Advanced Engineering Technology and Application
An International Journal

http://dx.doi.org/10.12785/aeta/040301

A Fuzzy Linear Programming Approach for Resource-Constrained Project Scheduling

Daniyar Artykov and Lyazzat Atymtayeva*

Department of Information Systems Management, Kazakh-British Technical University, 050000 Almaty, Kazakhstan

Received: 13 May 2015, Revised: 15 May 2015, Accepted: 18 May 2015
Published online: 1 Sep. 2015

Abstract: In this paper, first the problems that might be faced by project managers during project management period presented. The main goal of our research is development of intelligent application that based on a fuzzy linear programming approach. Intelligent application should find the optimal scheduling of project activities. We take duration of the project activity as a fuzzy random variable, and then we generate an integer programming model using approach proposed by Alvarez-Valdes and Tamarit.

Keywords: project management, fuzzy logic and sets, resource-constrained project scheduling problem, fuzzy random variable, integer programming

1 Introduction

The world has long been recognized that project management is a special area of management, the use of which gives tangible results. Professionals in this field are highly valued, and project management methodology itself became the de facto standard for managing many thousands of companies and is used to some extent in almost all large corporations. It helps project managers to support and control the projects life cycle project management program. Today, there are various programs of project management. They are designed to support and control the projects life cycle. However, as existing programs of project management are considered only tools that dont facilitate the work of project managers.

It is reported that in 2012 over $300 billion was spent on software projects, only 39 percent of software projects were completed successfully, and only 82 percent of projects were completed at all, either successfully or not. Poor project planning and often too late to correct the problems (risks) by the time they are detected the main reasons behind this unfruitful development. It clearly indicates the need for intelligent application for project management. Intelligent application can help project managers to schedule tasks, provide timely advice to optimize the scheduling of the project.

This paper is organized as follows: in the first section, overview of previous researches is presented. Then, the fuzzy project scheduling approaches are discussed. The integer programming model and an algorithm to solve the resource-constrained project scheduling problems are presented in Sections 4 and 5. In the next section, we describe our approach and architecture of our intelligent system. The concluding remarks and suggestions for further research are discussed in the last section.

2 Overview of previous researches

Numerous authors have contributed to the development of project scheduling, including:

1. Arvind Sathi, Thomas E. Morton, Stephen F. Roth (1986), they were among the first who started the deployment of intelligent components in the project management environment. They have developed an intelligent tool for project management called Callisto [2]. Callisto project was started as a research project to study the project planning, management, and configuration problems in the development of the engineering prototype of large computer systems and ultimately on the basis of the study developed an intelligent tool for project management, which contributes to the introduction of the documentation project management knowledge and this knowledge can be used for future projects;

* Corresponding author e-mail: da_artykov@kbtu.kz

2. Xiaoqing (Frank) Liu, Gautam Kane, Monu Bambroo (2006) [1], they have developed an intelligent system based on fuzzy logic using an integrated set of software metrics. This system able to warn project managers and developers about potential problems and risks of the project;
3. Wang (2002) [3] used Possibility Theory to model the uncertain and flexible temporal information and developed a fuzzy beam search algorithm based on fuzzy scheduling methodology to determine a schedule with the minimum schedule risk;
4. Herroelen, W., and R. Leus. (2005) [4], they reviewed the main approaches for planning under uncertainty. According to W. Herroelen and R. Leus include the following five main types of approach in the face of uncertainty: reactive scheduling, stochastic project planning, fuzzy project scheduling, reliable (active) planning and sensitivity analysis. In the article they offer an overview of each approach and how to reduce the risks of planning with the help of these approaches in the face of uncertainty projects deterministic evolution of the network structure.
5. Javad Nematian, Kourosh Eshghi, Abdolhamid Eshragh-Jahromi [11], they have developed an algorithm for solving Fuzzy Random Resource-Constrained Project Scheduling problem.
6. Mirseidova Sh. , Atymtayeva L. (2012), they have developed intelligent system based on fuzzy rules, which describes a direct dependency between software metrics and resulting cost of the project [14].

But the problems of project management such as poor planning and resource management are still not completely disclosed. After analyzing these problems it was revealed that for most problems of project management could be applied fuzzy logic, because in project management is handled incomplete, unreliable and inaccurate information in an uncertain environment. Fuzzy logic has found a lot of successful applications in risk assessment, security audit [13], health care and etc.

3 Fuzzy project scheduling

After the aim of the project is clear, begins the main work of the project manager. This work begins with planning objectives of the project. W. Herroelen and R. Leus described the main approaches planning under uncertainty, we have chosen fuzzy project planning approach for our intelligent system. The advocates of fuzzy project planning, as well as literature project planning using fuzzy sets is recommended to use fuzzy numbers for the duration of modeling activities in situations that include inaccuracy. This type of scheduling uses of membership functions based on possibility theory [4]. A fuzzy set is a function that measures the degree of membership to a set. Under the fuzzy set A is the set of ordered pairs composed of elements x universal set X and the corresponding degrees of affiliation $A = \{(x, \mu_A(x)) | x \in X\}$, $\mu_A(x)$ the membership function, indicating the extent to which element x belongs to the fuzzy set A. Function $\mu_A(x)$ takes values in a linearly ordered set M. The set M is called a membership set, often M is selected interval as [0, 1]. A high value of this membership function implies a high possibility, while a low value implies a poor possibility. If $M = 0, 1$ (i.e., consist of only two elements, $\mu_A(x) = 0$ or 1), the fuzzy set can be regarded as crisp set [4]. A fuzzy number $\tilde{A} = \{(x, \mu_{\tilde{A}}(x)) | x \in X\}$, where $\mu_{\tilde{A}}$ is the membership function of $\tilde{A}$, is a special kind of a fuzzy set defined as a fuzzy subset of the real line $\Re$ that is convex, which means that $\forall a, b \in \Re$, $\forall c \in [a, b]$, $\mu_{\tilde{A}}(c) \geq \min(\mu_{\tilde{A}}(a), \mu_{\tilde{A}}(b))$. It is also required that $\exists a \in \Re : \mu_{\tilde{A}}(a) = 1$. The advocates of fuzzy scheduling admit that the precise form of a fuzzy number is difficult to describe by an expert [5]. A practical way of getting suitable membership functions of fuzzy data has been proposed by Rommelfanger [6]. He recommends that the expert express his optimistic and pessimistic 3 intervals on $\Re$: the smallest interval $[\underline{m}, \overline{m}]$ for which $\mu(x) = 1$, meaning that x certainly belongs to the set of possible values; a larger interval $\left[\underline{m}^{\lambda}, \overline{m}^{\lambda}\right]$, containing, for which it holds that values x have a good chance $\geq \lambda$ of belonging to the set of possible values; and a third interval $[\underline{m}^{\varepsilon}, \overline{m}^{\varepsilon}]$, containing the second, for which all values x have $\mu(x) < \varepsilon$. Values x with $\mu(x) < \varepsilon$ have a very small chance of belonging to the set of possible values; i.e. the expert is willing to neglect the corresponding values of x. Using a six-point representation, a fuzzy number $\tilde{M}$ is then represented by the list of symbols $\tilde{M} = \left(\underline{m}^{\varepsilon}, \underline{m}^{\lambda}, \underline{m}, \overline{m}, \overline{m}^{\lambda}, \overline{m}^{\varepsilon}\right)$ as shown in Fig. 1.

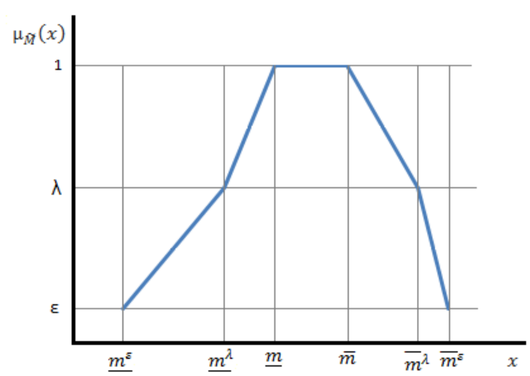

Fig. 1: Fuzzy number in six-point representation [5]

The output of a fuzzy scheduling pass will be a fuzzy schedule with fuzzy starting and ending times for the activities. Such fuzzy time instances may be interpreted as start or completion to a certain extent only. A fuzzy schedule assists in the explicit representation of certain degrees of freedom in the predictive schedule to represent the discretion management has to start certain activities a little earlier or later when duly propagating certain hard and soft constraints that may be imposed [7]. In this sense, a fuzzy schedule comprises multiple crisp schedules.

Fuzzy arithmetics. Let A and B be fuzzy numbers of the universe X and Y. Let * denote any basic arithmetic operations (+, -, x, /). Then any operation $A*B$ can be characterized by Zadehs extension principle [9]:

$$\mu_{\tilde{A}*\tilde{B}}(z) = \max_{x*y=z}\{\min[\mu_{\tilde{A}}(x),\ \mu_{\tilde{B}}(y)]\} \ (1)$$

In the case of arithmetic operations on fuzzy numbers in piece-wise representation the equation (1) corresponds to

$$\tilde{C}_\alpha = \left(\tilde{A}*\tilde{B}\right)_\alpha = \tilde{A}_\alpha * \tilde{B}_\alpha, \text{ for any } \alpha \in (0,1] \ (2)$$

Equation (2) shows that the α-cut on a general arithmetic operation on two fuzzy numbers is equivalent to the arithmetic operation on the respective α-cuts of the two fuzzy numbers. Both $\left(\tilde{A}*\tilde{B}\right)_\alpha$ and $\tilde{A}_\alpha * \tilde{B}_\alpha$ are interval quantities the operations on which can make use of classical interval analysis [10]. Thus, one can define the following arithmetic operations for fuzzy numbers in six-point representation:

$$\tilde{A}\oplus\tilde{B} = \left(\underline{a}^\varepsilon+\underline{b}^\varepsilon, \underline{a}^\lambda+\underline{b}^\lambda, \underline{a}+\underline{b}, \overline{a}+\overline{b}, \overline{a}^\lambda+\overline{b}^\lambda, \overline{a}^\varepsilon+\overline{b}^\varepsilon\right) \ (3)$$

$$\tilde{A}\ominus\tilde{B} = \left(\underline{a}^\varepsilon+\overline{b}^\varepsilon, \underline{a}^\lambda+\overline{b}^\lambda, \underline{a}+\overline{b}, \underline{a}+\overline{b}, \underline{a}^\lambda+\overline{b}^\lambda, \underline{a}^\varepsilon+\overline{b}^\varepsilon\right) \ (4)$$

$$\tilde{A}\times\tilde{B} = \left(\underline{a}^\varepsilon\times\underline{b}^\varepsilon, \underline{a}^\lambda\times\underline{b}^\lambda, \underline{a}\times\underline{b}, \overline{a}\times\overline{b}, \overline{a}^\lambda\times\overline{b}^\lambda, \overline{a}^\varepsilon\times\overline{b}^\varepsilon\right) \ (5)$$

$$\tilde{A}/\tilde{B} = \left(\underline{a}^\varepsilon/\underline{b}^\varepsilon, \underline{a}^\lambda/\underline{b}^\lambda, \underline{a}/\underline{b}, \overline{a}/\overline{b}, \overline{a}^\lambda/\overline{b}^\lambda, \overline{a}^\varepsilon/\overline{b}^\varepsilon\right) \ (6)$$

and moreover:

$$m\tilde{a}x(\tilde{A},\tilde{B}) = (max(\underline{a}^\varepsilon,\underline{b}^\varepsilon), max(\underline{a}^\lambda,\underline{b}^\lambda), max(\underline{a},\underline{b}),$$
$$max(\overline{a},\overline{b}), max(\overline{a}^\lambda,\overline{b}^\lambda), max(\overline{a}^\varepsilon,\overline{b}^\varepsilon)) \ (7)$$
$$m\tilde{i}n(\tilde{A},\tilde{B}) = (min(\underline{a}^\varepsilon,\underline{b}^\varepsilon), min(\underline{a}^\lambda,\underline{b}^\lambda), min(\underline{a},\underline{b}),$$
$$min(\overline{a},\overline{b}), min(\overline{a}^\lambda,\overline{b}^\lambda), min(\overline{a}^\varepsilon,\overline{b}^\varepsilon)) \ (8)$$

4 Fuzzy Linear Programming Approach

J. Nematian et al. (2010) [11] have developed an algorithm for solving Fuzzy Random Resource-Constrained Project Scheduling problem on the basis of previous researches of Wang, Hapke and Slowinski, and Alvarez-Valdes and Tamarit. Their algorithm is based on linear programming approach, which was presented by Alvarez-Valdes and Tamarit (1993). But J. Nematian et al. proposed using of trapezoidal fuzzy random variables as expected values.

$$\text{Max } f_n$$
$$\text{subject to } x_{ij} = 1; x_{ji} = 0 \text{ for all } (i,j) \in A \ (9)$$
$$x_{ij} + x_{ji} \le 1 \text{ for } i,j = 1,\dots,n \text{ and } i \ne j \ (10)$$
$$x_{ij} + x_{jk} - x_{ik} \le 1 \text{ for } i,j,k = 1,\dots,n \text{ and } i \ne j,\ j \ne k,\ i \ne k \ (11)$$
$$\sum_{i,jS,\ i\ne j} x_{ij} \ge 1 \text{ for all } S \in IS \ (12)$$
$$f_{i2} \le f_{j2} - x_{ij}\left(d_{j2}+M\right) + M \text{ for } i,j = 1,\dots,n \text{ and } i \ne j \ (13)$$
$$f_{i3} \le f_{j3} - x_{ij}\left(d_{j3}+M\right) + M \text{ for } i,j = 1,\dots,n \text{ and } i \ne j \ (14)$$
$$f_{i1} \le f_{j1} - x_{ij}\left(d_{j1}+M\right) + M \text{ for } i,j = 1,\dots,n \text{ and } i \ne j \ (15)$$
$$f_{i4} \le f_{j4} - x_{ij}\left(d_{j4}+M\right) + M \text{ for } i,j = 1,\dots,n \text{ and } i \ne j \ (16)$$
$$\left(f_{11}, f_{12}, f_{13}, f_{14}\right) = \left(b_1, b_2, b_3, b_4\right) + \left(d_{11}, d_{12}, d_{13}, d_{14}\right) \ (17)$$
$$x_{ij} \in \{0,1\} \text{ for } i,j = 1,\dots,n \text{ and } i \ne j \ (18)$$
$$f_{i1} \le f_{i2} \le f_{i3} \le f_{i4} \text{ for } i = 1,\dots,n \ (19)$$
$$f_{i1}, f_{i2}, f_{i3}, f_{i4} \ge 0 \text{ for } i = 1,\dots,n \ (20)$$
$$0 \le \eta \le 1$$

The objective function f_n minimises the completion time of the dummy end activity and thus the completion of the project [12]. Eq. (9) shows the precedence relations: the binary decision variable x_{ij} is set to 1 if a precedence relation exists between activities i and j and to 0 otherwise [12]. Eq. (10) specifies that no cycle be allowed in the network: at most one precedence relation can be introduced between any pair of activities. Transitivity of the precedence relations is guaranteed by Eq. (11): if activity i precedes activity j and if activity j precedes activity k, then activity i must precede activity k. The resource constraints are introduced in Eq. (12): at least one precedence relation should be specified between the activities of every minimal resource incompatible set. The completion times f_i are set in Eqs (13 - 17): the dummy start activity is started (and completed) at time 0 and for all the original or introduced precedence relation (x_{ij}=1) it is specified that the start time of the succeeding activity should at least equal the completion time of its predecessor (M denotes an arbitrary large number). If x_{ij}=0, no additional constraint is introduced because the completion time of the first activity will always be smaller than the sum of the start time of the second activity and an arbitrary large number [12]. Eq. (18) indicates that x_{ij} variables should be binary.

The above model is a mixed integer programming model and can be solved by one of the mixed integer programming solution approaches such as branch and bound [11]. Expected-optimal solution of generated model will be trapezoidal fuzzy number $f_i^* = (f_{i1}^*, f_{i2}^*, f_{i3}^*, f_{i4}^*)$ for $i = 1,\dots,n$.

In the given model, finish times of activities are assumed to be fuzzy random variables, which are more suitable to real world problems. However, it is not possible to determine the finish time of project theoretically in this case due to fuzzy randomness of activity duration [11]. In the above model used the concept of expected value of fuzzy random variables to overcome this problem. Therefore, it enabled to convert original complex model to a mixed integer programming model.

5 Intelligent add-in for Microsoft Project

For implementation of proposed idea we have developed an intelligent add-in for Microsoft Project. The system architecture contains following primary components: 1) Intelligent Component; 2) Input data; 3) Output data. Our system as input data takes expected durations of tasks. As output data the system gives the average value of the duration of project activities with precedence relations. Intelligent components responsible for generating the above described mixed-integer programming model and for solving generated MIP. The system architecture is shown in Fig. 2.

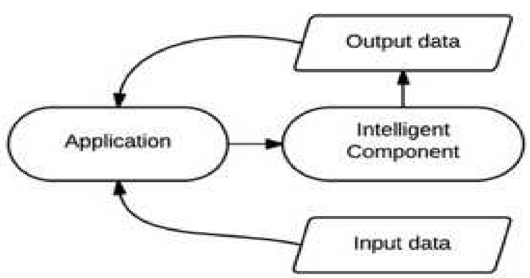

Fig. 2: System architecture

Example: Assume that a project consists of seven activities and as it is represented by the precedence graph shown in Fig. 3. The corresponding activity information is listed in Table 1 and supposed that the resource usage of each activity is 1. The fuzzy project ready-time and deadline are set to (0, 1, 1, 1) and (59, 59, 59, 65) respectively. Only one type of resource is required for the project and its resource availability is 2. Each activity resource usage is 1.

Table 1: Activity information in Fig. 3

Activity	Expected of Duration
a_1	(6, 8, 11, 13)
a_2	(7, 9, 12, 15)
a_3	(15, 18, 23, 27)
a_4	(5, 7, 9, 10)
a_5	(8, 10, 13, 16)
a_6	(17, 19, 25, 29)
a_7	(13, 15, 17, 20)

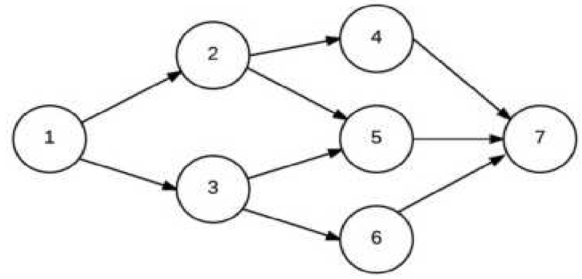

Fig. 3: Precedence graph with 7 activities

Apply the proposed algorithm and solve the obtained MIP problem by one of the MIP solver. The following MIP model is generated

$$
\begin{gathered}
\text{Max } [f_{72}, f_{73}, f_{74}-f_{73}, f_{71}-f_{72}] \\
\text{s.t. } x_{ij}=1; x_{ji}=0 \text{ for all } (i,j) \in A \\
x_{ij}+x_{ji} \le 1 \text{ for } i, j = 1, \dots, 7 \text{ and } i \ne j \\
x_{ij}+x_{jk}-x_{ik} \le 1 \text{ for } i, j, k = 1, \dots, 7 \text{ and } i \ne j, j \ne k, i \ne k \\
x_{45}+x_{54}+x_{46}+x_{64}+x_{56}+x_{65} \ge 1 \\
f_{ik} \le f_{jk} - x_{ij}\left(d_{jk}+M\right)+M \text{ for } i, j = 1, \dots, 7 \text{ and } i \ne j, k = 1, \dots, 4 \\
(f_{11}, f_{12}, f_{13}, f_{14}) = (0,\ 1,\ 1,\ 1)+(6,\ 7,\ 10,\ 13) = (6,\ 8,\ 11,\ 14) \\
x_{ij} \in \{0,1\} \text{ for } i, j = 1, \dots, 7 \text{ and } i \ne j \\
f_{i1} \le f_{i2} \le f_{i3} \le f_{i4} \text{ for } i = 1, \dots, 7 \\
f_{i1}, f_{i2}, f_{i3}, f_{i4} \ge 0 \text{ for } i = 1, \dots, 7
\end{gathered}
$$

The optimal solution of the model is summarized in the second column of Table 2.

Table 2: Schedule generated for the problem

	FR-RCPS	RCPSP	
Activity	$\underline{f}_i^*$	f_i^*	Precedence relations
1	(6, 8, 11, 14)	9.5	
2	(13, 17, 23, 29)	20	1→3→6→7
3	(21, 26, 34, 41)	30	
4	(34, 43, 56, 67)	49.5	2→5→4
5	(29, 36, 47, 57)	41.5	
6	(37, 45, 59, 60)	52	3→5
7	$\underline{f}_i^*$=(50, 60, 76, 80)	f_7^*=68	

Fig. 4 indicates the optimal precedence relations of activities according to resource constraints. As this result shows, precedence relations have not been changed but finish time of activity of our model are more confident. There is no difference between optimal precedence relations of two models because we have used exact linear programming in both fuzzy and real models. In the fuzzy random model the finish time of activates and the project makespan are trapezoidal fuzzy variables which are critical for a top manager of a project and he/she can apply them in order to schedule more confidently in compare with the real model which has real variables and has more risk for the manager.

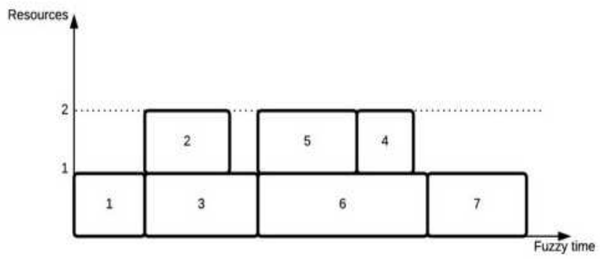

Fig. 4: Schedule with 7 activities

The developed system takes input data such as activity information, precedence relations from opened file in MS Project. It then generates a mixed integer programming model and solves it. The output parameters of the system are the optimal precedence relations of activities according to resource constraints, confident finish time of every activity and the whole project. So we expanded the functionality of MS Project with very convenient add-in that helps to introduce a project in the conditions of limited resources and inaccurate data. The current versions of Microsoft Project do not have this capability.

6 Conclusion

In this paper, we discussed the problems of the project management. We also made a brief review of previous researches, and finally have taken the approach proposed by J. Nematian, K. Eshghi, A. Eshragh-Jahromi. Selected method is based on linear programming formulation and uses the expected value of fuzzy random variables, fuzzy inequality approaches. In the fuzzy random model the finish time of activities is a trapezoidal fuzzy variable which is more flexible for a project manager. Finally, we have developed an intelligent add-in for Microsoft Project using the method described above.

As further research, neuro-fuzzy approach can be chosen to get the expected values of activity duration. Furthermore, the usage of resource availability can also be considered as fuzzy random numbers. Also resource availability can be taken into account.

Acknowledgement

This research paper is made possible through the help and advisory from members of Faculty of Information Technology. Also the authors express their gratitude to the researchers, the approaches which were taken as the basis for this paper.

References

[1] Xiaoqing (Frank) Liu, Gautam Kane, Monu Bambroo. An intelligent early warning system for software quality improvement and project management. Journal of Systems and Software. **79(11)**, 15521564 (2006).
[2] Arvind Sathi, Thomas E. Morton, and Steven F. Roth. Callisto: An Intelligent Project Management System. Journal AI Magazine. **7(5)**, 34-52 (1986).
[3] Wang, J. A fuzzy project scheduling approach to minimize schedule risk for product development. Fuzzy Sets and Systems, **127**, 99116 (2002).
[4] Herroelen W., and R. Leus. Project Scheduling under Uncertainty: Survey and Research Potentials. European Journal of Operational Research **165**, 289306 (2005).
[5] Hapke M., Jaskievicz A., Slowinski R. Fuzzy multimode resource-constrained project scheduling with multiple objectives. Weglarz, J. (Ed.), Project SchedulingRecent Models, Algorithms and Applications. Kluwer Academic Publishers, 355382 (1999).
[6] Rommelfanger, H. FULPAL: An Interactive Method for Solving (Multiobjective) Fuzzy Linear Programming Problems. In: Slowinski, R., Teghem, J. (Eds.), Stochastic Versus Fuzzy Approaches to Multiobjective Mathematical Programming Under Uncertainty. Kluwer Academic Publishers, Dordrecht. 279299 (1990).
[7] Dorn J., Kerr R., Thalhammer G. Reactive scheduling: improving robustness of schedules and restricting the effects of shop floor disturbances by fuzzy reasoning. International Journal of HumanComputer Studies 42, 687704 (1995).
[8] H. J. Zimmermann, Fuzzy set theory and its applications, Kluwer Academic (1991).
[9] Zadeh. Fuzzy sets as a basis for a theory of possibility. Fuzzy Sets and Systems, **1**, 328 (1978).
[10] Timothy J. Ross. Fuzzy Logic for Engineering Applications, McGraw-Hill Book Company. ISBN 0-07-053917-0 (1995).
[11] Javad Nematian, Kourosh Eshghi, Abdolhamid Eshragh-Jahromi. A Resource-Constrained Project Scheduling Problem with Fuzzy Random Duration. Journal of Uncertain Systems **Vol.4, No.2**, pp.123-132, 2010
[12] Erik Leuven Demeulemeester,Willy S. Herroelen. Project Scheduling: A Research Handbook, 2002
[13] Atymtayeva L., K. Kozhakhmet, G. Bortsova, A. Inoue. Expert System for Security Audit Using Fuzzy Logic. // Proceedings of The 23rd midwest artificial intelligence and cognitive science conference, MAICS, April 21-22, 2012, Cincinnati, USA, **Vol. 841** pp. 146-151
[14] Mirseidova Sh. , Atymtayeva L. Definition of Software Metrics for Software Project Development by Using Fuzzy Sets and Logic // Proceedings of the 6th International Conference on Soft Computing and Intelligent Systems and the 13th International Symposium on Advanced Intelligent Systems, 20-24 November 2012, Kobe, Japan, pp. 272-276

Daniyar Artykov is a second year master student at Kazakh-British Technical University, Kazakhstan. He is research interests focus on fuzzy sets and logic and applying it in project management. He likes to apply theoretical researches in practice, in particular in the development of intelligent applications and this paper is the first publication of him.

Lyazzat Atymtayeva received the PhD and Doctor of Science degree in Mechanics, Mathematics and Computer Science at al-Farabi Kazakh National University, Kazakhstan. Her research interests are in the areas of mechanics, applied mathematics and computer science including the numerical and rigorous mathematical methods and models for mechanical engineering and computer science, intelligent and expert systems in Information Security, Project Management and HCI. She has published research papers in reputed international journals of mathematical and computer sciences. She is reviewer and editor of international journals in mathematics and information sciences.