

An Iteration Algorithm based on Mixed FEM and DEM for Safety Factor of Slope Stability with Determined Sliding Surface

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Abstract: An iteration algorithm based on mixed FEM and DEM is proposed for safety factor of slope stability with determined sliding surface. In this method, the system of forces acting on the block is divided into two parts: external forces and contact forces. The displacements of block are chosen as the basic variables and the nodal contact forces on possible sliding surface and rigid motions for possibly moving blocks are chosen as iteration variables. The relation between displacements of contactor and contact forces on possible contact regions is solved by general finite element method while the rigid motions are fixed. The contact forces and rigid motions for possibly moving blocks are solved through equations of global force equilibrium for each possibly moving block and contact equations which are modified according to contact conditions (such as opening, bonding and sliding). Thus the iterative procedure becomes easily to be carried out and is much more economical for this kind of local nonlinear problems. The iteration algorithm based on the limit equilibrium on sliding surface is used to determine the safety factor for evaluating the slope stability. The safety factor for typical slope stability is analyzed by the algorithm presented in this paper.

Keywords: Iteration algorithm, mixed FEM and DEM, slope stability, safety factor.

1. Introduction

An analysis method for slope safety factor through c, φ reduction algorithm [1] by FEM is widely used now. When the system reaches instability, the numerical non-convergence occurs simultaneously. The safety factor is then obtained by c, φ reduction algorithm. The same time, the critical failure surface is found automatically. The traditional limit equilibrium method can't get the safety factor and failure surface of jointed rock slope. Strength Reduction FEM (SRFEM) presents a powerful alternative approach for slope stability analysis, especially to jointed rock slope. But, because of the numerical non-convergence, the solving of the equations for the sliding surface being near limit state is very difficult.

In slope stability analysis with FEM, the sliding surface could be determined or unknown. For the later, the sliding surface can be approximately searched by elasto-plastic FE analysis and then the problem is translated to the former [2]. For slope stability with determined sliding surface, nonlinear contact between blocks is the main nonlinear source. It belongs to the local discontinuous scope. For this kind of problem, the corresponding solving

methods can be thin layer element method (TLEM)[6], discrete element method (DEM)[4] discontinuous deformation analysis (DDA)[5]. If it is taken as contact problem, the method suggested by Haug[7] and Peric[8] can be used.

When the number of contact surfaces exists only one or two, the nonlinear happens on a local field. Zhao and Li [10] propose a mixed method using contact force and contact displacement as unknown variables. This method takes advantage of FEM to solve the contact boundary to get the flexibility matrix according to the possible contact body. The advantage of this method is that the overall matrix is symmetrical and the convergence is relatively fast. If the contact body has the rigid motions, Chen [9] gives the solving method of plane problem. Two points are chosen from the contact body to consider the rigid motion. The disadvantage of the method is that the stiffness matrix of the coupling equation between unknown contact forces and rigid displacements is asymmetric.

The combined solution by FEM and DEM is suggested in this paper. The relation between displacements of contactor and contact forces on

possible contact regions are solved by general finite element method while the displacements for rigid bodies are fixed. The contact forces and displacements for rigid bodies of possibly moving rock blocks are solved through equations of global force equilibrium for possible moving block and contact equations which are modified according to contact conditions (such as opening, bonding and sliding). Thus the iterative procedure became easily to be carried out and much more economical.

Finite element iteration method for stability safety factor of slopes against sliding is very effective for determined sliding surface [3]. With this method, the contact surface between landslide body and rock foundation is simulated by rectangle element for plane problems or cubic element for three dimensional problems with small thickness. A nonlinear iterative computation method is suggested to meet Mohr-Coulomb criterion for the stresses on the surface of sliding. On the basis of the nonlinear iterative method, a finite element iteration method of safety factor against sliding of the landslide body is suggested to meet the limit equilibrium condition. This method can be used for 2D and 3D problems and the results include displacement field and the stress field. It is extended to solve the safety factor with determined sliding surface based on mixed FEM and DEM in this paper.

2. Static Contact Equations for Two Blocks

A. Mechanical Descriptions

With no loss of generality, consider two elastic bodies Ω_1, Ω_2 , as shown in Figure 1, which are brought into contact by the external force F . Each boundary of the two bodies Γ^I is divided into three disjoint parts: $\Gamma^I = \Gamma_t^I + \Gamma_u^I + \Gamma_c^I$. Γ_c^I is the potential contact region. Here the superscript $I=1, 2$ denotes the two bodies, respectively. Other notations used in this section are given as follow: f^I : contact forces on Γ_c^I ; f_ξ, f_η, f_ζ : component of contact forces f in local coordinate $\xi\eta\zeta$, respectively; u^I : displacement vector on Γ_c^I ; δ is the gaps between Ω_1 and Ω_2 measured in the normal direction, $\delta = (u_1 - u_2) \cdot \bar{\xi} + \delta_0$, where $\bar{\xi}$ is the normal direction to the contact region, δ_0 is the initial normal gaps.

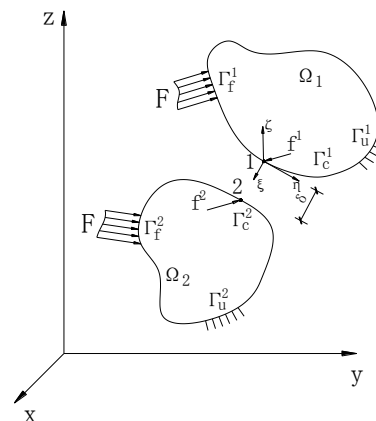


Fig. 1 Mechanical model for contact problems

For contact problems, three important principles, the non-penetration condition, normal traction condition and frictional condition, must be taken into account. When the three contact statuses (stick, slip and separation) are involved, those conditions mentioned above can be summarized as TABLE 1.

TABLE 1 CONTACT CONDITIONS FOR CONTACT PROBLEMS WITH FRICTION AND INITIAL GAPS

| Contact status | Equality constraint | Inequality constraint |
|----------------|--|--|
| Separation | $f^1 = f^2 = 0$ | $\delta > 0$ |
| Stick | $\delta = 0, f^1 = -f^2$ | $f_\xi < \sigma_t \cdot A$, $\sqrt{f_\eta^2 + f_\zeta^2} < -\mu \cdot f_\xi + A \cdot c$ |
| Slip | $\delta = 0$, $\sqrt{f_\eta^2 + f_\zeta^2} = -\mu \cdot f_\xi + A \cdot c$ | $f_\xi < \sigma_t \cdot A$ |

B. STATIC CONTACT EQUATIONS

If the contact body is rock block, the rigid motion should be considered. Assuming the rigid displacement at centroid point is γ , the rigid displacement at arbitrary point with relative coordinate of $(\Delta x, \Delta y, \Delta z)$ to centroid point can be expressed as

$$(1) \quad u_g = \begin{bmatrix} 1 & 0 & 0 & 0 & \Delta z & -\Delta y \\ 0 & 1 & 0 & -\Delta z & 0 & \Delta x \\ 0 & 0 & 1 & \Delta y & -\Delta x & 0 \end{bmatrix} \begin{Bmatrix} \bar{u} \\ \bar{v} \\ \bar{w} \\ \theta_x \\ \theta_y \\ \theta_z \end{Bmatrix} = \omega \gamma$$

For contact bodies of Ω_1, Ω_2 , the equilibrium equation for FEM at step $(n+1)$ without considering rigid motion is:

$$K\Delta u_n = R_{n+1} + \Delta f_n \quad (2)$$

where

$$R_{n+1} = F_{n+1} + f_n - \int B^T \sigma_n d\Omega$$

K is the global stiffness matrix; Δu_n is displacement increment matrix, F_{n+1} is load vector, f_n contact internal force vector at step n , Δf_n contact internal force increment at step $(n+1)$, σ_n stress vector at step n .

Remembering $\Delta \bar{u}_n = K^{-1}R_{n+1}$, and using $C = K^{-1}$ to represent flexibility matrix, formula (2) can be written as,

$$\Delta u_n = \Delta \bar{u}_n + C\Delta f_n \quad (3)$$

When the rigid displacements are considered for contact bodies, the total displacements for points on contact boundaries of Ω_1 and Ω_2 are

$$\Delta u_{n_n}^1 = \Delta \bar{u}_n^1 + C^1 \Delta f_n^1 + \omega_1 \gamma^1 \quad (4)$$

$$\Delta u_{n_n}^2 = \Delta \bar{u}_n^2 + C^2 \Delta f_n^2 + \omega_2 \gamma^2 \quad (5)$$

Because of $\Delta f_n^1 = \Delta f_n^2 = \Delta f_n$, and considering $C = C^1 + C^2$, the following formula can be obtained from formula of (4) and (5),

$$C\Delta f_n + \Delta u_g^2 - \Delta u_g^1 = (\Delta u_n^2 - \Delta u_n^1) - (\Delta \bar{u}_n^2 - \Delta \bar{u}_n^1) \quad (6)$$

For each contact body, the static equilibrium equation related to centroid point for all forces acted on it could be written as follows.

$$\omega_1^T \Delta f_n^1 + \omega_1^T \Delta F_n^1 = 0 \quad (7)$$

$$\omega_2^T \Delta f_n^2 + \omega_2^T \Delta F_n^2 = 0 \quad (8)$$

Finally, for two contacted bodies, the equations included the contact forces and rigid displacements of each contact body are:

$$\begin{bmatrix} C & -\omega^1 & \omega^2 \\ -\omega_1^T & 0 & 0 \\ \omega_2^T & 0 & 0 \end{bmatrix} \begin{Bmatrix} \Delta f_n \\ \Delta \gamma^1 \\ \Delta \gamma^2 \end{Bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \omega_1^T & 0 \\ 0 & 0 & -\omega_2^T \end{bmatrix} \begin{Bmatrix} \Delta \bar{\delta}_n \\ \Delta F_n^1 \\ \Delta F_n^2 \end{Bmatrix} \quad (9)$$

3. Iteration Algorithm for Safety Factor of Slope with Determined Sliding Surface

The iteration steps are as follows.

1) Assuming safety factor K (usually $K=1$ for the first iteration), the c', φ' on the sliding surface are substituted as follows.

$$c' = c/K, \quad \tan \varphi' = \tan \varphi/K \quad (10)$$

2) Using mixed FEM and DEM, the stress and deformation of the slope is analyzed. The contact forces on the sliding surface can be obtained.

3) Computing the sliding force F_τ and Shearing resistance F_R ,

$$F_\tau = \sum_{i=1}^{nct} f_\tau^i, \quad F_R = \sum_{i=1}^{nct} (\tan \varphi' f_n^i + c' A_i) \quad (11)$$

Where nct is the total number of contact element on the sliding surface, f_n^i is the normal force at ith element, f_τ^i is the sliding force at ith element and A_i is the controlling area at ith element.

4) Calculating the new safety factor for next iteration,

$$K^* = KF_R / F_\tau \quad (12)$$

5) Checking the convergence,

$$\frac{K^* - K}{K^*} \leq \varepsilon \quad (13)$$

ε is the given tolerance.

When (13) is met, the iteration will stop. Otherwise, repeating 1) to 5).

4. Examples

A. Classical contact problem

To verify the mixed FEM and DEM, the classical Hertz contact problem is taken as the first example.

Figure 2 shows the contact between an infinite cylinder and half space. Assuming the cylinder is acted by uniform pressure p on the neutral axis, the theoretical solution (Herz solution) will be [12]:

$$\text{Half width of contact: } a = \sqrt{4pr / \pi E'}$$

Contact pressure distribution:

$$q = 2p(1.0 - (x/a)^2) / (\pi a)$$

where,

$$\frac{1}{E'} = \frac{1 - \mu_1^2}{E_1} + \frac{1 - \mu_2^2}{E_2}$$

E_1, E_2, μ_1, μ_2 are elastic modulus and poisson's ratio of the cylinder and the foundation. Considering symmetry, half part is taken to compute as a plane strain problem. 20 contact nodal pairs are arranged. Taking $E_1 = E_2 = 1.0 \times 10^5 \text{MPa}$, $\mu_1 = \mu_2 = 0.3$, $r = 100 \text{mm}$, $p = 2.0 \times 10^4 \text{N/mm}$, the theoretical solution of $a = 6.808 \text{mm}$, the maximum contact pressure $q_{\max} = 1870.277 \text{N/mm}^2$. The result with mixed FEM and DEM are: $a_0 = 6.63 \text{mm}$, $q_{\max} = 1881.25 \text{N/mm}^2$. Figure 3 shows the comparison between theoretical solution and numerical results. The two results are almost the same.

B. Safety factor for determined sliding surface

A typical slope stability problem is shown in Figure 4. The main coordinates is marked in the figure. The example in [11] is used to check the sliding modes and the safety factor with three kinds of soils. Here, it is used to check the method presented in this paper. Assuming the sliding surface is known as an arc with centre coordinate (19.61, 46.32) and radius of 21.32. The elastic modulus of the sliding block is $1.0 \times 10^4 \text{kPa}$ and the density is 20kN/m^3 . c, φ on the sliding surface are assumed as 3.0kN/m^2 and 19.6° . The finite mesh is shown in Figure 5. Normal constraint conditions are used for the later and bottom boundary.

With the method mentioned above, three iterations is made for obtaining the safety factor $K = 1.37$. Figure 6 gives the deformation mode and the displacement vector for the slope on the critical sliding state.

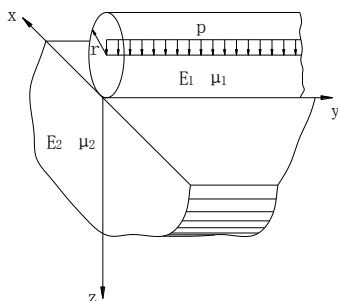


Fig. 2 Contact between cylinder and semi-space

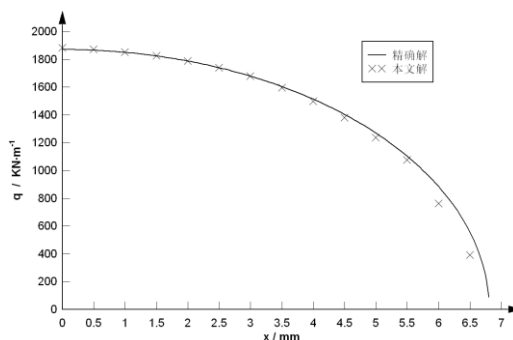


Fig. 3 Distribution of contact pressure

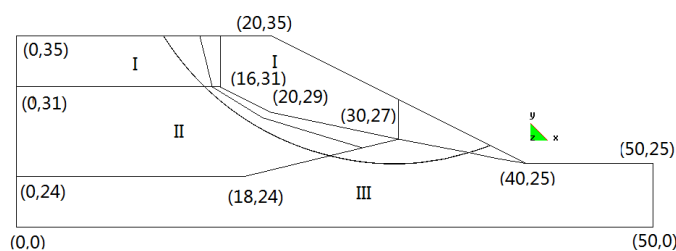


Fig. 4 Typical slope problem

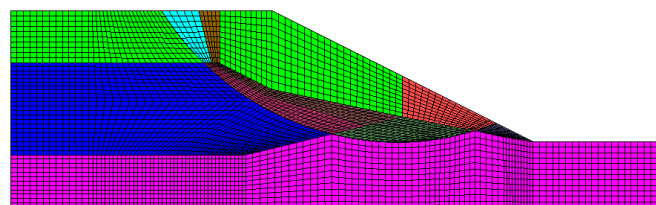


Fig. 5 FE mesh for the slope

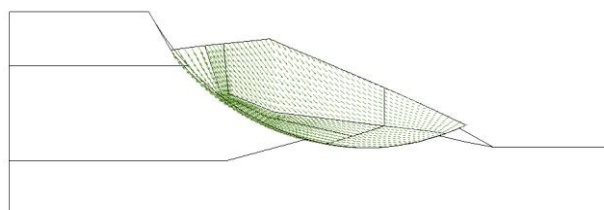


Fig. 6 Deformation and the displacement vector for the slope being on critical state

5. Conclusion

The iteration algorithm based on mixed FEM and DEM proposed in this paper can be effectively used to solve for safety factor for slope with determined sliding surface. The evaluation with theoretical solution and the evaluation with typical slope stability support this conclusion.

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