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# Blow-up solutions to a class of generalized Nonlinear Schrodinger equations

**Ning Chen*, Baodan Tian and Jiqian Chen**
*School of Science, Southwest University of Science and Technology, Mianyang 621010, P. R. China*
*Email: chenning783@163.com

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**Abstract:** In this paper, we'll present some new results of blow-up solution to some higher-order nonlinear Schrodinger equations. The initial boundary value problem is a generalized nonlinear Schrodinger equation $u_t - i\Delta^3 u = f(u, D_x u, D_x^2 u) + \Delta^3 g(u)$, $u(x,0) = u_0(x), u|_{\partial\Omega} = 0$ is studied. As an extension of $u_t - i\Delta u = f(u, D_x u, D_x^2 u)$ and $u_t - i\Delta u = -\Delta g(u)$, the global non-existence and blow-up infinite time of solutions to this problems are proved. The conclusions are complementary to expound the blow-up of solution to nonlinear Schrodinger equations by using eigen-function method. Main results can be found in theorem 3.1 and theorem 4.1.

**Keywords:** Operator equation; nonlinear equation; He's iterative method; initial boundary value; multi-valued map.

## 1 Introduction

The nonlinear Schrodinger equation is the basic equation in nonlinear science and widely applied in natural science such as the chemistry, biology, communication and almost all branches of physics such as the fluid mechanics, plasma physics, and nonlinear optics as well as the condensed matter physics. We study this equation to extend in some generalized equation form are with important meaning .Now, we may extend some results in [4] by using eigen-function method in this paper.

As we know the solution of initial problem for Schrodinger equation bellow,

$$\begin{cases} u_t - ia^2\Delta u = f(x,t), x \in R^n, t > 0, \\ u(x,0) = \varphi(x), x \in R^n. \end{cases}$$

Assume that real part and imaginary part of $\varphi(x), f(x,t)$ are real analytical function for $x \in R^n$, then this solution of the problem may express in form,

$$u(x,t) = \sum_{k=0}^{\infty} \frac{(ia^2)^k}{k!} (t^k \Delta^k \varphi(x) + \int_0^t (t-\tau)^k \Delta_x^k f(x,\tau) d\tau).$$

Let $Du = (u_t, D_x u) = (u_t, u_{x_1}, u_{x_2} \cdots, u_{x_n})$, $(Du)_{x_i} = (u_{tx_i}, u_{x_1x_i}, \cdots, u_{x_nx_i})$ $(i = 1, 2, \cdots, n)$,

$$D_x Du = ((Du)_{x_1}, (Du)_{x_2}, \cdots, (Du)_{x_n}), (i = 1, 2, \cdots, n)$$

then we consider higher order nonlinear equation. We consider the initial boundary value of higher order nonlinear Schrodinger equation:

$$u_t - i\Delta^k u = f(u, D_x u, D_x^2 u) + \Delta^k g(u), x \in \Omega, t > 0 \quad (1')$$

$$u(x,0) = u_0(x), x \in \Omega \quad (2')$$

$$u|_{\partial\Omega} = 0, x \in \partial\Omega, t > 0 \quad (3')$$

Let $k = 2$, we have the simple case.

$$u_t - i\Delta^2 u = f(u, D_x u, D_x^2 u) + \Delta^2 g(u), x \in \Omega, t > 0 \quad (1'')$$

$$u(x,0) = u_0(x), x \in \Omega \quad (2'')$$

$$u|_{\partial\Omega} = 0, x \in \partial\Omega, t > 0 \quad (3'')$$

And $k = 3$, we have the simple case bellow.

$$u_t - i\Delta^3 u = f(u, D_x u, D_x^2 u) + \Delta^3 g(u), x \in \Omega, t > 0 \quad (1.1)$$

$$u(x,0) = u_0(x), x \in \Omega \quad (1.2)$$

$$u|_{\partial\Omega} = 0, x \in \partial\Omega, t > 0 \quad (1.3)$$

As convenient in first, we consider that case of $k = 3$. Where $\Omega$ a bounded domain in $R^n$ with suite smooth boundary $\partial\Omega$, $f, g$ are complex value function, $u_0(x)$ is also enough smooth complex value function.

By using of eigen-function method, we can get new results bellow. In first, stating that lemma1.

$$\begin{cases}\Delta\varphi+\lambda\varphi=0, x\in R^n,\\ \partial\varphi|_{\partial\Omega}=0.\end{cases} \quad (*)$$

As we all know the first eigen valu1e $\lambda_1>0$ of (*), the corresponding eigen-function $\varphi_1(x)>0$, assume it with $\int_\Omega \varphi_1(x)dx=1$.

## 2 Several theorems

**Theorem 2.1** Assume that problem (1)-(3) satisfy,

$(i)$ $G(0)=0, \frac{\partial G}{\partial n}|_{\partial\Omega}=0, G=\mathrm{Re}\,\Delta^2 g(u)-\Delta^2\,\mathrm{Im}\,u$,

$(ii)$ $A=\mathrm{Re}(f(u,D_x u,D_x^2 u)-\lambda^3 g(u))-\mathrm{Im}\,u-CF(\mathrm{Re}\,u)$, $\alpha=\int_\Omega \varphi\,\mathrm{Re}\,u_0 dx, A\cdot\alpha\ge 0$, $F(u)$ is continuous, convex, and even function.

$$C=\begin{cases}1,(\alpha>0)\\ -1,(\alpha<0)\end{cases}.$$

$(iii) F(s)>0 (s>\alpha)$ and $\int_\alpha^\infty \frac{ds}{F(s)}<\infty$.

then the blow-up of classical solution for this problem (1)-(3) at some time.

**Proof.** Step I. When $A\ge 0, \alpha>0$ and $A\cdot\alpha\ge 0$.

In the same way, from

$$u_t-i\Delta^3 u=f(u,D_x u,D_x^2 u)+\Delta^3 g(u), x\in\Omega. \quad (2.1)$$

We take the real part of both sides of (2.1),

$$\mathrm{Re}\,u_t-\mathrm{Re}\,i\Delta^3 u=\mathrm{Re}\,f(u,D_x u,D_x{}^2 u)+\mathrm{Re}\,\Delta^3 g(u),$$

$$\mathrm{Re}\,u_t-\Delta^3\,\mathrm{Im}\,u=\mathrm{Re}\,f(u,D_x u,D_x^2 u)+\mathrm{Re}\,\Delta^3 g(u),$$

Multiplying by $\varphi(x)$ the both sides of (2.1) and integrate on $\Omega$ for $x$, then

$$\int_\Omega \varphi\,\mathrm{Re}\,u_t dx=\int_\Omega \varphi[-\Delta^3\,\mathrm{Im}\,u +\mathrm{Re}\,f(u,D_x u,D_x{}^2 u)+\Delta^3\,\mathrm{Re}\,g(u)]dx$$

Taking $a(t)=\int_\Omega \varphi\,\mathrm{Re}\,u dx$, then

$$a'(t)=\int_\Omega \varphi\,\mathrm{Re}\,u_t dx,$$

and that

$$\begin{aligned}a'(t)&=\int_\Omega \varphi[-\Delta^3\,\mathrm{Im}\,u+\mathrm{Re}\,f(u,D_x u,D_x^2 u) +\Delta^3\,\mathrm{Re}\,g(u)]dx\\ &=\int_\Omega[\varphi\Delta(\Delta^2\,\mathrm{Re}\,g(u)-\Delta^2\,\mathrm{Im}\,u) -\Delta\varphi(\mathrm{Re}\,\Delta^2 g(u)-\Delta^2(\mathrm{Im}\,u)]dx\\ &=\int_\Omega[\varphi\,\mathrm{Re}\,f(u,D_x u,D_x^2 u) -\Delta\varphi(\Delta^2\,\mathrm{Im}\,u-\Delta^2\,\mathrm{Re}\,g(u))]dx \end{aligned} \quad (2.2)$$

By $(i)$ and Green's second formula, we have

$$\int_\Omega \varphi\Delta\left(\Delta^2\,\mathrm{Re}\,g(u)-\Delta^2\,\mathrm{Im}\,u\right)dx =\int_\Omega \Delta\varphi\left(\Delta^2\,\mathrm{Re}\,g(u)-\Delta^2\,\mathrm{Im}\,u\right)dx \quad (2.3)$$

Substituting (2.3) into (2.2), we get

$$\begin{aligned}a'(t)&=\int_\Omega[\varphi\,\mathrm{Re}\,f(u,D_x u,D_x^2 u) -\Delta\varphi(\Delta^2\,\mathrm{Im}\,u-\Delta^2\,\mathrm{Re}\,g(u))]dx\\ &=\int_\Omega \varphi[\mathrm{Re}\,f(u,D_x u,D_x^2 u) +\lambda(\Delta^2\,\mathrm{Im}\,u-\Delta^2\,\mathrm{Re}\,g(u))]dx\\ &=\int_\Omega \lambda\varphi[\mathrm{Re}\,\Delta g(u)-\Delta\,\mathrm{Im}\,u] -\lambda\Delta(\Delta\,\mathrm{Im}\,u-\mathrm{Re}\,\Delta g(u))]dx\\ &=+\int_\Omega \varphi[\mathrm{Re}\,f(u,D_x u,D_x^2 u)] -\lambda\Delta\varphi(\mathrm{Re}\,\Delta g(u)-\Delta\,\mathrm{Im}\,u)]dx \end{aligned} \quad (2.4)$$

Hence,

$$\begin{aligned}a'(t)&=\int_\Omega \varphi[\mathrm{Re}\,f(u,D_x u,D_x^2 u) +(-\lambda)^3 g(u)-(-\lambda)^3\,\mathrm{Im}\,u)]dx\\ &=\int_\Omega \varphi[\mathrm{Re}\,f(u,D_x u,D_x^2 u) -\lambda^3 g(u)+\lambda^3\,\mathrm{Im}\,u)]dx \end{aligned} \quad (2.5)$$

From $A\ge 0$, $A=\mathrm{Re}(f(u,D_x u,D_x^2 u)$, then

$$\mathrm{Re}(f(u,D_x u,D_x^2 u)-\lambda^3 g(u))\ge\lambda^3\,\mathrm{Im}\,u+CF(\mathrm{Re}\,u) \quad (2.6)$$

Combing (2.5)-(2.6) and using Jensen inequality, we obtain that

$$a'(t)\ge\varphi F(\mathrm{Re}\,u)\ge F(\int_\Omega \varphi\,\mathrm{Re}\,u dx)=F(a(t)) \quad (2.7)$$

Here, $F(a(t)) \le \frac{da}{dt}$.

Thus, $t \le \int_{\alpha}^{a(t)} \frac{da}{F(a)}$,

and there exists $T \le \int_{\alpha}^{\infty} \frac{da}{F(a)} < +\infty$ such that

$$\lim_{t\to T} a(t) = +\infty. \tag{2.8}$$

From $a(t) = \int_{\Omega} \varphi \operatorname{Re} u dx$ and Holder inequality, we get $(1/p + 1/q/ = 1)$.

$$a(t) = \int_{\Omega} \varphi \operatorname{Re} u dx \le \|\varphi\|_{L^q(\Omega)} \cdot \|\operatorname{Re} u\|_{L^p(\Omega)},$$

that is $\frac{a(t)}{\|\varphi\|_{L^q(\Omega)}} \le \lim_{t\to T} \|\operatorname{Re}(u)\|_{L^p(\Omega)}$

Therefore, $\lim_{t\to T} \frac{a(t)}{\|\varphi\|_{L^q(\Omega)}} \le \lim_{t\to T} \|\operatorname{Re} u\|_{L^p(\Omega)}$.

Hence,

$$\lim_{t\to T} \|\operatorname{Re} u\|_{L^p(\Omega)} = +\infty, \forall 1 \le p \le +\infty.$$

Step II. When $A \le 0, \alpha < 0$, taking that $u(x,t) = -u_1(x,t)$, then $\operatorname{Re} u = -\operatorname{Re} u_1$.

Therefore, let $a(t) = \int_{\Omega} \varphi \operatorname{Re}(u_1) dx$, we have

$$a_1(t) = -a(t), a_1'(t) = -a'(t), \alpha_1 = -\alpha > 0.$$

Combine (1.1)-(2.5) and $A \le 0, (C = -1)$, we obtain that

$$-a_1'(t) \le -\int_{\Omega} \varphi F(-\operatorname{Re} u_1) dx \tag{2.9}$$

That is also $a_1'(t) \ge \int_{\Omega} \varphi F(-\operatorname{Re} u_1) dx$.

From Jensen inequality and is even function, we have $F(a_1) = F(-a_1) \le (da_1 / dt)$, then

$$dt \le \frac{da_1}{F(a_1)} \tag{2.10}$$

From (2.10) and similar step I, we can get

$$\lim_{t\to T} \|\operatorname{Re} u_1\|_{L^p(\Omega)} = +\infty, \forall 1 \le p \le +\infty.$$

$$\lim_{t\to T} \|\operatorname{Re} u(t)\|_{L^p(\Omega)} = \lim_{t\to T} \|-\operatorname{Re} u(t)\|_{L^p(\Omega)} = +\infty, \forall 1 \le p \le +\infty.$$

Combine step $I$ and $II$, we complete the proof of theorem 1.

**Theorem 2.2** Assume that probelem (1)-(3) satisfy,

$(i)$ $G(0) = 0, \frac{\partial G}{\partial n}|_{\partial\Omega} = 0, G = \operatorname{Re} \Delta^2 g(u) - \Delta^2 \operatorname{Im} u$;

$(ii)$ $B = \frac{\operatorname{Im}(f(u, D_x u, D_x{}^2 u) - \lambda^3 g(u)) + \lambda^3 (\operatorname{Re} u)}{F(\operatorname{Im} u)}$,

and $|B| - 1 \ge 0, \beta = \int_{\Omega} \varphi \operatorname{Im} u_0 dx < 0$; where $F(s)$ is continuous, convex and even function;

$(iii)$ $F(s) > 0, (s > \beta)$ and $\int_{\beta}^{+\infty} \frac{ds}{F(s)} < +\infty$,

then the classical solution for the problem (1)-(3) is blow-up in finite time.

**Proof**. From $|B| - 1 \ge 0$, we discuss two case,

$(I)$ $B - 1 \ge 0, \beta < 0, u(x,t) = i\bar{u}_2(x,t)$,

then $\operatorname{Im} u = \operatorname{Re} \bar{u}_2$.

Taking the imaginary part for both sides of (1.1), similar the method of proof for Theorem 1, we can easy have

$$\lim_{t\to T} \|\operatorname{Re} u_2\|_{L^p(\Omega)} = +\infty, \forall 1 \le p \le +\infty.$$

So we get that

$$\lim_{t\to T} \|\operatorname{Im} u(t)\|_{L^p(\Omega)} = +\infty, \forall 1 \le p \le +\infty.$$

$(II)$ $|B| - 1 \le 0, \beta < 0$, we may let

$$u(x,t) = -\bar{u}_3(x,t),$$

then

$$\operatorname{Im} u = \operatorname{Im} u_3.$$

Thus, $\lim_{t\to T} \|\operatorname{Im} u_3(t)\|_{L^p(\Omega)} = +\infty, \forall 1 \le p \le +\infty.$

Taking the imaginary part for both sides of (1.1), by (II) and similar the method of proof for theorem 1, we can easy have

$$\lim_{t\to T} \|\operatorname{Re} u_3\|_{L^p(\Omega)} = +\infty, \forall 1 \le p \le +\infty.$$

We get that

$$\lim_{t\to T} \|\operatorname{Im} u(t)\|_{L^p(\Omega)} = +\infty, \forall 1 \le p \le +\infty.$$

Combine $I$ and $II$, we can complete the proof of theorem 2.2.

We consider that problem:

$$u_t - i\Delta^{2k-1} u = f(u, D_x u, D_x^2 u) + \Delta^{2k-1} g(u), x \in \Omega, t > 0 \tag{2.11}$$

$$u(x,0)=u_0(x), x\in\Omega \qquad (2.12)$$

$$u|_{\partial\Omega}=0, x\in\partial\Omega, t>0 \qquad (2.13)$$

**Theorem 2.3** Assume that problem (1)-(3) satisfy,

$(i)$ $G(0)=0, \frac{\partial G}{\partial n}|_{\partial\Omega}=0,$

$G=\mathrm{Re}\,\Delta^{2k-2}g(u)-\Delta^{2k-2}\,\mathrm{Im}\,u;$

(ii)Let $A=\mathrm{Re}(f(u,D_xu,D_x^2u))+(-\lambda)^{2k-1}g(u))$ $+(-\lambda)^{2k-1}\,\mathrm{Im}\,u-CF(\mathrm{Re}\,u),\ a(t)=\int_\Omega\varphi\,\mathrm{Re}\,u_0dx,$

and $A\cdot\alpha\ge 0$, $F(u)$ is continuous, convex and even function,

$$C=\begin{cases}1,\alpha>0,\\-1,\alpha<0.\end{cases}$$

$(iii)F(s)>0(s>\alpha)$ and $\int_\alpha^\infty\frac{ds}{F(s)}<\infty.$

then the classical solution of problem (1.1)- (1.3) is blow-up in finite time.
(We omit this similar proof).

## 3 Some notes

Without loss of generality, we can along the direction of [4] by using similar method to further given that new conclusion as follows.

By inductively in the same way, we may consider that

$$u_t-i\Delta^{2k}u=f(u,D_xu,D_x^2u)+\Delta^{2k}g(u), x\in\Omega, t>0 \qquad (3.1)$$

$$u(x,0)=u_0(x), x\in\Omega \qquad (3.2)$$

$$u|_{\partial\Omega}=0, x\in\partial\Omega, t>0 \qquad (3.3)$$

Taking $a(t)=\int_\Omega\varphi\,\mathrm{Re}\,udx,$ then

$$a'(t)=\int_\Omega\varphi\,\mathrm{Re}\,u_tdx,$$

we have that

$$a'(t)=\int_\Omega\varphi[\mathrm{Re}\,f(u,D_xu,D_x^2u)+$$
$$(-\lambda)^{2k}g(u)-(-\lambda)^{2k}\,\mathrm{Im}\,u)]dx$$

We easy obtain following conclusion.

**Theorem 3.1** Assume that problem (1)-(3) satisfy,

$(i)$ $G(0)=0, \frac{\partial G}{\partial n}|_{\partial\Omega}=0, G=\mathrm{Re}\,\Delta^{2k-1}g(u)-\Delta^{2k-1}\,\mathrm{Im}\,u;$

(ii)Let $A=\mathrm{Re}(f(u,D_xu,D_x^2u))+(-\lambda)^{2k}g(u))$ $+(-\lambda)^{2k}\,\mathrm{Im}\,u-CF(\mathrm{Re}\,u),\ a(t)=\int_\Omega\varphi\,\mathrm{Re}\,u_0dx,$

and $A\cdot\alpha\ge 0$,

$F(u)$ is continuous ,convex and even function ,

$$C=\begin{cases}1,\alpha>0,\\-1,\alpha<0.\end{cases}$$

$(iii)$ $F(s)>0(s>\alpha)$ and $\int_\alpha^\infty\frac{ds}{F(s)}<\infty.$

Then the classical solution of problem (3.1)-(3.3) is blow-up in finite time. (We omit this similar proof)

## 4 Main results

We consider the initial boundary value of higher order nonlinear Schrodinger equation as form, ($k\ge 1-$integer)

$$u_t-i\Delta^ku=f(u,D_xu,D_x^2u)+\Delta^kg(u), x\in\Omega, t>0 \qquad (4.1)$$

$$u(x,0)=u_0(x), x\in\Omega \qquad (4.2)$$

$$u|_{\partial\Omega}=0, x\in\partial\Omega, t>0 \qquad (4.3)$$

Taking $a(t)=\int_\Omega\varphi\,\mathrm{Re}\,udx,$ then

$$a'(t)=\int_\Omega\varphi\,\mathrm{Re}\,u_tdx,$$

and we obtain that

$$a'(t)=\int_\Omega\varphi[\mathrm{Re}\,f(u,D_xu,D_x^2u)$$
$$+(-\lambda)^kg(u)-(-\lambda)^k\,\mathrm{Im}\,u)]dx$$
$$=\int_\Omega\varphi[\mathrm{Re}\,f(u,D_xu,D_x^2u)$$
$$-\lambda^kg(u)+\lambda^k\,\mathrm{Im}\,u)]dx.$$

By inductively in the same way for generalized form bellow.

$$u_t-i\Delta^ku=f(u,D_xu,D_x^2u)+\Delta^kg(u), x\in\Omega, t>0,$$
$$a'(t)=\int_\Omega\varphi[\mathrm{Re}\,f(u,D_xu,D_x^2u)$$
$$+(-\lambda)^kg(u)-(-\lambda)^k\,\mathrm{Im}\,u)]dx$$

Let $k=5,6,$ therefore, then we shall obtain that form in this case.

$$a'(t)=\int_\Omega\varphi[\mathrm{Re}\,f(u,D_xu,D_x^2u)$$
$$-\lambda^5g(u)+\lambda^5\,\mathrm{Im}\,u)]dx$$

$$a'(t) = \int_\Omega \varphi[\mathrm{Re}\, f(u, D_x u, D_x^2 u) + (-\lambda)^6 g(u) - (-\lambda)^6 \,\mathrm{Im}\, u)]dx$$

**Theorem 4.1** The problem (4.1)-(4.3) satisfy

$(i)$ $G(0) = 0, \frac{\partial G}{\partial n}|_{\partial\Omega} = 0, G = \mathrm{Re}\,\Delta^{k-1} g(u) - \Delta^{k-1}\,\mathrm{Im}\,u;$

$(ii)$ $B = \frac{\mathrm{Im}(f(u, D_x u, D_x{}^2 u) + (-\lambda)^k g(u)) - (-\lambda)^k (\mathrm{Re}\,u)}{F(\mathrm{Im}\,u)},$

$|B| - 1 \geq 0, \quad \beta = \int_\Omega \varphi\,\mathrm{Im}\,u_0 dx < 0,$ where $F(s)$ is continuous convex and couple function.

$(iii)$ $F(s) > 0, (s > \beta)$ and $\int_\beta^{+\infty} \frac{ds}{F(s)} < +\infty.$

then the classical solution of problem (4.1)-(4.3), with blow-up in finite time.

**Proof.** From $|B| - 1 \geq 0,$ we discuss two case,

$(I)$ $B - 1 \geq 0, \beta < 0, u(x,t) = i\bar{u}_2(x,t),$ then

$$\mathrm{Im}\,u = \mathrm{Re}\,\bar{u}_2.$$

Taking the imaginary part of (1), similar as the proof for method of theorem 1, we can easy have

$$\lim_{t\to T} \|\mathrm{Re}\,u_2\|_{L^p(\Omega)} = +\infty, \forall 1 \leq p \leq +\infty.$$

So we get that

$$\lim_{t\to T} \|\mathrm{Im}\,u(t)\|_{L^p(\Omega)} = +\infty, \forall 1 \leq p \leq +\infty.$$

$(II)$ $B + 1 \leq 0, \beta < 0,$ we may let

$$u(x,t) = -\bar{u}_3(x,t),$$

then $\mathrm{Im}\,u = \mathrm{Im}\,\bar{u}_3.$

Thus,

$$\lim_{t\to T} \|\mathrm{Im}\,u_3(t)\|_{L^p(\Omega)} = +\infty, \forall 1 \leq p \leq +\infty.$$

We get that

$$\lim_{t\to T} \|\mathrm{Im}\,u(t)\|_{L^p(\Omega)} = +\infty, \forall 1 \leq p \leq +\infty.$$

Combine $(I) - (II)$, we can complete the proof of theorem 4.

**Remark.** The system of two equations may be considered and will be proved to it in more holding meaning by variable method. In [11], the author discuss the Cauchy problem of the fourth order nonlinear Schrodinger equation.

$$\begin{cases} iu_t - \Delta^2 + |u|^2 = 0; t \geq 0, x \in R^4, \\ u(0,x) = u_0. \end{cases}$$

holds new meaning. Note fourth order Schrodinger equation are introduced by Karpman. Such fourth–order Schrodinger equation are written as

$$i\varphi_t + \varepsilon\Delta^2\varphi + |\varphi|^{p-1} = 0, \varphi = \varphi(t,x) : I \times R^d \to C.$$

## 5. Concluding Remarks

Recently, the higher-order Schrodinger equations is also a very interesting topic and we may see them in [3] and [4] etc. It applies some physics and mechanics to some fields with nonlinear Schrodinger equations and some compute methods.

In our future work, we may try to do some research in this field and may obtain some good results. Recently, the author gives some extending conditions for it, and it proves to be a very interesting topic, too. It applies some physics and mechanics to some more fields with some model equations. It is need to be noted that the above equation is a special case of last equation by taking parameter $\varepsilon = -1, \mu = 0,$ and $p = 1 + 8/d = 3.$

Moreover, as regards the solution $u(t,x)$ to the Cauchy problem, there are two conservation laws in $H^2$ (see [11]), which express as follows.

(ii) Conservation of energy

$$E(u(t)) := \frac{1}{2}\int_{R^4} |\Delta u(t,x)|^2 dx - \frac{1}{4}\int_{R^4} |u(t,x)|^4 dx = E(u_0).$$

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**Ning Chen** is presently a associate professor at southwest University of Science and Technology to teach lesson as: Numerical analysis; Linear algebra; partial differential equation; Stochastic process; Complex analysis; etc. He is famous teacher of the Southwest University of Science and Technology.

His research Interests: Long-term differential equations, dynamical systems, nonlinear analysis and numerical analysis of the study, participated in many international academic conferences,mutually beneficial exchanges with foreign counterparts.

**Baodan Tian** is a Ph.D. candidate in school of mathematical sciences, University of Electronic Science and Technology of China. He is also a lecturer in Southwest University of Science and Technology.

His current interests include nonlinear differential equations and their applications, impulsive differential equations and their applications, mathematical biology, and nonlinear functional analysis and so on.

**Jiqian Chen** is professor at Southwest University of Science and Technology. His research Interests: long-term partial differential equations, nonlinear analysis of the study, participated in many international academic conferences.