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Realizing an N Quantum Controlled Phase Gate In a Cavity QED With Dipole-Dipole Interaction

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Abstract: We propose an effective method to realize a multiqubit controlled-phase gate with one qubit simultaneously controlling N target qubits based on the dipole-dipole (atom-atom) interaction (DDI) and the qubit-cavity interaction (ACI) in a cavity QED. In our scheme, the operation time of this phase gate is independent of the number N of qubits. On the other hand, the three operational steps are required to realize the N target-qubit controlled-phase gate. This gate can be realized in a time much shorter than radiative time and lifetime of the cavity photon. Also, we give a discussion on the total operation time of the proposed gate and on the role of DDI.

Keywords: Phase gate, dipole-dipole interaction, qubit, cavity QED, NTCP gate.
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1 Introduction

Over the past decade, various physical systems have been considered for building up quantum information processors [1,2]. The cavity QED with neutral atoms is a very promising approach for quantum information processing, because a cavity can manipulate as a quantum bus the coupled qubits efficiently, and information can be stored in certain atomic energy levels with long coherence time. So far, a large number of theoretical proposals for realizing two-qubit gates in many physical systems have been proposed. Moreover, two-qubit controlled phase gates have been experimentally demonstrated in, for example, cavity QED [3,4], ion traps [5,6], nuclear magnetic resonance NMR [7,8], quantum dots [9,10], and superconducting qubits [11,12]. Recently, Yang et al. proposed a scheme for implementing multiqubit tunable phase gate (NTCP gate) of one qubit simultaneously controlling n qubits selected from N qubits $(1 < n < N)$ in a cavity [13]. In ref. [14], the authors proposed a method for realizing a multiqubit gate, the procedure will become complicated as the number of qubits increases. Furthermore, it is signicant to realize multiqubit gates directly[15,16].

In this paper, we present and demonstrate a method for realizing a NTCP gate [13] by using dipole-dipole interaction (DDI) with one control qubit simultaneously controlling N target qubits in cavity QED by adding a strong resonant classical field. This gate can be realized in a time much shorter than radiative time and lifetime of the cavity photon. Thus, the operation time required for the gate implementation is independent of the number N of qubits, this type of controlled gate with N target qubits is useful in quantum information processing. For instance, it has applications in entanglement preparation [17,18], error correction [19], Grover search algorithm [20,21], quantum discrete Fourier transform [22,23], Deutsch-Jozsa algorithm [24,25], quantum dense coding [26,27], and quantum cloning [28,29].The unitary operator representing this type of multiqubit gate is given by: [30]

$$U_p = \prod_{j=2}^{N+1} (I_j - 2|-_1\rangle|-_j\rangle\langle-_1|\langle-_j|), \tag{1}$$

where the subscript 1 represents the control qubit 1, while j represents the target qubit j, and I_j is the identity operator for the qubit pair $(1, j)$, which is given by $I_j = \sum_{rs} |r_1 s_j\rangle\langle r_1 s_j|$, with $r, s \in \{+,-\}$. From the operator of

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the Eq. (1.1), it can be seen that the operator U_p induces a phase flip (from the $+$ sign to the $-$ sign) to the logical state $|-\rangle$ of each target qubit when the control qubit 1 is initially in the state $|-\rangle$, and nothing happens otherwise.

In the following, we will present a way for realizing NTCP gate by introducing a qubit-qubit interaction. We calculated the evolution operator a three-step from the Jaynes-Cummings model Hamiltonian, we used the overall evolution operator to obtain a NTCP gate. We also calculated the total operation time of this gate, which is smaller than the total operation time calculated in the ref [13].

2 Basic theory of the NTCP gate

We consider $(N+1)$ qubits each having two levels, whose states are designated by a ground state $|g_j\rangle$ and an excited state $|e_j\rangle$, interacting with a single mode cavity simultaneously, and driven by a conventional field. The N qubits are very close together, then the qubit-qubit interaction should be included in the cavity QED. The Hamiltonian of the whole system in the rotating wave approximation (assuming $\hbar = 1$) [31,32] is given by

$$H = \sum_{j=1}^{N+1}[\omega_0 S_{z,j} + \omega_a a^+ a + \Omega(S_j^+ e^{-i(\omega t+\varphi)} + S_j^- e^{i(\omega t+\varphi)}) + g(a^+ S_j^- + a S_j^+)] + \Gamma \sum_{\substack{i,j=1\\ i\neq j}}^{N+1} S_i^+ S_j^-, \quad (2)$$

$$= H_0 + H_1 + H_2 + H_3$$

with

$$H_0 = \sum_{j=1}^{N+1}\omega_0 S_{z,j} + \omega_a a^+ a, \quad (3)$$

$$H_1 = \Omega\sum_{j=1}^{N+1}(S_j^+ e^{-i(\omega t+\varphi)} + S_j^- e^{i(\omega t+\varphi)}), \quad (4)$$

$$H_2 = g\sum_{j=1}^{N+1}(a^+ S_j^- + a S_j^+), \quad (5)$$

$$H_3 = \Gamma\sum_{\substack{i,j=1\\ i\neq j}}^{N+1} S_i^+ S_j^-, \quad (6)$$

where φ the initial phase of the pulse, H_0 is the free Hamiltonian of the qubits and the cavity mode, H_1 is the interaction Hamiltonian between the qubits and the classical pulse, H_2 is the interaction Hamiltonian between the qubits and the cavity mode, and H_3 is the interaction Hamiltonian between qubits, $S_{z,j}$, S_j^-, and S_j^+ are the collective operators for the $(1,2,...,N+1)$ qubits, where $S_{z,j} = \frac{1}{2}(|e_j\rangle\langle e_j| - |g_j\rangle\langle g_j|), S_j^+ = |e_j\rangle\langle g_j|, S_j^- = |g_j\rangle\langle e_j|$ with $|e_j\rangle(|g_j\rangle)$ is the excited state (ground state) of the qubit, $\omega_0, \omega_a, \omega$,are the transition frequency between the two levels $|g\rangle$ and $|e\rangle$ of each qubit, the cavity mode, and the conventional field, respectively. a^+, a are the creation and annihilation to the cavity mode, g is intensity qubit-cavity coupling, Ω the Rabi frequency of the classical field, and Γ is the force dipole-dipole coupling. Assuming that $\omega_0 = \omega$ we have the following Hamiltonian in the interaction picture [26,33]

$$H_I = \sum_{j=1}^{N+1}[g(e^{-i\delta t} a^+ S_j^- + e^{i\delta t} a S_j^+) + \Omega(e^{i\varphi} S_j^- + e^{-i\varphi} S_j^+)] + \Gamma\sum_{\substack{i,j=1\\ i\neq j}}^{N+1} S_i^+ S_j^-, \quad (7)$$

$$= H_1 + H_{I1}, \quad (8)$$

with

$$H_1 = \sum_{j=1}^{N+1}\Omega(e^{i\varphi} S_j^- + e^{-i\varphi} S_j^+), \quad (9)$$

$$H_{I1} = \sum_{j=1}^{N+1}[g(e^{-i\delta t} a^+ S_j^- + e^{i\delta t} a S_j^+)] + \Gamma\sum_{\substack{i,j=1\\ j\neq j}}^{N+1} S_i^+ S_j^-, \quad (10)$$

where $\delta = \omega_0 - \omega_a$ is the detuning (for the states $|g\rangle$ and $|e\rangle$) between the atomic transition frequency ω_0 and the frequency of the cavity mode ω_a.

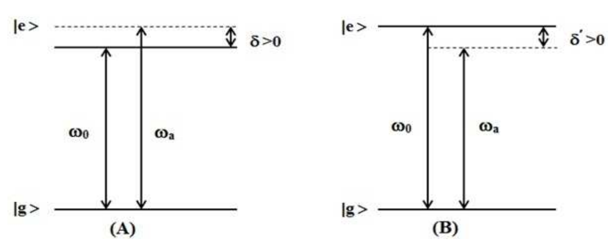

Fig. 1: Representation of different detunings $\delta = \omega_0 - \omega_a$. In (A), the detuning $\delta < 0$. In (B), the detuning $\delta' > 0$. We will use the first case (A) with $\varphi = \pi$ and the second case (B) with $\varphi = 0$ to obtain the NTCP gate. We use the same symbols ω, ω_0, and ω_a for the pulse frequency, the atom transition frequency, and the cavity mode frequency. The two horizontal solid lines represent the qubit energy levels for the states $|g>$ and $|e>$ [13].

We will work on two special cases: ($\varphi = \pi$ and $\delta < 0$) and ($\varphi = 0$ and $\delta > 0$). The operators of developments that will get these two particular cases will be used in Sec.3 for obtained NTCP gate in the case of the qubit-qubit interaction.

2.1 Evolution operator in the case of $\varphi = \pi$ and $\delta < 0$

Let us now consider the $(N+1)$ qubits placed in a single-mode cavity, where the first qubit as the controlling qubit and the other N qubits as the target qubits. In the case of pulse phase $\varphi = \pi$ and the negative detuning $\delta = (\omega_0 - \omega_a) < 0$, the Hamiltonian H_I becomes

$$\begin{aligned} H_I &= \sum_{j=1}^{N+1} [g(e^{-i\delta t} a^+ S_j^- + e^{i\delta t} a S_j^-) - \Omega(S_j^- + S_j^+)] \\ &\quad + \Gamma \sum_{\substack{i,j=1 \\ i\neq j}}^{N+1} S_i^+ S_j^- \qquad (11) \\ &= H_1 + H_{I1} \qquad (12) \end{aligned}$$

with

$$H_1 = -\sum_{j=1}^{N+1} \Omega(S_j^+ + S_j^-), \qquad (13)$$

$$H_{I1} = \sum_{j=1}^{N+1} [g(e^{-i\delta t} a^+ S_j^- + e^{i\delta t} a S_j^+)] + \Gamma \sum_{\substack{i,j=1 \\ j\neq j}}^{N+1} S_i^+ S_j^-, \qquad (14)$$

S_j^-, and S_j^+ are the collective operators for the $(1,2,...,N+1)$ qubits. Define the new basis [15,34,35] $|+_j\rangle = \frac{1}{\sqrt{2}}(|g_j\rangle + |e_j\rangle), |-_j\rangle = \frac{1}{\sqrt{2}}(|g_j\rangle - |e_j\rangle)$. Then, H_I becomes

$$\begin{aligned} H_I &= \sum_{j=1}^{N+1} g\left[e^{-i\delta t} a^+ (\sigma_{x,j} - \frac{1}{2}\sigma_j^+ + \frac{1}{2}\sigma_j^-) + e^{i\delta t} a(\sigma_{x,j} + \frac{1}{2}\sigma_j^+ - \frac{1}{2}\sigma_j^-)\right] \qquad (15) \\ &\quad + \Gamma \sum_{\substack{i,j=1 \\ i\neq j}}^{N+1} (\sigma_{x,i} + \frac{1}{2}\sigma_i^+ - \frac{1}{2}\sigma_i^-)(\sigma_{x,j} - \frac{1}{2}\sigma_j^+ + \frac{1}{2}\sigma_j^-) - 2\Omega \sum_{j=1}^{N+1} \sigma_{x,j}, \end{aligned}$$

with

$$\begin{aligned} H_{I1} &= \sum_{j=1}^{N+1} g[e^{-i\delta t} a^+ (\sigma_{x,j} - \frac{1}{2}\sigma_j^+ + \frac{1}{2}\sigma_j^-) + e^{i\delta t} a(\sigma_{x,j} + \frac{1}{2}\sigma_j^+ - \frac{1}{2}\sigma_j^-)] \\ &\quad + \Gamma \sum_{\substack{i,j=1 \\ i\neq j}}^{N+1} (\sigma_{x,i} + \frac{1}{2}\sigma_i^+ - \frac{1}{2}\sigma_i^-)(\sigma_{x,j} - \frac{1}{2}\sigma_j^+ + \frac{1}{2}\sigma_j^-), \qquad (16) \\ H_1 &= -2\Omega S_x, \qquad (17) \end{aligned}$$

where $\sigma_{x,j} = \frac{1}{2}(|+_j\rangle\langle +_j| - |-_j\rangle\langle -_j|)$, $\sigma_j^+ = |+_j\rangle\langle -_j|$, $\sigma_j^- = |-_j\rangle\langle +_j|$, and

$$S_x = \sum_{j=1}^{N+1} \sigma_{x,j}. \qquad (18)$$

By solving the Schrodinger equation

$$i\frac{d|\Psi(t)\rangle}{dt} = H_I|\Psi(t)\rangle, \qquad (19)$$

with

$$|\Psi(t)\rangle = e^{-iH_0 t}|\Psi'(t)\rangle, \qquad (20)$$

we obtain

$$i\frac{d|\Psi'(t)\rangle}{dt} = H_I^{'}|\Psi'(t)\rangle, \qquad (21)$$

with

$$H_I^{'} = e^{iH_1 t} H_{I1} e^{-iH_1 t}, \qquad (22)$$

where

$$\begin{aligned} H_I^{'} &= \sum_{j=1}^{N+1} g\left[a^+ e^{-i\delta t}(\sigma_{x,j} - \frac{1}{2}\sigma_j^+ e^{2i\Omega t} + \frac{1}{2}\sigma_j^- e^{-2i\Omega t}) + a e^{i\delta t}(\sigma_{x,j} + \frac{1}{2}\sigma_j^+ e^{2i\Omega t} - \frac{1}{2}\sigma_j^- e^{-2i\Omega t})\right] \\ &\quad + \Gamma \sum_{\substack{i,j=1 \\ i\neq j}}^{N+1} (\sigma_{x,i} + \frac{1}{2}\sigma_i^+ e^{2i\Omega t} - \frac{1}{2}\sigma_i^- e^{-2i\Omega t})(\sigma_{x,j} - \frac{1}{2}\sigma_j^+ e^{2i\Omega t} + \frac{1}{2}\sigma_j^- e^{-2i\Omega t}). \end{aligned}$$

In the strong driving region $2\Omega \gg \delta, g, \Gamma$,when the evolution time $t = \tau = \frac{\pi}{2\delta}$,we can eliminate the terms oscillating fast. Then the Hamiltonian $H_I^{'}$ reduces to [36, 37]

$$H_I^{'} = g(a^+ e^{i\delta t} + a e^{-i\delta t}) S_x + \Gamma \sum_{\substack{i,j=1 \\ i\neq j}}^{N+1} \sigma_{xi}\sigma_{xj}. \qquad (23)$$

The evolution operator for the Hamiltonian $H_I^{'}$ can be written as [38,39]

$$U^{'}(t) = e^{-iA(t)S_x^2} e^{-iB(t)aS_x} e^{-iB^*(t)a^+S_x} e^{-iC(t)X}, \ (X = \textstyle\sum_{\substack{i,j=1 \\ i\neq j}}^{N+1} \sigma_{xi}\sigma_{xj}). \qquad (24)$$

By solving the Schrodinger equation

$$i\frac{dU^{'}(t)}{dt} = H_I^{'} U^{'}(t), \qquad (25)$$

we obtain

$$\begin{aligned} C(t) &= \frac{1}{2}\int_0^t \Gamma dt^{'} = \Gamma t, \\ B(t) &= g\int e^{i\delta t^{'}} dt^{'} = \frac{g}{i\delta}(e^{i\delta t} - 1), \\ A(t) &= ig\int_0^t B(t^{'}) e^{-i\delta t^{'}} dt^{'} = \frac{g^2}{\delta}\left[t + \frac{1}{i\delta}(e^{-i\delta t} - 1)\right]. \qquad (26) \end{aligned}$$

Setting $t = \tau = -\frac{2\pi}{\delta}$, we have $B(t) = B^*(t) = 0$. Then, the evolution operator $U'(t)$ become

$$U^{'}(\tau_1) = e^{i\lambda S_x^2 \tau} \prod_{\substack{i,j=1 \\ i\neq j}}^{N+1} e^{-i\Gamma \sigma_{xi}\sigma_{xj}\tau}, \qquad (27)$$

where $\lambda = \frac{-g^2}{\delta} > 0$, Then, we obtain the evolution operator of the system as

$$\begin{aligned} U(\tau_1) &= e^{-iH_0\tau} U'(\tau_1) \\ &= e^{2i\Omega\tau S_x} e^{i\lambda\tau S_x^2} \prod_{\substack{i,j=1 \\ i\neq j}}^{N+1} e^{-i\Gamma\sigma_{z,i}\sigma_{z,j}\tau}, \qquad (28) \end{aligned}$$

The evolution operator $U(\tau)$ will be needed in the first step for realizing the NTCP gate.

2.2 Evolution operator in the case of $\varphi = 0$ and $\delta > 0$

We consider the positive detuning case $\delta > 0$ and $\varphi = 0$, where the atom-cavity coupling constant varies when the detuning δ changes. We suppose that the qubit 1 is decoupled the cavity and the pulse. In this case, we will adjusted the level spacing of qubit 1 [13].

Replace now the notation Ω, δ, g and Γ by $\Omega^{'}$, $\delta^{'}$, $g^{'}$ and $\Gamma^{'}$, respectively, for distinguish the $\varphi = 0$ and $\delta > 0$ from the case $\varphi = \pi$ and $\delta < 0$. In the following, and for simplicity of this work, we use the same symbols ω_0, and ω_a [see Fig.1]. Then, the Hamiltonian H_I can be written as

$$H_I^{'} = \sum_{\hat{j}=2}^{N+1}[g^{'}(e^{-i\delta^{'}t}a^{+}S_j^{-} + e^{i\delta^{'}t}aS_j^{+}) + \Omega^{'}(S_j^{-} + S_j^{+})] + \Gamma^{'}\sum_{\substack{i,j=2 \\ i\neq j}}^{N+1} S_i^{+}S_j^{-}, \tag{29}$$

S_j^{-} and S_j^{+} are the operators for the qubits $(2,...,N+1)$. In the case $2\Omega' \gg g^{'}, \delta^{'}$, when the evolution time $t = \tau^{'} = \frac{2\pi}{\delta^{'}}$, the Hamiltonian H_{I1}' is

$$H_{I1}' = g'(a^{+}e^{i\delta' t} + ae^{-i\delta' t})S_x' + \Gamma' \sum_{\substack{i,j=2 \\ i\neq j}}^{N+1} \sigma_{xi}\sigma_{xj}, \tag{30}$$

and

$$H_1' = 2\Omega' S_x', \tag{31}$$

then the evolution operator $U'(\tau')$ is

$$U'(\tau_1) = e^{-2i\Omega'\tau^{'}S_x'}e^{-i\lambda'\tau^{'}S_x'^2}\prod_{\substack{i,j=2 \\ i\neq j}}^{N+1} e^{i\Gamma'\tau^{'}\sigma_{z,i}\sigma_{z,j}}, \tag{32}$$

where $\lambda' = \frac{g^2}{\delta^{'}} > 0$, and $S_x' = \sum_{j=2}^{N+1}\sigma_{x,j}$, with $\sigma_{x,j} = \frac{1}{2}(S_j^{+} + S_j^{-})$.Then, the evolution operator $U^{'}(\tau_1)$ will be needed for the second step for realizing the NTCP gate.

For realizing the NTCP gate, we will have the qubits decoupled from the cavity, and applying a resonant pulse to each qubit. Therefore, we assumed that the Rabi frequency of the pulse applied to qubit 1 is Ω_1 and the Rabi frequency of the pulse applied to qubits $(2,...,N+1)$ is Ω_r [see Fig.2], where the initial phase for each pulse is $\varphi = 0$. so, in the interaction picture, we have the following interaction Hamiltonian for the qubit system and the pulses as

$$H_\Omega = 2\Omega_1\sigma_{z,1} + 2\Omega_r S_x'. \tag{33}$$

The evolution operator for the Hamiltonian H_Ω in the evolution time $\tau = -\frac{2\pi}{\delta}$ is

$$U_\Omega(\tau) = e^{-2i\Omega_1\tau\sigma_{z,1}}e^{-i2\Omega_r\tau S_x'}. \tag{34}$$

The evolution operator $U_\Omega(\tau)$ given here, will be needed in the next section for realizing the NTCP gate.

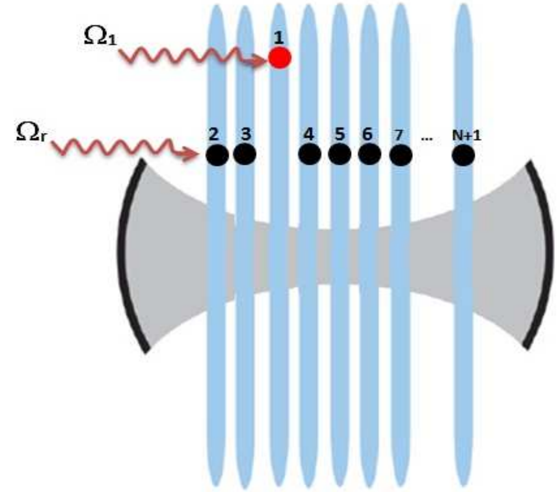

Fig. 2: Proposed the control qubit (the red dot), the N identical target qubits (the black dots), and a cavity, where the N qubits are very close together . The Rabi frequency for the pulse applied to qubit 1 is Ω_1, while the Rabi frequency for the pulses applied to qubits $(2,3,\ldots N+1)$ is Ω_r.

3 Preparation of the NTCP gate

In this section, We will demonstrate how the NTCP gate can be realized based on the evolution operators $U(\tau)$, $U^{'}(\tau^{'})$,and $U_\Omega(\tau)$.

We consider $N+1$ qubits moved to a cavity QED. The operations for the NTCP gate Realization and the evolutions operators after each step of operation are as follows:

First Step: With a detuning $\delta < 0$ [Fig.3(A_1) and (A_2)], we Apply a resonant pulse (with $\varphi = \pi$) to each qubit. The pulse Rabi frequency is Ω. Thus, $U(\tau)$ is the evolution operator for the $N+1$ qubits system, where the interaction time $\tau = -\frac{2\pi}{\delta}$.

Second Step: Apply a resonant pulse (with $\varphi = 0$) to each of the qubits $(2,3,...,N+1)$ with a detuning $\delta' > 0$. Adjust the qubit transition frequency for qubits $(2,3,...,N+1)$ [13], such that the cavity mode is coupled to qubits $(2,3,...,N+1)$ [see Fig.3 (B_2)]. The pulse Rabi frequency is Ω', and $U'(\tau')$ is the evolution operator for the qubit system. Thus, adjust the transition frequency of qubit 1 [13], such that qubit 1 is decoupled from the cavity mode and the pulses applied to qubits $(2,3,...,N+1)$ [see Fig.3 (B_1)]. In addition, the interaction time $\tau^{'} = \frac{2\pi}{\delta^{'}}$.

We choose $\delta = -4g$ ($\delta^{'} = 4g$), where the reason for us to set $\delta = -4g$ is to get a shorter operation time. We notice that δ satisfies the equation $16g^2/\delta^2 = (2k+1)$, where k is an integer. So, when $k = 0$, δ takes maximum $\delta = -4g$. In this case, the operation time will be the shortest, where, the equation $4g^2/\delta^2 = (2k+1)$ is the condition to implement the NTCP gate given below. So, the combined time after these two steps, is

$$\tau + \tau' = \frac{\pi}{g} \tag{35}$$

We suppose that $\Omega'\tau' = -\Omega\tau$, $\lambda'\tau' = \lambda\tau$, and $\Gamma'\tau' = \Gamma\tau$, that can be achieved by adjusting the δ and δ' (changing the ω_0 and ω_a), the Ω and Ω' (changing the intensity of the pulses), the Γ and Γ'. Then $U(\tau+\tau_1)$ becomes

$$\begin{aligned} U(\tau+\tau') &= e^{2i\Omega\tau(S_x-S'_x)} e^{i\lambda\tau(S_x^2-S'^2_x)} \prod_{j=2}^{N+1} e^{i\Gamma\tau\sigma_{z,1}\sigma_{z,j}} \\ &= e^{-2i\Omega\tau\sigma_{z,1}} e^{-2i\lambda\tau\sigma_{z,1}S'_x} \prod_{j=2}^{N+1} e^{i\Gamma\tau\sigma_{z,1}\sigma_{z,j}}, \end{aligned} \tag{36}$$

with $S_x - S'_x = \sigma_{z,1}$, $S_x^2 - S'^2_x = I + 2\sigma_{z,1}S'_x$ (I is the identity operator for qubit1), where $S'_x = \sum_{j=2}^{N+1}\sigma_{x,j}$, with $\sigma_{x,j} = \frac{1}{2}(S_j^+ + S_j^-)$.

The third step: In the case of $\varphi = 0$, we applied the Rabi frequency for the pulse Ω_1 to qubit 1 [see Fig.3(C_1)], and also, applied the Rabi frequency of the pulse Ω_r to qubits $(2,\ldots,N+1)$ [see Fig.3(C_2)], then, we will obtained the time evolution operator $U_\Omega(\tau)$ with τ is evolution time given above.

After this three step operation, the combined time evolution operator of the $N+1$ qubits system is

$$\begin{aligned} U(2\tau+\tau') &= U_\Omega(\tau)U(\tau+\tau') \\ &= e^{-2i\sigma_{z,1}\tau(\Omega+\Omega_1)} . e^{-2i\Omega_r S'_x\tau} \prod_{j=2}^{N+1} e^{-2i\sigma_{z,1}\sigma_{z,j}\tau(\lambda-\frac{\Gamma}{2})} \end{aligned} \tag{37}$$

With the conditions

$$\lambda = 4\lambda N + \frac{\Gamma}{2} = -4Ng^2/\delta + \Gamma/2,$$
$$\Omega_1 = 2\lambda - \Omega = -2g^2/\delta - \Omega, \tag{38}$$
$$\Omega_r = 2\lambda = -2g^2/\delta, \tag{39}$$

which can be achieved by adjusting the Rabi frequencies Ω, $\Omega_1$and Ω_r,so the time evolution operator $U(2\tau+\tau')$ becomes

$$U(2\tau+\tau') = \prod_{j=2}^{N+1} e^{-4i\lambda\tau(\sigma_{z,1}+\sigma_{z,2}+2\sigma_{z,1}\sigma_{z,2})} \tag{40}$$
$$= \prod_{j=2}^{N+1} U_P(1,j). \tag{41}$$

Where $U_p(1,j) = e^{-2i\lambda''\tau(\sigma_{z,1}+\sigma_{z,j}+2\sigma_{z,1}\sigma_{z,j})}$. According to the evolution operator $U(2\tau+\tau')$ above, on the basis $|+_1\rangle = \frac{1}{\sqrt{2}}(|g_1\rangle + |e_1\rangle)$ and $|-_1\rangle = \frac{1}{\sqrt{2}}(|g_1\rangle - |e_1\rangle)$ of the Pauli operator $\sigma_{x,1}$ for qubit 1,so the basis $|+_j\rangle = \frac{1}{\sqrt{2}}(|g_j\rangle + |e_j\rangle)$ and $|-_j\rangle = \frac{1}{\sqrt{2}}(|g_j\rangle - |e_j\rangle)$ of the Pauli operator $\sigma_{x,j}$ for qubits $(2,3,\ldots,N+1)$, we can obtain following evolutions

$$\begin{aligned} U_p|+_1\rangle|+_j\rangle &= e^{-8i\lambda\tau}|+_1\rangle|+_j\rangle, \\ U_p|+_1\rangle|-_j\rangle &= |+_1\rangle|-_j\rangle, \\ U_p|-_1\rangle|+_j\rangle &= |-_1\rangle|+_j\rangle, \\ U_p|-_1\rangle|-_j\rangle &= |-_1\rangle|-_j\rangle. \end{aligned} \tag{42}$$

Where the term $e^{i2\lambda\tau}$ is omitted. By selecting $8\lambda\tau = (2k+1)\pi$, i.e., $16g^2/\delta^2 = (2k+1)$ (with k being an integer) ,we have

$$\begin{aligned} U_p|+_1\rangle|+_j\rangle &= -|+_1\rangle|+_j\rangle, \\ U_p|+_1\rangle|-_j\rangle &= |+_1\rangle|-_j\rangle, \\ U_p|-_1\rangle|+_j\rangle &= |-_1\rangle|+_j\rangle, \\ U_p|-_1\rangle|-_j\rangle &= |-_1\rangle|-_j\rangle. \end{aligned} \tag{43}$$

Obviously, one can see that we obtain a NTCP gate. Hence, it is clear that the NTCP gate can be realised after the three-step process.

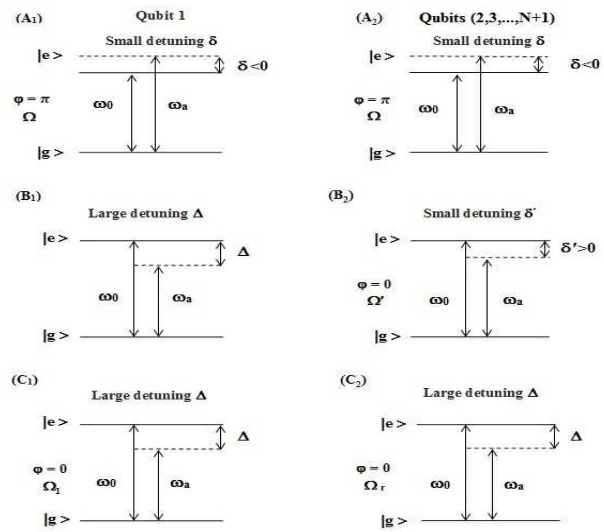

Fig. 3: Representation of the three steps: The first step (A_1 and A_2), the second step (B_1 and B_2), and the third step (C_1 and C_2), where the figures (A_1),(B_1) and (C_1) correspond to qubit 1, and the other figures (A_2),(B_2) and (C_2) correspond to qubit $(2,3,\ldots,N+1)$. in these figures, δ,and δ' are small detuning between the cavity mode frequency ω_a, and the qubit transition frequency ω_0, $\Delta = \omega_0 - \omega_a$ is large detuning of the cavity mode, φ is the initial stage of the pulse, and the Rabi frequencies of various Applied pulses are $\Omega, \Omega', \Omega_1$, and Ω_r [13].

4 Discussion

Now, we give a brief discussion about our proposal. Principal quantum numbers 50 and 51, the radiative time is $T_r = 3.0 \times 10^{-2}s$ and the coupling strength is

$g = 2\pi \times 50$ kHz [13,40], with $-\delta = \delta' = 2g$ and $\Gamma = (2k+1)/\delta$ $(k = 0, 1, ..., n)$ [31,41]. The total operation time $T_{op} = \tau + \tau' + \tau_a + 4\tau_m$ is independent of the number of target qubits N, where τ_a is the typical time required for adjusting the cavity mode frequency during first Step above, and τ_m is the typical time required for moving atoms into or out of the cavity. So, the direct calculation shows that the time required to implement the NTCP gate with atoms is $T_{op} = 15\mu s$ (with $\tau_a = \tau_m = 1\mu s$), which is much shorter than T_r. In recent experiments, the decay time of the cavity field (the lifetime of the cavity photon) was $T_c = 1.0 \times 10^{-3} s$ [31, 41], which is much longer than the total operation time T_{op}.

Furthermore, to implement the NTCP gate, we must precisely control dipole-dipole interaction (DDI) between the atoms. In recent years, the (DDI) has been studied as an emergence of collective scattering [42], entanglement of two artificial two-level atoms with degenerate two-photon transitions [43], nanoparticle ensembles [44]. The DDI depends upon three following factors: (i) the length of the cavity, (ii) the positions of the atoms and (iii) the polarization of dipoles. In order to achieve the tight localization required for DDI, the atoms must be trapped and cooled. Moreover, the dipolar interaction energy of the tightly trapped atoms can be much larger than the photon scattering rate and the atoms are cooled simultaneously by a one-dimensional cooling field [45, 46]. The DDI could be used for quantum computation by using an experimental apparatus that was realizable with presently available technology [45,46]. Based on the above technology, therefore, the dipole-dipole coupling strength between the atoms can be adjusted to satisfy the condition $\Gamma = (2k+1)/\delta$ $(k = 0, 1, ..., n)$.

5 Conclusion

In conclusion, we have proposed a simple scheme for realizing NTCP gate in a cavity QED with a single mode and a standing-wave pattern along the cavity axis by using atom-atom DDI and ACI. In our scheme, we have applied an inhomogeneous field to distinguish between the $N+1$ qubits. The scheme is insensitive to the thermal field. Furthermore, the gate operation time is independent of the number of qubits, and the qubit definitions are the same, which makes the work easier. In addition, the operation time can be controlled by adjusting the frequency between the $|g_j\rangle$ and $|e_j\rangle$. Thus, the gate operation is independent of the initial state of the cavity mode. However, we have presented an effective method for obtained a NTCP gate in a cavity QED, we have calculated an evolution operator with the three steps in the case of the DDI and ACI. Finally, we have applied the overall evolution operator to working basis of the qubit 1 and the qubits j $(j = 2, \ldots, N+1)$ for find the logical gate. this gate can be realized in a time much shorter than radiative time and lifetime of the cavity photon. The essential advantage of the scheme is that this gate can be realized in a time much shorter than radiative time and lifetime of the cavity photon. Therefore, the present scheme is simple and is feasible with cavity QED techniques.

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