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Point and Interval Estimation of $R = P(Y > X)$ for Generalized Inverse Weibull Distribution by Transformation Method

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Abstract: The problem of estimating the $R = P(Y > X)$ through point and interval estimation is considered for the generalized inverse Weibull distribution. In order to obtain these estimators, the major role is played by the transformation method.

Keywords: Generalized Inverse Weibull distribution, Stress-Strength reliability, MLE, UMVUE, Confidence interval, Transformation method.

1 Introduction

A lot of work has been done in the literature to deal with various inferential problems related to $R = P(Y > X)$, which represents the reliability of an item of random strength Y subject to a random stress X. For a brief review, one may refer to Church and Harris (1970)[6], Enis and Geisser (1971)[9], Downton (1973)[8], Tong (1974)[15], Kelly, Kelly and Schucany (1976)[11], Sathe and Shah (1981)[13], Chao (1982)[5], Awad and Gharraf (1986)[1], Chaturvedi and Rani (1997)[2], Chaturvedi and Surinder (1998)[4], Chaturvedi and Sharma (2007)[3], Constantine Karson and Tse (1986)[7], Surinder and Mayank (2014)[14].

In the present paper, we have considered the generalized inverse Weibull distribution proposed by Keller and Kanath (1982)[10], which covers many lifetime distributions as specific cases. In section 2, the MLE and UMVUE of $'R'$ are derived, when the random variables (rv's)X and Y follows generalized inverse Weibull distribution. In section 3, we construct the confidence interval for $'R'$. In order to derive the MLE, UMVUE and confidence interval for $'R'$ the major role is played by the transformation method.

2 MLE and UMVUE of $R = P(Y > X)$ for Generalized Inverse Weibull distributions

The probability density function (pdf) of generalized inverse Weibull distribution is given by

$$f(x;\alpha,\beta,\gamma) = \gamma\beta\alpha^{\beta}x^{-(\beta+1)}\exp[-\gamma(\frac{\alpha}{x})^{\beta}];,x > 0 \tag{2.1}$$

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On considering different values for α,β,γ the pdf's of different continuous distributions such as one-parameter Inverse exponential distribution, Inverse Weibull distribution, Inverse Rayleigh distribution etc. can be obtained. Let the rv's X and Y follows the generalized inverse Weibull distribution given at (2.1) with the parameters (α,β,γ) and (θ,μ,χ), respectively.

Theorem 1: The MLE of $R = P(Y > X)$ is given by

$$\tilde{R} = \frac{\overline{T_Y}}{T_X + T_Y} \tag{2.2}$$

where $\overline{\varepsilon} = \frac{1}{n_1}\sum_{i=1}^{n_1} X_i^{-\beta_1} = \overline{T_X}$(say) and, $\overline{\eta} = \frac{1}{n_2}\sum_{j=1}^{n_2} Y_j^{-\beta_2} = \overline{T_Y}$(say).

Proof: Let us consider the transformation $x^{-\beta} = \varepsilon$ in(2.1) we get

$$f(\varepsilon;\lambda) = \lambda \exp[-\lambda\varepsilon]; \quad \varepsilon,\lambda > 0 \tag{2.3}$$

which is exponential distribution with parameter λ, where $\lambda = \alpha^{\beta}\gamma$.

Now , let us considered ε and η two independent rv's which follows exponential distribution λ_1 and λ_2 parameters respectively, where $\varepsilon = x^{-\beta_1}$ and $\eta = y^{-\beta_2}$.

Thus for $R = P(\eta > \varepsilon)$, we have

$$R = P(\eta > \varepsilon) = \int_0^{\infty}\int_{\varepsilon=0}^{\eta} f(\varepsilon|\lambda_1)d\varepsilon f(\eta|\lambda_2)d\eta$$

where $f(\varepsilon|\lambda_1) = \lambda_1 \exp(-\lambda_1\varepsilon)$ and $f(\eta|\lambda_2) = \lambda_2 \exp(-\lambda_2\eta)$,

$$R = \int_0^{\infty}(1-\exp(-\lambda_1\varepsilon))\lambda_2 \exp(-\lambda_2\varepsilon)d\varepsilon$$

$$R = \frac{\lambda_1}{\lambda_1+\lambda_2} \tag{2.4}$$

If $\varepsilon_1.....\varepsilon_{n_1}$ and $\eta_1.....\eta_{n_2}$ are two independent random samples of size n_1 and n_2 from the pdf's $f(\varepsilon|\lambda_1)$ and $f(\eta|\lambda_2)$ respectively, then the joint pdf is given by

$$f(\varepsilon,\eta|\lambda_1,\lambda_2) = {\lambda_1}^{n_1}{\lambda_2}^{n_2}\exp(-n_1\lambda_1\overline{\varepsilon} - n_2\lambda_2\overline{\eta}) \tag{2.5}$$

Taking likelihood function of (2.5) and derivatives w.r.to λ_1 and λ_2 and equating to zero, we get MLE's of λ_1 and λ_2 respectively i.e.

$$\frac{dL}{d\lambda_1} = \frac{n_1}{\lambda_1} - n_1\overline{\varepsilon} = 0 \Rightarrow \tilde{\lambda_1} = \frac{1}{\overline{\varepsilon}}$$
$$\frac{dL}{d\lambda_2} = \frac{n_2}{\lambda_2} - n_2\overline{\varepsilon} = 0 \Rightarrow \tilde{\lambda_2} = \frac{1}{\overline{\eta}}$$

The reliability function $\tilde{R} = \frac{\overline{\eta}}{\varepsilon+\eta}$,can be written as

$$\tilde{R} = \frac{\overline{T_Y}}{T_X + T_Y},$$

hence, the theorem follows.

Corollary 1

1. On substituting $\gamma = 1$ in (2.1), we get the pdf of Inverse Weibull distribution and subsequently

$$\tilde{R} = \frac{\overline{T_Y}}{T_X + T_Y},$$

where $\overline{T_X} = \frac{1}{n_1}\sum_{i=1}^{n_1} X_i^{-\beta}$ and, $\overline{T_Y} = \frac{1}{n_2}\sum_{j=1}^{n_2} Y_j^{-\beta}$, which is MLE of $R = P(Y > X)$ when X and Y follows Inverse Weibull distribution.

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2. On substituting $\gamma = \beta = 1$ in (2.1), we get the pdf of Inverse exponential distribution and subsequently

$$\tilde{R} = \frac{\overline{T_Y}}{T_X + T_Y},$$

where $\overline{T_X} = \frac{1}{n_1}\sum_{i=1}^{n_1} X_i^{-1}$ and, $\overline{T_Y} = \frac{1}{n_2}\sum_{j=1}^{n_2} Y_j^{-1}$, which is MLE of $R = P(Y > X)$ when X and Y follows Inverse exponential distribution.

3. On substituting $\gamma = 1, \beta = 2$ in (2.1), we get the pdf of Inverse Rayleigh distribution and subsequently

$$\tilde{R} = \frac{\overline{T_Y}}{T_X + T_Y},$$

where $\overline{T_X} = \frac{1}{n_1}\sum_{i=1}^{n_1} X_i^{-2}$ and, $\overline{T_Y} = \frac{1}{n_2}\sum_{j=1}^{n_2} Y_j^{-2}$, which is MLE of $R = P(Y > X)$ when X and Y follows Inverse Raleigh distribution.

Theorem 2: The UMVUE of $R = P(Y > X)$ is given by

$$\hat{R} = \begin{cases} \sum_{i=0}^{n_2-1} (-1)^i \frac{\Gamma(n_1)\Gamma(n_2)}{\Gamma(n_2-i)\Gamma(n_1+i)} (\frac{n_1\bar{\varepsilon}}{n_2\eta})^i; & \text{if} n_1\bar{\varepsilon} < n_2\bar{\eta} \\ \sum_{i=0}^{n_1-2} (-1)^i \frac{\Gamma(n_1)\Gamma(n_2)}{\Gamma(n_1-i-1)\Gamma(n_2+i+1)} (\frac{n_2\bar{\eta}}{n_1\varepsilon})^{i+1}; & \text{if} n_1\bar{\varepsilon} > n_2\bar{\eta} \end{cases} \tag{2.6}$$

where $\bar{\varepsilon} = \frac{1}{n_1}\sum_{i=1}^{n_1} X_i^{-\beta_1}$, and $\bar{\eta} = \frac{1}{n_2}\sum_{j=1}^{n_2} Y_j^{-\beta_2}$.

Proof: Let us consider the transformation $x^{-\beta} = \varepsilon$ in(2.1), we get

$$f(\varepsilon|\alpha,,\beta,\gamma) = \lambda \exp(-\lambda\varepsilon); \quad \varepsilon, \lambda > 0$$

Now we obtain$P(\eta > \varepsilon)$, we required to obtain UMVUE of $f(\varepsilon;\lambda_1)$ and $f(\eta;\lambda_2)$ i.e. $\hat{f}(\varepsilon;\lambda_1)$ and $\hat{f}(\eta;\lambda_2)$ which is given by

$$\hat{f}(\varepsilon;\lambda_1) = \frac{(n_1-1)}{n_1\bar{\varepsilon}^{n_1-1}}[\bar{\varepsilon} - \frac{\varepsilon}{n_1}]^{n_1-2}; \quad \varepsilon < n_1\bar{\varepsilon} \tag{2.7}$$

Similarly on replacing ε by η and n_1 by n_2 in (2.7), we get the UMVUE of $f(\eta;\lambda_2)$.

$$\hat{f}(\eta;\lambda_2) = \frac{(n_2-1)}{n_2\bar{\eta}^{n_2-1}}[\bar{\eta} - \frac{\eta}{n_2}]^{n_2-2}; \quad \eta < n_2\bar{\eta} \tag{2.8}$$

Now, let us consider ε and η be the two random variables follows exponential distribution with the parameters λ_1 and λ_2 respectively, where $\varepsilon = x^{-\beta_1}$ and $\eta = y^{-\beta_2}$

$$\hat{R} == \int_0^{n_1\bar{\varepsilon}} \int_{\varepsilon}^{n_2\bar{\eta}} \hat{f}(\varepsilon;\lambda_1)\hat{f}(\eta;\lambda_2) d\eta d\varepsilon$$

$$\hat{R} = \int_0^{n_1\bar{\varepsilon}} \int_{\varepsilon}^{n_2\bar{\eta}} \frac{(n_1-1)}{n_1\bar{\varepsilon}^{n_1-1}}[\bar{\varepsilon} - \frac{\varepsilon}{n_1}]^{n_1-2} \frac{(n_2-1)}{n_2\bar{\eta}^{n_2-1}}[\bar{\eta} - \frac{\eta}{n_2}]^{n_2-2} d\eta d\varepsilon$$

Let $t = (1 - \frac{\eta}{n_2\eta})$,

$$\hat{R} = \frac{(n_1-1)}{n_1\bar{\varepsilon}} \int_0^{n_1\bar{\varepsilon}} [1 - \frac{\varepsilon}{n_1\bar{\varepsilon}}]^{n_1-2} [1 - \frac{\varepsilon}{n_2\bar{\eta}}]^{n_2-1} d\varepsilon$$

$$= \frac{(n_1-1)}{n_1\bar{\varepsilon}} \int_0^{min(n_1\bar{\varepsilon}, n_2\bar{\eta})} [1 - \frac{\varepsilon}{n_1\bar{\varepsilon}}]^{n_1-2} \sum_{i=0}^{n_2-1} (-1)^i \binom{n_2-1}{i} (\frac{\varepsilon}{n_2\bar{\eta}})^i d\varepsilon$$

Now,consider the case (i) when, $n_1\bar{\varepsilon} < n_2\bar{\eta}$,

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$$\hat{R}=\frac{(n_1-1)}{n_1\overline{\varepsilon}}\sum_{i=0}^{n_2-1}(-1)^i\binom{n_2-1}{i}\int_0^{min(n_1\overline{\varepsilon},n_2\overline{\eta})}\left[1-\frac{\varepsilon}{n_1\overline{\varepsilon}}\right]^{n_1-2}(\frac{\varepsilon}{n_2\overline{\eta}})^i d\varepsilon$$

Let $(1-\frac{\varepsilon}{\overline{\varepsilon}n_1})=z$,

$$\hat{R}=(n_1-1)\sum_{i=0}^{n_2-1}(-1)^i\binom{n_2-1}{i}(\frac{\overline{\varepsilon}n_1}{n_2\overline{\eta}})\int_0^1 z^{n_1-2}(1-z)^i dz$$

$$\hat{R}=\sum_{i=0}^{n_2-1}(-1)^i\frac{\Gamma(n_1)\Gamma(n_2)}{\Gamma(n_2-i)\Gamma(n_1+i)}(\frac{n_1\overline{\varepsilon}}{n_2\overline{\eta}})^i;\text{if} n_1\overline{\varepsilon}<n_2\overline{\eta} \tag{2.9}$$

Case (ii) when, $n_1\overline{\varepsilon}>n_2\overline{\eta}$,

$$=\frac{(n_1-1)}{n_1\overline{\varepsilon}}\int_0^{n_1\overline{\eta}}\left[1-\frac{\varepsilon}{n_1\overline{\varepsilon}}\right]^{n_1-2}\left[1-\frac{\varepsilon}{n_2\overline{\eta}}\right]^{n_2-1}d\varepsilon$$

$$\hat{R}=(n_1-1)\sum_{i=0}^{n_1-2}(-1)^i\binom{n_1-2}{i}(\frac{\overline{\eta}n_2}{n_1\overline{\varepsilon}})^{i+1}\int_0^1 z^{n_2-1}(1-z)^i dz$$

$$\hat{R}=\sum_{i=0}^{n_1-2}(-1)^i\frac{\Gamma(n_1)\Gamma(n_2)}{\Gamma(n_1-i-1)\Gamma(n_2+i+1)}(\frac{n_2\overline{\eta}}{n_1\overline{\varepsilon}})^{i+1};\text{if} n_1\overline{\varepsilon}>n_2\overline{\eta} \tag{2.10}$$

which is the UMVUE for $R=P(Y>X)$ where $\overline{\varepsilon}=\frac{1}{n_1}\sum_{i=1}^{n_1}X_i^{-\beta_1}$ and $\overline{\eta}=\frac{1}{n_2}\sum_{j=1}^{n_2}Y_j^{-\beta_2}$ in (2.9) and (2.10), when X and Y follows Generalized Inverse Weibull distribution, hence the theorem follows.

Corollary 2

1. On substituting $\gamma=1$ in (2.1), we get the pdf of Inverse Weibull distribution and subsequently

$$\hat{R}=\begin{cases}\sum_{i=0}^{n_2-1}(-1)^i\frac{\Gamma(n_1)\Gamma(n_2)}{\Gamma(n_2-i)\Gamma(n_1+i)}(\frac{n_1\overline{\varepsilon}}{n_2\eta})^i; & \text{if} n_1\overline{\varepsilon}<n_2\overline{\eta}\\ \sum_{i=0}^{n_1-2}(-1)^i\frac{\Gamma(n_1)\Gamma(n_2)}{\Gamma(n_1-i-1)\Gamma(n_2+i+1)}(\frac{n_2\overline{\eta}}{n_1\varepsilon})^{i+1}; & \text{if} n_1\overline{\varepsilon}>n_2\overline{\eta}\end{cases} \tag{2.11}$$

where $\overline{\varepsilon}=\frac{1}{n_1}\sum_{i=1}^{n_1}X_i^{-\beta}$ and $\overline{\eta}=\frac{1}{n_2}\sum_{j=1}^{n_2}Y_j^{-\beta}$, which is UMVUE of $R=P(Y>X)$ when X and Y follows Inverse Weibull distribution.

2. On substituting $\gamma=\beta=1$ (2.1), we get the pdf of Inverse exponential distribution and subsequently

$$\hat{R}=\begin{cases}\sum_{i=0}^{n_2-1}(-1)^i\frac{\Gamma(n_1)\Gamma(n_2)}{\Gamma(n_2-i)\Gamma(n_1+i)}(\frac{n_1\overline{\varepsilon}}{n_2\eta})^i; & \text{if} n_1\overline{\varepsilon}<n_2\overline{\eta}\\ \sum_{i=0}^{n_1-2}(-1)^i\frac{\Gamma(n_1)\Gamma(n_2)}{\Gamma(n_1-i-1)\Gamma(n_2+i+1)}(\frac{n_2\overline{\eta}}{n_1\varepsilon})^{i+1}; & \text{if} n_1\overline{\varepsilon}>n_2\overline{\eta}\end{cases} \tag{2.12}$$

where $\overline{\varepsilon}=\frac{1}{n_1}\sum_{i=1}^{n_1}X_i^{-1}$ and $\overline{\eta}=\frac{1}{n_2}\sum_{j=1}^{n_2}Y_j^{-1}$, which is UMVUE of $R=P(Y>X)$ when X and Y follows Inverse exponential distribution.

3. On substituting $\gamma=1,\beta=2$ we get the pdf of Inverse Rayleigh distribution and subsequently

$$\hat{R}=\begin{cases}\sum_{i=0}^{n_2-1}(-1)^i\frac{\Gamma(n_1)\Gamma(n_2)}{\Gamma(n_2-i)\Gamma(n_1+i)}(\frac{n_1\overline{\varepsilon}}{n_2\eta})^i; & \text{if} n_1\overline{\varepsilon}<n_2\overline{\eta}\\ \sum_{i=0}^{n_1-2}(-1)^i\frac{\Gamma(n_1)\Gamma(n_2)}{\Gamma(n_1-i-1)\Gamma(n_2+i+1)}(\frac{n_2\overline{\eta}}{n_1\varepsilon})^{i+1}; & \text{if} n_1\overline{\varepsilon}>n_2\overline{\eta}\end{cases} \tag{2.13}$$

where $\overline{\varepsilon}=\frac{1}{n_1}\sum_{i=1}^{n_1}X_i^{-2}$ and $\overline{\eta}=\frac{1}{n_2}\sum_{j=1}^{n_2}Y_j^{-2}$, which is UMVUE of $R=P(Y>X)$ when X and Y follows Inverse Rayleigh distribution.

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3 Interval estimation of $R = P(Y > X)$

Theorem 3: The confidence interval for $R = P(Y > X)$ is given by

$$P[\frac{n_2\tilde{R}a}{n_1(1-\tilde{R})(1-a)+n_2\tilde{R}a} < R < \frac{n_2\tilde{R}b}{n_1(1-\tilde{R})(1-b)+n_2\tilde{R}b}] = 1-\gamma \tag{3.1}$$

where $\tilde{R} = \frac{\bar{\eta}}{\varepsilon+\eta}$, a and b are random quantities.

Proof: We know that $R = \frac{\lambda_1}{\lambda_1+\lambda_2}$ and the MLE of R is$\tilde{R} = \frac{\bar{\eta}}{\varepsilon+\eta}$

where $\bar{\varepsilon} = \frac{1}{n_1}\sum_{i=1}^{n_1} X_i^{-\beta_1}$ and, $\bar{\eta} = \frac{1}{n_2}\sum_{j=1}^{n_2} Y_j^{-\beta_2}$. Here $\bar{\varepsilon}n_1$ and $\bar{\eta}n_2$ follows gamma distribution with parameters $(\lambda_1, n_1)$and (λ_2, n_2) respectively. In order to obtain exact confidence interval for $R = P(Y > X)$, we derive the exact distribution of

$$\zeta = \frac{\lambda_1 n_1 \bar{\varepsilon}}{(\lambda_1 n_1 \bar{\varepsilon} + \lambda_2 n_2 \bar{\eta})}$$

On substituting $\phi = \lambda_1 n_1 \bar{\varepsilon}$ and $\psi = \lambda_2 n_2 \bar{\eta}$, we observe that ϕ and ψ have gamma distribution with parameters $(1, n_1)$ and $(1, n_2)$. We can write

$$\zeta = \frac{\phi}{\phi+\psi}$$

On taking $\tau = \psi$ and expressing the old variables in terms of new set of variables then $\phi = \frac{\zeta\tau}{1-\zeta}$, we find joint probability density function of (ζ, τ).

$$\rho(\zeta,\tau) = \frac{e^{-(\frac{\tau}{1-\zeta})}\tau^{n_1+n_2-1}\zeta^{n_1-1}}{\Gamma n_1 \Gamma n_2 (1-\zeta)^{n_1+1}}$$

The marginal distribution of ζ,

$$p(\zeta) = \frac{1}{[B(n_1,n_2)]}\zeta^{n_1-1}(1-\zeta)^{n_2-1}; \quad 0 < \zeta < 1$$

here n_1 ,n_2 are the known parameters for any value $0 < a < b$.

$$P(a < \zeta < b) = I_b(n_1,n_2) - I_a(n_1,n_2)$$

where $I_x(n_1,n_2) = \frac{1}{[B(n_1,n_2)]}\int_0^x z^{n_1-1}(1-z)^{n_2-1}dz$ is incomplete beta function. We know that $R = \frac{\lambda_1}{\lambda_1+\lambda_2}$ and $\tilde{R} = \frac{\bar{\eta}}{\varepsilon+\eta}$, we get

$$\zeta = [1+\frac{n_2\tilde{R}(1-R)}{n_1R(1-\tilde{R}}]^{-1} \tag{3.2}$$

The right hand side pivotal quantities is a and b such that

$I_b(n_1,n_2) - I_a(n_1,n_2) = 1-\gamma$, then

$$P(a < \zeta < b) = 1-\gamma$$

On substituting the value from (3.2), get

$$P(a < [1+\frac{n_2\tilde{R}(1-R)}{n_1R(1-\tilde{R}}]^{-1} < b) = 1-\gamma.$$

Hence

$$P[\frac{n_2\tilde{R}a}{n_1(1-\tilde{R})(1-a)+n_2\tilde{R}a} < R < \frac{n_2\tilde{R}b}{n_1(1-\tilde{R})(1-b)+n_2\tilde{R}b}] = 1-\gamma$$

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where $\tilde{R} = \frac{\bar{\eta}}{\varepsilon+\eta}$ and $\bar{T}_X = \frac{1}{n_1}\sum_{i=1}^{n_1} X_i^{-\beta_1} = \bar{\varepsilon}$, $\bar{T}_Y = \frac{1}{n_2}\sum_{j=1}^{n_2} Y_j^{-\beta_2} = \bar{\eta}$, a and b random quantity and the theorem follows.

Corollary 3

1. On substituting $\gamma = 1$ in (2.1), we get the pdf of Inverse Weibull distribution and subsequently

$$P[\frac{n_2\tilde{R}a}{n_1(1-\tilde{R})(1-a)+n_2\tilde{R}a} < R < \frac{n_2\tilde{R}b}{n_1(1-\tilde{R})(1-b)+n_2\tilde{R}b}] = 1-\gamma$$

where $\tilde{R} = \frac{\bar{\eta}}{\varepsilon+\eta}$ and $\bar{T}_X = \frac{1}{n_1}\sum_{i=1}^{n_1} X_i^{-\beta} = \bar{\varepsilon}$, $\bar{T}_Y = \frac{1}{n_2}\sum_{j=1}^{n_2} Y_j^{-\beta} = \bar{\eta}$, a and b are the random quantities which is the confidence interval for $R = P(Y > X)$ when X and Y follows Inverse Weibull distribution.

2. On substituting $\gamma = \beta = 1$ in (2.1), we get the pdf of Inverse exponential distribution and subsequently

$$P[\frac{n_2\tilde{R}a}{n_1(1-\tilde{R})(1-a)+n_2\tilde{R}a} < R < \frac{n_2\tilde{R}b}{n_1(1-\tilde{R})(1-b)+n_2\tilde{R}b}] = 1-\gamma$$

where $\tilde{R} = \frac{\bar{\eta}}{\varepsilon+\eta}$ and $\bar{T}_X = \frac{1}{n_1}\sum_{i=1}^{n_1} X_i^{-1} = \bar{\varepsilon}$, $\bar{T}_Y = \frac{1}{n_2}\sum_{j=1}^{n_2} Y_j^{-1} = \bar{\eta}$, a and b are the random quantities which is the confidence interval for $R = P(Y > X)$ when X and Y follows Inverse exponential distribution.

3. On substituting $\gamma = 1, \beta = 2$ in (2.1), we get the pdf of Inverse Rayleigh distribution and subsequently

$$P[\frac{n_2\tilde{R}a}{n_1(1-\tilde{R})(1-a)+n_2\tilde{R}a} < R < \frac{n_2\tilde{R}b}{n_1(1-\tilde{R})(1-b)+n_2\tilde{R}b}] = 1-\gamma$$

where $\tilde{R} = \frac{\bar{\eta}}{\varepsilon+\eta}$ and $\bar{T}_X = \frac{1}{n_1}\sum_{i=1}^{n_1} X_i^{-2} = \bar{\varepsilon}$, $\bar{T}_Y = \frac{1}{n_2}\sum_{j=1}^{n_2} Y_j^{-2} = \bar{\eta}$, a and b are the random quantities which is the confidence interval for $R = P(Y > X)$ when X and Y follows Inverse Rayleigh distribution.

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