\left(p+\frac{c+2}{a},q\right)\beta\left(p+\frac{c}{a},q\right)+(n-1)\beta^2\left(p+\frac{c+1}{a},q\right)\right]}$$

Furthermore, if WGBD1 distribution, we have

$$\frac{\sigma^2}{\mu^2}=\frac{\beta\left(p+\frac{c+2}{a},q\right)\beta\left(p+\frac{c}{a},q\right)}{\beta^2\left(p+\frac{c+1}{a},q\right)}-1 \tag{3.3}$$

And it can be show that

$$E\left(\frac{S_n{}^2}{\bar{X}_n{}^2}\right)\rightarrow\frac{\beta\left(p+\frac{c+2}{a},q\right)\beta\left(p+\frac{c}{a},q\right)}{\beta^2\left(p+\frac{c+1}{a},q\right)}-1 \tag{3.4}$$

Comparing above two equations, we have

$$E\left(\frac{S_n{}^2}{\bar{X}_n{}^2}\right)\rightarrow\frac{\sigma^2}{\mu^2}$$

Note that $E\left(\frac{S_n{}^2}{\bar{X}_n{}^2}\right)\rightarrow\frac{\sigma^2}{\mu^2}$ as $n\rightarrow\infty$ and that this limit is the square of the population coefficient of variation. Thus, $\frac{S_n{}^2}{\bar{X}_n{}^2}$ is an asymptotically unbiased estimator of the square of the population coefficient of variation.

4 Likelihood Ratio Test for weighted generalized beta distribution of first kind

Let x_1, x_2, x_3… x_n be random samples can be drawn from generalized beta distribution of first kind or weighted generalized beta distribution of first kind. We test the hypothesis

$$H_o: f(x)=f(x;a,b,p,q) \text{ vs } H_1: f(x)=f_w{}^*(x;a,b,p,q).$$

To test whether the random sample of size n comes from the generalized beta distribution of first kind or weighted generalized beta distribution of first kind the following test statistic is used.

$$\Delta=\frac{L_1}{L_0}=\prod_{i=1}^{n}\left(\frac{f_w{}^*(x;a,b,p,q)}{f(a,b,p,q)}\right)$$

$$\Delta=\frac{L_1}{L_0}=\prod_{i=1}^{n}\left(\frac{\frac{ax^{ap+c-1}}{b^{ap+c}\beta\left(p+\frac{c}{a},q\right)}\left[1-\left(\frac{x}{b}\right)^a\right]^{q-1}}{\frac{ax^{ap-1}}{b^{ap}\beta(p,q)}\left[1-\left(\frac{x}{b}\right)^a\right]^{q-1}}\right)$$

$$\Delta=\frac{L_1}{L_0}=\prod_{i=1}^{n}\frac{\beta(p,q)}{b^c\beta\left(p+\frac{c}{a},q\right)}.x^c$$

$$\Delta = \left[\frac{\beta(p,q)}{b^c \beta\left(p+\frac{c}{a},q\right)} \right]^n \prod_{i=1}^{n} x_i^{\,c} \tag{4.2}$$

We reject the null hypothesis.

$$\left[\frac{\beta(p,q)}{b^c \Gamma\left(p+\frac{c}{a},q\right)} \right]^n \prod_{i=1}^{n} x_i^{\,c} > k$$

Equivalently, we rejected the null hypothesis where

$$\Delta^* = \prod_{i=1}^{n} x_i^{\,c} > k^*, where k^* = k \left[\frac{b^c \beta\left(p+\frac{c}{a},q\right)}{\beta(p,q)} \right]^n > 0$$

For a large sample size of n, $2\log\Delta$ is distributed as a Chi-square distribution with one degree of freedom. Thus, the p-value is obtained from the Chi-square distribution. Also, we can reject the reject the null hypothesis, when probability value s given by:

$P(\Delta^* > \lambda^*), Where\ \lambda^* = \prod_{i=1}^{n} x_i$ is less than a specified level of significance, where $\prod_{i=1}^{n} x_i$ is the observed value of the test statistic Δ^*.

5 Introduction of Size Biased Generalized Beta Distribution of Second Kind (SBGBD2)

The Size biased generalized beta distribution of second kind (SBGBD2) is very versatile and a variety of uncertainties can be usefully modeled by them Size biased generalized beta distribution of second kind (SBGBD2) was introduced by Ahmed et.al (2013b). It captures the characteristics of income distribution including skewness, peakedness in low-middle range, and long right hand tail. The biased generalized beta distribution of second kind (SBGBD2) includes Size-biased beta distribution of second kind and area-biased beta distribution of second kind as special cases. The probability density function (pdf) of the Size biased generalized beta distribution of second kind (SBGBD2) is given by:

$$f^*(x;a,b,p,q) = \frac{a\ \ x^{ap}}{b^{ap+1}\ \beta\left(p+\frac{1}{a},q-\frac{1}{a}\right)\left[1+\left(\frac{x}{b}\right)^a\right]^{p+q}} \tag{5.1}$$

Where a, p, q are shape parameters and b is a scale parameter, $\beta\left(p+\frac{1}{a},q-\frac{1}{a}\right) = \frac{\Gamma\left(p+\frac{1}{a}\right)\Gamma\left(q-\frac{1}{a}\right)}{\Gamma(p+q)}$ is a beta function, a, b, p, q are positive real values.

The distribution like the Size-biased beta distribution of second kind as special case ($a = b = 1$), then the probability density function is given as:

$$f^*(x;p,q)=\frac{x^p}{\beta(p+1,q-1)[1+x]^{p+q}} \quad ; p,\, q>0$$

The rth moment of Size biased generalized beta distribution of second kind is given by:

$$\mu_r' = \frac{b^r}{\beta\left(p+\frac{1}{a},q-\frac{1}{a}\right)}\beta\left(p+\frac{r}{a}+\frac{1}{a},q-\frac{r}{a}-\frac{1}{a}\right) \tag{5.2}$$

Using the equation (5.2), the mean, variance and coefficient of variation of the SBGBD2 is given by

$$\mu_1' = \frac{b\,\beta\left(p+\frac{2}{a},q-\frac{2}{a}\right)}{\beta\left(p+\frac{1}{a},q-\frac{1}{a}\right)}$$

$$\mu_2 = b^2\left[\frac{\beta\left(p+\frac{3}{a},q-\frac{3}{a}\right)}{\beta\left(p+\frac{1}{a},q-\frac{1}{a}\right)}-\left[\frac{\beta\left(p+\frac{2}{a},q-\frac{2}{a}\right)}{\beta\left(p+\frac{1}{a},q-\frac{1}{a}\right)}\right]^2\right]$$

$$CV=\frac{\sqrt{V(X)}}{E(X)}=\sqrt{\frac{\beta\left(p+\frac{3}{a}.q-\frac{3}{a}\right)\beta\left(p+\frac{1}{a},q-\frac{1}{a}\right)}{\beta^2\left(p+\frac{2}{a},q-\frac{2}{a}\right)}-1}$$

The mode and harmonic mean of weighted generalized beta distribution of first kind are given as:

$$x_0 = b\left(\frac{p}{q}\right)^{\frac{1}{a}}$$

$$H = \frac{b\beta\left(p+\frac{1}{a}\right)\beta\left(q-\frac{1}{a}\right)}{\beta(p,q)}$$

6 Estimation of Parameters in the Size-Biased Generalized Beta Distribution of Second Kind

In this section, we obtain estimates of the parameters for the Size-biased Generalized Beta distribution of second kind by employing the new method of moment (MOM) estimator.

6.1 New Method of Moment Estimators

Let X_1, X_2, X_3… Xn be an independent random samples from the SBGBD2 with weight c=1. The method of moment estimators are obtained by setting the raw moments equal to the sample moments, that is $E(X^r) = M_r$, where M_r is the sample moment M_r corresponding to the .$E(X^r)$ The following equations are obtained using the first and second sample moments.

$$\frac{1}{n}\sum_{j=1}^{n} X_j = \frac{b\,\beta\left(p+\frac{2}{a}, q-\frac{2}{a}\right)}{\beta\left(p+\frac{1}{a}, q-\frac{1}{a}\right)} \tag{6.1}$$

$$\frac{1}{n}\sum_{j=1}^{n} X_j^{\,2} = \frac{b\,\beta\left(p+\frac{3}{a}, q-\frac{3}{a}\right)}{\beta\left(p+\frac{1}{a}, q-\frac{1}{a}\right)} \tag{6.2}$$

Case 1: When p and q are fixed and a=1, then

$$\bar{X} = \frac{b\,\beta\left(p+\frac{2}{a}, q-\frac{2}{a}\right)}{\beta\left(p+\frac{1}{a}, q-\frac{1}{a}\right)}$$

$$\hat{b} = \bar{X}\frac{(q-2)}{p+1} \tag{4.14.3}$$

Case 2: When p and b are fixed and a=1, then dividing equation (6.2) by (6.1), we have:

$$\frac{\bar{X}}{M_2} = \frac{\Gamma(p+2)\Gamma(q-2)}{b\Gamma(p+3)\Gamma(q-3)}$$

$$\hat{q} = (p+2)b\frac{\bar{X}}{M_2} + 3 \tag{6.4}$$

Case 3: When b and q are fixed and a=1, then dividing equation (6.1) by (6.2), we have:

$$\frac{\bar{X}}{M_2} = \frac{\Gamma(p+2)\Gamma(q-2)}{b\Gamma(p+3)\Gamma(q-3)}$$

$$\frac{\bar{X}}{M_2} = \frac{q-3}{b(p+2)}$$

$$\hat{p} = \frac{M_2(q-3)}{b\bar{X}} - 2 \tag{6.5}$$

Case 4: When p and q are fixed, b =1 then we can calculate the value of $\hat{a}$ estimator by numerical methods.

7 Characterization of Size- Biased Generalized Beta Distribution of Second Kind

This section is based on a new moment estimation method of parameters of SBGB2 family using its characterization. The characterization is used to derive the expectation and the variance of $V_n^{\,2}$ and then the new estimators for the four parameters of size-biased generalized Beta distribution of second kind are proposed. For deriving new moment estimators of four parameters of the size-biased generalized Beta distribution of second kind, we need the following theorem obtained by using the similar approach of Ahmed *et al* (2013).

Theorem 7.1: Let $n \geq 3$ and let $x_1, x_2, x_3 \ldots x_n$ be an n positive identical independently distributed random samples drawn from a population having a Size- biased generalized Beta distribution of second kind.

$$f^{*}(x;a,b,p,q)=\frac{a\ \ x^{ap}}{b^{ap+1}\ \beta\left(p+\frac{1}{a},q-\frac{1}{a}\right)\left[1+\left(\frac{x}{b}\right)^{a}\right]^{p+q}}$$

Then
$$E(S_n^{\ 2})=\frac{b^2\left[\beta\left(p+\frac{3}{a},q-\frac{3}{a}\right)\beta\left(p+\frac{1}{a},q-\frac{1}{a}\right)-\beta^2\left(p+\frac{2}{a},q-\frac{2}{a}\right)\right]}{\beta^2\left(p+\frac{1}{a},q-\frac{1}{a}\right)}$$

Proof: Here,
$$E(X)^r=\frac{b^r}{\beta\left(p+\frac{1}{a},q-\frac{1}{a}\right)}\beta\left(p+\frac{r}{a}+\frac{1}{a},q-\frac{r}{a}-\frac{1}{a}\right)$$

$$E(\overline{X}_n)=\frac{b\,\beta\left(p+\frac{2}{a},q-\frac{2}{a}\right)}{\beta\left(p+\frac{1}{a},q-\frac{1}{a}\right)}$$

$$E(X_n^{\ 2})=\frac{b^2\left[\beta\left(p+\frac{3}{a},q-\frac{3}{a}\right)\beta\left(p+\frac{1}{a},q-\frac{1}{a}\right)-(n-1)\,\beta^2\left(p+\frac{2}{a},q-\frac{2}{a}\right)\right]}{n\beta^2\left(p+\frac{1}{a},q-\frac{1}{a}\right)}$$

And
$$E(S_n^{\ 2})=\frac{b^2\left[\beta\left(p+\frac{3}{a},q-\frac{3}{a}\right)\beta\left(p+\frac{1}{a},q-\frac{1}{a}\right)-\beta^2\left(p+\frac{2}{a},q-\frac{2}{a}\right)\right]}{\beta^2\left(p+\frac{1}{a},q-\frac{1}{a}\right)} \quad (7.1)$$

Where $\overline{X}_n$ and $S_n^{\ 2}$ are respectively their sample mean and sample variance.

Theorem 7.2: Let $n\geq 3$ and let $x_1,x_2,x_3...x_n$ be an n positive identical independently distributed random samples drawn from a population having a Size- biased generalized Beta distribution of second kind

$$f^{*}(x;a,b,p,q)=\frac{a\ \ x^{ap}}{b^{ap+1}\ \beta\left(p+\frac{1}{a},q-\frac{1}{a}\right)\left[1+\left(\frac{x}{b}\right)^{a}\right]^{p+q}}$$

Then
$$E\left(\frac{S_n^{\ 2}}{\overline{X}_n^{\ 2}}\right)=\frac{n\left[\beta\left(p+\frac{3}{a},q-\frac{3}{a}\right)\beta\left(p+\frac{1}{a},q-\frac{1}{a}\right)-\beta^2\left(p+\frac{2}{a},q-\frac{2}{a}\right)\right]}{\left[\beta\left(p+\frac{3}{a},q-\frac{3}{a}\right)\beta\left(p+\frac{1}{a},q-\frac{1}{a}\right)-(n-1)\,\beta^2\left(p+\frac{2}{a},q-\frac{2}{a}\right)\right]}$$

Where $\overline{X}_n$ and $S_n^{\ 2}$ are respectively their sample mean and sample variance.

Proof: By using theorem 7.1, we have

$$E(S_n^{\ 2}) = E\left(\frac{S_n^{\ 2}}{\overline{X}_n^{\ 2}}.\overline{X}_n^{\ 2}\right) = E\left(\frac{S_n^{\ 2}}{\overline{X}_n^{\ 2}}\right)..E\left(\overline{X}_n^{\ 2}\right)$$

And hence

$$E\left(\frac{S_n^{\ 2}}{\overline{X}_n^{\ 2}}\right) = \frac{E\left(S_n^{\ 2}\right)}{E\left(\overline{X}_n^{\ 2}\right)}$$

Applying theorem 7.1to the above identity yields that

$$E\left(\frac{S_n^{\ 2}}{\overline{X}_n^{\ 2}}\right) = \frac{n\left[\beta\left(p+\frac{c+2}{a},q\right)\beta\left(p+\frac{c}{a},q\right)-\beta^2\left(p+\frac{c+1}{a},q\right)\right]}{\left[\beta\left(p+\frac{c+2}{a},q\right)\beta\left(p+\frac{c}{a},q\right)+(n-1)\beta^2\left(p+\frac{c+1}{a},q\right)\right]} \qquad (7.2)$$

Thus, theorem7.2 is established.

Theorem 7.3: Let $n \geq 3$ and let $x_1, x_2, x_3 \ldots x_n$ be an n positive identical independently distributed random samples drawn from a population having a Size- biased generalized Beta distribution of second kind

$$f^{\ *}(x;a,b,p,q) = \frac{a \quad x^{ap}}{b^{ap+1}\ \beta\left(p+\frac{1}{a},q-\frac{1}{a}\right)\left[1+\left(\frac{x}{b}\right)^a\right]^{p+q}}$$

$$E(S_n^{\ 2}) = \frac{b^2\left[\beta\left(p+\frac{3}{a},q-\frac{3}{a}\right)\beta\left(p+\frac{1}{a},q-\frac{1}{a}\right)-\beta^2\left(p+\frac{2}{a},q-\frac{2}{a}\right)\right]}{\beta^2\left(p+\frac{1}{a},q-\frac{1}{a}\right)}$$

$$E\left(\frac{S_n^{\ 2}}{\overline{X}_n^{\ 2}}\right) = \frac{n\left[\beta\left(p+\frac{c+2}{a},q\right)\beta\left(p+\frac{c}{a},q\right)-\beta^2\left(p+\frac{c+1}{a},q\right)\right]}{\left[\beta\left(p+\frac{c+2}{a},q\right)\beta\left(p+\frac{c}{a},q\right)+(n-1)\beta^2\left(p+\frac{c+1}{a},q\right)\right]}$$

Furthermore, if SBGBD2 distribution, we have

$$\frac{\sigma^2}{\mu^2} = \frac{\beta\left(p+\frac{3}{a}.q-\frac{3}{a}\right)\beta\left(p+\frac{1}{a},q-\frac{1}{a}\right)}{\beta^2\left(p+\frac{2}{a},q-\frac{2}{a}\right)} - 1 \qquad (7.3)$$

And it can be show that

$$E\left(\frac{S_n^{\ 2}}{\overline{X}_n^{\ 2}}\right) \to \frac{\beta\left(p+\frac{3}{a}.q-\frac{3}{a}\right)\beta\left(p+\frac{1}{a},q-\frac{1}{a}\right)}{\beta^2\left(p+\frac{2}{a},q-\frac{2}{a}\right)}-1 \tag{7.4}$$

Comparing above two equations, we have

$$E\left(\frac{S_n^{\ 2}}{\overline{X}_n^{\ 2}}\right) \to \frac{\sigma^2}{\mu^2}$$

Note that $E\left(\frac{S_n^{\ 2}}{\overline{X}_n^{\ 2}}\right) \to \frac{\sigma^2}{\mu^2}$ as $n \to \infty$ and that this limit is the square of the population coefficient of variation. Thus, $\frac{S_n^{\ 2}}{\overline{X}_n^{\ 2}}$ is an asymptotically unbiased estimator of the square of the population coefficient of variation.

8 Conclusion

In this paper, we have proposed a new class of weighted Generalized Beta distribution of first kind.The several structural properties of these probability models includes mean, variance, coefficient of variation, mode and harmonic mean has been studied and defined. The estimation of parameters of this new model is obtained by employing the new methods of moments Also, a likelihood ratio test of Weighted and size biased probability distributions are to be conducted. Some important theorems have been derived to estimates the parameters of four parametric weighted beta distribution of first kind. It was found that the square of the sample coefficient of variation is asymptotically unbiased estimator of square of the population coefficient of variation.

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