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Analysis of Retail Facility with Correlated Service Times using Linear Programming

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Abstract: This paper studies a problem faced by a retail service facility with cross trained servers who can switch between the front and back room operations. The servers in the front room deal with serving customers, perhaps from a queue and those in the back room serve a job which is generated by the front room. In this facilities, there are two types of jobs a server can do is serving customers in the front room and the service times in the front room and the back room operations are correlated. The aim of this study is to minimize the mean number of customers in the front room subject to the back room constraint and the model is solved using linear programming technique. A numerical study is conducted to analyze the impact of various parameters on the performance of servers.

Keywords: Linear Programming, Correlated, Markovian decision process, Retail facility

1 Introduction

We consider a service facility with cross-trained servers who can perform operations in both the front room and back room. The servers who directly serve the customer are known as front room servers and the servers who serve the job generated by front room are known as back room servers. The back room work is much less time-sensitive and may include tasks such as sorting of materials, processing paperwork, updating customer information, performing credit checks, etc. Retail service facilities of this kind include banks, insurance agencies, retail stores, post offices, etc. For example in banks, services to the customers in the front room are granting a loan, opening a new account, etc and updating customer information, performing credit checks, etc are done in the back room.

The management can improve the quality of service by switching the servers from front room to back room and vice versa. There is no loss of productivity due to movement of servers between front and back rooms. In these facilities, there are two types of jobs a server can do: (i) serving customers in the front room and (ii) the service times in the front room and the back room operations are correlated.

2 Literature Review

Queueing models have been extensively applied in the last decades as a powerful tool for modeling and prediction of computer systems, as well as production and manufacturing systems. In most of the queueing models, the number of servers is assumed to be constant. However, in many real life situations, the number of servers varies, depending on the number of customers waiting in the queue. Moder and Phillips [16] dealt with controlling the number of servers based on the queue length by increasing a server one at a time when the queue length becomes greater. Yadin and Naor [23] first introduced the concept of a single threshold control policy which turns the server on whenever $N (N \geq 1)$ or more customers are present, the server is turned off when none is present and is not activated until N customers are present in the system. Two essential models are proposed by Arivudainambi and Poongothai [4]. The main objective is to minimize the mean number of customer waiting in the queue subject to the back room constraint and both the models are solved using linear programming.

The control of arrival and service rates in a single server queue without switching costs is studied by Serfozo [19]. The work of Serfozo and Lu [20] is interesting as it characterizes the optimal policy in case of more service levels. Terekhov and Beck [21] considered

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the problem of finding the optimal combination of cross-trained and specialized servers. A queueing control model for retail facility having back room operations and cross-trained workers are considered in Berman and Larson [6]. Kim [12] considered a queueing model for an automated workstation receiving jobs from an automated workstation and found queues size distribution. Two essential models for service facility are discussed by Arivudainambi and Poongothai [3]. The models are represented as a Markovian decision problem and solved using mixed integer programming techniques.

A queueing theory approach with a variable number of servers in which a loss of productivity occurs when a server returns to the back room is considered in Wang [22]. Luh [13] constructed an embedded Markov chain to obtain the steady-state probability distribution for finite queueing model. Arivudainambi and Godhandaraman [2] proposed a single server retrial queueing system with balking, second optional service and single vacation. In the work of Cezik and L'Ecuyer [9], the goal of the problem is to determine the number of agents of each possible type that should be scheduled to work during a particular time period in order to minimize the operating costs of the call center. Mayorga et al [14] proposed tandem queuing system in which flexible servers can be allocated across stations and they assumed that a switching cost is charged when servers move between stations. Queueing networks with finite capacity queues have been introduced to represent system with semi-Markov analysis of a loss tandem with blocking by Oniszczuk [17].

Loss of productivity due to switching of servers from front room to back room and a non deterministic policy with cross trained servers is taken into account by Berman and Ianovsky [5]. Berman et al [8] considered a retail service facility with cross trained workers who can switch between the front room and back room depending on the size of the queue in the front room and deal with the problem of minimizing the total number of servers. Arivudainambi and Godhandaraman [1] discussed a batch arrival queueing system with two phases of service, balking, feedback and K optional vacations under a classical retrial policy. Berman and Sapna [7] considered a retail service facility with cross trained workers who can switch between the front room and back room depending on the size of the queue in the front room and deal with the problem of minimising the total number of servers.

Lot of work have been done in the literature for various queueing models but not for the service facility in front and back room operations. Upto our knowledge, this work is the first one which consider queueing models with correlated service times in front and back room operations. The goal is to find the mean number of customers waiting in the front room and the mean number of jobs accumulated in the back room do not exceed predetermined levels for every possible state of the system.

The outline of the paper is as follows. A brief mathematical description and Markovian decision problem for the model are given in section 3 and 4 respectively. In section 5, we derive the steady-state balance equations for all possible state of the system. Using linear programming technique, we obtain optimal solution to our model in section 6. A brief numerical study is done in section 7 for the various parameters of the system.

3 Description of the model

Let N be the number of servers in the facility. Customers arrive according to a Poisson process with rate λ. The service time in the front room and back room are assumed to be independent exponential distributions with parameters μ and γ respectively. We consider our queueing model under the restriction of limited waiting room capacities. Assume that the front room has a maximum capacity of L and that the back room has a maximum capacity of M jobs. When the back room reaches the threshold limit of M then the service is stopped in the front room.

The aim of this study is to determine the number of servers to be used in the front room and back rooms at each instant of time, so as to minimize the mean number of customers in the front room ensuring that the mean number of jobs in the back room does not exceed $\alpha M\,(\alpha \in (0,1])$. The back room work must not reach its maximum capacity, so we multiply by α with the capacity M, in order to decrease the stoppage of service in the front room. We also assume that the maximum value that N can take is the minimum of L and M, *i.e* $N \leq min(L,M)$.

4 The Markovian Decision Problem

Let $J(t)$ and $K(t)$ be the number of customers in the front room and the number of jobs in the back room at time t. From our assumptions, the controlled process (J^S,K^S) is a finite state semi-Markov decision process. A policy S is called a stationary policy if it is randomized, time invariant and Markovian. Further, a process is said to be ergodic, if every stationary policy gives rise to an irreducible Markov chain. From our assumptions it can be seen that for every stationary policy h, (J^h,K^h) is completely ergodic. Since the action space is also finite by Guo et al [11], a stationary optimal policy exists.

Let μ_{ij} be the service rate in the front room when there are i customers in the front room and j jobs in the back room and γ_{ij} be the service rate in the back room when there are i customers in the front room and j jobs in the back room. The class H of all stationary policies is considered and the service rate in front room and back room is denoted as $\mu_{ij} = m\mu$ and $\gamma_{ij} = r\gamma$ $(m = 0,1,2,..,N;\ r = N-m$, where m and r are non-negative integers).

Let B represent the set of all possible actions of the system and $B = \{(m,r) : m+r \leq N$. The values μ_{ij} and γ_{ij} are expressed in terms of decision variables $n_{ij}, \mu_{ij} = min(n_{ij}\mu, i\mu) = m\mu, \gamma_{ij} = min([N-n_{ij}]\gamma, j\gamma) = r\gamma$, $n_{ij} = 0,1,...N$. A function $h : A \to B$ which is equivalent to the class H is given by $h(i,j) = \{(m,r);(i,j) \in A, (m,r) \in B\}$. For a fixed $h \in H$, $(i,j), (k,v) \in A$, the following definitions are adopted: $P^h_{kv}(i,j,t) = \mathrm{P}[J^h(t) = i, K^h(t) = j \mid J^h(0) = k, K^h(0) = v]$, (i,j) and $(k,v) \in A$. Each policy h, is an irreducible Markov chain and $P^h(i,j) = \lim_{t\to\infty} P^h_{kv}(i,j,t)$ exists and is independent of the initial conditions.

5 The steady state equations

To derive the steady state equations, we shall make use of the method of differential difference equations. This method is widely used when we have a model consisting of Poisson components. Consider the possible state changes that may occur in our queueing system for $1 \leq i \leq L-1,\ 1 \leq j \leq M-1$ then $P^h(i,j)$ could have arisen from one of the 6 possibilities. We obtain the following system of difference equations for the model $1 \leq i \leq L,\ 1 \leq j \leq M$.

$$\lambda P^h(0,0) = \mu_{10}P^h(1,0) + \gamma_{01}P^h(0,1) \tag{1}$$

$$(\mu_{LM} + \gamma_{LM})P^h(L,M) = \lambda P^h(L-1,M) \tag{2}$$

$$(\lambda + \gamma_{0M})P^h(0,M) = \mu_{1M}P^h(1,M) \tag{3}$$

$$\mu_{L0}P^h(L,0) = \lambda P^h(L-1,0) + \gamma_{L1}P^h(L,1) \tag{4}$$

$$\begin{aligned}(\lambda + \gamma_{0j})P^h(0,j) &= \gamma_{0\,j+1}P^h(0,j+1) \\ &\quad + \mu_{1\,j}P^h(1,j)\end{aligned} \tag{5}$$

$$\begin{aligned}(\mu_{Lj} + \gamma_{Lj})P^h(L,j) &= \lambda P^h(L-1,j) \\ &\quad + \gamma_{L\,j+1}P^h(L,j+1)\end{aligned} \tag{6}$$

$$\begin{aligned}(\lambda + \mu_{iM} + \gamma_{iM})P^h(i,M) &= \lambda P^h(i-1,M) \\ &\quad + \mu_{i+1\,M}P^h(i+1,M)\end{aligned} \tag{7}$$

$$\begin{aligned}(\lambda + \mu_{ij} + \gamma_{ij})P^h(i,j) &= \lambda P^f(i-1,j) + \mu_{i+1\,j}P^h(i+1,j) \\ &\quad + \gamma_{i\,j+1}P^h(i,j+1)\end{aligned} \tag{8}$$

$$\begin{aligned}(\lambda + \mu_{i0})P^h(i,0) &= \lambda P^h(i-1,0) + \mu_{i+10}P^h(i+1,0) \\ &\quad + \gamma_{i+10}P^h(i+1,0)\end{aligned} \tag{9}$$

The above set of balance equations (1) to (9) for model together with the normalized condition $\sum_{(i,j)\in D} P^h(i,j) = 1$ determines the steady state probabilities uniquely.

The objective is to find an optimal policy h^* that minimizes the number of customers in the queue L_q.

$$\begin{aligned}L_q &= E\left[i - \frac{\mu_{ij}}{\mu}\right] \\ &= \sum_{i=0}^{L}\sum_{j=0}^{M}\left[i - \frac{\mu_{ij}}{\mu}\right]P^h(i,j) \\ &= \sum_{i=0}^{L}\sum_{j=0}^{M} iP^h(i,j) - \frac{\lambda}{\mu}\left[1 - \sum_{j=0}^{M} P^h(L,j)\right]\end{aligned}$$

The mean number of jobs in the back room is given by

$$B = \sum_{i=0}^{L}\sum_{j=0}^{M} jP^h(i,j) \leq \alpha M$$

The model of minimizing the mean number of customers waiting in the queue L_q subject to the back room constraint is

$$\text{Minimize}\, L_q = \sum_{i=0}^{L}\sum_{j=0}^{M} iP^h(i,j) - \frac{\lambda}{\mu}\left[1 - \sum_{j=0}^{M} P^h(L,j)\right]$$

$$\text{subject to } B = \sum_{i=0}^{L}\sum_{j=1}^{M} jP^h(i,j) \leq aM$$

$$\frac{\mu_{ij}}{\mu} + \frac{\gamma_{ij}}{\gamma} \leq N,\ \forall i,j \in D$$

The above Markovian decision process can be solved using various methods. In this study linear programming technique was used to solve the problem.

6 The Linear Programming Problem

The probability function $E(i,j,r)$ is defined as the number of servers in the back room for the given state (i,j), where i is the number of customers in the front room and j is the number of jobs in the back room, i.e $E(i,j,r)$= P[there are r servers in the back room | state is (i,j)].

For any stationary policy h, we have $E(i,j,r) = 0$ or 1. Suppose probability function $E(i,j,r)$ were continuous variables, then the semi-Markov decision process can be reformulated as a linear programming problem.

Consider the class of all randomized, time-invariant Markovian policies [15] for which the probability function $E(i,j,r)$ satisfies the condition $0 \leq E(i,j,r) \leq 1$ and the total probability is given by $\sum_{(N-r,\ r)\in C} E(i,j,r) = 1, 0 \leq j \leq M;\ 0 \leq i \leq L$. For any given $h, z(i,j,r)$=P[state is in (i,j) and there are r servers in the back room], the linear programming model is expressed as

$$z(i,j,r) = P^h(i,\ j)E(i,j,r)$$

Taking summation on both sides and expressing $P^h(i,j)$ in terms of $z(i,j,r)$

$$P^h(i,j) \times \sum_{(N-r,\ r)\in C} E(i,j,r) = \sum_{(N-r,\ r)\in C} z(i,j,r)$$

$$P^h(i,j) = \sum_{(N-r,\ r)\in C} z(i,j,r), \forall(i,j) \in D$$

$$P^h(i,j) = \sum_{r=0}^{N} z(i,j,r), \forall(i,j) \in D$$

The linear programming model can be written in terms of the probability function $E(i,j,r)$

$$z(i,j,r) = \sum_{r=0}^{N} z(i,j,r)E(i,j,r) \tag{10}$$

Lemma 6.1

The optimal solution for linear programming problem yields a deterministic policy.

Proof

From equation (10) the probability function can be written as,

$$E(i,j,r) = \frac{z(i,j,r)}{\sum_{r=0}^{N} z(i,j,r)}$$

Since the decision model is completely ergodic, every basic feasible solution to the linear programming problem has the property [18] that for each $(i,j) \in D, z(i,j,r) > 0$, for exactly one $(N-r,r) \in C$ [15]. Hence any basic feasible solution of the linear programming problem yields a deterministic policy [10]. Using μ_{ij} and γ_{ij} in the Markovian decision process, we get the LPP as

$$\text{Minimize } L_q = \sum_{i=0}^{L}\sum_{j=0}^{M}\sum_{r=0}^{N} iz(i,j,r) - \frac{\lambda}{\mu}\left[1 - \sum_{j=0}^{M}\sum_{r=0}^{N} z(L,j,r)\right]$$

subject to the constraints

$$z(i,j,r) \geq 0$$

$$\sum_{i=0}^{L}\sum_{j=0}^{M}\sum_{r=0}^{N} jz(i,j,r) \leq \alpha M$$

$$\sum_{i=0}^{L}\sum_{j=0}^{M}\sum_{r=0}^{N} z(i,j,r) = 1$$

The remaining constraints of the linear programming models are the balance equations which are given by the set of equations (1) to (9) for model.

6.2 Optimization for the Model

Applying linear programming technique for the balance equations (1) to (9), we obtain

$$\sum_{r=0}^{N}[(N-r)\mu + r\gamma]z(L,M,r) = \lambda \sum_{r=0}^{N} z(L-1,M,r)$$

$$\sum_{r=0}^{N}[\lambda + (N-r)\mu + r\gamma]z(i,j,r) = \sum_{r=0}^{N} r\gamma z(i,j+1,r) + \lambda \sum_{r=0}^{N} z(i-1,j,r) + \sum_{r=0}^{N}(N-r)\mu z(i+1,j,r)$$

$$\sum_{r=0}^{N}[\lambda + r\gamma]z(0,j,r) = \sum_{r=0}^{N} r\gamma z(0,j+1,r) + \sum_{r=0}^{N}(N-r)\mu z(1,j,r)$$

$$\sum_{r=0}^{N}(N-r)\mu z(L,0,r) = \lambda \sum_{r=0}^{N} z(L-1,0,r) + \sum_{r=0}^{N} r\gamma z(L,1,r)$$

$$\sum_{r=0}^{N}[(N-r)\mu + r\gamma]z(L,j,r) = \sum_{r=0}^{N} r\gamma z(L,j+1,r) + \lambda \sum_{r=0}^{N} z(L-1,j,r)$$

$$\sum_{r=0}^{N}[\lambda + r\gamma]z(0,M,r) = \sum_{r=0}^{N}(N-r)\mu z(1,M,r)$$

$$\sum_{r=0}^{N}[\lambda + (N-r)\mu + r\gamma]z(i,M,r) = \lambda \sum_{r=0}^{N} z(i-1,M,r) + \sum_{r=0}^{N}(N-r)\mu z(i+1,M,r)$$

$$\sum_{r=0}^{N} \lambda z(0,0,r) = \sum_{r=0}^{N} r\gamma z(0,1,r) + \sum_{r=0}^{N}(N-r)\mu z(1,0,r)$$

$$\sum_{r=0}^{N}[\lambda + (N-r)\mu]z(i,0,r) = \lambda \sum_{r=0}^{N} z(i-1,0,r) + \sum_{r=0}^{N} r\gamma z(i,1,r) + \sum_{r=0}^{N}(N-r)\mu z(i+1,0,r)$$

Solving the above linear programming gives the optimal solution to the model.

7 Numerical Results

The interest in the current study is to illustrate the effect of various parameters on the mean number of customers in the queue L_q with respect to the parameters λ, μ, γ and N. Table 1 clearly shows that, if the arrival rate increases then the mean number of customers in the queue also increases and when we increase the number of servers for fixed value of arrival rate, the mean number of customers in the queue decreases. On increasing the service rate in the front room and the back room, the mean number of customers in the queue decreases are given in Table 2 and 3 respectively.

Two dimensional and three dimensional graphs are plotted. On increasing the values of arrival rate, the mean number of customers in the queue also increases as shown in Figure 1. When the service rate in the front room increases, the mean number of customers in the queue decreases as shown in Figure 2. When the service rate in the back room increases, the mean number of customers in the queue decreases as shown in Figure 3. On increasing the values of service rate in the front room, the mean number of customers in the queue decreases and service rate in the back room increases, the mean number of customers in the queue decreases.

For the various values, the mean number of customers in the queue with respect to γ and N are displayed for the set of parameters (λ, μ) = (5, 2) as plotted. The surface displays a downward trend for increasing the service rate in the back room γ and N as shown in Figure 4. For the

Table 1 L_q versus λ and N

$\lambda \diagdown N$	1	2	3	4	5	6	7	8	9	10
1	1	0.667	0.500	0.400	0.300	0.287	0.250	0.222	0.2	0.182
2	2	1.500	1.200	1.000	0.857	0.750	0.667	0.600	0.545	0.500
3	3	2.400	2.000	1.714	1.500	1.333	1.200	1.091	1.000	0.923
4	4	3.333	2.857	2.500	2.222	2.000	1.818	1.667	1.538	1.429
5	5	4.286	3.750	3.333	3.000	2.721	2.500	2.308	2.143	1.814
6	6	5.250	4.667	4.200	3.818	3.500	3.231	3.000	2.80	2.754
7	7	6.222	5.600	5.091	4.667	4.308	4.000	3.733	3.547	3.294
8	8	7.200	6.545	6.000	5.538	5.143	4.800	4.500	4.235	4.100
9	9	8.182	7.500	6.923	6.519	6.000	5.625	5.294	5.000	4.737
10	10	9.167	8.462	7.857	7.333	6.875	6.471	6.111	5.784	5.500

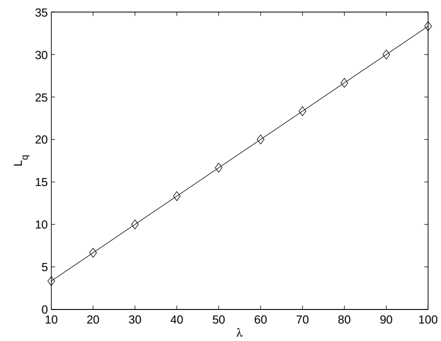

Fig 1: L_q versus λ

Table 2 L_q versus μ and N

$\mu \diagdown N$	1	2	3	4	5	6	7	8	9	10
1	10.000	9.167	8.462	7.857	7.333	6.875	6.471	6.111	5.914	5.500
2	5.000	4.286	3.750	3.333	3.000	2.727	2.528	2.308	2.143	2.000
3	3.333	2.708	2.514	1.981	1.733	1.548	1.378	1.274	1.174	1.083
4	2.500	1.944	1.591	1.346	1.149	1.029	0.971	0.833	0.761	0.683
5	2.000	1.500	1.200	1.000	0.857	0.750	0.667	0.600	0.545	0.500
6	1.667	1.212	0.953	0.783	0.667	0.580	0.513	0.481	0.417	0.381
7	1.429	1.012	0.784	0.632	0.540	0.467	0.412	0.368	0.324	0.304
8	1.250	0.865	0.663	0.538	0.450	0.388	0.341	0.314	0.274	0.250
9	1.180	0.754	0.595	0.459	0.384	0.314	0.287	0.257	0.234	0.211
10	1.000	0.667	0.500	0.400	0.333	0.286	0.250	0.222	0.200	0.182

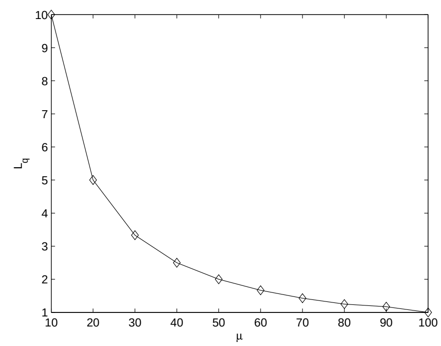

Fig 2: L_q versus μ

Table 3 L_q versus γ and N

$\gamma \diagdown N$	1	2	3	4	5	6	7	8	9	10
1	1.000	0.667	0.500	0.400	0.300	0.286	0.250	0.222	0.200	0.182
2	0.500	0.300	0.214	0.167	0.136	0.115	0.100	0.088	0.079	0.071
3	0.333	0.190	0.133	0.103	0.083	0.070	0.061	0.053	0.048	0.043
4	0.250	0.139	0.096	0.074	0.060	0.050	0.043	0.038	0.034	0.031
5	0.200	0.109	0.075	0.057	0.046	0.039	0.033	0.029	0.026	0.024
6	0.167	0.089	0.061	0.047	0.038	0.032	0.027	0.024	0.021	0.019
7	0.143	0.076	0.052	0.039	0.032	0.027	0.023	0.020	0.018	0.016
8	0.125	0.066	0.045	0.034	0.027	0.024	0.020	0.017	0.015	0.014
9	0.111	0.058	0.040	0.030	0.024	0.020	0.017	0.015	0.014	0.012
10	0.100	0.052	0.035	0.027	0.022	0.018	0.015	0.014	0.012	0.011

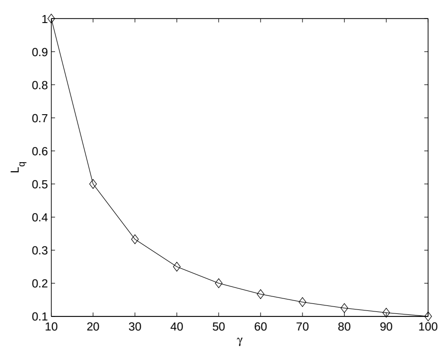

Fig 3: L_q versus γ

various values, the mean number of customers in the queue with respect to λ and N are displayed for the set of parameters (μ, γ) = (2, 5) as plotted.The surface displays a upward trend for L_q against increasing arrival rate λ and number of servers N as shown in Figure 5. For the various values, the mean number of customers in the queue with respect to μ and N are displayed for the set of parameters (λ, γ) = (5, 5) as plotted. In Figure 6, the surface displays a downward trend for L_q against increasing the service rate in the front room μ and N.

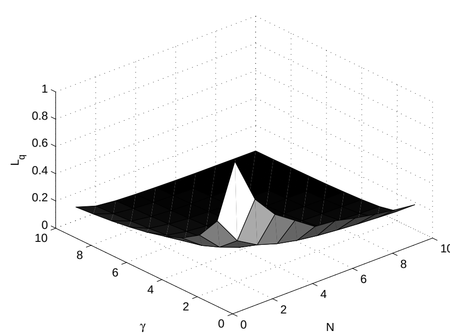

Fig 4: L_q versus γ and N

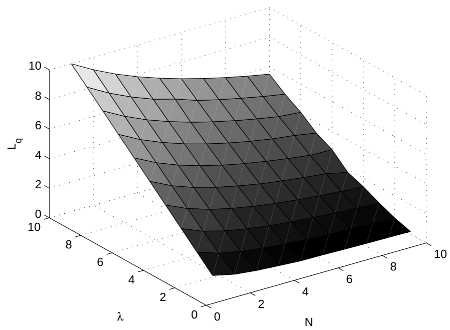

Fig 5: L_q versus λ and N

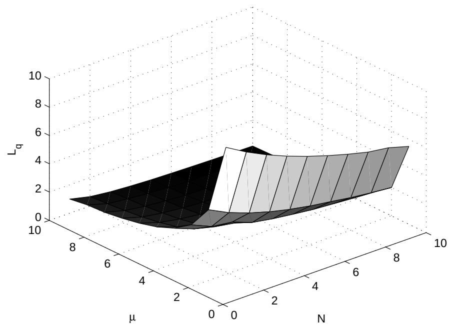

Fig 6: L_q versus μ and N

8 Conclusion

Assuming that servers in a retail facility can perform operations in both the front room and the back room, a model was developed to characterize a service policy, which controls the number of servers serving customers in the front room depending on the number of customers waiting for service. To investigate the systems, we consider queueing models with correlated service times in front and backroom operations. The main objective of this paper is to minimize the mean number of customer waiting in the queue subject to the back room constraint. A numerical study is conducted to analyze the impact of various parameters on the performance of servers. It is interesting to analyze the cost function for the given problems, such as when the work in the back room is generated by another arrival process.

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