Mathematical Sciences Letters
An International Journal

http://dx.doi.org/10.18576/msl/050214

Solving Linear Least Squares Problems Based on Improved Cuckoo Search Algorithm

Mohamed Abdel-Baset[1,*] *and Ibrahim M. Hezam*[2].

[1] Department of Operations Research, Faculty of Computers and Informatics, Zagazig University, El-Zera Square, Zagazig, Sharqiyah, Egypt.
[2] Department of Computer, Faculty of Education, Ibb University, Ibb city, Yemen.

Received: 18 Apr. 2015, Revised: 22 Oct. 2015, Accepted: 29 Oct. 2015.
Published online: 1 May 2016.

Abstract: In This paper, we propose a new algorithm that encompasses the features of cuckoo search and Newton method. It combines the fast convergence and the global search of the Cuckoo Search (CS), and the strong local search ability of Newton method (NM). The Compound Algorithm (CSNM) will be used to solve linear least squares problem. The linear least squares problem is transformed into an optimization problem, where the fitness function is defined as the sum of squared residuals. The proposed algorithm retained the global search capability with more accurate and faster convergence. The experimental results show that the proposed algorithm proved to be superior in convergence efficiency and computational accuracy.

Keywords: Cuckoo Search Algorithm; Meta-heuristics; Optimization; Newton Method; Linear Least Squares

1 Introduction

The general ratios optimization problems ROP the method of least squares is a standard approach to the approximate solution of over determined systems, i.e. sets of equations in which there are more equations than unknowns [1]. Least squares problems fall into two categories: linear least squares and non-linear least squares, depending on whether or not the residuals are linear in all unknowns [2]. The linear least-squares problem occurs in statistical regression analysis; it has a closed-form solution. The non-linear problem has no closed-form solution and is usually solved by iterative refinement; at each iteration the system is approximated by a linear one, thus the core calculation is similar in both cases [3].

In statistics and mathematics, linear least squares is an approach to fitting a mathematical or statistical model to data in cases where the idealized value provided by the model for any data point is expressed linearly in terms of the unknown parameters of the model. The resulting fitted model can be used to summarize the data, to predict unobserved values from the same system, and to understand the mechanisms that may underlie the system. Over the past few years, a number of approaches have been developed for solving the linear least squares using classical mathematical programming methods[4–6]. These methods require matrix factorizations or updates, and can become overly expensive for very large-scale problems.

Gradient-type methods, such as gradient projection methods [7], require matrix-vector multiplications, but typically have very slow convergence. Meanwhile, classical optimization methods are highly sensitive to starting points and frequently converge to local optimum solution or diverge altogether. Linear programming methods are fast and reliable but the main disadvantage associated with the piecewise linear cost approximation. Nonlinear programming methods have a problem of convergence and algorithmic complexity. Newton based algorithm have a problem in handling large number of inequality constraints [6], [8].

Mathematically, linear least squares is the problem of approximately solving an over determined system of linear equations, where the best approximation is defined as that which minimizes the sum of squared differences between the data values and their corresponding modeled values. The approach is called “linear” least squares since the solution depends linearly on the data. Linear least squares problems are convex and have a closed-form solution that is unique, provided that the number of data-points used for fitting equals or exceeds the number of unknown parameters, except in special degenerate situations[9], [10]. In contrast, non-linear least squares problems generally must be solved by an iterative procedure and the problems can be non-convex with multiple optima for the objective function. A simple data set consists of n points (data pairs) (x_i, y_i), $i = 1, 2, \cdots, n$, where x_i is an independent variable and y_i is a dependent variable whose value is found by

*Corresponding author E-mail: analyst_mohamed@yahoo.com

observation. The model function has the form *f(x, β)* where the m adjustable parameters are held in the vector *β*. The goal is to find the parameter values for the model which best fits the data. The least squares method finds its optimum when the sum *S* of squared residuals

$$S = \sum_{i=1}^{n} r_i^2 \quad (1)$$

It is a minimum. A residual is defined as the difference between the values predicted by the model and the actual value of the dependent variable

$$r_i = f(x_i, \beta) - y_i \quad (2)$$

Linear least squares model is that of the straight line. Denoting the intercept as β_0 and the slope as β_1 the model function is given by

$$f(x, \beta) = \beta_0 + \beta_1 x \quad (3)$$

Let

$$A^T = \begin{pmatrix} 1 & 1 \ldots & 1 \\ x_1 & x_2 \ldots & x_n \end{pmatrix}, \quad \beta = \begin{pmatrix} \beta_0 \\ \beta_1 \end{pmatrix}, \; b^T = (y_1 \; y_2 \ldots y_n) \quad (4)$$

Then linear least squares problem (1) is transformed into following optimization problem

$$\min S = \sum_{i=1}^{n} r_i^2 = (A\beta - b)^T (A\beta - b) = \|A\beta - b\|^2. \quad (5)$$

This paper is organized as follows: the original cuckoo search algorithms briefly introduced in section 2. In section 3, Newton method is introduced. In section 4, the proposed algorithm is described, while the results are discussed in section 5. Finally, conclusions and future work are presented in last section.

2 The Original Cuckoo Search Algorithm

The Cuckoo search algorithm is a Meta heuristic search algorithm, which has been proposed recently by Yang and Deb [11], it was based on the following idealized rules: (1) Each cuckoo lays one egg at a time, and dumps it in a randomly chosen nest. (2) The best nests with high quality of eggs (solutions) will carry over to the next generations. (3) The number of available host nests is fixed, and a host can discover an alien egg with a probability $p_a \in [0,1]$.

In this case, the host bird can either throw the egg away or abandon the nest so as to build a completely new nest in a new location. The main steps of cuckoo search algorithm are summarized in algorithm 1. When generating new solutions $x_i(t+1)$ for the i^{th} cuckoo, the following Lévy flight is performed

$$x_i^{(t+1)} = x_i^{(t)} + \alpha \oplus Levy(\lambda) \quad (6)$$

Where $\alpha > 0$ is the step size, which should be related to the scale of the problem of interest. The product $\oplus$ means entry-wise multiplications. We consider a Lévy flight in which the step-lengths are distributed according to the following probability distribution

$$Levy\, u = t^{-\lambda}, 1 < \lambda \le 3 \quad (7)$$

This has an infinite variance. Here the consecutive jumps/steps of a cuckoo essentially form a random walk process which obeys a power-law step length distribution with a heavy tail [12–14].

Algorithm 1: Cuckoo search algorithm

Define Objective function f (x), x = $(x_1, x_2, \ldots, x_d)$

Initial a population of n host nests $x_i \; (i = 1, 2, \ldots, d)$

while *(t < MaxGeneration) or (stop criterion);*

Get a cuckoo (say i) randomly and generate a new solution by Lévy flights;

Evaluate its quality/fitness; F_i

Choose a nest among n (say j) randomly;

if $(F_i > F_j)$,

Replace j by the new solution;

end

Abandon a fraction (P_a) *of worse nests*

[and build new ones at new locations via Lévy flights];

Keep the best solutions (or nests with quality solutions);

Rank the solutions and find the current best;

end while

Post process results and visualization;

End

3 Newton Method (NM)

Newton method iterative scheme as follows[9]:

$$X_{k+1} = X_k - \left[Hf(X_k)\right]^{-1} \nabla f(X_k) \quad (8)$$

Where $\nabla f(X_k)$ is the gradient function and Hf*(X)* is Hessian matrix function.

Usually Newton method is modified to include a small step size $\gamma(\gamma > 0)$

$$X_{k+1} = X_k - \gamma \left[Hf(X_k)\right]^{-1} \nabla f(X_k) \quad (9)$$

The computational procedure of Newton method can be summarized as follows:

Algorithm2: Newton Method

Step 1: *Set the parameters* ϵ *and* γ.
Step 2: *Give starting points X0 and Let k=0.*
Step 3: *Calculate* $\nabla f\left(X_k\right)$. *If* $\left\|\nabla f\left(X_k\right)\right\| \le \varepsilon$ *output the result; otherwise, go to Step4.*
Step 4: *Calculate* $\left[Hf\left(X_k\right)\right]^{-1}$ *and let* $X_{k+1} = X_k - \gamma\left[Hf\left(X_k\right)\right]^{-1}\nabla f\left(X_k\right)$
Step 5: *Let k = k + 1, then go back to Step 3.*

Where applicable, Newton method converges much faster towards a local minimum than gradient descent. In fact, every local minimum has a neighborhood *U* such that, if we start with $x_0 \in U$, Newton method with step size γ=1 converges quadratically (if the Hessian is invertible and a Lipschitz continuous function of *X* in that neighborhood). If *Hf(X)* is not positive or a minimization problem has saddle points, it sometimes gone wrong, for example the iterations are converging to a saddle point and not a minimum.

4 The Proposed Algorithm (CSNM) for Linear Least Squares

The main ideas of improving the CS algorithm based on Newton method for solving system of linear least squares are as follows: After converting the system of linear least squares into fitness function as (5). The CS is employed to optimize the problem (5); and then the Newton method is employed to solve the optimization problem (5) using the solution obtained by the CS as an initial guess; this process is repeated until an accurate solution is located.

The steps of the proposed hybrid cuckoo search algorithm based on Newton method for solving a system of linear least squares are summarized as follows:

Algorithm 3: Cuckoo Search Algorithm based on Newton method

Step1: *Assignment of parameters of Cuckoo Search and Newton method*
Step 2: *Randomly given* $x_1; x_2; \ldots; x_m$*, called initial cuckoo.*
Step 3: *Apply CS to optimize the problem (5). x* is the optimization solution at that time.*
Step 4: *Apply Newton method to optimize problem (5) with initial solution x*. For the single dimension optimization problems that is brought out by using Newton method, CS is used to optimize it. Let the final solution be x**.*
Step 5: *If the termination condition is satisfied, then stop, if not, substitute x** then go to step 2.*

5 Numerical Results and Discussion

The following test problem shown in table (1) were taken from literature [9-10] to demonstrate the efficiency and reliability of the proposed algorithms. The simulation parameter settings results of CS, are as follows: Number of nests *n=50*, Discovery rate of alien eggs/solutions *pa=0.25;* The data pairs (x_i, y_i) are listed in Table 1, where x_i is an independent variable and y_i is a dependent variable whose value is found by observation, $i = 1, 2, \ldots, 24$.

Table 1: Data pairs

i	1	2	3	4	5	6	7	8	9	10	11	12
x_i	1.9	2	2.1	2.5	2.7	2.7	3.5	3.5	4	4	4.5	4.6
y_i	1.4	1.3	1.8	2.5	2.8	2.5	3	2.7	4	3.5	4.2	3.5
i	13	14	15	16	17	18	19	20	21	22	23	24
x_i	5	5.2	5	6.3	6.5	7.1	8	8	8.9	9	9.5	10
y_i	5.5	5	5.5	6.4	6	5.3	6.5	7	8.5	8	8.1	8.1

Table 2: Computation results

Method	β0	β1	S
[9]	0.1502	0.8587	5.661375330899998
CS	0.150474085526993	0.858734289429005	5.661374565933022
CSNM	0.150474089952635	0.858734288488779	5.661374565933021

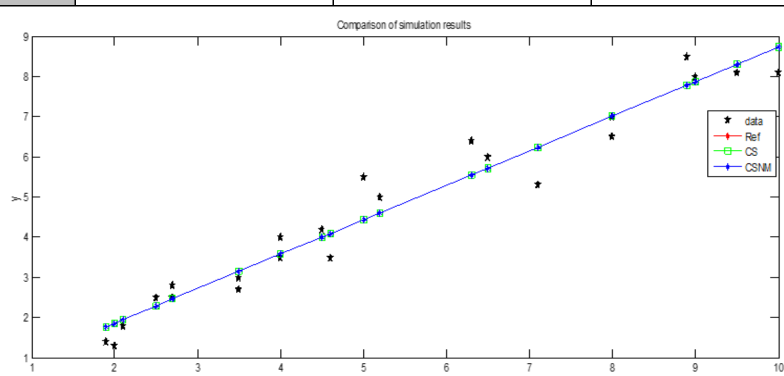

Fig. 1: Results for data fitting

6 Conclusions and Future Work

This paper introduced an improved cuckoo search algorithm based on Newton method for solving the linear least squares. Simulation example show that the algorithm can converge to be the best solution, and it has a high convergence rate and high accuracy. The proposed algorithm has been provided the better and clear way to optimize linear least squares problem. The results proved the superiority of the proposed methodology. The aim of future works includes investigating the use of CSNM for non-linear least squares or other practical application.

Acknowledgements

The authors would like to thank the anonymous reviewers for their careful reading of this paper and for their helpful comments

References

[1] S. J. Wright and J. Nocedal, *Numerical optimization*, vol. 2. Springer New York, 1999.

[2] C. L. Lawson and R. J. Hanson, *Solving least squares problems*, vol. 161. SIAM, 1974.

[3] A. Björck, *Numerical methods for least squares problems*. Siam, 1996.

[4] W. Gander, “Least squares with a quadratic constraint,” *Numerische Mathematik*, vol. 36, pp. 291–307, 1980.

[5] T. Yang and H. X. Lin, “Solving sparse least squares problems with preconditioned CGLS method on parallel distributed memory computers,” *PARALLEL ALGORITHMS AND APPLICATION*, vol. 13, pp. 289–305, 1999.

[6] L. Yong, “A feasible interior point algorithm for a class of nonnegative least squares problems,” in *Future Computer and Communication, 2009. FCC’09. International Conference on*, 2009, pp. 157–159.

[7] G. McCormick and R. Tapia, “The gradient projection method under mild differentiability conditions,” *SIAM Journal on Control*, vol. 10, pp. 93–98, 1972.

[8] Y. Zhang, “Interior-point gradient methods with diagonal-scalings for simple-bound constrained optimization,” Citeseer, 2004.

[9] Y. H. Dengxu HE Liangdong QU, “Harmony Search Algorithm Based on Newton Method for Linear Least Squares,” *Journal of Computational Information Systems*, vol. 8: 22, pp. 9441–9448, 2012.

[10] L. Yong, S. Liu, J. Zhang, and Q. Feng, “An improved harmony search algorithm for linear least squares,” *Journal of Computational Information Systems*, vol. 8, pp. 4217–4224, 2012.

[11] A. H. Gandomi, X.-S. Yang, and A. H. Alavi, “Cuckoo search algorithm: a metaheuristic approach to solve structural optimization problems,” *Engineering with Computers*, vol. 29, pp. 17–35, 2013.

[12] M. Abdel-Baset, and Ibrahim Hezam, “Cuckoo Search and Genetic Algorithm Hybrid Schemes for Optimization Problems,” Applied Mathematics & Information Sciences An International Journal, vol. 10, No. 3pp. 1–13, 2016.

[13] I. M.Hezam and M. Abdel-Baset “An Improved Flower Pollination Algorithm for Ratios optimization Problems,” Applied Mathematics & Information Sciences Letters An International Journal, vol. 3, No. 2, pp. 83–91,2015.

[14] I. M.Hezam , M. Abd-El Baset and I. Selem, “Cuckoo Search Algorithm for Stellar Population Analysis of Galaxies”, International Journal of Information Technology and Computer Science, vol. 7, pp.29-33,2015.

Mohamed A. Metwally Received his B.Sc., M.Sc. and the Ph.D in information technology from Zagazig University. He is a lecturer in the operations Research Department, Faculty of Computers and Informatics, Zagazig University. His current research interests are Optimization, Operations Research, Data Mining, Computational Intelligence, Applied Statistics and Decision support systems. He is also a reviewer in different international journals and conferences. He has published more than 70 articles in international journals and conference proceedings.

Ibrahim M. Hezam obtained his B.Sc. in 2003 in Mathematics from IBB University, IBB, Yemen; His M.Sc. in Pure Mathematics (Operations Research) from Helwan University, Egypt, in 2011. Currently, he is a Ph.D. Student in Department of Operations Research and Decision Support, Faculty of Computers & Information, Menoufia University, Egypt. His research interests are metaheuristic optimization methods, artificial intelligence, and decision support systems.