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A Method for Aggregating Group Preference Based on Pair-wise Comparison with Random Binary Relations under Interval Belief Structures

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Abstract: The aim of this paper is to develop an approach to solve the random lattice order group decision-making problem, where the preference information on alternatives pair provided by experts is in the form of uncertain binary preference relations. In this paper, firstly, the preference characterization of decision makers is extended from four various binary relations to seven various binary relations. We give the definition and properties of significance degree, interval belief and interval-valued distribution preference vector. Then, to process uncertain binary preference relations, a comparison matrix about interval significance degree of preference relations is constructed. Based on the interval-valued comparison matrix, the interval significance degree by approximation models is driven, so we need to aggregate group preference to interval-valued distribution preference vector of decision group. Furthermore, the comparison principle of interval number and the method to determine the binary relation are presented. Finally, example is used to illustrate the use of the proposed approach.

Keywords: group decision-making (GDM), lattice order, uncertain binary relation, interval-valued distribution preference vector, significance degree

1 Introduction

Group decision-making (GDM) is decision-making in groups consisting of multiple members/ entities[1]. In recent years, how to solve group ranking problems has became an important issue and the method has been widely used in many applications, such as selection of advanced manufacturing technology (Chuu,2009) [2],selection of industrial robotic (Kahraman, cevik, Ates, & Gülbay, 2007)[3] ,web search strategies and so on [2,8]. In the actual GDM, decision-makers need to give alternatives and decision-making target, and provide preference information of alternatives about each evaluation target in the light of different decision environment (Hwang & Lin,1987)[4]. At this time, the preference information provided by decision-makers can be expressed in multiple formats, such as utility values, multiplicative preference relations, fuzzy preference relations, linguistic variables, interval numbers, and preference rankings or ranking ordinals etc [5-10].

Although the decision making problems with the above types preference information have got relatively abundant research achievements[2-14], they share a common weakness that most of them assume that the preferences of decision makers with total order properties, at this time the preference results of decision makers often have “artificially” and “barely” feature, but not entirely meet the realistic situation[15,16]. In fact, the study indicated that using ordinal preferences to process the inaccuracy, badness and unclear problem is more appropriate. More and more authors have been concerning particularly partial preorders (including the indifference, preference and incomparability relations) [16-22]. Such as, González-Pachón & Romero(2001) aggregating partial ordinal rankings using an interval goal programming method[17];Khaled Jabeur (2004, 2007, 2010,2012) has studied aggregation procedure, ordinal sorting and weight determining method etc.; about decision problem with collective preorder (or reference preorder) preference structure based on the distance measure suggested by Roy Slowinski et al[18-22]; Khaled Jabeur(2010) proposed an index to measure the agreement level of an individual preorder with respect to a collective preorder (or reference preorder)[21];Khaled Jabeur(2012) derived a binary mathematical programming based on the minimum distance of collective preorder [22];Wade.D. Cook (2006) and Abdelwaheb Rebai(2006) etc. also have driven much research achievement[23]; Seok Kee Lee(2010) proposed collaborative filtering with ordinal scale-based implicit ratings for mobile music recommendations[24].

On the other hand, T.L. Saaty (1970) pointed out that the simplest and most direct methods are pair-wise comparing alternatives when decision makers express their preferences about schemes, and the comparison result is presented using binary preference relation. But, in complex changeableness decision-making environments, the implementation environment, implementation tache and element of decision project are all involved in stochastic uncertainty elements. Such impact making the results could also be random uncertainty, decision makers may be difficult to have a clear preference relation, only can express their uncertain binary preference ordinals on alternatives [25-27].

Hence, it is necessary to develop a straightforward and friendly method to solve GDM problems with uncertain preference ordinals. The existing approaches have significant contributions to solve the GDM problems with uncertain preference ordinals on alternatives[25-27]. However, the approaches of González-Pachón and Romero(2001), González-Pachón et al. (2003) are based on the interval goal programming models[17,25]. If the number of alternatives or experts is great, the number of constraints of the models is also great. It may be somewhat difficult to solve the models. Furthermore, the approach of Wang et al. (2005) focuses on solving the GDM problem with preference ranking or partial preference ranking and can not be used to solve the GDM problem with uncertain preference ordinals directly [6]. For this problem, Fan Zhi-ping(2010) develop an approach to solving group decision-making problems, where the preference information on alternatives provided by experts is in the form of uncertain preference ordinals [26].

The study of previous scholars is worthy of threshing because of the following reasons:

(1) For some reason, it should be noted that A_i is incomparable to A_k (denoted as $A_i \parallel A_k$), but most of the major decision making problems can gain the least upper bound or the greatest lower bound or both about alternative pair by mining some other information between A_i and A_k .So, only using ordinal preference formal information depicting decision makers' preferences are broad with some redundancy.

At this point, it is more realistic to describe the preferences of decision makers by using lattice order structure. For group decision-making problems with lattice order structure, Yao-Huang Guo , Jiacheng Liu and Chun-Xiang Guo et al have driven some research achievement [15,16,28]. Yao-Huang Guo and Jiacheng Liu made the total order of Von Neumann-Morgenstern rational behavior axiom system wide expanse to the lattice-ordered sequence, and correspondingly weaken the continuity axiom, while maintaining rational decision-making behavior should be guided by the independence axiom as same, and established the axiom system of lattice-ordered decision-making [15]. Qiang GUO and Chun-Xiang Guo proposed the method of group decision making based on the distance of lattice order preferences [16], Chun-Xiang Guo proposed the method random lattice order group decision-making based on interval probability preferences [28].

(2) In the uncertain decision process, the probability or possibility of binary preference relations about alternative pairs can not be expressed by certain numeral.

In fact, in response to complex and uncertain environment, in the judgment and decision making process people might be used to use direct and simple thinking way, also hope to get the information of program's supremum or infimum. So, it is significant by using uncertain variable to replace accurate number of the binary relation's possibility.

In this paper, we propose an interval estimation procedure of binary preference relations based on random lattice order group decision making problem with uncertain binary preference relations. We extend preference relations from $\{\succ, \parallel, \approx, \prec\}$ to $\{\succ, \parallel, \overset{\vee}{\parallel}, \underset{\wedge}{\parallel}, \overset{\vee}{\underset{\wedge}{\parallel}}, \approx, \prec\}$. The proposed method has three advantages: (1)the belief degree of binary preference relations is interval number,(2) it deals group decision-making problem with lattice order characteristic, and (3) the relative importance of the members is explicitly considered. This paper is organized as follows. In section 2, we briefly review some basic concepts that will be used in this paper. In section 3, we driven the concept of interval-valued distribution preference vector, and the interval-valued comparison matrix and the interval significance degree by approximation models are presented. The interval number comparison principle and binary relation determination method are presented in section 4, section 5 is reserved for an illustrative example of our method. Some concluding remarks are summarized in section 6.

2 Basic concepts and definitions

2.1 Preference description

Definition 1 [15] A poset is a set in which a binary relation $x \succcurlyeq y$ is defined, which satisfies for all x,y and z the following conditions:

P1 For all x, $x \succcurlyeq x$;

P2 If $x \succcurlyeq y$ and $y \succcurlyeq x$, then $x \parallel y$;

P3 If $x \succcurlyeq y$ and $y \succcurlyeq z$,then $x \succcurlyeq z$.

$\succ, \sim$ respectively denote "superior" and "no difference" relations in options set A, and "$\succcurlyeq$" $=\succ\cup\approx$.

Definition 2 [15] A lattice is a poset X with any two of whose elements have a least upper bound（l.u.b.）or "meet" denoted by $x\wedge y$,and a greatest lower bound（g.l.b.）or "join" denoted by $x\vee y$.

In determining lattice order decision-making circumstance, each of decision makers expresses his preference by giving one of the four following relations:

A_i is preferred to A_k ($A_i\succ A_k$); A_k is preferred to A_i ($A_i\prec A_k$); A_i is indifferent to A_k ($A_i\approx A_k$); A_i is incomparable to A_k ($A_i\,\|\,A_k$).

But in random lattice order decision-making circumstance, it is not enough to describe decision makers' random preferences only by such four preference relations. For the sake of expressing and describing decision makers' random preferences relations better, we extend preference relations from four preferences relations $\{\succ,\|,\approx,\prec\}$ to seven preferences relations $\{\succ,\|,\overset{\vee}{\|},\underset{\wedge}{\|},\overset{\vee}{\underset{\wedge}{\|}},\approx,\prec\}$ (A_i is preferred to A_k（$A_i\succ A_k$）; A_k is preferred to A_i（$A_i\prec A_k$）; A_i is indifferent to A_k（$A_i\approx A_k$）; A_i is incomparable to A_k（$A_i\,\|\,A_k$）; A_i is incomparable to A_k ,but A_i and A_k have l.u.b.（$A_i\overset{\vee}{\|}A_k$）; A_i is incomparable to A_k ,but A_i and A_k have g.l.b.（$A_i\underset{\wedge}{\|}A_k$）; A_i is incomparable to A_k , A_i and A_k not only have l.u.b. but aslo g.l.b.（$A_i\overset{\vee}{\underset{\wedge}{\|}}A_k$）).

2.2 Basics of the evidence theory

As a conceptual framework for modeling and reasoning under uncertainty, Dempster-Shafer theory (DST), also named as the theory of belief functions[30,31], has so far found many applications in many areas, such as expert systems, diagnosis and reasoning, pattern classification, information fusion, audit risk assessment, environmental impact assessment and contractor selection etc[32-35].

Let $H=\{H_1,H_2,\cdots,H_N\}$ be a collectively exhaustive and mutually exclusive set of hypotheses or propositions, which is called the frame of discernment. A basic probability assignment (BPA) (also called a belief structure) is a function $m:2^H\to[0,1]$, which is called a mass function and satisfies:

$$m(\phi)=0$$

$$\sum_{A\subseteq H} m(A)=1$$

Where ϕ is the null set, A is any subset of H, and 2^H is the power set of H, which consists of all the subsets of H, i.e.

$$2^{H=\{\Phi,\{H_1\},\cdots,\{H_N\},\{H_1,H_2\},\cdots,\{H_1,H_N\},\cdots,\{H\}\}}.$$

The assigned probability (also called probability mass) $m(A)$ measures the belief exactly assigned to A and represents how strongly the evidence supports A.

A belief measure (Bel) and a plausibility measure (Pl) are associated with each BPA and they are both functions: $m:2^H\to[0,1]$ defined by the following equations, respectively:

$$Bel(A)=\sum_{B\subseteq A} m(B)=1 \qquad \forall A\subseteq H$$

$$Pl(A)=\sum_{B\cap A\neq\phi} m(B)=1$$

Where A and B are subsets of H. $Bel(A)$ represents the exact support for A, i.e. the belief of the hypothesis of A being true; $Pl(A)$ represents the possible support for A, i.e. the total amount of belief that could be potentially placed in A.

In real situations, such as many decision situations, and group decision analysis, however, assigning the precise (crisp) number to every individual hypothesis is often regarded as too restrictive, due to the incompleteness or the lack of information, which results in partial or total ignorance. The interval-valued belief structure (IBS) is a sensible option in these situations, in which the belief degree of each individual hypothesis lies within a certain interval (Denoeux, 1999; Wang, Yang, Xu, & Chin, 2007) [29-33].

Below is a brief summary of the interval-valued belief structure (IBS), we start with the definition of interval belief structure defined by Denoeux (1999,2008) [25, 29-32].

Definition3[30,31] Let $H=\{H_1,H_2,\cdots,H_N\}$ be the frame of discernment, $F_1,F_2,\cdots,F_n$ be n subsets of H and $[a_i,b_i]$ be n intervals with $0\le a_i\le b_i\le 1(1\le i\le n)$. An interval belief structure is a set of belief structures on H such that

(1) $a_i\le m(F_i)\le b_i,(1\le i\le n)$;

(2) $\sum_{i=1}^{n} a_i\le 1$, $\sum_{i=1}^{n} b_i\ge 1$;

(3) $m(A)=0,\forall A\notin\{F_1,F_2,\cdots F_n\}$.

Remark1. If $\sum_{i=1}^{n} a_i > 1$ or $\sum_{i=1}^{n} b_i < 1$, then the interval belief structure m is said to be invalid. Invalid interval belief structures need to be revised.

Remark2. For a valid interval belief structure, we can always obtain a particular belief structure by selecting a value $m(F_i) \in [a_i, b_i]$ for each $i = 1,2,\cdots,n$ such that $\sum_{i=1}^{n} m(F_i) = 1$.

Definition4[30,31] Let m be a valid interval belief structure with interval probability masses $a_i \le m(F_i) \le b_i$ for $i = 1,2,\cdots,n$ .If a_i and b_i satisfy $\sum_{j=1}^{n} b_j - (b_i - a_i) \ge 1$ and $\sum_{j=1}^{n} a_j - (b_i - a_i) \le 1$ then m is said to be a normalized interval belief structure.

Remark3. Normalized interval belief structures are in fact the compact and equivalent form of valid interval belief structures. An interval belief structure may be valid, but may not necessarily be normalized [30,32,34].

Definition 5[30,31] Let m be a normalized interval belief structure with interval probability masses a $a_i \le m(F_i) \le b_i$ for $i = 1,2,\cdots,n$ and A be a subset of $H = \{H_1, H_2, \cdots, H_N\}$. The belief measure (Bel) and the plausibility measure (Pl) of A are the closed intervals defined respectively by:

$$Bel_m(A) = [Bel_m^-(A), Bel_m^+(A)],$$

$$Pl_m(A) = [Pl_m^-(A), Pl_m^+(A)]. \qquad (1)$$

where

$$Bel_m^-(A) = \min \sum_{F_i \subset A} m(F_i) = \max[\sum_{F_i \subset A} a_i, (1 - \sum_{F_i \not\subset A} b_j)]$$

$$Bel_m^+(A) = \max \sum_{F_i \subset A} m(F_i) = \min[\sum_{F_i \subset A} b_i, (1 - \sum_{F_i \not\subset A} a_j)]$$

$$Pl_m^-(A) = \min \sum_{F_i \cap A \ne \phi} m(F_i) = \max[\sum_{F_i \cap A \ne \phi} a_i, (1 - \sum_{F_i \cap A = \phi} b_j)]$$

$$Pl_m^+(A) = \max \sum_{F_i \cap A \ne \varphi} m(F_i) = \max[\sum_{F_i \cap A \ne \varphi} b_i, (1 - \sum_{F_i \cap A = \varphi} a_j)]$$

3 Interval belief by approximation models

3.1 Interval-valued distribution preference vector

Let $D = \{D_1, \cdots, D_i, \cdots, D_m\}$ be the set of m decision makers, the relative weights of m decision makers are denoted by $w = (w^1, \cdots, w^i \cdots, w^m)$, $w^i = [\underline{w}^i, \overline{w}^i]$ which are known or given and satisfy the following condition: $\sum_{i=1}^{m} w^i = 1, 1 \ge w^i \ge 0$.

The possible binary relations between alternatives A, B are $\succ, \|, \overset{\vee}{\|}, \underset{\wedge}{\|}, \overset{\vee}{\underset{\wedge}{\|}}, \approx$ or $\prec$, we denote this by $H = \{r_j \mid j = 1,\cdots,7\} = \{\succ, \|, \overset{\vee}{\|}, \underset{\wedge}{\|}, \overset{\vee}{\underset{\wedge}{\|}}, \approx, \prec\}$,which are mutually exclusive and collectively exhaustive. The 7 species binary relations form the frame of discernment in the D-S theory, the preference of decision maker D_i is evidence. If the relation of alternative pair (A,B) is $r_j \in \{\succ, \|, \overset{\vee}{\|}, \underset{\wedge}{\|}, \overset{\vee}{\underset{\wedge}{\|}}, \approx, \prec\}$ with an interval belief degree of $m_i(r_j) = [m_i^-(r_j), m_i^+((r_j)]$ $(r_j \in H)$,we denote this by $S(D_i(A,B) = \{r_j, [m_i^-(r_j), m_i^+(r_j)]\}$,which is an interval-valued distribution preference vector, where $m_i^-(r_j) \ge 0$ .Note that precise belief degree is a special case of interval belief degree with $m_i^-(r_j) = m_i^+(r_j)$, the decision preference system is denoted as S=(H,D).

Definition6 Let $S(D_i(A,B) = \{r_j, [m_i^-(r_j), m_i^+(r_j)], \quad r_j \in \{\succ, \|, \overset{\vee}{\|}, \underset{\wedge}{\|}, \overset{\vee}{\underset{\wedge}{\|}}, \approx, \prec\}\}$ be an interval-valued distribution preference vector of the preference system S=(H,D).If the interval belief degrees $[m_i^-(r_j), m_i^+(r_j)]$ satisfy $\sum_{r_j \in \{\succ, \|, \overset{\vee}{\|}, \underset{\wedge}{\|}, \overset{\vee}{\underset{\wedge}{\|}}, \approx, \prec\}} m_i^-(r_j) \le 1$, then $S(D_j(A,B)$ is said to be valid; otherwise, it is invalid.

For an invalid interval-valued distribution assessment vector, it needs to be revised or adjusted before it can be used to conduct decision analysis.

Definition7 Let $S(D_i(A,B) = \{r_j, [m_i^-(r_j), m_i^+(r_j)]$, $r_j \in \{\succ, \|, \overset{\vee}{\|}, \underset{\wedge}{\|}, \overset{\vee}{\underset{\wedge}{\|}}, \approx, \prec\}$ be a valid interval-valued distribution preference vector. If the interval belief degrees $m_i(r_j) = [m_i^-(r_j), m_i^+(r_j)]$ $(m_j^-(r_j) \le m_j^+(r_j)$ of D_i always satisfy $\sum_{r_j \in \{\succ, \|, \overset{\vee}{\|}, \underset{\wedge}{\|}, \overset{\vee}{\underset{\wedge}{\|}}, \approx, \prec\}} m_i(r_j) = 1$ in any circumstances, where $m_i(r_j) \in [m_i^-(r_j), m_i^+(r_j)]$,for each $r_j \in \{\succ, \|, \overset{\vee}{\|}, \underset{\wedge}{\|}, \overset{\vee}{\underset{\wedge}{\|}}, \approx, \prec\}$,then $S(D_i(A,B)$ is said to be a complete interval-valued distribution preference vector; otherwise, it is incomplete. Especially, when $\sum_{r\in\{\succ,\|,\underset{\wedge}{\|},\overset{\vee}{\|},\overset{\vee}{\underset{\wedge}{\|}},\approx,\prec\}} m_j^+(r)=0$, the preference is said to be totally ignorant.

For a complete interval-valued distribution preference vector, the preference relations between A,B are for sure to one or more of the defined binary relations and there is no remaining belief degree assigned to the whole set H. However, if an interval-valued distribution preference vector is incomplete, then there might be an interval belief degree that is unassigned to any of the defined binary relations. This unassigned interval belief degree should be assigned to the whole set H.

3.2 Interval-valued comparison matrix

Let the BPA of $A\succ B$, $A\|B$, $A\overset{\vee}{\|}B$, $A\underset{\wedge}{\|}B$, $A\overset{\vee}{\underset{\wedge}{\|}}B$, $A\approx B$ and $A\prec B$ are separately $m_i(\succ)$, $m_i(\|)$, $m_i(\overset{\vee}{\|})$, $m_i(\underset{\wedge}{\|})$, $m_i(\overset{\vee}{\underset{\wedge}{\|}})$, $m_i(\approx)$ and $m_i(\prec)$, defined it as "significance degree" of binary relations $r_j\in\{\succ,\|,\underset{\wedge}{\|},\overset{\vee}{\|},\overset{\vee}{\underset{\wedge}{\|}},\approx,\prec\}$,

satisfying $\sum_{r_j\in\{\succ,\|,\underset{\wedge}{\|},\overset{\vee}{\|},\overset{\vee}{\underset{\wedge}{\|}},\approx,\prec\}} m_i(r_j)\le 1$.

Where $m_i(r_j)=[m_i^-(r_j),m_i^+(r_j)]$ $\quad (m_j^-(r_j)\le m_j^+(r_j))$ is the interval belief degrees.

Suppose the decision maker D_i compares a pair of BPA of binary relations for all possible pairs to obtain a pair-wise comparison A_7^i as follows:

$$A_7^i=(a_{RR'}^i)_{7\times 7}=\begin{array}{c|ccccccc} & m_i(\succ) & m_i(\|) & m_i(\underset{\wedge}{\|}) & m_i(\overset{\vee}{\|}) & m^i(\overset{\vee}{\underset{\wedge}{\|}}) & m^i(\approx) & m_i(\prec)\\ \hline m_i(\succ) & a^i_{\succ\succ} & a^i_{\succ\|} & a^i_{\succ\underset{\wedge}{\|}} & a^i_{\succ\overset{\vee}{\|}} & a^i_{\succ\overset{\vee}{\underset{\wedge}{\|}}} & a^i_{\succ\approx} & a^i_{\succ\prec}\\ m_i(\|) & a^i_{\|\succ} & a^i_{\|\|} & a^i_{\|\underset{\wedge}{\|}} & a^i_{\|\overset{\vee}{\|}} & a^i_{\|\overset{\vee}{\underset{\wedge}{\|}}} & a^i_{\|\approx} & a^i_{\|\prec}\\ m_i(\underset{\wedge}{\|}) & a^i_{\underset{\wedge}{\|}\succ} & a^i_{\underset{\wedge}{\|}\|} & a^i_{\underset{\wedge}{\|}\underset{\wedge}{\|}} & a^i_{\underset{\wedge}{\|}\overset{\vee}{\|}} & a^i_{\underset{\wedge}{\|}\overset{\vee}{\underset{\wedge}{\|}}} & a^i_{\underset{\wedge}{\|}\approx} & a^i_{\underset{\wedge}{\|}\prec}\\ m_i(\overset{\vee}{\|}) & a^i_{\overset{\vee}{\|}\succ} & a^i_{\overset{\vee}{\|}\|} & a^i_{\overset{\vee}{\|}\underset{\wedge}{\|}} & a^i_{\overset{\vee}{\|}\overset{\vee}{\|}} & a^i_{\overset{\vee}{\|}\overset{\vee}{\underset{\wedge}{\|}}} & a^i_{\overset{\vee}{\|}\approx} & a^i_{\overset{\vee}{\|}\prec}\\ m_i(\overset{\vee}{\underset{\wedge}{\|}}) & a^i_{\overset{\vee}{\underset{\wedge}{\|}}\succ} & a^i_{\overset{\vee}{\underset{\wedge}{\|}}\|} & a^i_{\overset{\vee}{\underset{\wedge}{\|}}\underset{\wedge}{\|}} & a^i_{\overset{\vee}{\underset{\wedge}{\|}}\overset{\vee}{\|}} & a^i_{\overset{\vee}{\underset{\wedge}{\|}}\overset{\vee}{\underset{\wedge}{\|}}} & a^i_{\overset{\vee}{\underset{\wedge}{\|}}\approx} & a^i_{\overset{\vee}{\underset{\wedge}{\|}}\prec}\\ m_i(\approx) & a^i_{\approx\succ} & a^i_{\approx\|} & a^i_{\approx\underset{\wedge}{\|}} & a^i_{\approx\overset{\vee}{\|}} & a^i_{\approx\overset{\vee}{\underset{\wedge}{\|}}} & a^i_{\approx\approx} & a\approx\prec\\ m_i(\prec) & a^i_{\prec\succ} & a^i_{\prec\|} & a^i_{\prec\underset{\wedge}{\|}} & a^i_{\prec\overset{\vee}{\|}} & a^i_{\prec\overset{\vee}{\underset{\wedge}{\|}}} & a_{\prec\approx}^{\ i} & a^i_{\prec\prec} \end{array} \quad (2)$$

Where $R,R'\in\{\succ,\|,\underset{\wedge}{\|},\overset{\vee}{\|},\overset{\vee}{\underset{\wedge}{\|}},\approx,\prec\}$ $(i=1,2,\cdots,m)$ shows the priority ratio of binary relation R comparing to binary relation R' and they satisfy the following relations so that the decision maker gives 21 comparisons.

$$a_{*RR'}^i=1/\mathrm{a}^{*\mathrm{i}}_{\mathrm{R'R}} \quad (3)$$

$$a_{*RR}^i=\mathrm{a}^{*\mathrm{i}}_{\mathrm{RR}} \quad (4)$$

$$\forall R,R'\in\{\succ,\|,\underset{\wedge}{\|},\overset{\vee}{\|},\overset{\vee}{\underset{\wedge}{\|}},\approx,\prec\}\quad (i=1,2,\cdots,m).$$

Where $a^i_{RR'}=[\mathrm{a}^{\mathrm{i}}_{*\mathrm{RR'}},a^{*i}_{RR'}]$, it follows from the reciprocal relation that the assumed model is ratio model.

Definition8 Given the interval comparison matrix (2), if the convex feasible set

$$\mathrm{M}=\{(\mathrm{m_i}(\succ),\mathrm{m_i}(\|),\cdots,\mathrm{m_i}(\prec))\,|\,\mathrm{a}^{\mathrm{i}}_{*\mathrm{RR'}}\le \mathrm{m_i}(\mathrm{R})/\mathrm{m_i}(\mathrm{R'})\le \mathrm{a}^{*\mathrm{i}}_{\mathrm{RR'}},\ \sum_{\mathrm{R}\in\{\succ,\|,\underset{\wedge}{\|},\overset{\vee}{\|},\overset{\vee}{\underset{\wedge}{\|}},\approx,\prec\}}\mathrm{m_i}(\mathrm{R})=1,\ m_i(R),m_i(R)\ge 0,\forall \mathrm{R},\mathrm{R'}\in\{\succ,\|,\underset{\wedge}{\|},\overset{\vee}{\|},\overset{\vee}{\underset{\wedge}{\|}},\approx,\prec\}\} \quad (5)$$

is nonempty, then A is said to be a consistent interval comparison matrix; otherwise, A is said to be an inconsistent interval comparison matrix.

Theorem 1 A_7^i is a consistent interval comparison matrix if and only if it satisfies the following inequality constraints:

$$\max_{\mathrm{R''}\in\{\succ,\|,\underset{\wedge}{\|},\overset{\vee}{\|},\overset{\vee}{\underset{\wedge}{\|}},\approx,\prec\}} \mathrm{a}^{\mathrm{i}}_{*\mathrm{RR''}}\ \mathrm{a}^{\mathrm{i}}_{*\mathrm{R''R'}}\le \min_{\mathrm{R''}\in\{\succ,\|,\underset{\wedge}{\|},\overset{\vee}{\|},\overset{\vee}{\underset{\wedge}{\|}},\approx,\prec\}} \mathrm{a}^{*\mathrm{i}}_{\mathrm{RR''}}\ \mathrm{a}^{*\mathrm{i}}_{\mathrm{R''R'}} \quad (6)$$

$$\forall \mathrm{R},\mathrm{R'}\in\{\succ,\|,\underset{\wedge}{\|},\overset{\vee}{\|},\overset{\vee}{\underset{\wedge}{\|}},\approx,\prec\}$$

Proof. If A is a consistent interval comparison matrix, then the convex feasible region M is nonempty, which means that there is no contradiction among the following inequality constraints:

$$a^{i}_{*RR'} \le m_i(R)/m_i(R') \le a^{*i}_{RR'} \quad (7)$$

$R,R' \in \{\succ,\|,\underset{\wedge}{\|},\overset{\vee}{\|},\overset{\vee}{\underset{\wedge}{\|}},\approx,\prec\}$

$$a^{i}_{*R'R''} \le m_i(R')/m_i(R'') \le a^{*i}_{R'R''} \quad (8)$$

$R',R'' \in \{\succ,\|,\underset{\wedge}{\|},\overset{\vee}{\|},\overset{\vee}{\underset{\wedge}{\|}},\approx,\prec\}$

Multiplying (7) by (8) leads to the following implied indirect inequalities

$$a^{i}_{*RR'}a^{i}_{*R'R''} \le m_i(R)/m_i(R'') \le a^{*i}_{RR'}a^{*i}_{R'R''} \quad (9)$$

$R,R',R'' \in \{\succ,\|,\underset{\wedge}{\|},\overset{\vee}{\|},\overset{\vee}{\underset{\wedge}{\|}},\approx,\prec\}$

Since (3) and (4) holds for any $R'' \in \{\succ,\|,\underset{\wedge}{\|},\overset{\vee}{\|},\overset{\vee}{\underset{\wedge}{\|}},\approx,\prec\}$,it follows that $\max\limits_{R''\in\{\succ,\|,\underset{\wedge}{\|},\overset{\vee}{\|},\overset{\vee}{\underset{\wedge}{\|}},\approx,\prec\}} a^{i}_{*RR'}a^{i}_{*R'R''} \le \min\limits_{R''\in\{\succ,\|,\underset{\wedge}{\|},\overset{\vee}{\|},\overset{\vee}{\underset{\wedge}{\|}},\approx,\prec\}} a^{*i}_{RR'}a^{*i}_{R'R''}$ holds for all $R,R',R'' \in \{\succ,\|,\underset{\wedge}{\|},\overset{\vee}{\|},\overset{\vee}{\underset{\wedge}{\|}},\approx,\prec\}$.

Conversely, if (6) holds for $R,R',R'' \in \{\succ,\|,\underset{\wedge}{\|},\overset{\vee}{\|},\overset{\vee}{\underset{\wedge}{\|}},\approx,\prec\}$ then $a^{i}_{*RR'} \le m_i(R)/m_i(R') \le a^{*i}_{RR'}\}$ holds for any $R,R' \in \{\succ,\|,\underset{\wedge}{\|},\overset{\vee}{\|},\overset{\vee}{\underset{\wedge}{\|}},\approx,\prec\}$. So, M cannot be empty. By definition 8, A is a consistent interval comparison matrix.

The above Theorem 1 can be used to judge whether or not an interval comparison matrix is consistent without solving any mathematical programming model. It only requires simple algebraic operations.

3.3 Interval normalization

The given pair-wise comparison $a^{i}_{RR'}$ is approximated by the ratio of significance degree, $m_i(R)$ and $m_i(R')$,symbolically written as $a^{i}_{RR'} \approx m_i(R)/m_i(R')$ .Where $m_i(R)=[m_i^-(R),m_i^+((R)] \quad (R\in H)$ is the significance degree of binary relation R. Then, the approximated pair-wise with the interval significance degree is defined as the following interval:

$$m_i(R)/m(R') = [\frac{m_i^-(R)}{m_i^+(R')},\frac{m_i^+(R)}{m_i^-(R')}] \quad (10)$$

Where the upper and lower bounds of the approximated comparison are defined as maximum range. It should be noted that the sum of significance degrees obtained by the conventional AHP is normalized to be one.

In this paper, supposing

$S(D_i(A,B)=\{r_j,[m_i^-(r_j),m_i^+(r_j)], \quad r_j\in\{\succ,\|,\underset{\wedge}{\|},\overset{\vee}{\|},\overset{\vee}{\underset{\wedge}{\|}},\approx,\prec\}\}$ be a complete interval-valued distribution assessment vector. Therefore, we consider interval belief proposed in so as to normalize the interval significance degrees. Their conditions are defined as follows.

Property 1 Let S=(H,D) be a decision preference system, and the muster of binary relation between alternative A and B is the frame of discernment $H=\{\succ,\|,\underset{\wedge}{\|},\overset{\vee}{\|},\overset{\vee}{\underset{\wedge}{\|}},\approx,\prec\}$,

$S(D_i(A,B)=\{r_j,[m_i^-(r_j),m_i^+(r_j)], \quad r_j\in\{\succ,\|,\underset{\wedge}{\|},\overset{\vee}{\|},\overset{\vee}{\underset{\wedge}{\|}},\approx,\prec\}\}$ be the complete interval-valued distribution preference vector of S=(H,D), let m be a valid interval belief structure with interval probability masses $m_i(r_j)=[m_i^-(r_j),m_i^+(r_j)](m_i^-(r_j)\le m_i^+(r_j)$, then satisfied such condition:

$$\begin{cases} m_i^+(\succ)+m_i^-(\|)+m_i^-(\overset{\vee}{\|})+\cdots+m_i^-(\approx)+m_i^-(\prec)\le 1 \\ m_i^+(\|)+m_i^-(\succ)+m_i^-(\overset{\vee}{\|})+\cdots+m_i^-(\approx)+m_i^-(\prec)\le 1 \\ \vdots \\ m_i^+(\prec)+m_i^-(\succ)+m_i^-(\|)+m_i^-(\overset{\vee}{\|})+\cdots+m_i^-(\approx)\le 1 \end{cases} \quad (11)$$

$$\begin{cases} m_i^-(\succ)+m_i^+(\|)+m_i^+(\overset{\vee}{\|})+\cdots+m_i^+(\approx)+m_i^+(\prec)\ge 1 \\ m_i^-(\|)+m_i^+(\succ)+m_i^+(\overset{\vee}{\|})+\cdots+m_i^+(\approx)+m_i^+(\prec)\ge 1 \\ \vdots \\ m_i^-(\prec)+m_i^+(\succ)+m_i^+(\|)+m_i^+(\overset{\vee}{\|})+\cdots+m_i^+(\approx)\ge 1 \end{cases} \quad (12)$$

Property 1 can be driven by definition 4 and 5 (Denoeux,1999) [29].

It can be said that the conventional normalization which makes the sum of values be one is extended to the interval normalization by using the above conditions. The sums of the bounds of intervals are constrained by one in two ways. These conditions make the intervals meaningful in the sense that there are elements in the intervals so as to make the sum of them be one. In order to make intervals be normalized, their redundancy should be reduced by (11) and (12) as the constraint conditions in the optimization problem described in the section 3.4.

3.4 Upper approximations of pair-wise comparisons

The proposed approach is based on the view that the interval significance degrees are obtained so as to include the given interval. The obtained interval significance degrees satisfy the following inclusion relations:

$$a^i_{RR'} \subseteq m_i(R)/m(R') = [\frac{m_i^-(R)}{m_i^+(R')}, \frac{m_i^+(R)}{m_i^-(R')}] \quad (13)$$

$R, R' \in \{\succ, \|, \underset{\wedge}{\|}, \overset{\vee}{\|}, \overset{\vee}{\underset{\wedge}{\|}}, \approx, \prec\}$,

Which means $\frac{m_i^-(R)}{m_i^+(R')} \le a^i_{RR'} \le \frac{m_i^+(R)}{m_i^-(R')}$.

The approximated interval significance degrees should be estimated as closely as possible to the given comparisons subject to the above inclusion relations. The concept of the least upper approximation, that is the width of each interval significance degree must be minimized, is applied. In the following LP problem, simply the sum of widths of all significance degrees is minimized under the constraints.

$$\min \sum_{R\in\{\succ, \|, \underset{\wedge}{\|}, \overset{\vee}{\|}, \overset{\vee}{\underset{\wedge}{\|}}, \approx, \prec\}} (m_i^+(R) - m_i^-(R))$$

$$s.t. \begin{cases} \sum_{R\neq R'} m_i^+(R) + m_i^-(R') \ge 1 & \forall R' \\ \sum_{R\neq R'} m_i^-(R) + m_i^+(R') \le 1 & \forall R' \\ \frac{m_i^-(R)}{m_i^+(R')} \le a^i_{RR'} \le \frac{m_i^+(R)}{m_i^-(R')} & \forall (R, R') \\ m_i^+(R) \ge m_i(R) \ge m_i^-(R) \ge \varepsilon & \forall R \\ m_i(R) \in M & \forall R \end{cases} \quad (14)$$

Where constraint conditions consist of the interval normalization (11),(12) and inclusion relations (13). Eq. (14) is called the upper approximation model.

The interval significance degrees by (14) include the given inconsistent comparisons data. The width of the interval represents uncertainty of each significance degrees and the least uncertain significance degrees are obtained by solving (14).

So, the interval-valued distribution preference vector of decision maker D_i about (A,B) can be driven, we denote this by

$S(D_i(A,B) = \{R, [m_i^-(R), m_i^+(R)]\}$,

($R \in \{\succ, \|, \underset{\wedge}{\|}, \overset{\vee}{\|}, \overset{\vee}{\underset{\wedge}{\|}}, \approx, \prec\}$).

3.5 Interval global significance degrees

Decision makers give pair-wise comparison matrix for the significance degree of binary relation R respectively. By the proposed approximation model, the local significance degree of binary relation R under decision maker D_i is denoted as $m_i(R) = [m_i^-(R), m_i^+((R)]$ and the referenced priority weight of decision maker D_i is denoted as $w^i = [\underline{w}^i, \overline{w}^i]$ .The global significance degree of binary relation R is obtained as

$w_R = \sum_{i=1}^{m} w^i m_i(R)$ by interval arithmetic [35-37].

It is the sum of multiplications of the referenced priority weight and corresponding local significance degrees so that it represents the significance degree of binary relation R considering all decision makers. The global significance degree can be obtained by repeating the similar calculation. The local significance degrees and referenced priority weight are intervals so that the global significance degrees tend to be obtained as large intervals because of interval arithmetic. [38]

Therefore, we consider that the global significance degree is obtained by the crisp referenced priority weights, whose sum is one, within the interval significance degrees.

The upper bound of interval global significance degree of the binary relation R is obtained as follows.

$$\overline{w}_R = \max \sum_{i=1}^{m} w_R^{*i} (\alpha * m_i^+(R) + (1-\alpha) * m_i^-(R))$$

$$s.t. \quad \sum_{i=1}^{m} w_R^{*i} = 1 \quad (15)$$

$$\underline{w}^i \le w_R^{*i} \le \overline{w}^i \quad \forall i$$

$$0 \le \alpha \le 1$$

Where, $w^{*i} (i = 1, 2, \cdots, m)$ are decision variables for the crisp referenced priority weights of decision maker that maximize the upper bound of the interval global significance degree of R, and $0 \le \alpha \le 1$ are compromise coefficients.

Similarly, the problem to obtain the lower bound of interval global significance degree of the binary relation R is formulated as follows:

$$\underline{w}_R = \min \sum_{i=1}^{m} w^i_{*R} (\alpha * m_i^+(R) + (1-\alpha) * m_i^-(R))$$

$$s.t. \quad \sum_{i=1}^{m} w^i_{*R} = 1 \quad (16)$$

$$\underline{w}^i \le w^i_{*R} \le \overline{w}^i \quad \forall i$$

$$0 \le \alpha \le 1$$

Where, $w_*^i (i = 1, 2, \cdots, m)$ are decision variables for the crisp referenced priority weights of decision maker that minimize the lower bound of the interval global significance degree of R, and $0 \le \alpha \le 1$ are the coefficients in model (15), they are designed to reflect the preference attitude of different DMs in a group decision-making.

The interval global significance degree of the binary relation R can be denoted as

$w_R = [\underline{w}_R, \bar{w}_R]$ .The bounds satisfy $w_R^{*i} m_i^+(R) \geq w_{*R}^i m_i^-(R)$ because of maximizing and minimizing the objective functions, respectively. It represents the possible range under the condition that the sum of the interval global significance degrees is one.

The interval global significance degrees reflect inconsistency in the given comparisons for decision maker and alternatives under each decision maker. The crisp referenced priority weights are determined by maximizing and minimizing the global significance degrees of each binary relation R , respectively. The selected crisp weights for the upper and lower bounds of interval global significance degrees are different from each other. (15) and (16)are formulated for each binary relation R ,so that the crisp referenced priority weights are also different among binary relation. They depend on local significance degrees of the alternative under the decision maker. The obtained interval global significance degrees satisfy the conditions in (11) and (12).

By the proposed models for global significance degrees (15) and (16), the interval-valued distribution preference vector of decision group about (A,B) can be driven, we denote this by

$$S(D(A,B) = \{R, [m^-(R), m^+(R)]\}$$

$(R \in \{\succ, \|, \overset{\|}{\wedge}, \overset{\vee}{\|}, \overset{\vee}{\underset{\wedge}{\|}}, \approx, \prec\})$.

4 Comparison interval global significance degrees and determining binary relationships

It can be said that decision makers consider the possible binary relation between A and B are $\succ, \|, \overset{\|}{\wedge}, \overset{\vee}{\|}, \overset{\vee}{\underset{\wedge}{\|}}, \approx$ or $\prec$,and we have reason to believe that only the binary relation makes contribution to group decision-making results, it can be selected as a consequence value of group decision-making. This shows that on the one hand whether a weak significance degree binary relation will be removed depends on significance degree of evaluation target, the more important the less likely to be removed, vice versa.

So, the global significance degree that has maximum is the right binary relationships and it is chosen as compromise solution. But here the $m(R) = [m^-(R), m^+(R)]$ $(R \in \{\succ, \|, \overset{\|}{\wedge}, \overset{\vee}{\|}, \overset{\vee}{\underset{\wedge}{\|}}, \approx, \prec\})$ are interval numbers. To choose the maximum interval number they are compared with each other. So, we introduce a new method for comparison of interval numbers as follows:

Suppose that $m(r_1) = [m^-(r_1), m^+(r_1)]$ and $m(r_2) = [m^-(r_2), m^+(r_2)]$ $(r_1, r_2 \in \{\succ, \|, \overset{\|}{\wedge}, \overset{\vee}{\|}, \overset{\vee}{\underset{\wedge}{\|}}, \approx, \prec\})$ are two interval significance degrees of r_1 and r_2 , that we want to determine which binary relationship is more appropriate, that is need to choose maximum interval number between them. These two interval numbers can have four status:

(1) If these interval numbers have no intersection, the minimum interval number is the one that has lower values. In other words: If $m^+(r_1) \leq m^-(r_2)$ then, we choose $m(r_2) = [m^-(r_2), m^+(r_2)]$ as maximum interval number.

(2) If two interval numbers are the same, both of them have the same priority for us.

(3) In situations that $m^-(r_1) \leq m^-(r_2) < m^+(r_2) \leq m^+(r_1)$, we choose maximum interval number in this way:

If $\beta(m^-(r_2) - m^-(r_1)) < (1-\beta)(m^+(r_1) - m^+(r_2))$ then $m(r_2) = [m^-(r_2), m^+(r_2)]$ is our maximum interval number, otherwise $m(r_1) = [m^-(r_1), m^+(r_1)]$ is maximum interval number; If $\beta(m^-(r_2) - m^-(r_1)) = (1-\beta)(m^+(r_1) - m^+(r_2))$ then both of them have the same priority for us.

(4) In situations that $m^-(r_1) \leq m^-(r_2) < m^+(r_1) \leq m^+(r_2)$, if $\beta(m^-(r_2) - m^-(r_1)) \leq (1-\beta)(m^+(r_2) - m^+(r_1))$, then $m(r_2) = [m^-(r_2), m^+(r_2)]$ is maximum interval number, otherwise $m(r_1) = [m^-(r_1), m^+(r_1)]$ is maximum interval number; If $\beta(m^-(r_2) - m^-(r_1)) = (1-\beta)(m^+(r_1) - m^+(r_2))$ then both of them have the same priority for us.

Here, β is introduced as optimism level of the decision maker ($0 < \beta \leq 1$). The optimist decision maker has greater a value than the pessimist decision maker. For rational decision maker β = 0.5 and in this situation the result of the comparisons obtained by the introduced method is similar to interval numbers comparison that has been made on the basis of interval numbers means.

Through the above methods, we can determine the possible binary relation, and weak significance degrees binary relationships can be removed from $\{\succ, \|, \overset{\|}{\wedge}, \overset{\vee}{\|}, \overset{\vee}{\underset{\wedge}{\|}}, \approx, \prec\}$.

5 A numerical example

There are six alternatives $X = \{A_1, \cdots, A_i, \cdots, A_k, \cdots, A_6\}$ and five decision makers $D=\{D_1,\cdots,D_i,\cdots,D_5\}$,the referenced priority weight of decision maker D_i is denoted as $w^i=[\underline{w}^i,\overline{w}^i]$,w^1=[0.08,0.086] , w^2=[0.05,0.101] , w^3=[0.04,0.072], w^4=0.505, w^5=[0.253,0.272], and $1\geq w^i\geq 0,\sum_{i-1}^{5}w^i=1$ .A decision maker gives pairwise comparison matrices about $\succ,\|,\underset{\wedge}{\|},\overset{\vee}{\|},\underset{\wedge}{\overset{\vee}{\|}},\approx$ and $\prec$ for each alternative pairs. Then, the global significance degrees of $\forall R\in\{\succ,\|,\underset{\wedge}{\|},\overset{\vee}{\|},\underset{\wedge}{\overset{\vee}{\|}},\approx,\prec\}$ are calculated from them by the proposed models. $\forall A_i,A_k\in X$, the binary relation between A_i,A_k are $\succ,\|,\underset{\wedge}{\|},\overset{\vee}{\|},\underset{\wedge}{\overset{\vee}{\|}},\approx$ and $\prec$,the occurrence beliefs of $A_i\succ A_k$, $A_i\|A_k$, $A_i\underset{\wedge}{\|}A_k$, $A_i\overset{\vee}{\|}A_k$, $A_i\underset{\wedge}{\overset{\vee}{\|}}A_k$, $A_i\approx A_k$ and $A_i\prec A_k$ are separately $m(\succ)$, $m(\|)$, $m(\underset{\wedge}{\|})$, $m(\overset{\vee}{\|})$, $m(\underset{\wedge}{\overset{\vee}{\|}})$, $m(\approx)$ and $m(\prec)$. The decision makers give five pairwise comparison matrices for binary relation. As an example, the pairwise comparison matrix under the decision maker $D_i(i=1,2,3,4,5)$ about alternative pair (A_2,A_4) followed by (1) is as follows:

$$A_7^1=\begin{bmatrix} 1 & [1/6,1/4] & [2,3] & [1/4,1/3] & [1,1] & [1/3,1/2] & [1/3,1/2] \\ [4,6] & 1 & [6,8] & [5,6] & [8,9] & [8,9] & [6,7] \\ [1/3,1/2] & [1/8,1/6] & 1 & [2,3] & [1/3,1/2] & [4,5] & [3,6] \\ [3,4] & [1/6,1/5] & [1/3,1/2] & 1 & [1,2] & [2,3] & [1,1] \\ [1,1] & [1/9,1/8] & [2,3] & [1/2,1] & 1 & [1,1] & [1/4,1/2] \\ [2,3] & [1/9,1/8] & [1/5,1/4] & [1/3,1/2] & [1,1] & 1 & [1/2,1/2] \\ [2,3] & [1/7,1/6] & [1/6,1/3] & [1,1] & [2,4] & [2,2] & 1 \end{bmatrix}$$

$$A_7^2=\begin{bmatrix} 1 & [2,3] & [6,8] & [3/5,2/3] & [4,7] & [4,5] & [3/2,2] \\ [1/3,1/2] & 1 & [2,3] & [1/3,1/2] & [1,2] & [4/3,5/3] & [1/3,1/2] \\ [1/8,1/6] & [1/3,1/2] & 1 & [1/7,1/6] & [1/2,2/3] & [1/3,1/2] & [1/6,1/4] \\ [3/2,5/3] & [2,3] & [6,7] & 1 & [1/7,1/4] & [1/2,5/7] & [8,9] \\ [1/7,1/4] & [1/2,1] & [2,3] & [4,7] & 1 & [4,6] & [3,3] \\ [1/5,1/4] & [3/5,3/4] & [2,3] & [7/5,2] & [1/6,1/4] & 1 & [3/2,2] \\ [1/2,2/3] & [2,3] & [4,6] & [1/9,1/8] & [1/3,1/3] & [1/2,2/3] & 1 \end{bmatrix}$$

$$A_7^3=\begin{bmatrix} 1 & [4,5] & [2,3] & [6,8] & [6,7] & [6,6] & [1/4,1/3] \\ [1/5,1/4] & 1 & [2,3] & [7,9] & [1,3] & [3,6] & [5,6] \\ [1/3,1/2] & [1/3,1/2] & 1 & [1/7,1/4] & [1/2,2/3] & [1/5,1/3] & [1/8,1/7] \\ [1/8,1/6] & [1/9,1/7] & [4,7] & 1 & [2,3] & [2,3] & [1/8,1/7] \\ [1/7,1/6] & [1/3,1] & [2,3] & [1/3,1/2] & 1 & [1/4,1/3] & [1/6,1/4] \\ [1/6,1/6] & [1/6,1/3] & [3,5] & [1/3,1/2] & [3,4] & 1 & [1/3,1/2] \\ [3,4] & [1/6,1/5] & [7,8] & [7,8] & [4,6] & [2,3] & 1 \end{bmatrix}$$

$$A_7^4=\begin{bmatrix} 1 & [4,6] & [6,8] & [2,3] & [2,2] & [1,3] & [1/8,1/9] \\ [1/6,1/4] & 1 & [2/7,2/5] & [5,7] & [1,3] & [3,6] & [1/6,1/5] \\ [1/8,1/6] & [5/2,7/2] & 1 & [3/2,2] & [1/3,1/2] & [6/5,2] & [5,7] \\ [1/3,1/2] & [1/7,1/5] & [1/2,2/3] & 1 & [2,3] & [4/3,5/3] & [1,1] \\ [1/2,1/2] & [1/3,1] & [2,3] & [1/3,1/2] & 1 & [1,1] & [5/4,4/3] \\ [1/3,1] & [1/6,1/3] & [1/2,5/6] & [3/5,3/4] & [1,1] & 1 & [1,3/2] \\ [8,9] & [5,6] & [1/7,1/5 & [1,1] & [3/4,4/5] & [2/3,1] & 1 \end{bmatrix}$$

$$A_7^5=\begin{bmatrix} 1 & [1/6,1/5] & [6,8] & [1,5/3] & [1,1] & [1/4,1/3] & [1/6,1/4] \\ [1/5,1/4] & 1 & [7,8] & [5,7] & [8,9] & [1/3,1/2] & [1/8,1/5] \\ [1/3,1/2] & [1/3,1/2] & 1 & [4,5] & [1/3,1/2] & [5,6] & [3/8,1/2] \\ [1/8,1/6] & [1/9,1/7] & [4,7] & 1 & [2,3] & [1/7,1/5] & [1/8,1/8] \\ [1/7,1/6] & [1/3,1] & [2,3] & [1/3,1/2] & 1 & [2/9,2/7] & [1/9,1/8] \\ [1/6,1/6] & [1/6,1/3] & [1/6,1/5] & [5,7] & [7/2,9/2] & 1 & [1/3,1/2] \\ [4,6] & [5,8] & [2,8/3] & [8,8] & [8,9] & [2,3] & 1 \end{bmatrix}$$

The interval significance degrees obtained from the above matrix by the upper approximation model (14), the interval-valued distribution preference vector of decision maker $D_i(i=1,2,3,4,5)$ about (A_2,A_4) are shown in Table 1.

Table1 The interval-valued distribution preference vector under decision makers

	$\succ$	$\parallel$	$\underset{\wedge}{\parallel}$	$\overset{\vee}{\parallel}$	$\overset{\vee}{\underset{\wedge}{\parallel}}$	$\approx$	$\prec$
D1	[0.0313,0.09375]	0.5625	[0.05,0.0975]	[0.0313,0.1]	[0.0469,0.0625]	[0.02,0.0625]	[0.025,0.0937]
D2	0.3430	[0.1053,0.1143]	[0.0436,0.22871]	0.0381	[0.0572,0.1742]	[0.0348,0.0762]	[0.0254,0.2105]
D3	[0.0850,0.2952]	[0.0590,0.2692]	[0.0299,0.1476]	0.0319	[0.0492,0.0638]	[0.0492,0.1476]	[0.0449,0.2550]
D4	[0.0252,0.2153]	[0.0359,0.2259]	[0.0377,0.1506]	[0.0308,0.2208]	0.0753	[0.0452,0.0753]	[0.0368,0.2268]
D5	[0.0402,0.1006]	[0.0575,0.2011]	[0.0335,0.0503]	[0.0144,0.1724]	[0.0223,0.0447]	[0.0287,0.2011]	0.4022

Table 2 The global interval-valued distribution preference vector of binary relation

binary relation R	Global weight (α =1)	Global weight(α =1/2)	Global weight (α =1/4)	Global weight (α =0)
$\succ$	[0.1877,0.1943]	[0.1212,0.1298]	[0.0880,0.0975]	[0.0548,0.0652]
$\parallel$	[0.2366,0.2440]	[0.1642,0.1686]	[0.1281,0.1312]	[0.0905,0.0949]
$\underset{\wedge}{\parallel}$	[0.1236,0.1287]	[0.0805,0.0832]	[0.0589,0.0604]	[0.0374,0.0379]
$\overset{\vee}{\parallel}$	[0.1688,0.1719]	[0.0981,0.0995]	[0.0628,0.0633]	[0.0269,0.0274]
$\overset{\vee}{\underset{\wedge}{\parallel}}$	[0.0714,0.0758]	[0.0634,0.0660]	[0.0594,0.0611]	[0.0554,0.0562]
$\approx$	[0.1101,0.1138]	[0.0741,0.0760]	[0.0560,0.0571]	[0.0377,0.0383]
$\prec$	[0.2596,0.2648]	[0.1936,0.1999]	[0.1605,0.1674]	[0.1275,0.1350]

The interval global significance degrees obtained by (15) and (16), the interval-valued distribution preference vectors of decision group about (A_2,A_4) for α =1, 1/2, 1/4 and 0 are shown in Table 2. When α =1, using interval comparison method in Section 4, the group preference about A_2,A_4 can be obtained, that is $A_2 \prec A_4$; when α =1/2, 1/4 and 0, the same conclusion can be obtained. By the same method, the relationships between any two alternatives can be obtained, so final group ranking about $\{A_1,A_2,A_3,A_4,A_5,A_6\}$ can be obtained.

6 Conclusions

In the actual decision-making process, the implementation environment, implementation tache and element of decision project are all involved in stochastic uncertainty elements. Such impact makes the results that also have a random uncertainty, so its value will be showed on a particular range .The research results show that the point estimation method should not be used for the decision making consequences, and we should use interval estimation which is in line with the wishes of decision makers that they generally demand to know the worst, the best and the most likely result.

For the above, given that the decision maker can not give the interval belief of binary preference relations between alternative pairs, decision maker gives pairwise comparisons on the importance of occurrence belief of binary relations according to his intuitive judgments. This paper focuses on studying the method of determining interval endpoints of the consequences of decision-making programs. And the interval number comparison principle and binary relation determination method have been studied in this paper. The method is applicable not only for the preference structure of decision-makers without connectivity, but also for the preference information of decision makers with the supremum and the infimum. The feasibility and effectiveness of the approach proposed in this paper are illustrated with numerical examples.

In terms of future research, the proposed approach can be extended to multiple attribute GDM problems with uncertain binary preference relations. It can also be extended to support the circumstances where decision-makers' preferences are in the form of uncertain fuzzy variables.

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Appendix

The interval global significance degree of the binary relation R is obtained as $w_R = [\sum_{i=1}^{m} w_{*R}^{i} m_i^-(R), \sum_{i=1}^{m} w_R^{*i} m_i^+(R)]$ where w_{*R}^{i} and w_R^{*i} ($R \in \{\succ, \|, \overset{\vee}{\|}, \underset{\wedge}{\|}, \overset{\vee}{\underset{\wedge}{\|}}, \approx, \prec\}$) are the optimal solutions of (15) and (16) for each binary relation, respectively. The first condition of interval belief (11) is verified as follows:

$$\sum_{R \neq R'} \sum_{i=1}^{m} w_R^{*i} m_i^+(R) + \sum_{i=1}^{m} w_{R'*}^{i} m_i^-(R')$$

$$\geq \sum_{R \neq R'} \sum_{i=1}^{m} w_{*R'}^{i} m_i^+(R) + \sum_{i=1}^{m} w_{*R'}^{i} m_i^-(R') \qquad (\imath)$$

$$= \sum_{i=1}^{m} w_{*R'}^{i} (\sum_{R \neq R'} m_i^+(R) + m_i^-(R')) \qquad (ii)$$

$$\geq \sum_{i=1}^{m} w_{*R'}^{i} \qquad (iii)$$

$$= 1$$

(i) The referenced priority weights $w_{*R'}^{i}$ are the optimal solutions of (16) for binary relation R. They can be the possible solutions of (15) for binary relation R, since the constraint conditions of (15) and (16) are the same. It follows that $\sum_{i=1}^{m} w_R^{*i} m_i^+(R) \geq \sum_{i=1}^{m} w_{*R'}^{i} m_i^+(R)$.

(ii) The local significance degree $m_i(R) = [m_i^-(R), m_i^+((R)]$ for each decision maker D_i is interval belief so that it holds $\sum_{R \neq R'} m_i^+(R) + m_i^-(R') \geq 1$ from the condition of interval beliefs (12).

(iii) The sum of the referenced priority weights $w_{*R'}^{i}$ is normalized to be one in the constraint condition of (16).

The second condition in (15) is verified in the similar way as follows:

$$\sum_{R \neq R'} \sum_{i=1}^{m} w_{*R}^{i} m_i^-(R) + \sum_{i=1}^{m} w_{R'}^{*i} m_i^+(R')$$

$$\leq \sum_{R \neq R'} \sum_{i=1}^{m} w_{R'}^{*i} m_i^-(R) + \sum_{i=1}^{m} w_{R'}^{*i} m_i^+(R')$$

$$= \sum_{i=1}^{m} w_{R'}^{*i} (\sum_{R \neq R'} m_i^-(R) + m_i^+(R'))$$

$$\leq \sum_{i=1}^{m} w_{R'}^{*i}$$

$$= 1$$

Then, the interval global significance degree $w_R = [\underline{w}_R, \overline{w}_R]$ satisfies both of the two conditions in (11) and (12). Thus, they are interval beliefs.

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