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A Method of Proposing New Distribution and its Application to Bladder Cancer Patients Data

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Abstract: A new distribution is proposed by the use of some baseline distribution. As an application part, it is derived for the baseline distribution as exponential distribution. The new distribution, thus obtained have been shown to fit the bladder cancer patients data. Further, maximum likelihood estimator (MLE) and Bayes estimators under general entropy loss function (GELF) and squared error loss function (SELF) have been derived. The estimators have been compared through their Simulated risks.

Keywords: Life Time Distribution, Reliability Analysis, Maximum Likelihood Estimation and Bayesian Inferences.

1 Introduction

In statistical literature, there are several method to propose new distribution by the use of some baseline distribution. For example, Gupta et al. [4] proposed the cumulative distribution function (cdf) $G_1(x)$ of new distribution corresponding to the cdf $F(x)$ of baseline distribution as,

$$G_1(x) = (F(x))^{\alpha},$$

where, $\alpha > 0$ is the shape parameter of the proposed one.

Another idea of generalizing a baseline distribution is to transmute it by using quadratic rank transmutation map (QRTM) (see, Shaw and Buckley [6]). If $G_2(x)$ be the cdf of transmuted distribution corresponding to the baseline distribution having cdf $F(x)$, then

$$G_2(x) = (1+\lambda)F(x) - \lambda\{F(x)\}^2,$$

where $|\lambda| \leq 1$.

Recently, various generalizations has been introduced based on QRTM. For example, transmuted extreme value distribution (see, Aryal and Tsokos [8]), transmuted inverse Weibull distribution (see, Khan et al. [15]), transmuted modified Weibull distribution (see, Khan and King [14]), transmuted log-logistic distribution (see, Aryal [12]) and many more.

In the present study, we propose another method to get new distribution by the use of some baseline distribution. If $f(x)$ and $F(x)$ be the probability density function (pdf) and cdf of some baseline distribution, then the pdf $g(x)$ of new distrbution is proposed by,

$$g(x) = \frac{1}{e-1}\, f(x)\, e^{F(x)} \tag{1}$$

We will call the transformation (1) as DUS transformation for frequently used purpose in the present article or elsewhere. It is clearly a transformation, not a generalization, hence it will produce a parsimonious distribution in terms of

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computation and interpretation as it never contain any new parameter other than the parameter(s) involved in the baseline distribution.

The cdf and hazard rate function corresponding to the pdf $g(x)$ are given by,

$$G(x) = \frac{1}{e-1}\left[e^{F(x)} - 1\right] \tag{2}$$

and

$$h(x) = \frac{1}{e - e^{F(x)}}\, f(x)\, e^{F(x)} \tag{3}$$

respectively.

The rest of the paper is organized as follows: In section 2, we propose a new distribtion, as obtain by DUS transformation (1) by using Exp(θ)- distribution as the baseline distribution. Further, in section 3, we have shown the applicability of the new distribution obtained in the section 2, to the bladder cancer patients data in terms of assesing its fitting in comparison to some available distributions. In section 4, we have derived MLE and Bayes estimators of the parameter θ of this new distribution under GELF and SELF. Finally, comparison and conclusion has been shown in sections 5 and 6 respectively.

2 DUS transformation of Exp(θ)- distribution

Let the baseline distribution is exponential distribution with pdf,

$$f(x) = \theta\, e^{-\theta x} \quad ; \quad x > 0 \tag{4}$$

and the corresponding cdf is given by,

$$F(x) = 1 - e^{-\theta x} \tag{5}$$

Here, $\theta > 0$ is the rate parameter or inverse scale parameter of the exponential distribution.

Let $g(x)$ be the pdf of the new distribution; obtained by DUS transformation (1), corresponding to the baseline pdf (4), then

$$g(x) = \frac{1}{e-1}\, \theta\, e^{-\theta x}\, e^{1-e^{-\theta x}} \quad ; \quad x > 0 \tag{6}$$

For simplicity in terms of use, we name/call the distribution having pdf (6) as DUS transformation of Exp (θ)-distribution and will write it as $\mathrm{DUS}_E(\theta)$-distribution.

The cdf and hazard rate function of $\mathrm{DUS}_E(\theta)$-distribution are given by,

$$G(x) = \frac{1}{e-1}\left[e^{1-e^{-\theta x}} - 1\right] \tag{7}$$

and

$$h(x) = \theta e^{-\theta x}\left[e^{e^{-\theta x}} - 1\right]^{-1} \tag{8}$$

respectively.

The plots of pdf and hazard rate function of $\mathrm{DUS}_E(\theta)$-distribution for different values of θ are shown the Figures 1 and 2 respectively.

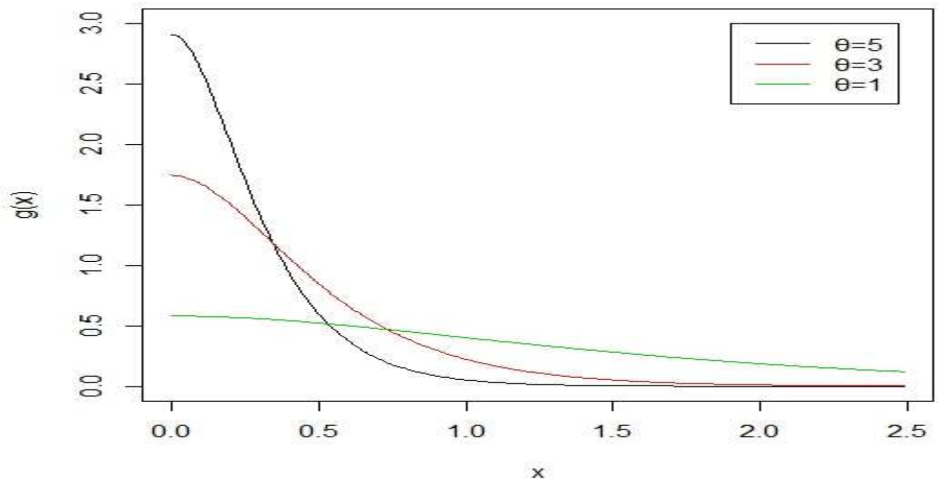

Fig. 1: Plots of pdf of $\text{DUS}_E(\theta)$-distribution for different values of θ

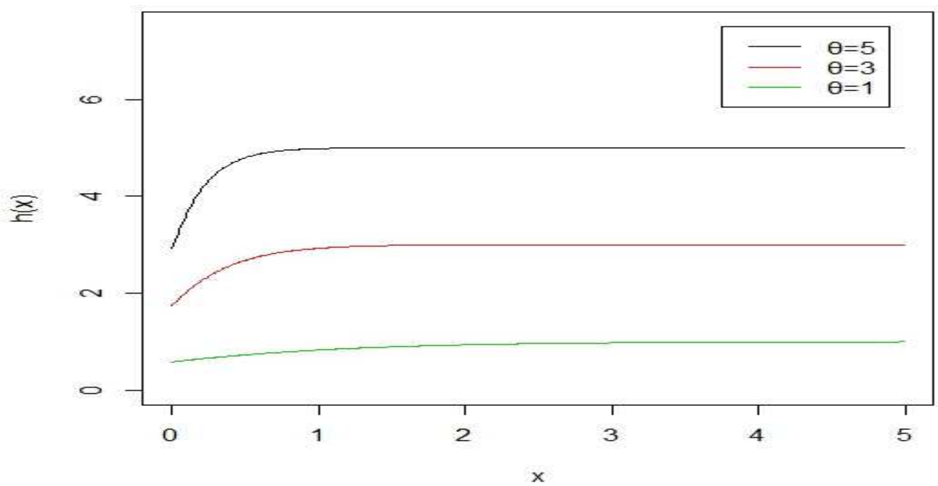

Fig. 2: Plots of hazard rate function of $\text{DUS}_E(\theta)$-distribution for different values of θ

MGF and Raw Moments of DUS$_E(\theta)$- distribution:

The momeng Generating Function (MGF) of DUS$_E(\theta)$-distribution having pdf (6) is obtained as follows,

$$\begin{aligned} M_X(t) &= E[e^{tX}] \\ &= \frac{e}{e-1} \sum_{k=0}^{\infty} \frac{(-1)^k}{(k+1)!} \left\{1 - \frac{t}{(k+1)\theta}\right\}^{-1} \end{aligned} \tag{9}$$

provided $t < \theta$.

The raw moments (i.e. r^{th} moment about origin) of DUS$_E(\theta)$- distribution is obtained as follows,

$$\begin{aligned} \mu_r^{'} &= \left[\frac{\partial^r M_X(t)}{\partial t^r}\right]_{t=0} \\ &= \frac{e}{e-1} \frac{r!}{\theta^r} \sum_{k=0}^{\infty} \frac{(-1)^k}{k!(k+1)^{r+1}} \end{aligned} \tag{10}$$

The infinite series representation of $\mu_r^{'}$ is convergent for every θ and r. Thus, $\mu_r^{'}$ exist for every r and for all θ.

Median of DUS$_E(\theta)$- distribution:

The median of DUS$_E(\theta)$-distribution is the solution of the following,

$$G(M) = \frac{1}{2}$$

for M and the same is obtained as follows,

$$M = -\frac{1}{\theta} \ln \left(1 - \ln \left(\frac{e+1}{2}\right)\right) \tag{11}$$

Mode of DUS$_E(\theta)$- distribution:

Differentiating (6) with respect to x, we get

$$g^{'}(x) = \frac{e}{e-1} \theta^2 e^{-\theta x} e^{-e^{-\theta x}} \left(e^{-\theta x} - 1\right) \tag{12}$$

Clearly $g^{'}(x) < 0 \; \forall \; r, \; \theta$, this shows that $g(x)$ is a decreasing function of x and hence $x = 0$ is the mode of DUS$_E(\theta)$-distribution.

A comparison between mean, median and mode of DUS$_E(\theta)$- distribution is shown in Figure 3 and it is clear that Mean $>$ Median $>$ Mode, i.e. DUS$_E(\theta)$- distribution is positively skewed.

3 Estimation of the parameter θ of DUS$_E(\theta)$- distribution

3.1 Maximum Likelihood Estimator

Let n identical items are put on life testing experiment and suppose $\underline{\mathbf{X}} = (X_1, X_2, \ldots, X_n)$ be their independent lives such that each X_i ($\forall \; i = 1[1]n$) follow DUS$_E(\theta)$-distribution having pdf (6). Then the likelihood function for $\underline{\mathbf{X}}$ is given by,

$$L_{\underline{\mathbf{X}}}(\theta) = \prod_{i=1}^{n} g(x_i) \tag{13}$$

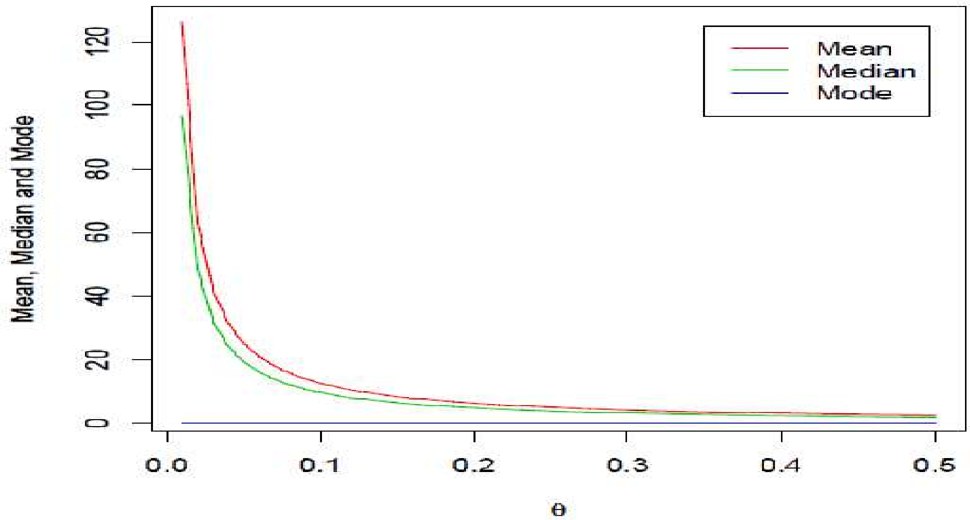

Fig. 3: Comparative plots of mean, median and mode of $\text{DUS}_E(\theta)$-distribution for different values of θ

Putting the value of g at x_i from (6) in (13), we get

$$\begin{aligned} L_{\underline{\mathbf{X}}}(\theta) &= \prod_{i=1}^{n}\left[\frac{1}{e-1}\ \theta\ e^{-\theta x_i}\ e^{1-e^{-\theta x_i}}\right] \\ &= \left(\frac{e}{e-1}\right)^n \theta^n\ e^{-\theta\sum\limits_{i=1}^{n} x_i}\ e^{-\sum\limits_{i=1}^{n} e^{-\theta x_i}} \end{aligned} \tag{14}$$

The log- likelihood function for $\underline{\mathbf{X}}$ is obtained as,

$$\begin{aligned} l &= \ln L_{\underline{\mathbf{X}}}(\theta) \\ &= K + n\ln\theta - \theta\sum_{i=1}^{n} x_i - \sum_{i=1}^{n} e^{-\theta x_i} \end{aligned} \tag{15}$$

where $K = n\ln\left(\frac{e}{e-1}\right)$ is a constant.

Hence, the log- likelihood equation for estimating θ is given by,

$$\begin{aligned} \frac{\partial l}{\partial\theta} &= 0 \\ \frac{n}{\theta} - \sum_{i=1}^{n} x_i + \sum_{i=1}^{n} x_i e^{-\theta x_i} &= 0 \end{aligned} \tag{16}$$

Above is an implicit equation in θ, hence it can not be solved analytically for θ. We propose Newton- Raphson method for its numerical solution.

3.2 Bayes Estimators

An important element in Bayesian estimation problem is the specification of the loss function. The choice is basically depends on the problem in hand. For more discussion on the choice of a suitable loss function, readers may refer to Singh et al. [10]. Another, important element is the choice of the appropriate prior distribution that covers all the prior knowledge regarding the parameter of interest. For the criteria of choosing an approprriate prior distribution, see Singh et al. [11]. With the above philosophical point of view, we are motivated to take the prior for θ as $G(\alpha,\beta)$-distribution with the pdf

$$\pi(\theta)=\frac{\alpha^{\beta}}{\Gamma(\beta)}e^{-\alpha\theta}\theta^{\beta-1}\quad;\quad \theta>0 \tag{17}$$

where $\alpha>0$ and $\beta>0$ are the hyper- parameters. These can be obtained, if any two independent informations on θ are available, say prior mean and prior variance are known (see, Singh et al. [11]). The mean and variance of the prior distribution (17) are $\frac{\beta}{\alpha}$ and $\frac{\beta}{\alpha^2}$ respectively. Thus, we may take $M=\frac{\beta}{\alpha}$ and $V=\frac{\beta}{\alpha^2}$, giving $\alpha=\frac{M^2}{V}$ and $\beta=\frac{M}{V}$. For any finite value of M and V to be sufficiently large, (17) behaves as like as non-informative prior. For more applications regarding the use of gamma prior, readers may refer to Singh et al. [16], Kundu [7] etc.

The posterior pdf of θ given $\underline{\mathbf{X}}$ corresponding to the considered prior pdf $\pi(\theta)$ of θ is given by,

$$\begin{aligned}\psi(\theta|\underline{\mathbf{X}})&=\frac{L_{\underline{\mathbf{X}}}(\theta)\ \pi(\theta)}{\int_0^\infty L_{\underline{\mathbf{X}}}(\theta)\ \pi(\theta)\partial\theta}\\&=\frac{e^{-\left(a+\sum_{i=1}^n x_i\right)\theta}\ \theta^{b+n-1}\ e^{-\sum_{i=1}^n e^{-\theta x_i}}}{\int_0^\infty e^{-\left(a+\sum_{i=1}^n x_i\right)\theta}\ \theta^{b+n-1}\ e^{-\sum_{i=1}^n e^{-\theta x_i}}\partial\theta}\end{aligned} \tag{18}$$

Now, to have an idea about the shapes of the prior and corresponding posterior pdfs for different confidence levels in the guessed value of θ as its true value, we randomly generate a sample from $\text{DUS}_E(\theta)$-distribution for fixed values $n=15$, $\theta=2$, $M=2$, $V=0.25$ (showing a higher confidence in the guessed value) and $V=1000$ (showing a weak confidence in the guessed value). The sample thus generated is,

$\underline{\mathbf{X}}=(0.06255701, 0.09990278, 0.11774334, 0.12931799, 0.18341711, 0.26283575, 0.34819087, 0.45747237, 0.51671856, 0.63666472, 0.89801464, 1.35858856, 1.45194963, 1.63335531, 2.11814344)$

The graphs are shown in Figures 4 and 5 respectively.

From the Figures 4 and 5 of the prior and corresponding posterior pdfs, it is quite clear that the posterior is bell shaped and showing concentration around the true value of the parameter θ for whatever may be the nature of the prior (informative/non-informative).

The loss functions considered here are general entropy loss function (GELF) and squared error loss function (SELF), which are defined by,

$$L_G(\hat{\theta},\ \theta)=\left(\frac{\hat{\theta}}{\theta}\right)^{\delta}-\delta\ \ln\left(\frac{\hat{\theta}}{\theta}\right)-1 \tag{19}$$

and

$$L_S(\hat{\theta},\ \theta)=(\hat{\theta}-\theta)^2 \tag{20}$$

respectively.

The Bayes estimators of θ under GELF (19) and SELF (20) are given by

$$\hat{\theta}_G=\left[E\left\{\theta^{-\delta}|\underline{\mathbf{X}}\right\}\right]^{-\frac{1}{\delta}} \tag{21}$$

and

$$\hat{\theta}_S=E\left[\theta|\underline{\mathbf{X}}\right] \tag{22}$$

respectively. It is easy to see that when $\delta=-1$, the Bayes estimator (21) under GELF reduces to the Bayes estimator(22) under SELF.

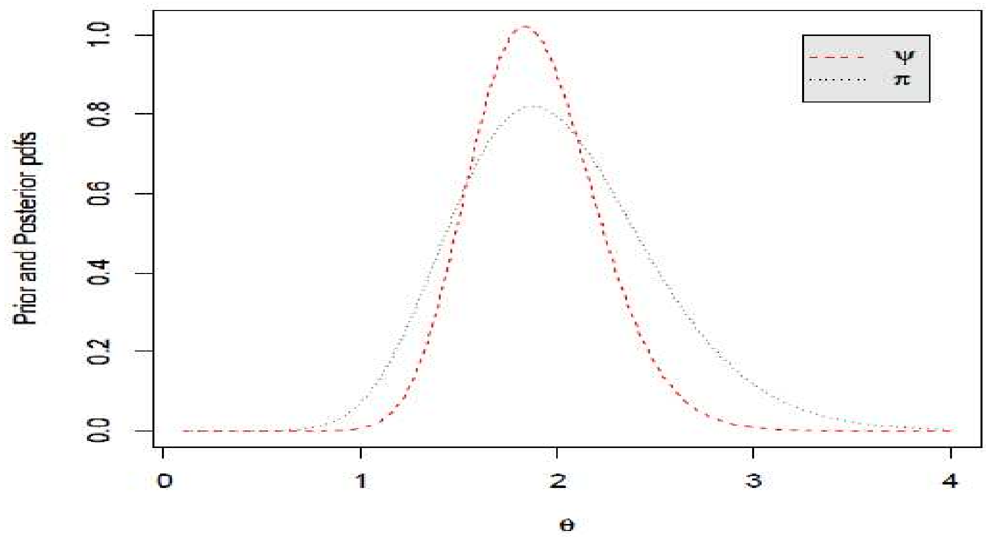

Fig. 4: Prior and Posterior pdfs of θ for a randomly generated sample from DUS$_E(\theta)$-distribution for fixed $n = 15$, $\theta = 2$, $M = 2$ and **V=0.25**

It is name-worthy to note here that GELF (19) was proposed by Calabria and Pulcini [3] and SELF (20) was proposed at first by Legendre [1] and Gauss [2] when he was developing the least square theory.

Now, the Bayes estimator of the parameter θ of DUS$_E(\theta)$-distribution having pdf (6) under GELF is obtained as follows'

$$\hat{\theta}_G = \left[E\left\{\theta^{-\delta}|\underline{\mathbf{X}}\right\}\right]^{-\frac{1}{\delta}}$$

$$= \left[\frac{\int_0^\infty e^{-\left(a+\sum_{i=1}^n x_i\right)\theta}\,\theta^{b-\delta+n-1}\,e^{-\sum_{i=1}^n e^{-\theta x_i}}\,\partial\theta}{\int_0^\infty e^{-\left(a+\sum_{i=1}^n x_i\right)\theta}\,\theta^{b+n-1}\,e^{-\sum_{i=1}^n e^{-\theta x_i}}\,\partial\theta}\right]^{-\frac{1}{\delta}} \tag{23}$$

Further, if $\hat{\theta}_S$ denotes the Bayes estimator of θ under SELF, then it can be obtained by putting $\delta = -1$ in (23) and therefore the same is given by,

$$\hat{\theta}_S = \frac{\int_0^\infty e^{-\left(a+\sum_{i=1}^n x_i\right)\theta}\,\theta^{b+n}\,e^{-\sum_{i=1}^n e^{-\theta x_i}}\,\partial\theta}{\int_0^\infty e^{-\left(a+\sum_{i=1}^n x_i\right)\theta}\,\theta^{b+n-1}\,e^{-\sum_{i=1}^n e^{-\theta x_i}}\,\partial\theta} \tag{24}$$

The integral involved in Bayes estimators do not solved analytically, therefore we propose Gauss - Lagurre's quadrature method for their numerical evaluation.

4 Bladder Cancer Data Application

To assess the applicability of DUS$_E(\theta)$-distribution, as obtained by DUS transformation (1), by using Exp(θ)-distribution as baseline distribution; in the real situations, we have considered a real data of the remission times of 128 bladder cancer patients. The data is extracted from Lee and Wang [5].

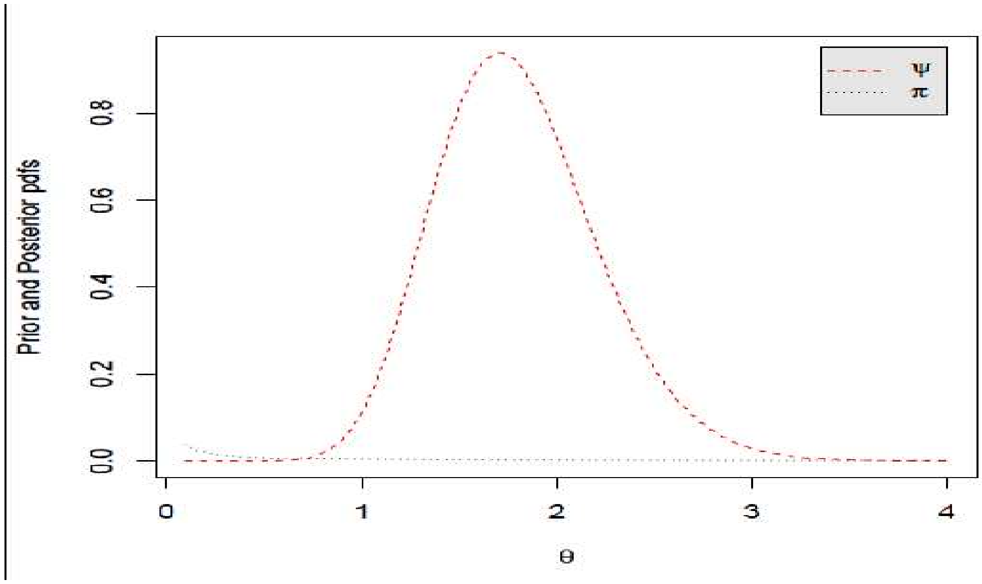

Fig. 5: Prior and Posterior pdfs of θ for a randomly generated sample from $\text{DUS}_E(\theta)$-distribution for fixed $n = 15$, $\theta = 2$, $M = 2$ and **V=1000**

Khan et al. [15] showed the applicability of transmuted inverse Weibull (TIW) distribution on this data by the fitting criteria in terms of Akaike information criteria (AIC), Bayesian information criteria (BIC), mean square error (MSE) and the associated Kolmogorov-Smirnov (KS) test value. They compared some life time distributions namely transmuted inverse Rayleigh (TIR) distribution, transmuted inverted exponential (TIE) distribution and inverse Weibull (IW) distribution in terms of their AIC, BIC, MSE and KS test value and found that the TIW distribution has the lowest AIC, BIC, MSE and KS test value, indicating that TIW distribution provides a better fit than the other three lifetime distributions.

We compute MLE of the parameter θ of $\text{DUS}_E(\theta)$- distribution having pdf (6) for the above data set and found it as 0.1341665. The AIC, BIC and KS test value for $\text{DUS}_E(\theta)$- distribution are calculated and we get their values as in Table 1. We have extracted the values of AIC, BIC and KS test value from Khan et al. [15] and present their values in the following comparative Table 1.

Table 1: AIC, BIC and KS test value for $\text{DUS}_E(\theta)$, TIW, TIR, TIE and IW distributions

Distributions	AIC	BIC	KS test value
$\text{DUS}_E(\theta)$	834.044	836.896	0.0812871
TIW	879.4	879.7	0.119
TIR	1424.4	1424.6	0.676
TIE	889.6	889.8	0.155
IW	892.0	892.2	0.131

The plots of empirical cdf F_n and fitted cdf $G(x)$ of $\text{DUS}_E(\theta)$- distribution for the data of remission times of 128 bladder cancer patients are shown in Figure 6.

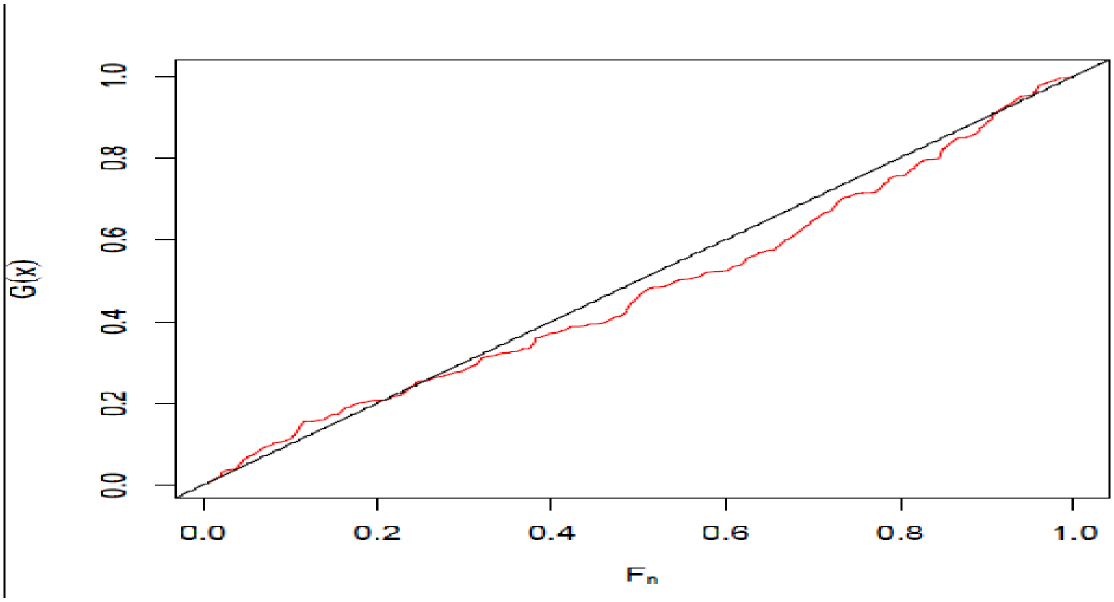

Fig. 6: Plots of empirical cdf F_n and fitted cdf $G(x)$ of $\text{DUS}_E(\theta)$- distribution for remission times of 128 bladder cancer patients data

$\text{DUS}_E(\theta)$- distribution having pdf (6) has the lowest AIC, BIC and KS test value in comparison to those of TIW, TIR, TIE and IW distributions (see, Table 1), indicating that $\text{DUS}_E(\theta)$- distribution having pdf (6) provides a better fit than the other four lifetime distributions namely TIW, TIR, TIE and IW distributions.

5 Comparison of the estimators

In this section, we compare the considered estimators i.e. $\hat{\theta}_M$, $\hat{\theta}_S$, $\hat{\theta}_G$ of the parameter θ of $\text{DUS}_E(\theta)$- distribution having pdf (6) in terms of simulated risks (average loss over sample space) under GELF. It is clear that the expressions for the risks cannot be obtained in nice closed form. So, the risks of the estimators are estimated on the basis of Monte Carlo simulation study of 5000 samples from $\text{DUS}_E(\theta)$- distribution. It may be noted that the risks of the estimators will be a function of number of items put on test n, parameter θ of the model, the hyper- parameters α and β of the prior distribution and the GELF parameter δ. In order to consider the variation of these values, we obtained the simulated risks for $n = 15$, $\theta = 2$, $\delta = \pm 3$, $M = 1,2,3$ and $V = 0.25, 0.5, 1, 2, 5, 10, 50, 100, 500, 1000$.

From Tables 2-4, we observed that when over estimation is more serious than under estimation, the estimator $\hat{\theta}_G$ performs better (in the sense of having smallest risk) in comparison to $\hat{\theta}_S$ and $\hat{\theta}_M$ for whatever confidence in the guessed value of θ as its true value. But if guessed value of θ is either more/less than its true value, the estimator $\hat{\theta}_G$ performs well for lower confidence in such guessed value of θ, otherwise $\hat{\theta}_M$ performs better (when guessed value of θ is less than its true value) or $\hat{\theta}_S$ performs better (when guessed value of θ is more than its true value). Further, when under estimation is more serious than over estimation, for whatever be the confidence in the guessed value of θ as less than its true value, the estimator $\hat{\theta}_G$ performs better than the estimators $\hat{\theta}_S$ and $\hat{\theta}_M$. But when guessed value of θ is same as its true value, the estimator $\hat{\theta}_G$ performs better for lower confidence; otherwise $\hat{\theta}_S$ performs better and when guessed value of θ is more than its true value, the estimator $\hat{\theta}_M$ performs well for higher confidence; $\hat{\theta}_S$, performs batter for moderate confidence and for lower confidence value, the estimator $\hat{\theta}_G$ performs better.

6 Conclusion

From the simulation study, it is clear that the estimators of the parametr θ of $\text{DUS}_E(\theta)$-distribution having pdf (6) may be recommended for their use as per confidence level in the guessed value of θ as discussed in the previous section. Further, DUS transformation (1) is full proof and by its use, the distribution, thus found may be appropriate for real life applications.

Table 2: Risks of the estimators of θ under GELF for fixed $n = 15$, $\theta = 2$, $M = 2$ and $\delta = \pm 3$

V	$\delta = -3$			$\delta = +3$		
	$R_G(\hat{\theta}_M)$	$R_G(\hat{\theta}_S)$	$R_G(\hat{\theta}_G)$	$R_G(\hat{\theta}_M)$	$R_G(\hat{\theta}_S)$	$R_G(\hat{\theta}_G)$
0.25	0.2351636	0.06202027	0.06927877	0.3087537	0.07142656	0.0544449
0.5	0.2397189	0.1000382	0.1040788	0.3150407	0.1249022	0.09999029
1	0.2366851	0.1456719	0.1444334	0.3316343	0.1887194	0.1573959
2	0.2295538	0.1735211	0.1710193	0.3040177	0.2228011	0.1879228
5	0.2404001	0.2159851	0.2039743	0.3353221	0.2863789	0.2327622
10	0.2383025	0.221264	0.2089175	0.3086364	0.281521	0.2308405
50	0.234083	0.2244676	0.2126007	0.3082911	0.2957092	0.2423513
100	0.2362913	0.2351454	0.2189663	0.305154	0.2935311	0.2382354
500	0.2373673	0.232264	0.2194286	0.3119443	0.3024726	0.2479735
1000	0.2320711	0.2264668	0.2143033	0.3217059	0.3127855	0.2530239

Table 3: Risks of the estimators of θ under GELF for fixed $n = 15$, $\theta = 2$, $M = 3$ and $\delta = \pm 3$

V	$\delta = -3$			$\delta = +3$		
	$R_G(\hat{\theta}_M)$	$R_G(\hat{\theta}_S)$	$R_G(\hat{\theta}_G)$	$R_G(\hat{\theta}_M)$	$R_G(\hat{\theta}_S)$	$R_G(\hat{\theta}_G)$
0.25	0.235754	0.3115501	0.3346941	0.33283	0.6272944	0.500172
0.5	0.2341955	0.2059068	0.2373002	0.3183992	0.4035556	0.2596403
1	0.2351689	0.1609973	0.1911085	0.3214862	0.3170205	0.1888534
2	0.2386029	0.1682289	0.1876279	0.3133841	0.2853366	0.1845553
5	0.2434708	0.1968458	0.1988416	0.3321115	0.3067552	0.2198998
10	0.2365215	0.2080114	0.2068211	0.3116328	0.2924269	0.2274174
50	0.2406898	0.2303895	0.219138	0.3079512	0.2969715	0.2395872
100	0.233762	0.227768	0.2148671	0.3180754	0.3079815	0.2461364
500	0.2413626	0.2377536	0.2248168	0.3177417	0.3084341	0.2519462
1000	0.2337319	0.2276503	0.2157224	0.3085333	0.299275	0.2428255

Table 4: Risks of the estimators of θ under GELF for fixed $n = 15$, $\theta = 2$, $M = 1$ and $\delta = \pm 3$

V	$\delta = -3$			$\delta = +3$		
	$R_G(\hat{\theta}_M)$	$R_G(\hat{\theta}_S)$	$R_G(\hat{\theta}_G)$	$R_G(\hat{\theta}_M)$	$R_G(\hat{\theta}_S)$	$R_G(\hat{\theta}_G)$
0.25	0.2339297	0.2674495	0.1838584	0.3240526	0.1325739	0.2429661
0.5	0.2353244	0.2165249	0.1653133	0.3000199	0.1422199	0.2037649
1	0.2349803	0.2136408	0.1824916	0.3341071	0.2056473	0.2145189
2	0.2307363	0.2117091	0.1908071	0.3238041	0.2456588	0.2272738
5	0.2397174	0.227333	0.2108287	0.3192615	0.2800625	0.2351175
10	0.2354567	0.2274237	0.2157414	0.3197352	0.2936598	0.2400396
50	0.238165	0.2335541	0.2171508	0.3302061	0.3181408	0.2533666
100	0.2445772	0.2413426	0.2231899	0.3223506	0.3108457	0.2510558
500	0.2344324	0.2279256	0.2144232	0.3154657	0.3052996	0.2479147
1000	0.2286325	0.2244536	0.2124129	0.3182051	0.3090924	0.2542603

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References

[1] Legendre, A. (1805): *New Methods for the Determination of Orbits of Comets*. Courcier, Paris.
[2] Gauss, C. F. (1810): *Least Squares method for the Combinations of Observations*. Translated by J. Bertrand 1955, Mallet-Bachelier, Paris.
[3] Calabria, R. and Pulcini, G. (1994): *An engineering approach to Bayes estimation for the Weibull distribution*. Micro Electron Reliab., 34, 789–802.

[4] Gupta, R. C., Gupta, R. D. and Gupta, P. L. (1998): *Modeling failure time data by Lehman alternatives*. Communication in Statistics-Theory and Methods, 27 (4), 887?-904.

[5] Lee, E. T. and Wang, J. W. (2003): *Statistical Methods for Survival Data Analysis*. Wiley, New York, DOI:10.1002/0471458546.

[6] Shaw, W. T. and Buckley, I. R. (2007): *The alchemy of probability distributions: beyond Gram-Charlier expansions and a skew-kurtotic-normal distribution from a rank transmutation map*. Research report.

[7] Kundu, D. (2008): *Bayesian Inference and Life Testing Plan for the Weibull Distribution in Presence of Progressive Censoring*. TECHNOMETRICS, 50 (2), 144–154.

[8] Aryal, G. R. and Tsokos, C. P. (2009): *On the transmuted extreme value distribution with application*. Nonlinear Analysis: Theory, Methods and Applications, 71 (12), 1401–1407,

[9] Singh, S. K., Singh, U. and Kumar, D. (2011): *Bayesian estimation of the exponentiated gamma parameter and reliability function under asymmetric loss function*. REVSTAT – Statistical Journal, 9 (3), 247–260.

[10] Singh, S. K., Singh, U. and Kumar, D. (2011): *Estimation of Parameters and Reliability Function of Exponentiated Exponential Distribution: Bayesian approach Under General Entropy Loss Function*. Pakistan journal of statistics and operation research, VII (2), 119–216.

[11] Singh, S. K., Singh, U. and Kumar, D. (2012): *Bayes estimators of the reliability function and parameter of inverted exponential distribution using informative and non-informative priors*. Journal of Statistical Computation and Simulation, online dated 29 May, 2012, DOI:10.1080/ 009496552012. 690156, 1–12.

[12] Aryal, G. R. (2013): *Transmuted log-logistic distribution*. Journal of Statistics Applications and Probability, 2 (1), 11?-20.

[13] Singh, S. K., Singh, U. and Kumar, D. (2013): *Bayesian estimation of parameters of inverse Weibull distribution*. Journal of Applied Statistics, 40 (7), 1597–1607.

[14] Khan, M. S. and King, R. (2013): *Transmuted modified Weibull distribution: A generalization of the modified Weibull probability distribution*. Europe. J. of Pure Appl. Math., 6 (1), 66–88

[15] Khan, M. S., King, R. and Hudson, I. L. (2014): *Characterisations of the transmuted inverse Weibull distribution*. ANZIAM J. 55 (EMAC-2013), C197–C217.

[16] Singh, S. K., Singh, U., Yadav, A. S. and Vishwkarma, P. K. (2015): *On the estimation of stress strength reliability parameter of inverted exponential distribution*. International Journal of Scientific World, 3 (1), 98–112.