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Bayesian Estimation for Extension of Exponential Distribution Under Progressive Type-II Censored Data Using Markov Chain Monte Carlo Method

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Abstract: In this paper, we consider the classical and Bayesian estimation of the parameters, reliability function and hazard function for a new extension of exponential distribution under progressive Type-II censored data using asymmetric loss functions. In most of the cases, it has been observed that the Classical and Bayes estimator of the parameters do not appear in explicit form. Therefore, Newton-Raphson (N-R) and Markov Chain Monte Carlo (MCMC) methods have been used to obtain the classical as well as Bayes estimates respectively. Further, we have also constructed the 95% asymptotic confidence interval based on maximum likelihood estimates (MLEs) and highest posterior density (HPD) credible intervals based on MCMC samples. A Monte Carlo simulation study is carried out to compare the performance of Bayes estimators with the corresponding classical estimators in terms of their simulated risk (average loss over whole sample space)

Keywords: Extension of exponential distribution, Maximum likelihood estimator, Bayes estimator, Reliability function, Asymmetric loss function, Gibbs sampling method (MCMC).

1 Introduction

In life time data analysis, the monotone hazard rate occurs most frequently in common practice. Such situations can be modelled by using the Weibull and gamma distributions. These distributions are generalized by using some modification and transformation in cumulative distribution function of the exponential distribution. Among these, Weibull distribution is more popular one. Murthy et al. [2] discussed about significance of Weibull distribution. Gupta and Kundu [9] introduced the exponentiated exponential distribution (EED) as an alternative to the gamma distribution. The EED distribution has many properties similar to the gamma distribution . Gupta and Kundu [10] provides a review and some developments on the exponentiated exponential distribution. In this paper, we consider a new extension of the exponential distribution introduced by Saralees Nadarajah and Firoozeh Haghighi in 2011 see, [1]. The considered model is also regarded as an alternative to the gamma, Weibull, exponentiated exponential and other exponentiated family of distributions because of its monotonic pattern of failure rate. Nadarajah et al. [1] have discussed the various properties related to this model. This distribution is a very new distribution in survival analysis having very small number of studies in both classical and Bayesian paradigm and no one has paid attention under progressive censoring. Therefore, here the authors are proposing a estimation procedure in both set up to estimate the unknown parameters, reliability function and hazard function of this distribution under progressive Type-II censored data. The probability density function, cumulative distribution function of the considered distribution are given as,

$$f(x|\alpha,\lambda) = \alpha\lambda(1+\lambda x)^{\alpha-1}e^{\{1-(1+\lambda x)^{\alpha}\}} \quad ;x \geq 0, \alpha, \lambda > 0 \tag{1}$$

$$F(x|\alpha,\lambda) = 1-e^{\{1-(1+\lambda x)^{\alpha}\}} \quad ;x \geq 0, \alpha, \lambda > 0 \tag{2}$$

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and the reliability function and hazard function of this model for specified value of t are given by following equations;

$$R(t|\alpha,\lambda) = e^{\{1-(1+\lambda t)^{\alpha}\}} \quad ;t \geq 0, \alpha, \lambda > 0 \tag{3}$$

$$H(t) = \frac{f(t)}{R(t)} = \alpha\lambda(1+\lambda t)^{\alpha-1} \quad ;t \geq 0, \alpha, \lambda > 0 \tag{4}$$

respectively.

In survival analysis, Type-I and Type-II censoring schemes are the two most popular censoring schemes which have been used in practice. These two censoring schemes are parallel to each other. Suppose, n identical items are put on test and exact time of failures are recorded. Usually, life tests are time consuming and costly. Therefore, at some predetermined fixed time T or after predetermined fixed number of failures r, the test may be terminated. In a censored case, where T is fixed and r is random, the censoring is said to be Type-I censoring scheme. On the other hand, when r is fixed and time of termination T is random, the censoring is said to be Type-II censoring scheme. For more details about the Type-I and Type-II censoring you may see Singh et al. [12],[22] and Kundu [21]. Unfortunately, none of these censoring schemes allows the removal of any experimental units during the experiment. Type-I and Type-II progressive censoring schemes allow the removal of experimental units during the experiment. Due to this flexibility, progressive censoring scheme has received considerable attention in the field of applied statistics. Progressive Type-II censoring scheme can be abbreviated as follows; Suppose n units are placed on experiment and m failures are going to be observed. When the first failure is observed, R_1 of the surviving units are randomly selected and removed. At the second observed failure, R_2 of the surviving units are randomly selected and removed. This experiment terminates at the time when the m-th failure is observed and the remaining $R_m = n - R_1 - R_2 - \cdots - m$ surviving units are all removed. Thus, progressive Type-II censoring scheme is the Generalization of the Type II censoring scheme. The statistical inference for life time distributions under progressive Type-II censoring scheme has been studied by several authors such as Cohen [6], Mann [7], Viveros and Balakrishnan [8], Balakrishnan and Aggarwala [5] and Krishna and Kumar [19], [20], Singh et al. [24], [25] and the references cited therein. Note that, in this scheme, $R_1, R_2, \cdots, R_m$ are all pre-fixed.

The aim of this paper is to consider the estimation of the unknown parameters, reliability and hazard functions of an extension of exponential distribution under progressive Type-II censoring scheme using asymmetric loss functions. In classical estimation we observed that the MLEs of the unknown parameters cannot be obtained in nice closed form, as expected, and It can be obtained by solving two non-linear equations simultaneously discussed in section 2. Further, Bayesian estimation under the assumption of independent gamma priors has been discussed in section 3 and 4 respectively. It is observed that the Bayes estimators are not in explicit form. Therefore, MCMC technique has been used to obtain the Bayes estimates based on posterior samples. In section 5, Monte Carlo simulations are conducted to compare the performances of the classical estimators with corresponding Bayes estimators obtained under both informative and non-informative prior set-up. Further, we have also constructed 95% asymptotic confidence intervals and highest posterior density (HPD) credible intervals for the parameters. A real data illustration and conclusions are presented in section 6 and 7 respectively.

2 Maximum Likelihood Estimation of the Parameters, Reliability and Hazard functions

In this section, we consider the classical estimation of the model parameters α and λ. It is well know that the theory of classical estimation concern with maximum likelihood principle. Let us suppose that n units are put on a test with corresponding life times being identically distributed with probability density function (1) and cumulative distribution function (2). If $x_1, x_2, x_3, ..., x_m$ are denoted as progressively Type-II ordered censored samples of size m observed from (1). Then, the likelihood function based on all m progressively Type-II censored sample is,

$$L(x|\alpha,\lambda) = C\alpha^m\lambda^m e^{\sum_{i=1}^{m}(1+R_i)\{1-(1+\lambda x_i)^{\alpha}\}} \prod_{i=1}^{m}(1+\lambda x_i)^{\alpha-1} \tag{5}$$

where, C is a constant and given as;

$$C = n(n-R_1-1), \cdots, (n-R_1-R_2-\cdots-R_{m-1}-m+1)$$

Therefore, the log likelihood $(LogL(x|\alpha,\lambda) = L_1)$ of this equation can be given by

$$L_1 = m\ln(\alpha) + m\ln(\lambda) + (\alpha-1)\sum_{i=1}^{m}\ln(1+\lambda x_i) + \sum_{i=1}^{m}(1+R_i)\{1-(1+\lambda x_i)^{\alpha}\} \tag{6}$$

Thus, the normal equations are

$$\frac{\partial L_1}{\partial \alpha} = \frac{m}{\alpha} + \sum_{i=1}^{m} \ln(1+\lambda x_i) - \sum_{i=1}^{m} (1+R_i)(1+\lambda x_i)^{\alpha} \ln(1+\lambda x_i) \tag{7}$$

$$\frac{\partial L_1}{\partial \lambda} = \frac{m}{\lambda} + (\alpha-1)\sum_{i=1}^{m} \frac{x_i}{1+\lambda x_i} - \alpha \sum_{i=1}^{m} x_i (1+R_i)(1+\lambda x_i)^{\alpha-1} \tag{8}$$

Maximum likelihood estimates can be obtained by solving above two equations but these equations can not be expressed in explicit form. Therefore, we use N-R method to obtain the MLEs. If $\hat{\alpha}$ and $\hat{\lambda}$ are the MLEs of the parameters then by using the invariance properties, the MLEs of reliability function and hazard function are given by;

$$\hat{R}_M = e^{\left\{1-(1+\hat{\lambda}t)^{\hat{\alpha}}\right\}} \tag{9}$$

and

$$\hat{H}(t) = \hat{\alpha}\hat{\lambda}(1+\hat{\lambda}t)^{\hat{\alpha}-1} \tag{10}$$

Now, we derive Fisher Information matrix to obtain asymptotic confidence intervals for the parameters. Thus, the Fisher information matrix can be obtained by using equation (4) as

$$I(\hat{\alpha},\hat{\lambda}) = \begin{pmatrix} -\frac{\partial^2 L_1}{\partial \alpha^2} & -\frac{\partial^2 L_1}{\partial \alpha \partial \lambda} \\ -\frac{\partial^2 L_1}{\partial \lambda \partial \alpha} & -\frac{\partial^2 L_1}{\partial \lambda^2} \end{pmatrix}_{(\hat{\alpha},\hat{\lambda})} \tag{11}$$

where,

$$\frac{\partial^2 L_1}{\partial \alpha^2} = -\frac{m}{\alpha^2} - \sum_{i=1}^{m} (1+R_i)(1+\lambda x)^{\alpha} \left\{\ln(1+\lambda x_i)\right\}^2$$

$$\frac{\partial^2 L_1}{\partial \lambda^2} = -\frac{m}{\lambda^2} - (\alpha-1)\sum_{i=1}^{m} \frac{x_i^2}{(1+\lambda x_i)^2} - \alpha(\alpha-1)\sum_{i=1}^{m} x_i^2 (1+R_i)(1+\lambda x_i)^{\alpha-2}$$

$$\frac{\partial^2 L_1}{\partial \alpha \partial \lambda} = \sum_{i=1}^{m} \frac{x_i}{1+\lambda x_i} - \sum_{i=1}^{m} x_i (1+R_i)(1+\lambda x_i)^{\alpha-1} \left\{\alpha \ln(1+\lambda x_i)+1\right\}$$

and

$$\frac{\partial^2 L_1}{\partial \lambda \partial \alpha} = \sum_{i=1}^{m} \frac{x_i}{1+\lambda x_i} - \sum_{i=1}^{m} x_i (1+R_i)(1+\lambda x_i)^{\alpha-1} \left\{\alpha \ln(1+\lambda x_i)+1\right\}$$

All the above derivatives are evaluated at $(\hat{\alpha},\hat{\lambda})$. The above matrix can be inverted to obtain the estimate of the asymptotic variance-covariance matrix of the MLEs and diagonal elements of $I^{-1}(\hat{\alpha},\hat{\lambda})$ provides the asymptotic variance of α and λ respectively. Then by using large sample theory a two sided $100(1-\beta)\%$ approximate confidence interval for α can be constructed as

$$\hat{\alpha} \mp Z_{1-\beta/2}\sqrt{var(\hat{\alpha})}$$

and similarly, for λ the two sided $100(1-\beta)\%$ approximate confidence interval can be obtained as

$$\hat{\lambda} \mp Z_{1-\beta/2}\sqrt{var(\hat{\lambda})}$$

3 Prior and Posterior distribution of the parameters

In this section, we provide prior and posterior distributions for the considered model. Let $\underline{x} = (x_1, x_2, x_3, \ldots, x_m)$ be a random sample of size m observed from (1) under progressive Type-II censoring, then the likelihood function is given as in (5). Here, we consider independent gamma prior for shape and scale parameter. This prior is very flexible in nature and accommodate different shapes. It is also converted as non-informative prior by setting the value of hyper-parameters are equal to zero. Therefore, the joint prior of (α,λ) is $\pi(\alpha,\lambda) \propto \pi_1(\alpha)\pi_2(\lambda)$, where

$$\pi_1(\alpha) \propto \alpha^{a-1}e^{-b\alpha} \quad ;a,b \geq 0 \tag{12}$$

$$\pi_2(\lambda) \propto \lambda^{c-1}e^{-d\lambda} \quad ;c,d \geq 0 \tag{13}$$

The joint posterior is written as,

$$p(\alpha,\lambda|\underline{x}) \propto C\alpha^{m+a-1}\lambda^{m+c-1}e^{\left\{\sum_{i=1}^{m}(1+R_i)[1-(1+\lambda x)^{\alpha}]-b\alpha-d\lambda\right\}}\prod_{i=1}^{m}(1+\lambda x_i)^{\alpha-1} \tag{14}$$

4 Bayes Estimators of the Parameters, Reliability and Hazard Functions Using General Entropy Loss Function:

In this part of paper, we have obtained the Bayes estimator under general entropy loss function (GELF). Calabria and Pulcini (1996) defined General Entropy loss function (GELF) as

$$L(\hat{\phi},\phi) = \delta\left\{\left(\frac{\hat{\phi}}{\phi}\right)^{\delta} - \delta\ln\left(\frac{\hat{\phi}}{\phi}\right) - 1\right\}$$

The constant δ involved in above equation is its shape parameter and it reflects the departure from symmetry. When $\delta > 0$, over estimation is considered to be more serious than under estimation of equal magnitude and vice versa. The above defined loss function is a generalization of the Entropy loss function. The Bayes estimator $\hat{\alpha}_{GE}$ of α for δ=1 under GELF is given by

$$\hat{\alpha}_{GE} = \left[E(\alpha^{-\delta}|\underline{x},\lambda)\right]^{(-1/\delta)} = \left\{K^{-1}\int_{\alpha=0}^{\infty}\int_{\lambda=0}^{\infty}\alpha^{m+a-\delta-1}\lambda^{m+c-1}E(\alpha,\lambda)P(\alpha,\lambda)d\alpha d\lambda\right\}^{-1/\delta} \tag{15}$$

and similarly the Bayes estimator $\hat{\lambda}_{GE}$ of λ is given by

$$\hat{\lambda}_{GE} = \left[E(\lambda^{-\delta}|\underline{x},\alpha)\right]^{(-1/\delta)} = \left\{K^{-1}\int_{\alpha=0}^{\infty}\int_{\lambda=0}^{\infty}\alpha^{m+a-1}\lambda^{m+c-\delta-1}E(\alpha,\lambda)P(\alpha,\lambda)d\alpha d\lambda\right\}^{-1/\delta} \tag{16}$$

and the form of reliability function and hazard function are given as,

$$\hat{R}_{GE} = \left\{K^{-1}\int_{\alpha=0}^{\infty}\int_{\lambda=0}^{\infty}\left\{e^{\{1-(1+\lambda t)^{\alpha}\}}\right\}^{-\delta}\alpha^{m+a-1}\lambda^{m+c-1}E(\alpha,\lambda)P(\alpha,\lambda)d\alpha d\lambda\right\}^{-1/\delta} \tag{17}$$

and

$$\hat{H}_{GE} = \left\{K^{-1}\int_{\alpha=0}^{\infty}\int_{\lambda=0}^{\infty}\left\{\alpha\lambda(1+\lambda t)^{\alpha-1}\right\}^{-\delta}\alpha^{m+a-1}\lambda^{m+c-1}E(\alpha,\lambda)P(\alpha,\lambda)d\alpha d\lambda\right\}^{-1/\delta} \tag{18}$$

Special Cases:

1. If $\delta = -1$. The above loss function is converted into squared error loss function and corresponding Bayes estimates are expressed as;

$$\hat{\alpha}_{BS} = E(\alpha|\underline{x},\lambda) = K^{-1}\int_{\alpha=0}^{\infty}\int_{\lambda=0}^{\infty}\alpha^{m+a}\lambda^{m+c-1}E(\alpha,\lambda)P(\alpha,\lambda)d\alpha d\lambda \tag{19}$$

$$\hat{\lambda}_{BS} = E(\lambda|\underline{x},\alpha) = K^{-1}\int_{\alpha=0}^{\infty}\int_{\lambda=0}^{\infty}\alpha^{m+a-1}\lambda^{m+c}E(\alpha,\lambda)P(\alpha,\lambda)d\alpha d\lambda \tag{20}$$

respectively. Now the reliability and hazard function under SELF are,

$$\hat{R}_{BS} = K^{-1}\int_{\alpha=0}^{\infty}\int_{\lambda=0}^{\infty}\left\{e^{\{1-(1+\lambda t)^{\alpha}\}}\right\}\alpha^{m+a-1}\lambda^{m+c-1}E(\alpha,\lambda)P(\alpha,\lambda)d\alpha d\lambda \tag{21}$$

and

$$\hat{H}_{BS} = K^{-1}\int_{\alpha=0}^{\infty}\int_{\lambda=0}^{\infty}(1+\lambda t)^{\alpha-1}\alpha^{m+a}\lambda^{m+c}E(\alpha,\lambda)P(\alpha,\lambda)d\alpha d\lambda \tag{22}$$

where, $E(\alpha,\lambda)$, $P(\alpha,\lambda)$ and K, are defined as

$$E(\alpha,\lambda) = e^{\left\{\sum_{i=1}^{m}(1+R_i)[1-(1+\lambda x_i)^{\alpha}]-b\alpha-d\lambda\right\}}$$

$$P(\alpha,\lambda) = \prod_{i=1}^{m}(1+\lambda x_i)^{\alpha-1}$$

$$K = \int_{\alpha=0}^{\infty}\int_{\lambda=0}^{\infty}\alpha^{m+a-1}\lambda^{m+c-1}E(\alpha,\lambda)P(\alpha,\lambda)d\alpha d\lambda$$

2.If $\delta = -2$. The above loss function is converted into precautionary loss function and corresponding Bayes estimates are given as;

$$\hat{\alpha}^2_{BP} = E(\alpha^2|\underline{x},\lambda) = K^{-1}\int_{\alpha=0}^{\infty}\int_{\lambda=0}^{\infty}\alpha^{m+a+1}\lambda^{m+c-1}E(\alpha,\lambda)P(\alpha,\lambda)d\alpha d\lambda \tag{23}$$

$$\hat{\lambda}^2_{BP} = E(\lambda^2|\underline{x},\alpha) = K^{-1}\int_{\alpha=0}^{\infty}\int_{\lambda=0}^{\infty}\alpha^{m+a-1}\lambda^{m+c+1}E(\alpha,\lambda)P(\alpha,\lambda)d\alpha d\lambda \tag{24}$$

respectively. Reliability and Hazard functions are expressed as,

$$\hat{R^2}_{BP} = K^{-1}\int_{\alpha=0}^{\infty}\int_{\lambda=0}^{\infty}\left\{e^{\{1-(1+\lambda t)^{\alpha}\}}\right\}^2\alpha^{m+a-1}\lambda^{m+c-1}E(\alpha,\lambda)P(\alpha,\lambda)d\alpha d\lambda \tag{25}$$

and

$$\hat{H^2}_{BP} = K^{-1}\int_{\alpha=0}^{\infty}\int_{\lambda=0}^{\infty}\left\{(1+\lambda t)^{\alpha-1}\right\}^2\alpha^{m+a+1}\lambda^{m+c+1}E(\alpha,\lambda)P(\alpha,\lambda)d\alpha d\lambda \tag{26}$$

3.If $\delta = 1$. The general entropy loss function is converted into entropy loss function and corresponding Bayes estimates are obtained as follows;

$$\hat{\alpha}_{BE} = \left[E\left(\frac{1}{\alpha}|\underline{x},\lambda\right)\right]^{-1} = \left\{K^{-1}\int_{\alpha=0}^{\infty}\int_{\lambda=0}^{\infty}\alpha^{m+a-2}\lambda^{m+c-1}E(\alpha,\lambda)P(\alpha,\lambda)d\alpha d\lambda\right\}^{-1} \tag{27}$$

$$\hat{\lambda}_{BE} = \left[E\left(\frac{1}{\lambda}|\underline{x},\alpha\right)\right]^{-1} = \left\{K^{-1}\int_{\alpha=0}^{\infty}\int_{\lambda=0}^{\infty}\alpha^{m+a-1}\lambda^{m+c-2}E(\alpha,\lambda)P(\alpha,\lambda)d\alpha d\lambda\right\}^{-1} \tag{28}$$

Reliability and hazard function can be written as;

$$\hat{R}_{BE}=\left\{K^{-1}\int\limits_{\alpha=0}^{\infty}\int\limits_{\lambda=0}^{\infty}\left\{e^{\{1-(1+\lambda t)^{\alpha}\}}\right\}^{-1}\alpha^{m+a-1}\lambda^{m+c-1}E(\alpha,\lambda)P(\alpha,\lambda)d\alpha d\lambda\right\}^{-1} \tag{29}$$

and

$$\hat{H}_{BE}=\left\{K^{-1}\int\limits_{\alpha=0}^{\infty}\int\limits_{\lambda=0}^{\infty}\left\{(1+\lambda t)^{\alpha-1}\right\}^{-1}\alpha^{m+a-2}\lambda^{m+c-2}E(\alpha,\lambda)P(\alpha,\lambda)d\alpha d\lambda\right\}^{-1} \tag{30}$$

4.1 Bayes Estimator Under Linex Loss Function (LLF)

The Linex loss function can be expressed as follows

$$L(\hat{\phi},\phi)=\delta_1\left\{e^{\delta_1(\hat{\phi}-\phi)}-\delta_1(\hat{\phi}-\phi)-1\right\}\quad;\delta_1\neq 0$$

where, δ_1 is the loss parameter. If $\delta_1>0$, then overestimation is more serious than under estimation and vice versa. Using this loss function the Bayes estimator $\hat{\alpha}_{BL}$ of α is written as

$$\hat{\alpha}_{BL}=-\frac{1}{\delta_1}\ln E\left[e^{(-\delta_1\alpha)}\right]=-\frac{1}{\delta_1}\ln\left\{K^{-1}\int\limits_{\alpha=0}^{\infty}\int\limits_{\lambda=0}^{\infty}e^{-\delta_1\alpha}\alpha^{m+a-1}\lambda^{m+c-1}E(\alpha,\lambda)P(\alpha,\lambda)d\alpha d\lambda\right\} \tag{31}$$

and similarly Bayes estimator $\hat{\lambda}_{BL}$ of λ is

$$\hat{\lambda}_{BL}=-\frac{1}{\delta_1}\ln E\left[e^{(-\delta_1\lambda)}\right]=-\frac{1}{\delta_1}\ln\left\{K^{-1}\int\limits_{\alpha=0}^{\infty}\int\limits_{\lambda=0}^{\infty}e^{-\delta_1\lambda}\alpha^{m+a-1}\lambda^{m+c-1}E(\alpha,\lambda)P(\alpha,\lambda)d\alpha d\lambda\right\} \tag{32}$$

Estimators of reliability and hazard function using this loss function are expressed as;

$$\hat{R}_{BL}=-\frac{1}{\delta_1}\ln\left\{K^{-1}\int\limits_{\alpha=0}^{\infty}\int\limits_{\lambda=0}^{\infty}e^{-\delta_1 e^{\{1-(1+\lambda t)^{\alpha}\}}}\alpha^{m+a-1}\lambda^{m+c-1}E(\alpha,\lambda)P(\alpha,\lambda)d\alpha d\lambda\right\} \tag{33}$$

and

$$\hat{H}_{BL}=-\frac{1}{\delta_1}\ln\left\{K^{-1}\int\limits_{\alpha=0}^{\infty}\int\limits_{\lambda=0}^{\infty}e^{-\delta_1\{\alpha\lambda(1+\lambda t)^{\alpha-1}\}}\alpha^{m+a-1}\lambda^{m+c-1}E(\alpha,\lambda)P(\alpha,\lambda)d\alpha d\lambda\right\} \tag{34}$$

4.2 Markov Chain Monte Carlo Method (MCMC)

From the above, we observed that the Bayes estimators obtained by using different loss functions can not be solved analytically. Therefore, in this subsection, we discuss about MCMC method to generate sample from posterior distribution. The MCMC method is one of the best and efficient method for exploring the posterior characteristics. For more detail about MCMC method; see Smith et al. [16], Hastings [17] and Singh et al. [22]. For this purpose, we consider Metropolis Hastings algorithm to extract the posterior samples from full conditional posterior densities. To implement this technique, the full conditional posterior densities of α and λ are given as;

$$p_1(\alpha|\lambda,\underline{x})\propto\alpha^{m+a-1}e^{\sum\limits_{i=1}^{m}(1+R_i)\{1-(1+\lambda x_i)^{\alpha}\}-b\alpha}\prod_{i=1}^{m}(1+\lambda x)^{\alpha-1} \tag{35}$$

$$p_2(\lambda|\alpha,\underline{x})\propto\lambda^{m+c-1}e^{\sum\limits_{i=1}^{m}(1+R_i)\{1-(1+\lambda x_i)^{\alpha}\}-b\lambda}\prod_{i=1}^{m}(1+\lambda x)^{\alpha-1} \tag{36}$$

Metropolis Algorithm: The algorithm consist the following steps.

1. Start with j=1 and initial values (α^0, λ^0)
2. Using M-H algorithm generate posterior sample for α and λ from (35) and (36) respectively, where asymptotic normal distribution of full conditional densities are considered as the proposal.
3. Repeat step 2, for all $j = 1, 2, 3, \cdots N$ and obtained $(\alpha_1, \lambda_1), (\alpha_2, \lambda_2), ...(\alpha_N, \lambda_N)$
4. After obtaining the posterior sample the Bayes estimate of the parameters, reliability function and hazard function under GELF and LLF are given as follows;
5. **Under General Entropy Loss Function:**

$$\hat{\alpha}_{BE} \approx \left\{ \frac{1}{N} \sum_{j=1}^{N} \alpha_j^{-\delta} \right\}^{-\frac{1}{\delta}}, \quad \hat{\lambda}_{BE} \approx \left\{ \frac{1}{N} \sum_{j=1}^{N} \lambda_j^{-\delta} \right\}^{-\frac{1}{\delta}} \text{ and}$$

$$\hat{R}_{BE} \approx \left\{ \frac{1}{N} \sum_{j=1}^{N} \left(e^{\left\{1-(1+\lambda_j t)^{\alpha_j}\right\}} \right)^{-\delta} \right\}^{-\frac{1}{\delta}}, \quad \hat{H}_{BE} \approx \left\{ \frac{1}{N} \sum_{j=1}^{N} \left[\alpha_j \lambda_j (1+\lambda_j t)^{\alpha_j - 1} \right]^{-\delta} \right\}^{-\frac{1}{\delta}}$$

Further, Bayes estimates of the parameters, reliability and hazard functions under special cases are obtained by putting respective values of δ.

6. **Under Linex Loss Function:**

$$\hat{\alpha}_{BL} \approx -\frac{1}{\delta_1} \ln \left\{ \frac{1}{N} \sum_{j=1}^{N} e^{-\delta_1 \alpha_j} \right\}, \quad \hat{\lambda}_{BL} \approx -\frac{1}{\delta_1} \ln \left\{ \frac{1}{N} \sum_{j=1}^{N} e^{-\delta_1 \lambda_j} \right\} \text{ and}$$

$$\hat{R}_{BL} \approx -\frac{1}{\delta_1} \ln \left\{ \frac{1}{N} \sum_{j=1}^{N} exp \left(-\delta_1 e^{\left\{1-(1+\lambda_j t)^{\alpha_j}\right\}} \right) \right\}, \quad \hat{H}_{BL} \approx -\frac{1}{\delta_1} \ln \left\{ \frac{1}{N} \sum_{j=1}^{N} exp \left(-\delta_1 e^{\alpha_j \lambda_j (1+\lambda_j t)^{\alpha_j - 1}} \right) \right\}$$

7. After extracting the posterior samples, we can easily construct the 95 % HPD credible intervals for α and λ. Therefore, for this purpose order $\alpha_1, \alpha_2, ..., \alpha_N$ as $\alpha_1 < \alpha_2 < ... < \alpha_N$ and $\lambda_1, \lambda_2, ..., \lambda_N$ as $\lambda_1 < \lambda_2 < ... < \lambda_N$. Then $100(1-\beta)\%$ credible intervals of α and λ are

$$(\alpha_1, \alpha_{[N(1-\beta)+1]}), ..., (\alpha_{[N\beta]}, \alpha_N)$$

and

$$(\lambda_1, \lambda_{[N(1-\beta)+1]}), ..., (\lambda_{[N\beta]}, \lambda_N)$$

Here, $[x]$ denotes the greatest integer less than or equal to x. Then the HPD credible interval is that interval which has the shortest length.

5 Comparison of Estimators

In this section, we compare the performances of the Bayes estimators with corresponding maximum likelihood estimators. From section 3, we observed that the obtained estimators under different loss functions are not in nice closed form. Therefore, MCMC technique has been used to compute them. In order to consider MCMC method for obtaining the Bayes estimates of the α, λ, $R(t)$ and $H(t)$, we generate 15000 deviates for the parameters α and λ by using the M-H algorithm discussed as in previous section. First thousand MCMC iterations (Burn-in period) have been discarded from the generated sequence and also checked the convergence of the chain. The estimators $\hat{\alpha}$, $\hat{\alpha}_{GE}$, $\hat{\alpha}_{BS}$, $\hat{\alpha}_{BP}$, $\hat{\alpha}_{BE}$ and $\hat{\alpha}_{BL}$ denote the MLE and Bayes estimators of α obtained under different loss functions, namely GELF, SELF, PLF, ELF, and LLF respectively and the similar notations have been used for scale parameter λ, reliability function $R(t)$ and hazard function $H(t)$. In order to consider GELF we have taken three choices of loss parameter δ say (δ=0.5, 1.0 and 2.0 when over estimation is more serious than under estimation and δ=-0.5, -1.0 and -2.0 when under estimation is more serious than over estimation) and for LLF we have chosen $\delta_1 = [0.5, -0.5]$. Both informative and non-informative prior information have been considered. Now in order to consider the informative prior role of the choice of hyper parameters has greater importance. Thus, in considered estimation procedure one choice has been taken in such a way that, prior mean (μ) is equal to the guess value and prior variance (ν) is very small say (a=0.5,b=1.0,c=8.0 and d=4.0). Further, the case of non-informative prior information has been obtained by setting the values of hyper-parameters are zero say, (a=b=c=d=0). The simulation study has been carried out for different variation of sample size (n), effective sample size (m) when random removals R_i are fixed with fixing values of α and λ say, 0.5 and 2.0 respectively. Three choices of sample size say (n=20(30)50) have been taken into account. In Tables 2-9, the estimated values and mean square error (MSE) of the corresponding estimators are tabulated for various choices of n and m. Further, we have also obtained Bayes estimators of the reliability and hazard function where the true values of $R(t)$ and $H(t)$ for specified time $t = 2.0$ are taken as $R(2.0) = 0.2905243$, $H(2.0) = 0.4472136$ respectively. The 95% asymptotic/HPD credible intervals for the parameters are also provided for the same. In all considered results Progressive Type-II censored samples from extension of exponential distribution are generated by using the algorithm introduced by Balakrishnan and Shandhu [3] and replicate the process 1000. Four schemes have taken to choose R_i, which are abbreviated in Table 1;

From this extensive study, we may conclude that the performances of Bayes estimators of the parameters, reliability

Table 1: This table represents the different scheme taken under study

n	m	Scheme	Removals (R_i)
20	12	I	(8,0,0,0,0,0,0,0,0,0,0,0)
		II	(0,0,0,0,0,0,0,0,0,0,0,8)
		III	(0,0,0,0,0,4,4,0,0,0,0,0)
		IV	(4,0,0,0,0,0,0,0,0,0,0,4)
30	18	I	(12,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0)
		II	(0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,12)
		III	(0,0,0,1,1,1,1,1,1,1,1,1,1,1,1,0,0,0)
		IV	(4,0,0,0,0,0,0,0,0,4,0,0,0,0,0,0,0,4)
50	30	I	(20,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0)
		II	(0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,20)
		III	(0,0,0,0,0,0,0,0,0,0,0,0,0,4,4,4,4,4,0,0,0,0,0,0,0,0,0,0,0,0)
		IV	(0,0,0,0,0,1,1,1,1,1,1,1,1,1,1,1,1,1,1,1,1,1,1,1,1,0,0,0,0,0)

and hazard functions are better than their respective maximum likelihood estimators in all considered loss functions when informative prior has been used see Tables (2-5). But in other hand the performances of the Bayes estimators as well as maximum likelihood estimators are quite similar in case of non-informative set-up see Tables (6-9). Furthermore, the width of the HPD intervals are much lesser than the length of asymptotic intervals see Tables (10,11) and finally, it is also noticed that mean square error (MSEs) of the estimators and average length of the classical as well as HPD credible intervals are decreases as increases percentage of n and m see, Tables (10,11). Also, the mean square error of the ML as well as Bayes estimators are least under the considered censoring scheme-I for each variation of n and m. Thus, scheme-I can be recommended for estimation procedure for the considered model under progressive Type-II censoring scheme.

6 Real Data Analysis

In this section, we have proposed a real data analysis to illustrate the proposed methodology in real life application. To check the validity of proposed model we provided empirical cumulative distribution function (ECDF) plot for considered real data set. The considered data set is taken form Linhart and Zucchini [23], which represents the failure times of the air conditioning system of an air plane: 23, 261, 87, 7, 120, 14, 62, 47, 225, 71, 246, 21, 42, 20, 5, 12, 120, 11, 3, 14, 71, 11, 14, 11, 16, 90, 1, 16, 52, 95. For the above complete data set progressive Type-II censored samples of different effective sample sizes with fixed removals schemes (see Table no. 12) are generated. In classical set-up the maximum likelihood estimates (MLEs) of α, λ, reliability function and hazard function $(R(t), H(t))$ are calculated see Table no.(12,13). The 95% asymptotic confidence intervals of α and λ based on fisher information matrix are also calculated see Table (14). To perform Bayesian analysis for this data set we have considered non-informative prior under general entropy loss function see, Table (12,13). In the schemes of removals $[a*k]$ represents that, the number a is repeated k times.

7 Conclusion

In this paper, we proposed the estimation of the unknown parameters as well as reliability and hazard functions of an extension of exponential distribution under progressive Type-II censored samples using different asymmetric loss functions. We observed that the exact mathematical expression for maximum likelihood estimators as well as Bayes estimators are not obtained. Therefore, N-R method and MCMC method are used to compute them respectively. The interval estimation of the parameters are also discussed. From this extensive study we observed that the Bayes estimates based on progressively Type-II censored data behave like the maximum likelihood estimates when non-informative prior is considered, but in case of informative prior the performances of Bayes estimates are much better than the maximum likelihood estimates.

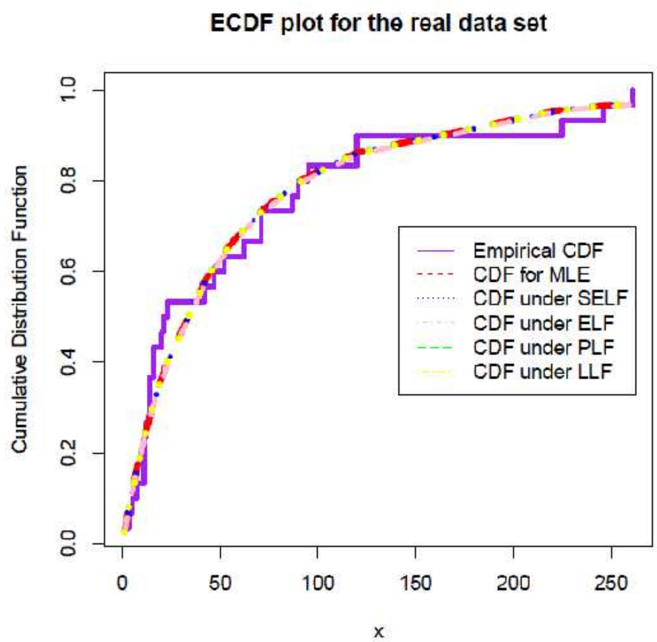

Fig. 1: ttttt

Table 2: Table represents the average estimates (AE) and corresponding mean square error (MSE) of the parameter α in case of informative prior

n	m	Scheme		$\hat{\alpha}$	GELF						Linex	
					$\delta=0.5$	$\delta=-0.5$	$\delta=1$	$\delta=-1$	$\delta=2$	$\delta=-2$	$\delta_1=0.5$	$\delta_1=-0.5$
					$\hat{\alpha}_{GE}$	$\hat{\alpha}_{GE}$	$\hat{\alpha}_{BE}$	$\hat{\alpha}_{BS}$	$\hat{\alpha}_{GE}$	$\hat{\alpha}_{BP}$	$\hat{\alpha}_{BL}$	$\hat{\alpha}_{BL}$
20	12	I	AE	0.6356	0.6216	0.6249	0.6200	0.6265	0.6167	0.6296	0.6251	0.6278
			MSE	0.0728	0.0613	0.0633	0.0602	0.0643	0.0582	0.0664	0.0632	0.0656
		II	AE	0.6861	0.6320	0.6443	0.6259	0.6504	0.6135	0.6626	0.6447	0.6565
			MSE	0.1464	0.0822	0.0921	0.0776	0.0973	0.0688	0.1084	0.0911	0.1045
		III	AE	0.6462	0.6293	0.6333	0.6273	0.6352	0.6233	0.6391	0.6336	0.6369
			MSE	0.0909	0.0663	0.0690	0.0649	0.0704	0.0622	0.0731	0.0688	0.0720
		IV	AE	0.6616	0.6277	0.6355	0.6238	0.6393	0.6159	0.6470	0.6359	0.6429
			MSE	0.1083	0.0733	0.0792	0.0705	0.0822	0.0650	0.0885	0.0787	0.0861
30	18	I	AE	0.6246	0.6161	0.6180	0.6151	0.6190	0.6131	0.6210	0.6183	0.6198
			MSE	0.0574	0.0514	0.0525	0.0509	0.0531	0.0498	0.0542	0.0525	0.0537
		II	AE	0.6528	0.6204	0.6279	0.6167	0.6317	0.6091	0.6392	0.6282	0.6353
			MSE	0.1155	0.0778	0.0839	0.0749	0.0870	0.0694	0.0936	0.0832	0.0913
		III	AE	0.6425	0.6302	0.6331	0.6287	0.6345	0.6258	0.6374	0.6333	0.6358
			MSE	0.0813	0.0706	0.0726	0.0696	0.0736	0.0676	0.0756	0.0724	0.0747
		IV	AE	0.6538	0.6352	0.6395	0.6330	0.6417	0.6286	0.6460	0.6399	0.6435
			MSE	0.0850	0.0689	0.0718	0.0674	0.0734	0.0645	0.0764	0.0717	0.0750
50	30	I	AE	0.5786	0.5750	0.5759	0.5745	0.5764	0.5736	0.5773	0.5761	0.5767
			MSE	0.0328	0.0310	0.0314	0.0309	0.0315	0.0305	0.0319	0.0314	0.0317
		II	AE	0.6264	0.6108	0.6146	0.6088	0.6166	0.6049	0.6204	0.6149	0.6182
			MSE	0.0994	0.0648	0.0675	0.0635	0.0688	0.0610	0.0716	0.0673	0.0704
		III	AE	0.6007	0.5960	0.5972	0.5954	0.5978	0.5943	0.5990	0.5974	0.5982
			MSE	0.0479	0.0449	0.0455	0.0446	0.0458	0.0440	0.0464	0.0455	0.0461
		IV	AE	0.6050	0.5995	0.6009	0.5988	0.6015	0.5974	0.6029	0.6010	0.6021
			MSE	0.0598	0.0463	0.0470	0.0459	0.0473	0.0452	0.0480	0.0470	0.0477

Table 3: Table represents the average estimates (AE) and corresponding mean square error (MSE) of the parameter λ in case of informative prior

n	m	Scheme		$\hat{\lambda}$	GELF						Linex	
					$\delta=0.5$	$\delta=-0.5$	$\delta=1$	$\delta=-1$	$\delta=2$	$\delta=-2$	$\delta_1=0.5$	$\delta_1=-0.5$
					$\hat{\lambda}_{GE}$	$\hat{\lambda}_{GE}$	$\hat{\lambda}_{BE}$	$\hat{\lambda}_{BS}$	$\hat{\lambda}_{GE}$	$\hat{\lambda}_{BP}$	$\hat{\lambda}_{BL}$	$\hat{\lambda}_{BL}$
20	12	I	AE	1.7207	1.6743	1.6874	1.6676	1.6939	1.6540	1.7066	1.6805	1.7073
			MSE	0.9130	0.8480	0.8301	0.8271	0.8314	0.8156	0.8096	0.7991	0.8245
		II	AE	1.7849	1.7190	1.7458	1.7053	1.7588	1.6772	1.7843	1.7333	1.7847
			MSE	0.9875	0.8075	0.8095	0.8067	0.8109	0.8060	0.8060	0.7937	0.8293
		III	AE	1.7555	1.7152	1.7272	1.7091	1.7331	1.6967	1.7448	1.7210	1.7453
			MSE	0.8828	0.7894	0.7909	0.7888	0.7917	0.7879	0.7879	0.7809	0.8032
		IV	AE	1.7525	1.6949	1.7148	1.6846	1.7246	1.6638	1.7438	1.7052	1.7443
			MSE	0.9645	0.8057	0.8054	0.8060	0.8053	0.8069	0.8069	0.7962	0.8150
30	18	I	AE	1.7134	1.6834	1.6919	1.6791	1.6961	1.6704	1.7044	1.6874	1.7048
			MSE	0.8864	0.8203	0.8217	0.8197	0.8225	0.8085	0.8085	0.7943	0.8110
		II	AE	1.8705	1.8160	1.8343	1.8067	1.8433	1.7878	1.8611	1.8249	1.8620
			MSE	0.9689	0.7718	0.7699	0.7735	0.7696	0.7786	0.7786	0.7516	0.7902
		III	AE	1.7297	1.7021	1.7108	1.6978	1.7150	1.6889	1.7235	1.7064	1.7236
			MSE	0.8374	0.7753	0.7757	0.7752	0.7759	0.7751	0.7751	0.7689	0.7832
		IV	AE	1.7256	1.6917	1.7035	1.6858	1.7093	1.6737	1.7208	1.6978	1.7209
			MSE	0.8954	0.8039	0.8038	0.8044	0.8042	0.8062	0.8062	0.7888	0.8012
50	30	I	AE	1.8486	1.8277	1.8329	1.8251	1.8355	1.8198	1.8407	1.8299	1.8412
			MSE	0.7927	0.7507	0.7527	0.7497	0.7537	0.7479	0.7479	0.7476	0.7599
		II	AE	1.8358	1.8060	1.8163	1.8008	1.8214	1.7903	1.8315	1.8111	1.8317
			MSE	0.8443	0.7649	0.7654	0.7648	0.7657	0.7647	0.7647	0.7501	0.7747
		III	AE	1.8181	1.8020	1.8065	1.7998	1.8087	1.7953	1.8132	1.8041	1.8134
			MSE	0.7164	0.6572	0.6580	0.6569	0.6584	0.6561	0.6561	0.6546	0.6623
		IV	AE	1.8103	1.7927	1.7978	1.7902	1.8003	1.7851	1.8052	1.7951	1.8055
			MSE	0.7487	0.6944	0.6953	0.6940	0.6957	0.6933	0.6933	0.6913	0.7003

Table 4: Table represents the average estimates (AE) and corresponding mean square error (MSE) of the Reliability function in the case of informative prior

n	m	Scheme		$\hat{R}_M$	GELF						Linex	
					$\delta=0.5$	$\delta=-0.5$	$\delta=1$	$\delta=-1$	$\delta=2$	$\delta=-2$	$\delta_1=0.5$	$\delta_1=-0.5$
					$\hat{R}_{GE}$	$\hat{R}_{GE}$	$\hat{R}_{BE}$	$\hat{R}_{BS}$	$\hat{R}_{GE}$	$\hat{R}_{BP}$	$\hat{R}_{BL}$	$\hat{R}_{BL}$
20	12	I	AE	0.2891	0.2906	0.2958	0.2879	0.2984	0.2824	0.3035	0.2977	0.2991
			MSE	0.0108	0.0108	0.0104	0.0110	0.0102	0.0116	0.0099	0.0102	0.0102
		II	AE	0.2695	0.2714	0.2846	0.2643	0.2908	0.2495	0.3027	0.2894	0.2923
			MSE	0.0132	0.0139	0.0120	0.0151	0.0113	0.0179	0.0103	0.0114	0.0113
		III	AE	0.2757	0.2770	0.2831	0.2739	0.2861	0.2675	0.2919	0.2854	0.2868
			MSE	0.0113	0.0115	0.0108	0.0118	0.0105	0.0127	0.0100	0.0105	0.0105
		IV	AE	0.2788	0.2807	0.2903	0.2756	0.2949	0.2650	0.3038	0.2938	0.2961
			MSE	0.0113	0.0116	0.0105	0.0123	0.0101	0.0139	0.0094	0.0101	0.0100
30	18	I	AE	0.2924	0.2934	0.2967	0.2917	0.2984	0.2882	0.3017	0.2979	0.2988
			MSE	0.0079	0.0079	0.0077	0.0080	0.0076	0.0082	0.0075	0.0076	0.0076
		II	AE	0.2721	0.2740	0.2823	0.2696	0.2863	0.2603	0.2941	0.2853	0.2873
			MSE	0.0099	0.0102	0.0092	0.0108	0.0088	0.0123	0.0082	0.0088	0.0088
		III	AE	0.2807	0.2817	0.2861	0.2795	0.2882	0.2750	0.2924	0.2876	0.2887
			MSE	0.0086	0.0076	0.0073	0.0078	0.0071	0.0083	0.0069	0.0071	0.0071
		IV	AE	0.2761	0.2773	0.2832	0.2743	0.2860	0.2680	0.2917	0.2853	0.2868
			MSE	0.0080	0.0081	0.0076	0.0085	0.0073	0.0093	0.0069	0.0073	0.0073
50	30	I	AE	0.2959	0.2968	0.2986	0.2959	0.2994	0.2941	0.3012	0.2992	0.2997
			MSE	0.0047	0.0046	0.0046	0.0046	0.0046	0.0047	0.0046	0.0046	0.0046
		II	AE	0.2799	0.2811	0.2857	0.2787	0.2880	0.2738	0.2924	0.2874	0.2885
			MSE	0.0065	0.0056	0.0052	0.0058	0.0051	0.0063	0.0048	0.0051	0.0051
		III	AE	0.2806	0.2813	0.2834	0.2802	0.2844	0.2781	0.2865	0.2841	0.2847
			MSE	0.0050	0.0050	0.0048	0.0051	0.0048	0.0052	0.0047	0.0048	0.0048
		IV	AE	0.2820	0.2827	0.2851	0.2816	0.2862	0.2792	0.2885	0.2859	0.2865
			MSE	0.0056	0.0046	0.0045	0.0047	0.0044	0.0048	0.0043	0.0044	0.0044

Table 5: Table represents the average estimates (AE) and corresponding mean square error (MSE) of the hazard function in the case of informative prior

					GELF						Linex	
n	m	Scheme		$\hat{H}_M$	$\delta = 0.5$	$\delta = -0.5$	$\delta = 1$	$\delta = -1$	$\delta = 2$	$\delta = -2$	$\delta_1 = 0.5$	$\delta_1 = -0.5$
					$\hat{H}_{GE}$	$\hat{H}_{GE}$	$\hat{H}_{BE}$	$\hat{H}_{BS}$	$\hat{H}_{GE}$	$\hat{H}_{BP}$	$\hat{H}_{BL}$	$\hat{H}_{BL}$
20	12	I	AE	0.5500	0.5264	0.5374	0.5209	0.5429	0.5100	0.5540	0.5372	0.5510
			MSE	0.1121	0.0830	0.0959	0.0771	0.1030	0.0663	0.1184	0.0879	0.1428
		II	AE	0.6518	0.5624	0.6016	0.5442	0.6234	0.5097	0.6740	0.5906	0.6235
			MSE	0.7064	0.1926	0.3477	0.1478	0.4930	0.0887	1.1527	0.1695	0.5178
		III	AE	0.5800	0.5510	0.5648	0.5442	0.5717	0.5306	0.5858	0.5643	0.5843
			MSE	0.1349	0.0979	0.1153	0.0900	0.1250	0.0757	0.1466	0.1049	0.2100
		IV	AE	0.5819	0.5343	0.5562	0.5235	0.5674	0.5021	0.5903	0.5562	0.5881
			MSE	0.1396	0.0833	0.1061	0.0735	0.1195	0.0569	0.1507	0.0975	0.2313
30	18	I	AE	0.5173	0.5047	0.5105	0.5018	0.5134	0.4959	0.5193	0.5114	0.5155
			MSE	0.0422	0.0367	0.0394	0.0354	0.0408	0.0329	0.0437	0.0392	0.0424
		II	AE	0.5855	0.5414	0.5611	0.5317	0.5713	0.5126	0.5921	0.5605	0.5648
			MSE	0.1649	0.0949	0.1197	0.0844	0.1346	0.0667	0.1713	0.1048	0.1121
		III	AE	0.5496	0.5313	0.5401	0.5269	0.5445	0.5182	0.5534	0.5406	0.5491
			MSE	0.0833	0.0679	0.0756	0.0642	0.0797	0.0574	0.0886	0.0725	0.0912
		IV	AE	0.5598	0.5347	0.5465	0.5288	0.5525	0.5171	0.5645	0.5478	0.5575
			MSE	0.0720	0.0556	0.0632	0.0521	0.0672	0.0455	0.0760	0.0626	0.0727
50	30	I	AE	0.4807	0.4747	0.4774	0.4734	0.4787	0.4707	0.4814	0.4780	0.4794
			MSE	0.0163	0.0152	0.0157	0.0149	0.0160	0.0144	0.0166	0.0158	0.0163
		II	AE	0.5326	0.5133	0.5223	0.5089	0.5268	0.5000	0.5359	0.5235	0.5303
			MSE	0.0642	0.0430	0.0478	0.0408	0.0503	0.0366	0.0557	0.0475	0.0534
		III	AE	0.5172	0.5095	0.5131	0.5077	0.5149	0.5040	0.5186	0.5137	0.5162
			MSE	0.0355	0.0323	0.0339	0.0315	0.0347	0.0300	0.0364	0.0339	0.0357
		IV	AE	0.5134	0.5048	0.5088	0.5029	0.5108	0.4989	0.5148	0.5095	0.5121
			MSE	0.0396	0.0265	0.0280	0.0258	0.0288	0.0244	0.0304	0.0281	0.0296

Table 6: Table represents the average estimates (AE) and corresponding mean square error (MSE) of the parameter α in the case of non-informative prior

					GELF						Linex	
n	m	Scheme		$\hat{\alpha}$	$\delta = 0.5$	$\delta = -0.5$	$\delta = 1$	$\delta = -1$	$\delta = 2$	$\delta = -2$	$\delta_1 = 0.5$	$\delta_1 = -0.5$
					$\hat{\alpha}_{GE}$	$\hat{\alpha}_{GE}$	$\hat{\alpha}_{BE}$	$\hat{\alpha}_{BS}$	$\hat{\alpha}_{GE}$	$\hat{\alpha}_{BP}$	$\hat{\alpha}_{BL}$	$\hat{\alpha}_{BL}$
20	12	I	AE	0.6356	0.6345	0.6380	0.6327	0.6397	0.6291	0.6432	0.6383	0.6413
			MSE	0.0728	0.0701	0.0727	0.0689	0.0740	0.0663	0.0765	0.0725	0.0755
		II	AE	0.6861	0.6638	0.6788	0.6562	0.6863	0.6411	0.7013	0.6777	0.6963
			MSE	0.1464	0.1296	0.1485	0.1209	0.1588	0.1048	0.1807	0.1429	0.1806
		III	AE	0.6462	0.6638	0.6681	0.6615	0.6703	0.6570	0.6746	0.6683	0.6724
			MSE	0.0909	0.0981	0.1020	0.0961	0.1039	0.0922	0.1079	0.1015	0.1065
		IV	AE	0.6616	0.6419	0.6501	0.6377	0.6541	0.6293	0.6621	0.6502	0.6583
			MSE	0.1083	0.0928	0.1005	0.0890	0.1045	0.0818	0.1127	0.0990	0.1108
30	18	I	AE	0.6246	0.6093	0.6112	0.6084	0.6121	0.6065	0.6140	0.6114	0.6129
			MSE	0.0574	0.0490	0.0501	0.0485	0.0506	0.0475	0.0517	0.0500	0.0512
		II	AE	0.6528	0.6525	0.6609	0.6483	0.6651	0.6399	0.6734	0.6610	0.6694
			MSE	0.1155	0.0991	0.1071	0.0952	0.1113	0.0877	0.1200	0.1057	0.1177
		III	AE	0.6425	0.6360	0.6389	0.6345	0.6404	0.6314	0.6434	0.6391	0.6418
			MSE	0.0813	0.0771	0.0797	0.0759	0.0810	0.0734	0.0836	0.0792	0.0830
		IV	AE	0.6538	0.6253	0.6296	0.6232	0.6317	0.6189	0.6358	0.6298	0.6335
			MSE	0.0850	0.0726	0.0758	0.0711	0.0774	0.0680	0.0806	0.0755	0.0794
50	30	I	AE	0.5786	0.5791	0.5800	0.5786	0.5805	0.5777	0.5814	0.5802	0.5808
			MSE	0.0328	0.0305	0.0308	0.0303	0.0310	0.0299	0.0313	0.0308	0.0311
		II	AE	0.6264	0.6356	0.6401	0.6333	0.6423	0.6287	0.6468	0.6402	0.6446
			MSE	0.0994	0.0889	0.0930	0.0869	0.0951	0.0830	0.0994	0.0924	0.0980
		III	AE	0.6007	0.5949	0.5961	0.5943	0.5967	0.5932	0.5978	0.5962	0.5971
			MSE	0.0479	0.0457	0.0463	0.0454	0.0466	0.0448	0.0472	0.0463	0.0469
		IV	AE	0.6050	0.6042	0.6056	0.6035	0.6063	0.6021	0.6077	0.6057	0.6068
			MSE	0.0598	0.0536	0.0544	0.0532	0.0548	0.0523	0.0556	0.0543	0.0552

Table 7: Table represents the average estimates (AE) and corresponding mean square error (MSE) of the parameter λ in the case of non-informative prior

n	m	Scheme		$\hat{\lambda}$	GELF						Linex	
					$\delta=0.5$	$\delta=-0.5$	$\delta=1$	$\delta=-1$	$\delta=2$	$\delta=-2$	$\delta_1=0.5$	$\delta_1=-0.5$
					$\hat{\lambda}_{GE}$	$\hat{\lambda}_{GE}$	$\hat{\lambda}_{BE}$	$\hat{\lambda}_{BS}$	$\hat{\lambda}_{GE}$	$\hat{\lambda}_{BP}$	$\hat{\lambda}_{BL}$	$\hat{\lambda}_{BL}$
20	12	I	AE	1.7207	1.6631	1.6771	1.6560	1.6840	1.6414	1.6975	1.6700	1.6981
			MSE	0.9130	0.9203	0.9217	0.9197	0.9227	0.9191	0.9191	0.9086	0.9377
		II	AE	1.7849	1.6941	1.7250	1.6782	1.7400	1.6455	1.7692	1.7114	1.7691
			MSE	0.9875	0.9403	0.9350	0.9439	0.9333	0.9530	0.9530	0.9092	0.9609
		III	AE	1.7555	1.6634	1.6755	1.6572	1.6815	1.6447	1.6933	1.6695	1.6935
			MSE	0.8828	0.9155	0.9158	0.9156	0.9160	0.9160	0.9160	0.9048	0.9279
		IV	AE	1.7525	1.6884	1.7107	1.6769	1.7215	1.6534	1.7427	1.7000	1.7433
			MSE	0.9645	0.9477	0.9476	0.9483	0.9480	0.9506	0.9506	0.9268	0.9712
30	18	I	AE	1.7134	1.6847	1.6933	1.6803	1.6976	1.6714	1.7061	1.6890	1.7063
			MSE	0.8864	0.7955	0.7960	0.7953	0.7963	0.7952	0.7952	0.7889	0.8039
		II	AE	1.8705	1.7090	1.7279	1.6994	1.7372	1.6797	1.7555	1.7195	1.7551
			MSE	0.9689	0.9274	0.9242	0.9294	0.9229	0.9341	0.9341	0.9082	0.9388
		III	AE	1.7297	1.7187	1.7277	1.7141	1.7322	1.7049	1.7410	1.7230	1.7414
			MSE	0.8374	0.8587	0.8597	0.8583	0.8603	0.8576	0.8576	0.8512	0.8697
		IV	AE	1.7256	1.7583	1.7711	1.7517	1.7775	1.7384	1.7900	1.7644	1.7906
			MSE	0.8954	0.8829	0.8845	0.8822	0.8855	0.8813	0.8813	0.8722	0.8995
50	30	I	AE	1.8486	1.8204	1.8257	1.8177	1.8284	1.8123	1.8337	1.8225	1.8342
			MSE	0.7927	0.7996	0.8017	0.7985	0.8028	0.7964	0.7964	0.7961	0.8097
		II	AE	1.8358	1.7881	1.7993	1.7825	1.8048	1.7711	1.8157	1.7939	1.8157
			MSE	0.8443	0.8499	0.8495	0.8502	0.8494	0.8510	0.8510	0.8398	0.8594
		III	AE	1.8181	1.8119	1.8165	1.8096	1.8188	1.8049	1.8233	1.8139	1.8236
			MSE	0.7164	0.7121	0.7132	0.7116	0.7137	0.7106	0.7106	0.7091	0.7184
		IV	AE	1.8103	1.8041	1.8092	1.8015	1.8118	1.7963	1.8169	1.8064	1.8172
			MSE	0.7487	0.7450	0.7460	0.7445	0.7466	0.7435	0.7435	0.7415	0.7519

Table 8: Table represents the average estimates (AE) and corresponding mean square error (MSE) of the reliability function in the case of non-informative prior

n	m	Scheme		$\hat{R}_M$	GELF						Linex	
					$\delta=0.5$	$\delta=-0.5$	$\delta=1$	$\delta=-1$	$\delta=2$	$\delta=-2$	$\delta_1=0.5$	$\delta_1=-0.5$
					$\hat{R}_{GE}$	$\hat{R}_{GE}$	$\hat{R}_{BE}$	$\hat{R}_{BS}$	$\hat{R}_{GE}$	$\hat{R}_{BP}$	$\hat{R}_{BL}$	$\hat{R}_{BL}$
20	12	I	AE	0.2891	0.2907	0.2963	0.2879	0.2990	0.2819	0.3043	0.2983	0.2997
			MSE	0.0108	0.0104	0.0100	0.0106	0.0098	0.0112	0.0095	0.0098	0.0098
		II	AE	0.2695	0.2744	0.2875	0.2674	0.2937	0.2527	0.3055	0.2922	0.2952
			MSE	0.0132	0.0141	0.0123	0.0152	0.0117	0.0178	0.0107	0.0117	0.0117
		III	AE	0.2757	0.2741	0.2805	0.2708	0.2836	0.2638	0.2896	0.2828	0.2843
			MSE	0.0113	0.0115	0.0107	0.0119	0.0104	0.0130	0.0098	0.0104	0.0104
		IV	AE	0.2788	0.2843	0.2939	0.2792	0.2985	0.2684	0.3074	0.2974	0.2997
			MSE	0.0113	0.0125	0.0114	0.0132	0.0110	0.0148	0.0104	0.0110	0.0110
30	18	I	AE	0.2924	0.2949	0.2982	0.2932	0.2999	0.2898	0.3031	0.2994	0.3003
			MSE	0.0079	0.0074	0.0073	0.0075	0.0072	0.0077	0.0071	0.0072	0.0072
		II	AE	0.2721	0.2741	0.2826	0.2696	0.2866	0.2600	0.2945	0.2856	0.2876
			MSE	0.0099	0.0100	0.0091	0.0106	0.0087	0.0122	0.0081	0.0087	0.0087
		III	AE	0.2807	0.2776	0.2836	0.2745	0.2864	0.2680	0.2921	0.2857	0.2872
			MSE	0.0086	0.0087	0.0082	0.0091	0.0079	0.0099	0.0075	0.0079	0.0079
		IV	AE	0.2761	0.2827	0.2884	0.2797	0.2912	0.2736	0.2966	0.2905	0.2919
			MSE	0.0080	0.0080	0.0075	0.0082	0.0073	0.0089	0.0070	0.0073	0.0073
50	30	I	AE	0.2959	0.2966	0.2984	0.2957	0.2993	0.2939	0.3011	0.2991	0.2996
			MSE	0.0047	0.0048	0.0048	0.0048	0.0048	0.0049	0.0048	0.0048	0.0048
		II	AE	0.2799	0.2741	0.2790	0.2716	0.2814	0.2664	0.2861	0.2808	0.2820
			MSE	0.0065	0.0068	0.0063	0.0071	0.0061	0.0077	0.0058	0.0061	0.0061
		III	AE	0.2806	0.2848	0.2869	0.2838	0.2879	0.2816	0.2899	0.2876	0.2882
			MSE	0.0050	0.0046	0.0045	0.0047	0.0045	0.0048	0.0044	0.0045	0.0045
		IV	AE	0.2820	0.2823	0.2846	0.2811	0.2858	0.2787	0.2881	0.2855	0.2861
			MSE	0.0056	0.0049	0.0047	0.0049	0.0047	0.0051	0.0045	0.0047	0.0047

Table 9: Table represents the average estimates (AE) and corresponding mean square error (MSE) of the hazard function in the case of non-informative prior

n	m	Scheme		$\hat{H}_M$	GELF						Linex	
					$\delta=0.5$	$\delta=-0.5$	$\delta=1$	$\delta=-1$	$\delta=2$	$\delta=-2$	$\delta_1=0.5$	$\delta_1=-0.5$
					$\hat{H}_{GE}$	$\hat{H}_{GE}$	$\hat{H}_{BE}$	$\hat{H}_{BS}$	$\hat{H}_{GE}$	$\hat{H}_{BP}$	$\hat{H}_{BL}$	$\hat{H}_{BL}$
20	12	I	AE	0.5500	0.5205	0.5310	0.5153	0.5363	0.5049	0.5470	0.5319	0.5411
			MSE	0.1121	0.0600	0.0675	0.0565	0.0715	0.0500	0.0802	0.0659	0.0788
		II	AE	0.6518	0.5751	0.6147	0.5563	0.6356	0.5198	0.6402	0.6041	0.6452
			MSE	0.7064	0.2005	0.3030	0.1627	0.3726	0.1059	0.3250	0.2083	0.3810
		III	AE	0.5800	0.5648	0.5789	0.5578	0.5861	0.5439	0.6006	0.5791	0.5941
			MSE	0.1349	0.0952	0.1093	0.0887	0.1170	0.0766	0.1335	0.1059	0.1317
		IV	AE	0.5819	0.5381	0.5604	0.5272	0.5717	0.5055	0.5951	0.5594	0.5585
			MSE	0.1396	0.1017	0.1304	0.0895	0.1473	0.0684	0.1881	0.1136	0.1701
30	18	I	AE	0.5173	0.5006	0.5063	0.4978	0.5092	0.4921	0.5150	0.5070	0.5115
			MSE	0.0422	0.0411	0.0445	0.0395	0.0462	0.0365	0.0500	0.0433	0.0502
		II	AE	0.5855	0.5501	0.5694	0.5405	0.5792	0.5217	0.5992	0.5695	0.5931
			MSE	0.1649	0.0902	0.1100	0.0815	0.1211	0.0660	0.1464	0.1025	0.1726
		III	AE	0.5496	0.5363	0.5453	0.5319	0.5498	0.5231	0.5589	0.5454	0.5559
			MSE	0.0833	0.0717	0.0813	0.0673	0.0865	0.0591	0.0979	0.0752	0.1164
		IV	AE	0.5598	0.5210	0.5322	0.5154	0.5378	0.5044	0.5492	0.5332	0.5430
			MSE	0.0720	0.0553	0.0631	0.0518	0.0673	0.0451	0.0763	0.0614	0.0760
50	30	I	AE	0.4807	0.4753	0.4780	0.4740	0.4794	0.4713	0.4821	0.4786	0.4801
			MSE	0.0163	0.0138	0.0143	0.0136	0.0146	0.0131	0.0152	0.0144	0.0148
		II	AE	0.5326	0.5370	0.5471	0.5320	0.5521	0.5220	0.5624	0.5480	0.5566
			MSE	0.0642	0.0616	0.0687	0.0583	0.0725	0.0520	0.0805	0.0677	0.0783
		III	AE	0.5172	0.5012	0.5046	0.4994	0.5064	0.4960	0.5099	0.5053	0.5075
			MSE	0.0355	0.0268	0.0281	0.0262	0.0288	0.0250	0.0301	0.0281	0.0294
		IV	AE	0.5134	0.5085	0.5125	0.5064	0.5146	0.5024	0.5187	0.5132	0.5160
			MSE	0.0396	0.0322	0.0339	0.0313	0.0349	0.0296	0.0368	0.0339	0.0359

Table 10: Table represents the Asymptotic confidence intervals and Highest posterior density (HPD) credible intervals of the parameter α and λ in the case of informative prior

n	m	Scheme	α_M^L	α_M^U	Length	α_B^L	α_B^U	length	λ_M^L	λ_M^L	length	λ_B^L	λ_M^U	length
20	12	I	0.0000	1.3575	1.3575	0.4085	0.7460	0.3375	0.0000	5.1916	5.1916	1.2973	2.0943	0.7970
		II	0.0000	2.6387	2.6387	0.4254	0.8909	0.4655	0.0000	7.1817	7.1817	1.1938	2.3355	1.1417
		III	0.0000	1.4557	1.4557	0.3746	0.7681	0.3934	0.0000	5.1104	5.1104	1.3467	2.1210	0.7743
		IV	0.0000	1.9847	1.9847	0.3595	0.8267	0.4672	0.0000	6.2270	6.2270	1.2359	2.2205	0.9846
30	18	I	0.0672	1.1819	1.1147	0.4627	0.7126	0.2499	0.0000	4.4885	4.4885	1.3739	2.0204	0.6465
		II	0.0000	2.0606	2.0606	0.4565	0.8163	0.3598	0.0000	6.3365	6.3365	1.3575	2.3340	0.9765
		III	0.0000	1.3435	1.3435	0.4523	0.7488	0.2965	0.0000	4.5477	4.5477	1.3898	2.0423	0.6525
		IV	0.0000	1.5866	1.5866	0.4504	0.7830	0.3326	0.0000	5.0761	5.0761	1.3311	2.0915	0.7604
50	30	I	0.2201	0.9370	0.7169	0.4515	0.6384	0.1869	0.0000	4.0290	4.0290	1.5698	2.1018	0.5320
		II	0.0000	1.5509	1.5509	0.4901	0.7480	0.2579	0.0000	5.1112	5.1112	1.4551	2.1908	0.7357
		III	0.1858	1.0156	0.8298	0.4527	0.6690	0.2162	0.0000	3.8928	3.8928	1.5646	2.0544	0.4898
		IV	0.1466	1.0634	0.9168	0.4525	0.6785	0.2261	0.0000	4.0041	4.0041	1.5427	2.0585	0.5158

Table 11: Table represents the Asymptotic confidence intervals and Highest posterior density (HPD) credible intervals of the parameter α and λ in the case of non-informative prior

n	m	Scheme	α_M^L	α_M^U	Length	α_B^L	α_B^U	length	λ_M^L	λ_M^L	length	λ_B^L	λ_M^U	length
20	12	I	0.0000	1.3575	1.3575	0.4159	0.7643	0.3484	0.0000	5.1916	5.1916	1.2776	2.0930	0.8154
		II	0.0000	2.6387	2.6387	0.3368	0.9565	0.6197	0.0000	7.1817	7.1817	1.1451	2.3457	1.2007
		III	0.0000	1.4557	1.4557	0.3295	0.8131	0.4835	0.0000	5.1104	5.1104	1.3009	2.2641	0.9632
		IV	0.0000	1.9847	1.9847	0.3673	0.8470	0.4797	0.0000	6.2270	6.2270	1.2082	2.2389	1.0307
30	18	I	0.0672	1.1819	1.1147	0.4221	0.7027	0.2806	0.0000	4.4885	4.4885	1.3712	2.0245	0.6533
		II	0.0000	2.0606	2.0606	0.4761	0.8641	0.3880	0.0000	6.3365	6.3365	1.1612	2.2167	1.0555
		III	0.0000	1.3435	1.3435	0.4268	0.7554	0.3286	0.0000	4.5477	4.5477	1.3983	2.0680	0.6697
		IV	0.0000	1.5866	1.5866	0.3967	0.7686	0.3719	0.0000	5.0761	5.0761	1.3735	2.1829	0.8093
50	30	I	0.2201	0.9370	0.7169	0.4177	0.6432	0.2254	0.0000	4.0290	4.0290	1.5600	2.0979	0.5379
		II	0.0000	1.5509	1.5509	0.4037	0.7854	0.3817	0.0000	5.1112	5.1112	1.4278	2.1851	0.7573
		III	0.1858	1.0156	0.8298	0.4260	0.6676	0.2416	0.0000	3.8928	3.8928	1.5696	2.0678	0.4982
		IV	0.1466	1.0634	0.9168	0.4286	0.6842	0.2556	0.0000	4.0041	4.0041	1.5504	2.0745	0.5241

Table 12: Estimates of the shape parameter α, scale parameter λ in the case of considered real data set.

Shape											
				GELF						Linex	
n	m	Scheme	$\hat{\alpha}$	$\delta = 0.5$	$\delta = -0.5$	$\delta = 1$	$\delta = -1$	$\delta = 2$	$\delta = -2$	$\delta_1 = 0.5$	$\delta_1 = -0.5$
				$\hat{\alpha}_{GE}$	$\hat{\alpha}_{GE}$	$\hat{\alpha}_{BE}$	$\hat{\alpha}_{BS}$	$\hat{\alpha}_{GE}$	$\hat{\alpha}_{BP}$	$\hat{\alpha}_{BL}$	$\hat{\alpha}_{BL}$
		5,0*24	0.8350	0.8290	0.8316	0.8276	0.8329	0.8250	0.8355	0.8318	0.8340
	25	0*10,1*5,0*10	0.6107	0.6068	0.6084	0.6060	0.6091	0.6044	0.6107	0.6087	0.6096
30		2,0*8,1,0*4,2,0*5	0.6903	0.6857	0.6873	0.6848	0.6882	0.6831	0.6898	0.6876	0.6887
		10,0*19	0.6517	0.6439	0.6469	0.6424	0.6484	0.6394	0.6513	0.6474	0.6493
	20	0*5,1*10,0*5	0.5968	0.5902	0.5927	0.5890	0.5939	0.5865	0.5963	0.5932	0.5946
		0*7,2*5,0*8	0.7102	0.7016	0.7056	0.6996	0.7076	0.6955	0.7115	0.7062	0.7090
Scale											
				GELF						Linex	
n	m	Scheme	$\hat{\lambda}$	$\delta = 0.5$	$\delta = -0.5$	$\delta = 1$	$\delta = -1$	$\delta = 2$	$\delta = -2$	$\delta_1 = 0.5$	$\delta_1 = -0.5$
				$\hat{\lambda}_{GE}$	$\hat{\lambda}_{GE}$	$\hat{\lambda}_{BE}$	$\hat{\lambda}_{BS}$	$\hat{\lambda}_{GE}$	$\hat{\lambda}_{BP}$	$\hat{\lambda}_{BL}$	$\hat{\lambda}_{BL}$
		5,0*24	0.0192	0.0190	0.0191	0.0189	0.0191	0.0188	0.0192	0.0191	0.0191
	25	0*10,1*5,0*10	0.0338	0.0333	0.0335	0.0332	0.0335	0.0331	0.0337	0.0335	0.0335
30		2,0*8,1,0*4,2,0*5	0.0268	0.0264	0.0265	0.0264	0.0266	0.0262	0.0267	0.0266	0.0266
		10,0*19	0.0255	0.0250	0.0252	0.0249	0.0253	0.0246	0.0256	0.0253	0.0253
	20	0*5,1*10,0*5	0.0275	0.0269	0.0271	0.0268	0.0272	0.0266	0.0274	0.0272	0.0272
		0*7,2*5,0*8	0.0200	0.0195	0.0197	0.0194	0.0198	0.0192	0.0199	0.0198	0.0198

Table 13: Estimates of the Reliability function and hazard function in the case of considered real data set.

Reliability											
				GELF						Linex	
n	m	Scheme	$\hat{R}_M$	$\delta = 0.5$	$\delta = -0.5$	$\delta = 1$	$\delta = -1$	$\delta = 2$	$\delta = -2$	$\delta_1 = 0.5$	$\delta_1 = -0.5$
				$\hat{R}_{GE}$	$\hat{R}_{GE}$	$\hat{R}_{BE}$	$\hat{R}_{BS}$	$\hat{R}_{GE}$	$\hat{R}_{BP}$	$\hat{R}_{BL}$	$\hat{R}_{BL}$
		5,0*24	0.9685	0.9688	0.9689	0.9688	0.9689	0.9688	0.9689	0.9689	0.9689
	25	0*10,1*5,0*10	0.9600	0.9605	0.9605	0.9605	0.9605	0.9605	0.9605	0.9605	0.9605
30		2,0*8,1,0*4,2,0*5	0.9639	0.9644	0.9644	0.9644	0.9644	0.9644	0.9644	0.9644	0.9644
		10,0*19	0.9675	0.9681	0.9681	0.9681	0.9681	0.9681	0.9681	0.9681	0.9681
	20	0*5,1*10,0*5	0.9681	0.9686	0.9686	0.9686	0.9686	0.9686	0.9686	0.9686	0.9686
		0*7,2*5,0*8	0.9721	0.9727	0.9727	0.9727	0.9727	0.9727	0.9727	0.9727	0.9727
Hazaed											
				GELF						Linex	
n	m	Scheme	$\hat{H}_M$	$\delta = 0.5$	$\delta = -0.5$	$\delta = 1$	$\delta = -1$	$\delta = 2$	$\delta = -2$	$\delta_1 = 0.5$	$\delta_1 = -0.5$
				$\hat{H}_{GE}$	$\hat{H}_{GE}$	$\hat{H}_{BE}$	$\hat{H}_{BS}$	$\hat{H}_{GE}$	$\hat{H}_{BP}$	$\hat{H}_{BL}$	$\hat{H}_{BL}$
		5,0*24	0.0160	0.0156	0.0157	0.0156	0.0158	0.0155	0.0159	0.0158	0.0158
	25	0*10,1*5,0*10	0.0202	0.0197	0.0198	0.0197	0.0199	0.0195	0.0200	0.0199	0.0199
30		2,0*8,1,0*4,2,0*5	0.0182	0.0178	0.0179	0.0178	0.0180	0.0177	0.0181	0.0180	0.0180
		10,0*19	0.0164	0.0158	0.0160	0.0158	0.0161	0.0156	0.0162	0.0161	0.0161
	20	0*5,1*10,0*5	0.0160	0.0156	0.0157	0.0155	0.0158	0.0154	0.0159	0.0158	0.0158
		0*7,2*5,0*8	0.0141	0.0135	0.0137	0.0135	0.0138	0.0133	0.0139	0.0138	0.0138

Table 14: Table represents the asymptotic and HPD confidence intervals and corresponding width of α and λ in the case of considered real data set.

n	m	Scheme	α_M^L	α_M^U	Length	α_B^L	α_B^U	length	λ_M^L	λ_M^L	length	λ_B^L	λ_M^U	length
		5,0*24	0.0700	1.6000	1.5299	0.6056	0.8299	0.2242	0.0000	0.0507	0.0507	0.0154	0.0226	0.0072
	25	0*10,1*5,0*10	0.1234	1.0980	0.9745	0.5232	0.6924	0.1692	0.0000	0.0811	0.0811	0.0269	0.0394	0.0125
30		2,0*8,1,0*4,2,0*5	0.1262	1.2545	1.1283	0.5861	0.7745	0.1884	0.0000	0.0683	0.0683	0.0216	0.0314	0.0098
		10,0*19	0.0000	1.5575	1.5575	0.5266	0.7698	0.2431	0.0000	0.0842	0.0842	0.0187	0.0322	0.0134
	20	0*5,1*10,0*5	0.0000	1.2246	1.2246	0.4897	0.6978	0.2081	0.0000	0.0812	0.0812	0.0111	0.0334	0.0223
		0*7,2*5,0*8	0.0000	1.6255	1.6255	0.5664	0.8575	0.2912	0.0000	0.0688	0.0688	0.0147	0.0249	0.0102

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