Journal of Statistics Applications & Probability Letters
An International Journal

http://dx.doi.org/10.18576/jsapl/030105

Bayesian Analysis of Lomax Distribution under Asymmetric Loss Functions

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Received: 29 Jun. 2015, Revised: 28 Aug.2015, Accepted: 2 Sep. 2015.
Published online: 1 Jan. 2016.

Abstract: In the present paper, a two parametric Lomax distribution is considered for Bayesian analysis. The estimation of the shape parameter of Lomax distribution is obtained by employing the classical and Bayesian paradigm. Bayes' estimators are obtained by using extension of Jeffrey's prior and Gamma prior under Entropy loss function and Precautionary loss function. Maximum likelihood estimation is also discussed. These methods are compared by using mean square error through simulation study with varying sample sizes.

Keywords: Lomax distribution, Informative and Non-informative Priors, Entropy Loss function, Precautionary Loss function, R software.

1 Introduction

The Lomax distribution also known as Pareto distribution of second kind has, in recent years, assumed opposition of importance in the field of life testing because of its uses to fit business failure data. It has been used in the analysis of income data, and business failure data. It may describe the lifetime of a decreasing failure rate component as a heavy tailed alternative to the exponential distribution. Lomax distribution was introduced by Lomax (1954), Abdullah and Abdullah (2010) estimated the parameters of Lomax distribution based on generalized probability weighted moment. Zangan (1999) deals with the properties of the Lomax distribution with three parameters. Abd-Elfatth and Mandouh (2004) discussed inference for R = Pr{Y<X} when X and Y are two independent Lomax random variables. Nasiri and Hosseini (2012) performs comparisons of maximum likelihood estimation (MLE) based on records and a proper prior distribution to attain a Bayes estimation (both informative and non-informative) based on records under quadratic loss and squared error loss functions. Afaq et al. (2014) estimates the parameters of Lomax distribution using Jeffery's and extension of Jeffery's prior under different loss functions. The cumulative distribution function of Lomax distribution is given by

$$F(x:\theta,\lambda)=1-\left(1+\frac{x}{\lambda}\right)^{-\theta} \tag{1.1}$$

Therefore, the corresponding probability density function is given by

$$f(x;\theta,\lambda)=\frac{\theta}{\lambda}\left(1+\frac{x}{\lambda}\right)^{-(\theta+1)} \qquad ; \quad x>0,\theta,\lambda>0 \tag{1.2}$$

Where θ and λ are shape and scale parameters, respectively.
The survival function is given by

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$$R(x;\theta,\lambda)=\left(1+\frac{x}{\lambda}\right)^{-\theta} \tag{1.3}$$

And the hazard function is given by

$$h(x;\theta,\lambda)=\frac{\theta}{\lambda}\left(1+\frac{x}{\lambda}\right)^{-(2\theta+1)} \tag{1.4}$$

2 Material and Method

2.1 Prior and Loss Functions

The Bayesian inference requires appropriate choice of prior(s) for the parameter(s). From the Bayesian viewpoint, there is no clear cut way from which one can conclude that one prior is better than the other. Nevertheless, very often priors are chosen according to one's subjective knowledge and beliefs. However, if one has adequate information about the parameter(s), it is better to choose informative prior(s); otherwise, it is preferable to use non-informative prior(s). In this paper we consider both types of priors: the extended Jeffreys' prior and the natural conjugate prior.
The extended Jeffreys' prior proposed by Al-Kutubi (2002)is given as

$$g_1(\theta)\propto\left[I(\theta)\right]^c \quad , \qquad c\varepsilon\, R^+$$

Where $\left[I(\theta)\right]=-nE\left[\frac{\partial^2 \log f(x;\theta,\alpha)}{\partial\theta^2}\right]$ is the Fisher's information matrix. For the model (1.1),

$$g_1(\theta)=\frac{1}{\theta^{2c}} \tag{2.1}$$

The gamma distribution is used as a conjugate prior for θ with hyper parameters a and b which is also a conjugate prior for the class of distribution, so the prior distribution is

$$g_2(\theta)=\frac{a^b}{\Gamma b}e^{-a\theta}\theta^{b-1}\ , \qquad a,b,\theta>0 \tag{2.2}$$

Where *a* and*b* are hyper-parameters.
With the above priors, we use two different loss functions for the model (1.2).

a) Entropy Loss Function

In many practical situations, it appears to be more realistic to express the loss in terms of the ratio $\hat{\theta}/\theta$. In this case, Calabria and Pulcini (1994) point out that a useful asymmetric loss function is the entropy loss function:

$$L(\delta^p)\propto[\delta^p-p\log(\delta)-1]$$

Where $\delta=\frac{\hat{\theta}}{\theta}$ and *p*>0, whose minimum occurs at $\hat{\theta}=\theta$. Also, the loss function $L(\delta)$ has been used in Dey et al (1987) and Dey and Liu (1992), in the original form having *p* =1. Thus, $L(\delta)$ can be written as

$$L(\delta)=d[\delta-\log(\delta)-1]\ ;d>0. \tag{2.3}$$

b) Precautionary Loss Function

The precautionary loss function given by:

$$l\left(\hat{\theta},\theta\right)=\frac{\left(\hat{\theta}-\theta\right)^2}{\hat{\theta}} \tag{2.4}$$

Which is an asymmetric loss function, for details, see Norstrom (1996). This loss function is interesting in the sense that a slight modification of squared error loss introduces asymmetry.

3 Maximum Likelihood Estimation

Let us consider a random sample $\underline{x}=(x_1,x_2,...,x_n)$ of size n from Lomax distribution. Then the log-likelihood function for the given sample observation is

$$\log L(\theta,\lambda)=n\log\theta-n\log\lambda-(\theta+1)\sum_{i=1}^{n}\log\left(1+\frac{x_i}{\lambda}\right)$$

As the parameter λ is assumed to be known, the ML estimator of θ is obtained by solving the equation

$$\frac{\partial\ln L(\theta,\lambda)}{\partial\theta}=0$$

$$\Rightarrow\frac{n}{\theta}-\sum_{i=1}^{n}\log\left(1+\frac{x_i}{\lambda}\right)=0\Rightarrow\hat{\theta}_{ML}=\frac{n}{\sum_{i=1}^{n}\log\left(1+\frac{x_i}{\lambda}\right)} \quad (3.1)$$

4 Bayesian Estimation of Lomax Distribution under The Extension of Jeffrey's prior by Using Different Loss Function

Combining the prior distribution in (2.1) and the likelihood function, the posterior density of θ is derived as follows:

$$\pi_1(\theta\,|\,\underline{x})\propto\left(\frac{\theta}{\lambda}\right)^{n}\prod_{i=1}^{n}\left(1+\frac{x_i}{\lambda}\right)^{-(\theta+1)}\frac{1}{\theta^{2c}}$$

$$\Rightarrow\pi_1(\theta\,|\,\underline{x})\propto\frac{\theta^{n-2c}}{\lambda^{n}}\exp\left(\left(-(\theta+1)\sum_{i=1}^{n}\log\left(1+\frac{x_i}{\lambda}\right)\right)\right)$$

$$\Rightarrow\pi_1(\theta\,|\,\underline{x})=k\,\theta^{n-2c}\exp\left(-\theta\sum_{i=1}^{n}\log\left(1+\frac{x_i}{\lambda}\right)\right)$$

Where k is independent of θ

And $$k^{-1}=\int_0^{\infty}\theta^{n-2c}\exp\left(-\theta\sum_{i=1}^{n}\log\left(1+\frac{x_i}{\lambda}\right)\right)d\theta$$

$$\Rightarrow k^{-1}=\frac{\Gamma(n-2c+1)}{\left[\sum_{i=1}^{n}\ln\left(1+\frac{x_i}{\lambda}\right)\right]^{n-2c+1}}$$

Hence posterior distribution of θ is given by

$$\pi_1(\theta\,|\,\underline{x})=\frac{\left[\sum_{i=1}^{n}\log\left(1+\frac{x_i}{\lambda}\right)\right]^{n-2c+1}}{\Gamma(n-2c+1)}\theta^{n-2c}\exp\left(-\left(\sum_{i=0}^{\infty}\left(1+\frac{x_i}{\lambda}\right)\right)\theta\right)$$

$$\pi_1(\theta\,|\,\underline{x})=\frac{t^{n-2c+1}}{\Gamma(n-2c+1)}\theta^{n-2c}e^{-t\theta} \quad (4.1)$$

Where $t=\sum_{i=1}^{n}\log\left(1+\frac{x_i}{\lambda}\right)$

4.1 Estimation under Entropy Loss Function

By using entropy loss function $L(\delta)=b[\delta-\log\delta-1]$ for some constant b the risk function is given by

$$R(\hat{\theta},\theta)=\int_0^{\infty} b\big(\delta-\log(\delta)-1\big)\pi_1(\theta\,|\,\underline{x})\,d\theta$$

$$=\int_0^{\infty} b\left(\frac{\hat{\theta}}{\theta}-\log\left(\frac{\hat{\theta}}{\theta}\right)-1\right)\frac{t^{n-2c+1}}{\Gamma(n-2c+1)}\theta^{n-2c}\;e^{-\theta t}\;d\theta$$

$$=b\frac{t^{n-2c+1}}{\Gamma(n-2c+1)}\int_0^{\infty}\left(\frac{\hat{\theta}}{\theta}-\log\left(\frac{\hat{\theta}}{\theta}\right)-1\right)\theta^{n-2c}\;e^{-\theta t}\;d\theta$$

$$=b\frac{t^{n-2c+1}}{\Gamma(n-2c+1)}\left[\begin{array}{l}\hat{\theta}\int_0^{\infty}\theta^{n-2c-1}\,e^{-\theta t}d\theta-\log(\hat{\theta})\int_0^{\infty}\theta^{n-2c}\,e^{-\theta t}d\theta+\int_0^{\infty}\log(\theta)\,\theta^{n-2c}\,e^{-\theta t}\,d\theta-\\ \int_0^{\infty}\theta^{n-2c}e^{-\theta t}\,d\theta\end{array}\right]$$

$$=b\frac{t^{n-c+1}}{\Gamma(n-2c+1)}\left[\hat{\theta}\frac{\Gamma(n-2c)}{t^{n-2c}}-\log(\hat{\theta})\frac{\Gamma(n-2c+1)}{t^{n-2c+1}}-\frac{\Gamma'(n-2c+1)}{t^{n-2c+1}}-\frac{\Gamma(n-2c+1)}{t^{n-2c+1}}\right]$$

$$=b\left[\frac{\hat{\theta}t}{n-2c}-\log(\hat{\theta})-\frac{\Gamma'(n-2c+1)}{\Gamma(n-2c+1)}-1\right]$$

Now solving $\frac{\partial R(\hat{\theta},\theta)}{\partial\hat{\theta}}=0$, we obtain the Bayes estimator as

$$\hat{\theta}=\frac{n-2c}{t} \tag{4.2}$$

Remark 1*:* We get the Jeffreys' non-informative prior for $c=1/2$ and the Hartigan's non-informative prior for $c=3/2$

4.2 Estimation under Precautionary loss function

By using precautionary loss function $l(\hat{\theta},\theta)=\frac{(\hat{\theta}-\theta)^2}{\hat{\theta}}$ the risk function is given by

$$R(\hat{\theta}) = \int_0^\infty \frac{(\hat{\theta}-\theta)^2}{\hat{\theta}} \frac{t^{n-2c+1}}{\Gamma(n-2c+1)} \theta^{n-2c} \exp(-\theta t) d\theta$$

$$= \frac{t^{n-2c+1}}{\hat{\theta}\,\Gamma(n-2c+1)} \int_0^\infty (\hat{\theta}-\theta)^2 \theta^{n-2c} \exp(-\theta t) d\theta$$

$$= \hat{\theta} + \frac{(n-2c+2)(n-2c+1)}{t^2 \hat{\theta}} - \frac{2(n-2c+1)}{t}$$

Now solving $\frac{\partial R(\hat{\theta})}{\partial \hat{\theta}} = 0$, we obtain the Baye's estimator as

$$\hat{\theta}_p = \frac{\left[(n-2c+2)(n-2c+1)\right]^{\frac{1}{2}}}{t} \tag{4.3}$$

Remark 2: We get the Jeffreys' non-informative prior for $c = 1/2$ and the Hartigan's non-informative prior for $c = 3/2$

5 Bayesian Estimation of Lomax Distribution under Conjugate Prior By Using Different Loss Function

Combining the prior distribution in (2.2) and the likelihood function, the posterior density of θ is derived as follows:

$$\pi_2(\theta \mid \underline{x}) \propto \left(\frac{\theta}{\lambda}\right)^n \prod_{i=1}^n \left(1+\frac{x}{\lambda}\right)^{-(\theta+1)} \frac{a^b}{\Gamma b} e^{-a\theta} \theta^{b-1}$$

$$\Rightarrow \pi_2(\theta \mid \underline{x}) \propto \frac{\theta^n}{\lambda^n} \exp\left(-(\theta+1)\sum_{i=1}^n \log\left(1+\frac{x}{\lambda}\right)\right) \frac{a^b}{\Gamma b} e^{-a\theta} \theta^{b-1}$$

$$\Rightarrow \pi_2(\theta \mid \underline{x}) = k\, \theta^{n+b-1} \exp\left(-\left(a+\sum_{i=1}^n \left(1+\frac{x_i}{\lambda}\right)\right)\right)$$

Where k is independent of θ

And $k^{-1} = \int_0^\infty \theta^{n+b-1} \exp\left(-\left(a+\sum_{i=1}^\infty \left(1+\frac{x_i}{\lambda}\right)\right)\right) d\theta$

$$\Rightarrow k^{-1} = \frac{\Gamma(n+b)}{\left[\left(a+\sum_{i=1}^n \ln\left(1+\frac{x_i}{\lambda}\right)\right)\right]^{n+b}}$$

Hence posterior distribution of θ is given by

$$\pi_2(\theta \mid \underline{x}) = \frac{\left[a + \sum_{i=1}^{n} \log\left(1 + \frac{x}{\lambda}\right)\right]^{n+b}}{\Gamma(n+b)} \;\; \theta^{n+b-1} \exp\left(-\left(a + \sum_{i=0}^{\infty}\left(1 + \frac{x_i}{\lambda}\right)\right)\theta\right)$$

$$\pi_2(\theta \mid \underline{x}) = \frac{t^{n+b}}{\Gamma(n+b)} \;\; \theta^{n+b-1} e^{-t\theta} \tag{5.1}$$

Where $t = \left[a + \sum_{i=1}^{n} \log\left(1 + \frac{x}{\lambda}\right)\right]$

Which is the probability density function of gamma distribution with parameters (t, $n+b$)

5.1 Estimation under Entropy Loss Function

$$R(\hat{\theta}, \theta) = \int_0^\infty b\left(\delta - \log(\delta) - 1\right)\pi_2(\theta \mid \underline{x})\, d\theta$$

$$= \int_0^\infty b\left(\frac{\hat{\theta}}{\theta} - \log\left(\frac{\hat{\theta}}{\theta}\right) - 1\right)\frac{t^{n+b}}{\Gamma(n+b)}\theta^{n+b-1} \; e^{-t\theta} \;\; d\theta$$

$$= b\frac{t^{n+b}}{\Gamma(n+b)}\int_0^\infty \left(\frac{\hat{\theta}}{\theta} - \log\left(\frac{\hat{\theta}}{\theta}\right) - 1\right)\theta^{n+b-1} \; e^{-t\theta} \;\; d\theta$$

$$= b\frac{t^{n+b}}{\Gamma(n+b)}\left[\begin{array}{l} \hat{\theta}\int_0^\infty \theta^{n+b-2} \, e^{-t\theta} \, d\theta - \log(\hat{\theta})\int_0^\infty \theta^{n+b-1} \, e^{-t\theta} \, d\theta + \int_0^\infty \log(\theta)\,\theta^{n+b-1} \, e^{-t\theta} \;\, d\theta - \\ \int_0^\infty \theta^{n+b-1} e^{-t\theta} \;\, d\theta \end{array}\right]$$

$$= b\frac{t^{n+b}}{\Gamma(n+b)}\left[\hat{\theta}\frac{\Gamma(n+b-1)}{t^{n+b-1}} - \log(\hat{\theta})\frac{\Gamma(n+b)}{t^{n+b}} - \frac{\Gamma'(n+b)}{t^{n+b}} - \frac{\Gamma(n+b)}{t^{n+b}}\right]$$

$$= b\left[\frac{\hat{\theta}t}{(n+b-1)} - \log(\hat{\theta}) - \frac{\Gamma'(n+b)}{\Gamma(n+b)} - 1\right]$$

Now solving $\dfrac{\partial R(\hat{\theta}, \theta)}{\partial \hat{\theta}} = 0$, we obtain the Bayes estimator as

$$\hat{\theta} = \frac{n+b-1}{t} \tag{5.2}$$

5.2 Estimation under Precautionary loss function

By using precautionary loss function $l(\hat{\theta},\theta)=\frac{(\hat{\theta}-\theta)^2}{\hat{\theta}}$ the risk function is given by

$$R(\hat{\theta},\theta)=\int_0^\infty \frac{(\hat{\theta}-\theta)^2}{\hat{\theta}}\frac{t^{n+b}}{\Gamma(n+b)}\theta^{n+b-1}e^{-t\theta}\,d\theta$$

$$=\frac{t^{n+b}}{\hat{\theta}\Gamma(n+b)}\left[\hat{\theta}^2\int_0^\infty \theta^{n+b-1}e^{-t\theta}\,d\theta-2\hat{\theta}\int_0^\infty \theta^{n+b}e^{-t\theta}\,d\theta+\int_0^\infty \theta^{n+b+1}e^{-t\theta}\,d\theta\right]$$

$$=\frac{t^{n+b}}{\hat{\theta}\Gamma(n+b)}\left[\hat{\theta}^2\frac{\Gamma(n+b)}{t^{n+b}}-2\hat{\theta}\frac{\Gamma(n+b+1)}{t^{n+b+1}}+\frac{\Gamma(n+b+2)}{t^{n+b+2}}\right]$$

$$=\hat{\theta}-\frac{2(n+b)}{t}+\frac{(n+b)(n+b+1)}{\hat{\theta}t^2}$$

Now solving $\frac{\partial R(\hat{\theta})}{\partial\hat{\theta}}=0$, we obtain the Baye's estimator as

$$\hat{\theta}_p=\frac{[(n+b)(n+b+1)]^{\frac{1}{2}}}{t} \tag{5.3}$$

6 Elicitation of Hyperparameters

When significant amount of information is available pertaining to the model, then it first become essential to quantify such information in the form of a (prior) probability distribution and then properly use this prior into the subsequent Bayesian analysis. The process of quantifying the prior information accurately is known as elicitation. Garthwaite et al. (2005), defined elicitation is the process of formulating a person's knowledge and attitude about one or more uncertain quantities into a probability distribution for those quantities. In the context of Bayesian statistical analysis, it arises most usually as a method for specifying the prior distribution for one or more unknown parameters of a statistical model. Aslam (2003) proposed some new methods in his paper based on prior predictive distribution. He develops four methods in this paper based on prior predictive distribution on elicit the hyperparameters of prior density for the parameters of Bradley Terry model for paired comparison data. Numerous method of elicitation are devised in Kazmi et al. (1993), Kadane (1980), Aslam (2003) and Gelman et al. (2004). The values of hyperparameters can also be taken directly by knowing the range of hyperparameters in prior distribution.

7 Simulation Study

In our simulation study, we chose a sample size of n=25, 50 and 100 to represent small, medium and large data set. The shape parameter is estimated for Lomax distribution with Maximum Likelihood and Bayesian using extension of Jeffrey's prior and gamma pior. For the shape parameter we have considered θ = 0.5 and 1.0. The scale parameter λ has been fixed at 0.5 and 1.0.The values of Jeffrey's extension were c = 0.5 and 1.5. The value of loss parameter *a* and*b* are (0.5, 0.1 and (1.0, 0.5) respectively. This was iterated 1000 times and the shape parameter for each method was calculated. A simulation study was conducted R-software to examine and compare the performance of the estimates for different sample sizes with different values of loss functions. The results are presented in tables for different selections of the parameters.

Table 7.1: Mean Squared Error for $(\hat{\theta})$ under extension of Jeffery's prior

n	θ	λ	C	$\hat{\theta}_{ML}$	$\hat{\theta}_E$	$\hat{\theta}_P$
25	0.5	0.5	0.5	0.0198 (0.5800)	0.0161 (0.5568)	0.0212 (0.5914)
	1.0	0.5	0.5	0.1079 (1.2202)	0.0864 (1.1714)	0.1167 (1.2443)
	0.5	1.0	1.5	0.0180 (0.5708)	0.0114 (0.5023)	0.0127 (0.5365)
	1.0	1.0	1.5	0.0438 (1.0646)	0.0438 (0.9369)	0.0399 (1.0005)
50	0.5	0.5	0.5	0.0056 (0.5174)	0.0052 (0.5071)	0.0057 (0.5226)
	1.0	0.5	0.5	0.0761 (1.2161)	0.0656 (1.1917)	0.0809 (1.2282)
	0.5	1.0	1.5	0.0054 (0.5142)	0.0051 (0.4833)	0.0049 (0.4987)
	1.0	1.0	1.5	0.0251 (1.0540)	0.0208 (0.9908)	0.0212 (1.0223)
100	0.5	0.5	0.5	0.0043 (0.5387)	0.0039 (0.5333)	0.0045 (0.5414)
	1.0	0.5	0.5	0.0134 (1.0497)	0.0124 (1.0392)	0.0139 (1.0550)
	0.5	1.0	1.5	0.0028 (0.5146)	0.0025 (0.4992)	0.0025 (0.5069)
	1.0	1.0	1.5	0.0102 (1.0099)	0.0102 (0.9796)	0.0104 (0.9742)

ML= Maximum Likelihood, E= Entropy loss function, P= precautionary loss function.

Table 7.2: Mean Squared Error for $(\hat{\theta})$ under Gamma prior

n	θ	λ	A	B	$\hat{\theta}_{ML}$	$\hat{\theta}_E$	$\hat{\theta}_P$
25	0.5	0.5	0.5	0.1	0.1550 (0.8547)	0.1252 (0.8101)	0.1589 (0.8604)
	1.0	0.5	0.5	0.1	0.6014 (1.1102)	0.0514 (1.0470)	0.0617 (1.1119)
	0.5	1.0	1.0	0.5	0.1877 (0.8947)	0.1495 (0.8465)	0.1880 (0.8982)
	1.0	1.0	1.0	0.5	0.0666 (1.1261)	0.0477 (1.0560)	0.0591 (1.1205)
50	0.5	0.5	0.5	0.1	0.0130 (0.5798)	0.0110 (0.5661)	0.0136 (0.5834)
	1.0	0.5	0.5	0.1	0.0322 (1.0927)	0.0275 (1.0614)	0.0325 (1.0938)
	0.5	1.0	1.0	0.5	0.0216 (0.6186)	0.0185 (0.6049)	0.0226 (0.6232)
	1.0	1.0	1.0	0.5	0.0326 (1.0934)	0.0259 (1.0593)	0.0307 (1.0913)
100	0.5	0.5	0.5	0.1	0.0025 (0.5023)	0.0025 (0.4966)	0.0025 (0.5041)
	1.0	0.5	0.5	0.1	0.0140 (1.0523)	0.0126 (1.0395)	0.0141 (1.0552)
	0.5	1.0	1.0	0.5	0.0039 (0.5334)	0.0035 (0.5279)	0.0040 (0.5359)
	1.0	1.0	1.0	0.5	0.0125 (1.0423)	0.0111 (1.0264)	0.0122 (1.0419)

ML= Maximum Likelihood, E= Entropy loss function, P= precautionary loss function.

It is marked from the above analysis that the increased samples size imposes a positive impact on the behavior of the estimators. On the other hand, the increasing true parametric values positively affected the performance of the estimators. The amount of over-estimation has been observed under each prior, sample size and loss function. In comparison of informative and non-informative priors it is assessed that the estimates under the informative priors are simply better than those under non-informative priors. Similarly, the estimates under entropy loss function provide the smallest values of mean square error under both Jeffery's and gamma priors.

8 Concluding Remarks

The study was conducted to find out an appropriate Bayes estimator for the parameter of Lomax distribution. Two informative and non-informative priors have been assumed under two loss functions for the posterior analysis. The performance of the different estimators has been evaluated under a detailed simulation study. The study proposed that in to order estimate the said parameter, the use of gamma prior under precautionary loss function can be preferred.

Acknowledgement

The authors are highly grateful to the editor and referees for their constructive comments and suggestions that helped in the improvement of the revised version of the paper.

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