Journal of Analysis & Number Theory
An International Journal

http://dx.doi.org/10.18576/jant/030211

Common Fixed Point Theorem for Weak Compatible Mappings of type (A) in Complex Valued Metric Space

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Received: 7 Jan. 2015, Revised: 9 Jun. 2015, Accepted: 10 Jun. 2015
Published online: 1 Jul. 2015

Abstract: In this paper, we prove some coincidence and common fixed point theorems for weak compatible mappings of type (A) in complex valued metric spaces.

Keywords: complex valued metric space, compatible maps, weak compatible mappings of type (A), common fixed point.

1 Introduction

In 2011, Azam *et al.* [1] introduced the notion of complex valued metric space which is a generalization of the classical metric spaces. They established some fixed point results for a pair of mapping satisfying a rational inequality.

A complex number $z \in \mathbb{C}$ is an ordered pair of real numbers, whose first co-ordinate is called $\mathrm{Re}(z)$ and second coordinate is called $\mathrm{Im}(z)$. A complex-valued metric d is a function from $X \times X$ into $\mathbb{C}$, where X is a nonempty set and $\mathbb{C}$ is the set of complex numbers.

Let $\mathbb{C}$ be the set of complex numbers and $z_1, z_2 \in \mathbb{C}$. Define a partial order $\precsim$ on $\mathbb{C}$ as follows:

$z_1 \precsim z_2$ if and only if $\mathrm{Re}(z_1) = \mathrm{Re}(z_2)$ and $\mathrm{Im}(z_1) = \mathrm{Im}(z_2)$, that is, $z_1 \precsim z_2$ if one of the following holds:

(C1) $\mathrm{Re}(z_1) = \mathrm{Re}(z_2)$ and $\mathrm{Im}(z_1) = \mathrm{Im}(z_2)$;
(C2) $\mathrm{Re}(z_1) < \mathrm{Re}(z_2)$ and $\mathrm{Im}(z_1) = \mathrm{Im}(z_2)$;
(C3) $\mathrm{Re}(z_1) = \mathrm{Re}(z_2)$ and $\mathrm{Im}(z_1) < \mathrm{Im}(z_2)$;
(C4) $\mathrm{Re}(z_1) < \mathrm{Re}(z_2)$ and $\mathrm{Im}(z_1) < \mathrm{Im}(z_2)$.

In particular, we will write $z_1 \precnsim z_2$ if $z_1 \neq z_2$ and one of (C2), (C3), and (C4) is satisfied and we will write $z_1 \prec z_2$ if only (C4) is satisfied.

Remark 1.1. *We note that the following statements hold:*

(i) $a, b \in R$ *and* $a = b \Rightarrow az \precsim bz \ \forall\ z \in C$.
(ii) $0 \precsim z_1 \precnsim z_2 \Rightarrow |z_1| < |z_2|$,
(iii) $z_1 \precsim z_2$ *and* $z_2 \prec z_3 \Rightarrow z_1 \prec z_3$.

Definition 1.2. *Let* X *be a nonempty set. Suppose that the mapping* $d : X \times X \to C$ *satisfies the following conditions:*

(i) $0 \precsim d(x,y)$, *for all* $x, y \in X$ *and* $d(x,y) = 0$ *if and only if* $x = y$;
(ii) $d(x,y) = d(y,x)$ *for all* $x, y \in X$;
(iii) $d(x,y) \precsim d(x,z) + d(z,y)$, *for all* $x, y, z \in X$.

Then d *is called a complex valued metric on* X *and* (X,d) *is called a complex valued metric space.*

Example 1.3. *Let* $X = \mathbb{C}$. *Define the mapping* $d : X \times X \to \mathbb{C}$ *by*

$$d(z_1, z_2) = 2i|z_1 - z_2|, \quad \textit{for all } z_1, z_2 \in X.$$

Then (X,d) *is a complex valued metric space.*

Definition 1.4. *Let* (X,d) *be a complex valued metric space and* $\{x_n\}$ *be a sequence in* X *and* $x \in X$. *If for every* $c \in \mathbb{C}$, *with* $0 \prec c$ *there is* $k \in \mathbb{N}$ *such that for all* $n > k$,

(i) $d(x_n, x) \prec c$, *then* $\{x_n\}$ *is said to be convergent to* x.
(ii) $d(x_n, x_{n+m}) \prec c$, *where* $m \in \mathbb{N}$, *then* $\{x_n\}$ *is said to be Cauchy sequence.*
(iii) *If every Cauchy sequence in* X *is convergent, then* (X,d) *is said to be a complete complex valued metric space.*

Lemma 1.5. *Let* (X,d) *be a complex valued metric space and* $\{x_n\}$ *be a sequence in* X. *Then* $\{x_n\}$ *converges to* x *if and only if* $|d(x_n, x)| \to 0$ *as* $n \to \infty$.

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Lemma 1.6. *Let (X,d) be a complex valued metric space and let $\{x_n\}$ be a sequence in X. Then $\{x_n\}$ is a Cauchy sequence if and only if $|d(x_n,x_{n+m})| \to 0$ as $n \to \infty$, where $m \in \mathbb{N}$.*

Definition 1.7. *Let (X,d) be a metric space, f and g be self maps on X. A point x in X is called a coincidence point of f and g iff $fx = gx$. In this case, $w = fx = gx$ is called a point of coincidence of f and g.*

Jungck [2] introduced the notion of weakly compatible maps as follows:

Definition 1.8. *Two maps f and g are said to be weakly compatible if they commute at their coincidence points.*

2 Main Results

Jungck *et al.* [3] introduced the concept of compatible mappings of type (A) in metric spaces. Pathak *et al.* [6] gives the concept of weak compatible mappings of type (A). One can refer [5,6,7] for more details.

In the same manner, we introduce the concept of weak compatible mappings of type (A) in complex valued metric spaces as follows:

Definition 2.1. *A mapping T from a complex valued metric space (X,d) into itself is said to be continuous at x if for every sequence $\{x_n\}$ in X such that $\lim_{n\to\infty} |d(x_n,x)| = 0$, $\lim_{n\to\infty} |d(Tx_n,Tx)| = 0$.*

Definition 2.2. *Let S and T be mapping from a complex valued metric space (X,d) into itself. The mapping S and T are said to be compatible if $\lim_{n\to\infty} |d(STx_n,TSx_n)| = 0$, whenever $\{x_n\}$ is a sequence in X such that*

$$\lim_{n\to\infty} Sx_n = \lim_{n\to\infty} Tx_n = u \text{ for some } u \in X.$$

Definition 2.3. *Let S and T be mappings from a complex valued metric space (X,d) into itself. The mappings S and T are said to be compatible of type (A) if $\lim_{n\to\infty} |d(TSx_n,SSx_n)| = 0$ and $\lim_{n\to\infty} |d(STx_n,TTx_n)| = 0$, whenever $\{x_n\}$ is a sequence in X such that*

$$\lim_{n\to\infty} Sx_n = \lim_{n\to\infty} Tx_n = u$$

for some $u \in X$.

Definition 2.4. *Let S and T be mappings from a complex valued metric space (X,d) into itself. The mappings S and T are said to be weak compatible of type (A), if*

$$\lim_{n\to\infty} |d(TSx_n,\ SSx_n)| \le \lim_{n\to\infty} |d(STx_n,SSx_n)|$$

and

$$\lim_{n\to\infty} |d(STx_n,TTx_n)| \le \lim_{n\to\infty} |d(TSx_n,TTx_n)|,$$

whenever $\{x_n\}$ is a sequence in X such that $\lim_{n\to\infty} Sx_n = \lim_{n\to\infty} Tx_n = u$ for some $u \in X$.

Proposition 2.5. *Let S and T be continuous self mapping of a complex valued metric space (X,d). If S and T are compatible, then they are compatible of type (A).*

Proof. Suppose S and T are compatible . Let $\{x_n\}$ be a sequence in X such that $\lim_{n\to\infty} x_n = \lim_{n\to\infty} Tx_n = u$ for some $u \in X$. Now,

$$d(SSx_n,TSx_n) \precsim d(SSx_n,STx_n) + d(STx_n,TSx_n).$$

Since S and T are compatible and S is continuous, we have $\lim_{n\to\infty} d(SSx_n,TSx_n)| = 0$.

Similarly , we have $\lim_{n\to\infty} d(TTx_n,STx_n)| = 0$.

Therefore, S and T are compatible of type (A).

Proposition 2.6. *Let (X,d) be complex valued metric space and $S,T : X \to X$ be compatible mapping of type (A). If one of S and T is continuous, then S and T are compatible .*

Proof. Without loss of generality, assume that T is continuous. To show that S and T are compatible, suppose that $\{x_n\}$ is a sequence in X such that $Sx_n, Tx_n \to u$ for some $u \in X$. Then $TSx_n \to Tu$ as $n \to \infty$.

Since T is continuous,

$$\lim_{n\to\infty} d(TSx_n,TTx_n)| = 0 \tag{2.1}$$

Now, we have

$$d(STx_n,TSx_n) \precsim d(STx_n,TTx_n) + d(TTx_n,TSx_n). \tag{2.2}$$

For all $n \in N$, since S and T are compatible mapping of type (A),

$$\lim_{n\to\infty} d(STx_n,TTx_n) = 0. \tag{2.3}$$

Using (2.1) and (2.3) in (2.2), we get $\lim_{n\to\infty} d(STx_n,TSx_n) \precsim 0$, that is, $\lim_{n\to\infty} |d(STx_n,TSx_n)| = 0$.

Therefore, we have

$$\lim_{n\to\infty} d(STx_n,TSx_n) = 0.$$

Hence S and T are compatible.

From above two prepositions, we have the following result:

Proposition 2.7. *Let S and T be continuous mappings from a complex valued metric space (X,d) into itself. Then S and T are compatible iff they are compatible of type (A).*

The following prepositions shows that Definitions 2.2, 2.3, 2.4 are equivalent under some conditions.

Proposition 2.8. *Every pair of compatible mappings of type (A) is weak compatible of type (A).*

*Proof.*Suppose that the pair (S,T) is compatible of type (A), so

$$\lim_{n\to\infty} d(STx_n, TTx_n) = 0$$

and

$$\lim_{n\to\infty} d(TSx_n, SSx_n) = 0,$$

Whenever $\{x_n\}$ is a sequence in X such that $\lim_{n\to\infty} Sx_n = \lim_{n\to\infty} Tx_n = u$ for some $u \in X$.

Now, we have

$$0 = \lim_{n\to\infty} d(TSx_n, SSx_n) \precsim \lim_{n\to\infty} d(STx_n, TTx_n)$$

and

$$0 = \lim_{n\to\infty} d(STx_n, TTx_n) \precsim \lim_{n\to\infty} d(TSx_n, SSx_n).$$

Hence the mappings S and T are weak compatible of type (A).

Proposition 2.9. *Let S and T be continuous mappings of a complex valued metric space (X,d) into itself. If S and T are weak compatible of type (A), then they are compatible of type (A).*

*Proof.*Suppose S and T are weak compatible of type (A).

Let $\{x_n\}$ be a sequence in X such that $\lim_{n\to\infty} Sx_n = \lim_{n\to\infty} Tx_n = u$ for some $u \in X$.

Since S and T are continuous mappings, we have

$$\begin{aligned}\lim_{n\to\infty} d(STx_n, TTx_n) &\precsim \lim_{n\to\infty} d(TSx_n, TTx_n)\\ &= d(Tu, Tu) = 0,\end{aligned}$$

that is,

$$|\lim_{n\to\infty} d(STx_n, TTx_n)| = 0,$$

implies that,

$$\lim_{n\to\infty} d(STx_n, TTx_n) = 0$$

and

$$\begin{aligned}\lim_{n\to\infty} d(TSx_n, SSx_n) &\precsim \lim_{n\to\infty} d(STx_n, SSx_n)\\ &= d(Su, Su) = 0,\end{aligned}$$

that is,

$$|\lim_{n\to\infty} d(TSx_n, SSx_n)| = 0,$$

implies that,

$$\lim_{n\to\infty} d(TSx_n, SSx_n) = 0.$$

Therefore S and T are compatible mappings of type (A).

Proposition 2.10. *Let S and T are compatible mappings of type (A) from a complex valued metric space (X,d) into itself. If one of S and T is continuous then S and T are compatible.*

*Proof.*Suppose that S is continuous. Let $\{x_n\}$ be a sequence in X such that $\lim_{n\to\infty} Sx_n = \lim_{n\to\infty} Tx_n = u$ for some $u \in X$. Since S is continuous, we have $\lim_{n\to\infty} STx_n = Su = \lim_{n\to\infty} SSx_n$.

Now, we have

$$d(STx_n, TSx_n) \precsim d(STx_n, SSx_n) + d(SSx_n, TSx_n).$$

Since S and T are weak compatible of type (A), we have

$$\lim_{n\to\infty} d(STx_n, TSx_n) \precsim 0,$$

that is,

$$|\lim_{n\to\infty} d(STx_n, TSx_n)| = 0,$$

implies that,

$$\lim_{n\to\infty} d(STx_n, TSx_n) = 0.$$

Therefore, S and T are compatible.

As a direct consequence of Preposition 2.9 and Preposition 2.10, we have the following:

Proposition 2.11. *Let S and T be continuous mappings from a complex valued metric space (X,d) into itself. Then*

(i) *S and T are compatible of type (A) iff they are weak compatible of type (A).*
(ii) *S and T are compatible iff they are weak compatible of type (A).*

Properties of weak compatible mappings of type (A) in complex valued metric spaces

Proposition 2.12. *Let S and T be compatible mappings of type (A) from a complex valued metric space (X,d) into itself. If $Su = Tu$ for some $u \in X$, then $STu = SSu = TTu = TSu$.*

*Proof.*Suppose that $\{x_n\}$ is a sequence in X defined by $x_n = u$, $n = 1,2,3,\ldots$ and $Su = Tu$.

Then we have $\lim_{n\to\infty} Sx_n = \lim_{n\to\infty} Tx_n = Su = Tu$.

Since S and T are compatible mappings of type (A), we have

$$\begin{aligned}d(STu, TTu) &= \lim_{n\to\infty} d(STx_n, TTx_n)\\ &\precsim \lim_{n\to\infty} d(TSx_n, TTx_n)\\ &= d(TSu, TTu) = 0,\end{aligned}$$

implies that,

$$|d(STu,TTu)| \leq 0, that is, d(STu,TTu) = 0.$$

Thus, we have

$$STu = SSu = TTu = TSu.$$

Proposition 2.13. *Let S and T be weakly compatible mappings of type (A) from a complex valued metric space (X,d) into itself.*

Suppose $\lim_{n\to\infty} Sx_n = \lim_{n\to\infty} Tx_n = u$ for some u in X.

Then we have the following:

(i) $\lim_{n\to\infty} TSx_n = Su$, *if S is continuous at u.*
(ii) $\lim_{n\to\infty} STx_n = Tu$, *if T is continuous at u.*
(iii) $STu = TSu$ *and* $Su = Tu$, *if S and T are continuous at u.*

Proof.(i) Suppose that $\lim_{n\to\infty} Sx_n = \lim_{n\to\infty} Tx_n = u$ for some $u \in X$. Since S is continuous, we have $\lim_{n\to\infty} STx_n = Su = \lim_{n\to\infty} SSx_n$.

Now, we have

$$d(TSx_n,Su) \precsim d(TSx_n,SSx_n) + d(SSx_n,Su).$$

Therefore, since S and T are weakly compatible of type (A),

$$\lim_{n\to\infty} d(TSx_n,Su) \precsim 0,$$

that is,

$$|\lim_{n\to\infty} d(TSx_n,Su)| = 0,$$

implies that,

$$\lim_{n\to\infty} d(TSx_n,Su) = 0.$$

Thus, we have $\lim_{n\to\infty} TSx_n = Su$.

(ii) Suppose that $\lim_{n\to\infty} Sx_n = \lim_{n\to\infty} Tx_n = u$ for some $u \in X$.

Since S is continuous, we have $\lim_{n\to\infty} TSx_n = Tu = \lim_{n\to\infty} TTx_n$.

Now, we have

$$d(STx_n,Tu) \precsim d(STx_n,TTx_n) + d(TTx_n,Tu).$$

Therefore, since S and T are weakly compatible of type (A),

$$\lim_{n\to\infty} d(STx_n,Tu) \precsim 0,$$

that is,

$$|\lim_{n\to\infty} d(STx_n,Tu)| = 0,$$

implies that,

$$\lim_{n\to\infty} d(STx_n,Tu) = 0$$

Thus, we get $\lim_{n\to\infty} STx_n = Tu$.

(iii) Since T is continuous at u we have $\lim_{n\to\infty} TSx_n = Tu$.

By (i), since S is continuous at u we have $\lim_{n\to\infty} TSx_n = Su$. Hence by the uniqueness of limit, we have $Su = Tu$ and so by Preposition 2.12, $STu = TSu$.

3 Coincidence point theorem

Let A, B, S and T be mappings from a complex valued metric space (X,d) into itself such that

$$AX \cup BX \subset SX \cap TX, \tag{3.1}$$

$$d(Ax,By) \precsim kd(Sx,Ty) \quad \text{for all } x,y \text{ in } X,\ 0 < k < 1. \tag{3.2}$$

The sequences $\{x_n\}$ and $\{y_n\}$ in X are such that $x_n \to x$, $y_n \to y$ implies that $d(x_n,y_n) \to d(x,y)$.

Then by (3.1), since $AX \subset TX$, for any arbitrary point $x_0 \in X$, there exists a point $x_1 \in X$ such that $Ax_0 = Tx_1$. Since $Bx \subset SX$, for this point x_1, we can choose a point $x_2 \in X$ such that $Bx_1 = Sx_2$ and so on. Inductively, we can define a sequence $\{y_n\}$ in X such that

$$y_{2n} = Tx_{2n+1} = Ax_{2n} \text{ and } y_{2n+1} = Sx_{2n+2} = Bx_{2n+1}, \tag{3.3}$$
$$\text{for every } n = 0,1,2,\ldots.$$

Lemma 3.1. *Let A, B, S and T be mappings from a complex valued metric space (X,d) into itself satisfying the conditions (3.1) and (3.2). Then the sequence $\{y_n\}$ defined by (3.3) is a Cauchy sequence in X.*

*Proof.*From (3.2) we have

$$\begin{aligned} d(y_{2n},y_{2n+1}) &= d(Ax_{2n},Bx_{2n+1}) \\ &\precsim kd(Sx_{2n},Tx_{2n+1}) \\ &= kd(y_{2n-1},y_{2n}). \end{aligned}$$

Consequently, it can be concluded that

$$\begin{aligned} d(y_n,y_{n+1}) &\precsim kd(y_{n-1},y_n) \\ &\precsim k^2 d(y_{n-2},y_{n-1}) \\ &\vdots \\ &\precsim k^n d(y_0,y_1) \end{aligned}$$

Now for all $m > n$

$$\begin{aligned} d(y_m,y_n) &\precsim d(y_n,y_{n+1}) + d(y_{n+1},y_{n+2}) + \cdots + d(y_m,y_{m-1}) \\ &\precsim k^n d(y_0,y_1) + k^{n+1} d(y_0,y_1) + \ldots + k^{m-1} d(y_0,y_1) \\ &\precsim \frac{k^n}{1-k} d(y_0,y_1). \end{aligned}$$

Therefore, we have

$$|d(y_m, y_n)| = \frac{k^n}{1-k}|d(y_0, y_1)|.$$

Hence, $\lim_{n\to\infty} |d(y_m, y_n)| = 0$.

Thus $\{y_n\}$ is a Cauchy sequence in X.

Theorem 3.2. *Let A, B, S and T be mappings from a complex valued metric space (X,d) into itself satisfying (3.1), (3.2) and the following:*

$$SX \cap TX \text{ is a complete subspace of } X. \tag{3.4}$$

Then the pairs (A,S) and (B,T) have a coincide point.

*Proof.*By Lemma 3.1, the sequence $\{y_n\}$ defined by (3.3) is a Cauchy sequence in $SX \cap TX$. Since $SX \cap TX$ is a complete subspace of X, $\{y_n\}$ converges to a point z in $SX \cap TX$. On the other hand, since the subsequences $\{y_{2n}\}$ and $\{y_{2n+1}\}$ of $\{y_n\}$ are also Cauchy sequences in $SX \cap TX$, they also converge to the same limit z. Hence, there exists two points u, v in X such that $Su = z$ and $Tv = z$ respectively.

From (3.2), we have

$$d(Au, Bx_{2n+1}) \precsim Kd(Su, Tx_{2n+1}).$$

Letting $n \to \infty$, we have

$$|d(Au, z)| = 0,$$

implies that,

$$d(Au, z) = 0.$$

Thus, $Au = z = Su$.

Hence u is a coincidence point of A and S.

Similarly, one can show that v is a coincidence point of B and T.

4 Common point fixed theorem

Theorem 4.1. *Let A, B, S and T be mappings from a complex valued metric space (X,d) into itself satisfying the conditions (3.1), (3.2), (3.4) and the following:*

$$\text{the pairs } (A,S) \text{ and } (B,T) \text{ areweakcompatibleoftype(A)}. \tag{4.1}$$

Then A, B, S and T have a unique common fixed point in X.

*Proof.*From Theorem 3.2, there exist two points u, v in X such that $Au = Su = z$ and $Bv = Tv = z$ respectively. Since A and S are weak compatible mapping of type (A), by Preposition 2.12, $ASu = SSu = SAu = AAu$, which implies that $Az = Sz$. Similarly, since B and T are weak compatible mappings of type A, we have $Bz = Tz$.

Now, we prove that $Az = z$.

If $Az \neq z$, then by (3.2), we have

$$d(Az, y_{2n+1}) = d(Az, Bx_{2n+1}) \precsim kd(Sz, Tx_{2n+1}).$$

Letting $n \to \infty$, we have

$$d(Az, z) \precsim kd(z, z) = 0,$$

that is,

$$|d(Az, z)| = 0,$$

implies that,

$$d(Az, z) = 0.$$

Thus, we get $Az = z$.

Hence, we have $Az = z = Sz$.

Similarly, we have $Bz = Tz = z$.

Hence z is a common fixed point of A, B, S and T. For the uniqueness, let y be another common fixed point of A, B, S and T such that $y \neq z$.

From (3.2), we have

$$d(y, z) = d(Ay, Bz) \precsim kd(Sy, Tz) = kd(y, z).$$

Thus, we have

$$|d(y, z)| = K|d(y, z)|,$$

a contradiction to $k < 1$.

Therefore, A, B, S and T have a unique common fixed point.

Acknowledgement

The authors are grateful to the anonymous referee for a careful checking of the details and for helpful comments that improved this paper.

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