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Fuzzy Soft α-Connectedness in Fuzzy Soft Topological Spaces

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Abstract: In the present paper, we study and investigate the properties of fuzzy soft α-connected sets, fuzzy soft α-separated sets and fuzzy soft α-*s*-connected sets and establish several interesting properties supported by examples. Moreover, we show that, a fuzzy soft α-disconnectedness is not an hereditary property in general. Finally, we show that the fuzzy α-irresolute surjective soft image of fuzzy soft α-connected (resp. fuzzy soft α-*s*-connected) is also fuzzy soft α-connected (resp. fuzzy soft α-*s*-connected).

Keywords: Fuzzy soft topological space, Fuzzy α-open soft, Fuzzy α-continuous soft functions, Fuzzy soft connected, Fuzzy soft α-connected.

1 Introduction

In real life situation, the problems in economics, engineering, social sciences, medical science etc. do not always involve crisp data. So, we cannot successfully use the traditional classical methods because of various types of uncertainties presented in these problems. To exceed these uncertainties, some kinds of theories were given like theory of fuzzy set, intuitionistic fuzzy set, rough set, bipolar fuzzy set, i.e. which we can use as mathematical tools for dealings with uncertainties. But, all these theories have their inherent difficulties. The reason for these difficulties Molodtsov [34] initiated the concept of soft set theory as a new mathematical tool for dealing with uncertainties which is free from the above difficulties. In [34,35], Molodtsov successfully applied the soft theory in several directions, such as smoothness of functions, game theory, operations research, Riemann integration, Perron integration, probability, theory of measurement, and so on. After presentation of the operations of soft sets [32], the properties and applications of soft set theory have been studied increasingly [7,27,35]. Xiao et al.[44] and Pei and Miao [38] discussed the relationship between soft sets and information systems. They showed that soft sets are a class of special information systems. In recent years, many interesting applications of soft set theory have been expanded by embedding the ideas of fuzzy sets [5,6,10, 16,25,30,31,32,33,35,36,47]. To develop soft set theory, the operations of the soft sets are redefined and a uni-int decision making method was constructed by using these new operations [11].
Recently, in 2011, Shabir and Naz [41] initiated the study of soft topological spaces. They defined soft topology on the collection τ of soft sets over X. Consequently, they defined basic notions of soft topological spaces such as open soft and closed soft sets, soft subspace, soft closure, soft nbd of a point, soft separation axioms, soft regular spaces and soft normal spaces and established their several properties. Min in [43] investigate some properties of these soft separation axioms. In [17], Kandil et. al. introduced some soft operations such as semi open soft, pre open soft, α-open soft and β-open soft and investigated their properties in detail. Kandil et al. [24] introduced the notion of soft semi separation axioms. In particular they study the properties of the soft semi regular spaces and soft semi normal spaces. The notion of soft ideal was initiated for the first time by Kandil et al.[20]. They also introduced the concept of soft local function. These concepts are discussed with a view to find new soft topologies from the original one, called soft topological spaces with soft ideal $(X,\tau,E,\tilde{I})$. Applications to various fields were further investigated by Kandil et al. [18,19,21,22,23,26]. The notion of b-open soft sets was initiated for the first time by El-sheikh and Abd El-latif [13]. Maji et. al. [30] initiated the study

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involving both fuzzy sets and soft sets. In [8] the notion of fuzzy soft set was introduced as a fuzzy generalization of soft sets and some basic properties of fuzzy soft sets are discussed in detail. Then, many scientists such as X. Yang et. al. [45], improved the concept of fuzziness of soft sets. In [4], Karal et al. defined the notion of a mapping on classes of fuzzy soft sets, which is fundamental important in fuzzy soft set theory, to improve this work and they studied properties of fuzzy soft images and fuzzy soft inverse images of fuzzy soft sets. Chang [12] introduced the concept of fuzzy topology on a set X by axiomatizing a collection $\mathfrak{T}$ of fuzzy subsets of X. Tanay et.al. [42] introduced the definition of fuzzy soft topology over a subset of the initial universe set while Roy and Samanta [40] gave the definition of fuzzy soft topology over the initial universe set. Some fuzzy soft topological properties based on fuzzy pre (resp. semi, β-) open soft sets, were introduced in [1,2,3,15,16,25].

In the present paper, we generalize the notion of fuzzy soft connectedness [29], by using the notions of fuzzy α-open soft sets.

2 Preliminaries

In this section, we present the basic definitions and results of fuzzy soft set theory which will be needed in the paper.

Definition 2.1.[46] A fuzzy set A of a non-empty set X is characterized by a membership function $\mu_A : X \longrightarrow [0,1] = I$ whose value $\mu_A(x)$ represents the "degree of membership" of x in A for $x \in X$. We denote family of all fuzzy sets by I^X.

Definition 2.2.[30] Let $A \subseteq E$. A pair (f,A), denoted by f_A, is called fuzzy soft set over X , where f is a mapping given by $f : A \to I^X$ defined by $f_A(e) = \mu^e_{f_A}$, where $\mu^e_{f_A} = \overline{0}$ if $e \notin A$ and $\mu^e_{f_A} \neq \overline{0}$ if $e \in A$, where $\overline{0}(x) = 0\ \forall\, x \in X$. The family of all these fuzzy soft sets over X denoted by $FSS(X)_A$.

Definition 2.3.[39] Let $\mathfrak{T}$ be a collection of fuzzy soft sets over a universe X with a fixed set of parameters E, then $\mathfrak{T} \subseteq FSS(X)_E$ is called fuzzy soft topology on X if

(1)$\tilde{1}_E, \tilde{0}_E \in \mathfrak{T}$, where $\tilde{0}_E(e) = \overline{0}$ and $\tilde{1}_E(e) = \overline{1}$, $\forall e \in E$,
(2)the union of any members of $\mathfrak{T}$ belongs to $\mathfrak{T}$,
(3)the intersection of any two members of $\mathfrak{T}$ belongs to $\mathfrak{T}$.

The triplet $(X,\mathfrak{T},E)$ is called fuzzy soft topological space over X. Also, each member of $\mathfrak{T}$ is called fuzzy open soft in $(X,\mathfrak{T},E)$. We denote the set of all open soft sets by $FOS(X,\mathfrak{T},E)$, or $FOS(X)$.

Definition 2.4.[39] Let $(X,\mathfrak{T},E)$ be a fuzzy soft topological space. A fuzzy soft set f_A over X is said to be fuzzy closed soft set in X, if its relative complement f^c_A is fuzzy open soft set. We denote the set of all fuzzy closed soft sets by $FCS(X,\mathfrak{T},E)$, or $FCS(X)$.

Definition 2.5.[37] Let $(X,\mathfrak{T},E)$ be a fuzzy soft topological space and $f_A \in FSS(X)_E$. The fuzzy soft closure of f_A, denoted by $Fcl(f_A)$ is the intersection of all fuzzy closed soft super sets of f_A. i.e.,
$Fcl(f_A) = \sqcap\{h_D : h_D\ is\ fuzzy\ closed\ soft\ set\ and\ f_A \sqsubseteq h_D\}$.
The fuzzy soft interior of g_B, denoted by $Fint(f_A)$ is the fuzzy soft union of all fuzzy open soft subsets of f_A.i.e.,
$Fint(g_B) = \sqcup\{h_D : h_D\ is\ fuzzy\ open\ soft\ set\ and\ h_D \sqsubseteq g_B\}$.

Definition 2.6.[29] The fuzzy soft set $f_A \in FSS(X)_E$ is called fuzzy soft point if there exist $x \in X$ and $e \in E$ such that $\mu^e_{f_A}(x) = \alpha$ $(0 < \alpha \leq 1)$ and $\mu^e_{f_A}(y) = \overline{0}$ for each $y \in X - \{x\}$, and this fuzzy soft point is denoted by x^e_α or f_e.

Definition 2.7.[29] The fuzzy soft point x^e_α is said to be belonging to the fuzzy soft set (g,A), denoted by $x^e_\alpha \tilde{\in} (g,A)$, if for the element $e \in A$, $\alpha \leq \mu^e_{g_A}(x)$.

Definition 2.8.[29] A fuzzy soft set g_B in a fuzzy soft topological space $(X,\mathfrak{T},E)$ is called fuzzy soft neighborhood of the fuzzy soft point x^e_α if there exists a fuzzy open soft set h_C such that $x^e_\alpha \tilde{\in} h_C \sqsubseteq g_B$. A fuzzy soft set g_B in a fuzzy soft topological space $(X,\mathfrak{T},E)$ is called fuzzy soft neighborhood of the soft set f_A if there exists a fuzzy open soft set h_C such that $f_A \sqsubseteq h_C \sqsubseteq g_B$. The fuzzy soft neighborhood system of the fuzzy soft point x^e_α, denoted by $N_{\mathfrak{T}}(x^e_\alpha)$, is the family of all its fuzzy soft neighborhoods.

Definition 2.9.[29] Let $(X,\mathfrak{T},E)$ be a fuzzy soft topological space and $Y \subseteq X$. Let h^Y_E be a fuzzy soft set over (Y,E) such that $h^Y_E : E \to I^Y$ such that $h^Y_E(e) = \mu^e_{h^Y_E}$,

$$\mu^e_{h^Y_E}(x) = \begin{cases} 1\ x \in Y, \\ 0,\ x \notin Y. \end{cases}$$

Let $\mathfrak{T}_Y = \{h^Y_E \sqcap g_B : g_B \in \mathfrak{T}\}$, then the fuzzy soft topology $\mathfrak{T}_Y$ on $(Y,E)$is called fuzzy soft subspace topology for (Y,E) and $(Y,\mathfrak{T}_Y,E)$ is called fuzzy soft subspace of $(X,\mathfrak{T},E)$. If $h^Y_E \in \mathfrak{T}$ (resp. $h^Y_E \in \mathfrak{T}^c$), then $(Y,\mathfrak{T}_Y,E)$ is called fuzzy open (resp. closed) soft subspace of $(X,\mathfrak{T},E)$.

Definition 2.10.[37] Let $FSS(X)_E$ and $FSS(Y)_K$ be families of fuzzy soft sets over X and Y, respectively. Let $u : X \to Y$ and $p : E \to K$ be mappings. Then the map f_{pu} is called fuzzy soft mapping from X to Y and denoted by $f_{pu} : FSS(X)_E \to FSS(Y)_K$ such that,

(1)If $f_A \in FSS(X)_E$. Then the image of f_A under the fuzzy soft mapping f_{pu} is the fuzzy soft set over Y defined by $f_{pu}(f_A)$, where $\forall\, k \in p(E)$, $\forall\, y \in Y$,
$$f_{pu}(f_A)(k)(y) = \begin{cases} \bigvee_{u(x)=y} [\vee_{p(e)=k}(f_A(e))](x) & if\ x \in u^{-1}(y), \\ 0 & otherwise. \end{cases}$$
(2)If $g_B \in FSS(Y)_K$, then the pre-image of g_B under the fuzzy soft mapping f_{pu} is the fuzzy soft set over X defined by $f^{-1}_{pu}(g_B)$, where $\forall\, e \in p^{-1}(K)$, $\forall\, x \in X$,
$$f^{-1}_{pu}(g_B)(e)(x) = \begin{cases} g_B(p(e))(u(x)) & for\ p(e) \in B, \\ 0 & otherwise. \end{cases}$$

The fuzzy soft mapping f_{pu} is called surjective (resp. injective) if p and u are surjective (resp. injective), also it is said to be constant if p and u are constant.

Definition 2.11.[37] Let $(X,\mathfrak{T}_1,E)$ and $(Y,\mathfrak{T}_2,K)$ be two fuzzy soft topological spaces and $f_{pu}: FSS(X)_E \to FSS(Y)_K$ be a fuzzy soft mapping. Then f_{pu} is called

(1)Fuzzy continuous soft if $f_{pu}^{-1}(g_B) \in \mathfrak{T}_1\ \forall\ (g_B) \in \mathfrak{T}_2$.
(2)Fuzzy open soft if $f_{pu}(g_A) \in \mathfrak{T}_2 \forall\ (g_A) \in \mathfrak{T}_1$.

Theorem 2.1.[4] Let $FSS(X)_E$ and $FSS(Y)_K$ be two families of fuzzy soft sets. For the fuzzy soft function $f_{pu}: FSS(X)_E \to FSS(Y)_K$, the following statements hold,

(a)$f_{pu}^{-1}((g,B)^c) = (f_{pu}^{-1}(g,B))^c \forall\ (g,B) \in FSS(Y)_K$.
(b)$f_{pu}(f_{pu}^{-1}((g,B))) \sqsubseteq (g,B) \forall\ (g,B) \in FSS(Y)_K$. If f_{pu} is surjective, then the equality holds.
(c)$(f,A) \sqsubseteq f_{pu}^{-1}(f_{pu}((f,A))) \forall\ (f,A) \in FSS(X)_E$. If f_{pu} is injective, then the equality holds.
(d)$f_{pu}(\tilde{0}_E) = \tilde{0}_K$, $f_{pu}(\tilde{1}_E) \sqsubseteq \tilde{1}_K$. If f_{pu} is surjective, then the equality holds.
(e)$f_{pu}^{-1}(\tilde{1}_K) = \tilde{1}_E$ and $f_{pu}^{-1}(\tilde{0}_K) = \tilde{0}_E$.
(f)If $(f,A) \sqsubseteq (g,A)$, then $f_{pu}(f,A) \sqsubseteq f_{pu}(g,A)$.
(g)If $(f,B) \sqsubseteq (g,B)$, then $f_{pu}^{-1}(f,B) \sqsubseteq f_{pu}^{-1}(g,B)\ \forall\ (f,B),(g,B) \in FSS(Y)_K$.
(h)$f_{pu}^{-1}(\sqcup_{j\in J}(f,B)_j) = \sqcup_{j\in J} f_{pu}^{-1}(f,B)_j$ and $f_{pu}^{-1}(\sqcap_{j\in J}(f,B)_j) = \sqcap_{j\in J} f_{pu}^{-1}(f,B)_j, \forall\ (f,B)_j \in FSS(Y)_K$.
(I)$f_{pu}(\sqcup_{j\in J}(f,A)_j) = \sqcup_{j\in J} f_{pu}(f,A)_j$ and $f_{pu}(\sqcap_{j\in J}(f,A)_j) \sqsubseteq \sqcap_{j\in J} f_{pu}(f,A)_j\ \forall\ (f,A)_j \in FSS(X)_E$. If f_{pu} is injective, then the equality holds.

Definition 2.12.[29] Let $(X,\mathfrak{T},E)$ be a fuzzy soft topological space. A fuzzy soft separation of $\tilde{1}_E$ is a pair of non null proper fuzzy open soft sets g_B, h_C such that $g_B \sqcap h_C = \tilde{0}_E$ and $\tilde{1}_E = g_B \sqcup h_C$.

Definition 2.13.[29] A fuzzy soft topological space $(X,\mathfrak{T},E)$ is said to be fuzzy soft connected if and only if there is no fuzzy soft separations of $\tilde{X}$. Otherwise, $(X,\mathfrak{T},E)$ is said to be fuzzy soft disconnected space.

Definition 2.14.[17] Let (X,τ,E) be a soft topological space and $F_A \in SS(X)_E$. If $F_A \tilde{\subseteq} int(cl(int(F_A)))$, then F_A is called α-open soft set. We denote the set of all α-open soft sets by $\alpha OS(X,\tau,E)$, or $\alpha OS(X)$ and the set of all α-closed soft sets by $\alpha CS(X,\tau,E)$, or $\alpha CS(X)$.

Definition 2.15.[25] Two fuzzy soft sets f_A and g_B are said to be disjoint, denoted by $f_A \sqcap g_B = \tilde{0}_E$, if $A \cap B = \varphi$ and $\mu^e_{f_A} \cap \mu^e_{g_B} = \overline{0}\ \forall e \in E$.

3 Fuzzy soft α-connected spaces

In this section, we introduce the notions of fuzzy soft α-connectedness in fuzzy soft topological space and examine its basic properties.

Definition 3.1. Let $(X,\mathfrak{T},E)$ be a fuzzy soft topological space and $f_A \in FSS(X)_E$. If $f_A \sqsubseteq Fint(Fcl(Fint(f_A)))$, then f_A is called fuzzy α-open soft set. We denote the set of all fuzzy α-open soft sets by $F\alpha OS(X,\mathfrak{T},E)$, or $FPOS(X)$ and the set of all fuzzy α-closed soft sets by $F\alpha CS(X,\mathfrak{T},E)$, or $F\alpha CS(X)$.

Definition 3.2. Let $(X,\mathfrak{T},E)$ be a fuzzy soft topological space. A fuzzy soft α-separation on $\tilde{1}_E$ is a pair of non null proper fuzzy α-open soft sets f_A, g_B such that $f_A \sqcap g_B = \tilde{0}_E$ and $\tilde{1}_E = f_A \sqcup g_B$.

Definition 3.3. A fuzzy soft topological space $(X,\mathfrak{T},E)$ is said to be fuzzy soft α-connected if and only if there is no fuzzy soft α-separations of $\tilde{1}_E$. Otherwise, $(X,\mathfrak{T},E)$ is said to be fuzzy soft α-disconnected space.

Examples 3.1.

(1)Let $X = \{a,b,c,d\}$, $E = \{e_1,e_2,e_3\}$ and $\mathfrak{T}$ be the discrete fuzzy soft topology on X. Then, $(X,\mathfrak{T},E)$ is not fuzzy soft α-connected.
(2)Let $X = \{a,b,c\}$, $E = \{e_1,e_2\}$ and $\mathfrak{T}$ be the indiscrete fuzzy soft topology on X. Then, $\mathfrak{T}$ is always fuzzy soft α-connected.

Definition 3.4. A fuzzy soft subspace $(Y,\mathfrak{T}_Y,E)$ of fuzzy soft topological space $(X,\mathfrak{T},E)$ is said to be fuzzy α-open soft (resp. α-closed soft, soft α-connected) subspace if $h^Y_E \in F\alpha OS(X)$ (resp. $h^Y_E \in F\alpha CS(X)$, h^Y_E is fuzzy soft α-connected).

Theorem 3.1. Let $(Y,\mathfrak{T}_Y,E)$ be a fuzzy soft semi connected subspace of fuzzy soft topological space $(X,\mathfrak{T},E)$ such that $h^Y_E \sqcap g_A \in FSOS(X)$ $g_A \in FSOS(X)$. If $\tilde{1}_E$ has a fuzzy soft semi separations f_A, g_B, then either $h^Y_E \sqsubseteq f_A$, or $h^Y_E \sqsubseteq g_B$.

Proof. Let $(Y,\mathfrak{T}_Y,E)$ be a fuzzy soft α-connected subspace of fuzzy soft topological space $(X,\mathfrak{T},E)$ such that $h^Y_E \sqcap g_A \in F\alpha OS(X)$ $g_A \in F\alpha OS(X)$. If $\tilde{1}_E$ has a fuzzy soft α-separations f_A, g_B, then either $h^Y_E \sqsubseteq f_A$, or $h^Y_E \sqsubseteq g_B$.

Theorem 3.2. If $(X,\mathfrak{T}_2,E)$ is a fuzzy soft α-connected space and $\mathfrak{T}_1$ is fuzzy soft coarser than $\mathfrak{T}_2$, then $(X,\mathfrak{T}_1,E)$ is also a fuzzy soft α-connected.

Proof. Let f_A, g_B be fuzzy soft α-separation on $(X,\mathfrak{T}_1,E)$. Then, $f_A, g_B \in \mathfrak{T}_1$. Since $\mathfrak{T}_1 \subseteq \mathfrak{T}_2$. Then, $f_A, g_B \in \mathfrak{T}_2$ such that f_A, g_B is fuzzy soft α-separation on $(X,\mathfrak{T}_2,E)$, which is a contradiction with the fuzzy soft α-connectedness of $(X,\mathfrak{T}_2,E)$. Hence, $(X,\mathfrak{T}_1,E)$ is fuzzy soft α-connected.

Remark 3.1 The converse of Theorem 3.2 is not true in general, as shown in the following example.

Example 3.1. Let $X = \{a,b,c\}$, $E = \{e_1,e_2,e_3,e_4\}$ and $A,B \subseteq E$ where $A = \{e_1,e_2\}$ and $B = \{e_3,e_4\}$. Let $\mathfrak{T}_1$ be the indiscrete fuzzy soft topology, then $\mathfrak{T}_1$ is fuzzy soft α-connected, on the other hand, let $\mathfrak{T}_2 = \{\tilde{1}_E, \tilde{0}_E, f_A, g_A, k_B, h_B, s_E, v_E\}$ where $f_A, g_A, k_B, h_B, s_E, v_E$ are fuzzy soft sets over X defined as follows:
$\mu^{e_1}_{f_A} = \{a_1,b_1,c_1\}$, $\mu^{e_2}_{f_A} = \{a_1,b_1,c_1\}$,

$\mu_{g_A}^{e_1} = \{a_{0.2}, b_{0.5}, c_{0.8}\}$, $\mu_{g_A}^{e_2} = \{a_{0.1}, b_{0.6}, c_{0.7}\}$,
$\mu_{k_B}^{e_3} = \{a_1, b_1, c_1\}$, $\mu_{k_B}^{e_4} = \{a_1, b_1, c_1\}$,
$\mu_{h_B}^{e_3} = \{a_{0.5}, b_0, c_{0.3}\}$, $\mu_{h_B}^{e_4} = \{a_1, b_{0.8}, c_{0.3}\}$,
$\mu_{s_E}^{e_1} = \{a_{0.2}, b_{0.5}, c_{0.8}\}$, $\mu_{s_E}^{e_2} = \{a_{0.1}, b_{0.6}, c_{0.7}\}$,
$\mu_{s_E}^{e_3} = \{a_1, b_1, c_1\}$, $\mu_{s_E}^{e_4} = \{a_1, b_1, c_1\}$,
$\mu_{v_E}^{e_1} = \{a_1, b_1, c_1\}$, $\mu_{v_E}^{e_2} = \{a_1, b_1, c_1\}$,
$\mu_{v_E}^{e_3} = \{a_{0.5}, b_0, c_{0.3}\}$, $\mu_{v_E}^{e_4} = \{a_1, b_{0.8}, c_{0.3}\}$,
Then $\mathfrak{T}_2$ defines a fuzzy soft topology on X such that $\mathfrak{T}_1 \subseteq \mathfrak{T}_2$. Now, f_A and k_B are fuzzy α-open soft sets in which form a fuzzy soft α-separation of $(X, \mathfrak{T}_2, E)$ where $f_A \sqcap k_B = \tilde{0}_E$ and $\tilde{1}_E = f_A \sqcup k_B$. Hence, $(X, \mathfrak{T}_2, E)$ is fuzzy soft α-disconnected.

Theorem 3.3. A fuzzy soft subspace $(Y, \mathfrak{T}_Y, E)$ of fuzzy soft α-disconnectedness space $(X, \mathfrak{T}, E)$ is fuzzy soft α-disconnected if $h_E^Y \sqcap g_A \in F\alpha OS(X)\ \forall\, g_A \in F\alpha OS(X)$.

Proof. Let $(Y, \mathfrak{T}_Y, E)$ be fuzzy soft α-connected space. Since $(X, \mathfrak{T}, E)$ is fuzzy soft α-disconnected. Then, there exist fuzzy soft α-separation f_A, g_B on $(X, \mathfrak{T}, E)$. By hypothesis, $f_A \sqcap h_E^Y \in F\alpha OS(X)$, $g_B \sqcap h_E^Y \in F\alpha OS(X)$ and $[g_B \sqcap h_E^Y] \sqcup [f_A \sqcap h_E^Y] = h_E^Y$, which is a contradiction with the fuzzy soft α-connectedness of $(Y, \mathfrak{T}_Y, E)$. Therefore, $(Y, \mathfrak{T}_Y, E)$ is fuzzy soft α-disconnected.

Remark 3.2 A fuzzy soft α-disconnectedness property is not hereditary property in general, as in the following example.

Example 3.2. In Example 3.1, let $Y = \{a, b\} \subseteq X$. We consider the fuzzy soft set h_E^Y over (Y, E) defined as follows:
$\mu_{h_E^Y}^{e_1} = \{a_1, b_1, c_0\}$, $\mu_{h_E^Y}^{e_2} = \{a_1, b_1, c_0\}$, $\mu_{h_E^Y}^{e_3} = \{a_1, b_1, c_0\}$,
$\mu_{h_E^Y}^{e_4} = \{a_1, b_1, c_0\}$.
Then we find $\mathfrak{T}_Y$ as follows, $\mathfrak{T}_Y = \{h_E^Y \sqcap z_E : z_E \in \mathfrak{T}\}$ where
$h_E^Y \sqcap \tilde{0}_E = \tilde{0}_E$, $h_E^Y \sqcap \tilde{1}_E = h_E^Y$, $h_E^Y \sqcap f_A = h_C$, where
$\mu_{h_C}^{e_1} = \{a_1, b_1, c_0\}$, $\mu_{h_C}^{e_2} = \{a_1, b_1, c_0\}$,

$h_E^Y \sqcap g_A = h_W$, where
$\mu_{h_W}^{e_1} = \{a_{0.2}, b_{0.5}, c_0\}$, $\mu_{h_W}^{e_2} = \{a_{0.1}, b_{0.6}, c_0\}$,

$h_E^Y \sqcap k_B = h_R$, where
$\mu_{h_R}^{e_3} = \{a_1, b_1, c_0\}$, $\mu_{h_R}^{e_4} = \{a_1, b_1, c_0\}$,

$h_E^Y \sqcap h_B = h_T$, where
$\mu_{h_T}^{e_3} = \{a_{0.5}, b_0, c_0\}$, $\mu_{h_T}^{e_4} = \{a_1, b_{0.8}, c_0\}$,

$h_E^Y \sqcap s_E = h_P$, where
$\mu_{h_P}^{e_1} = \{a_{0.2}, b_{0.5}, c_0\}$, $\mu_{h_P}^{e_2} = \{a_{0.1}, b_{0.6}, c_0\}$,
$\mu_{h_P}^{e_3} = \{a_1, b_1, c_0\}$, $\mu_{h_P}^{e_4} = \{a_1, b_1, c_0\}$.
Thus, the collection $\mathfrak{T}_Y = \{h_E^Y \sqcap z_E : z_E \in \mathfrak{T}\}$ is a fuzzy soft topology on (Y, E) in which there is no fuzzy soft α-separation on $(Y, \mathfrak{T}_Y, E)$. Therefore, $(Y, \mathfrak{T}_Y, E)$ is fuzzy soft α-connected, although $(X, \mathfrak{T}, E)$ is fuzzy soft α-disconnected as shown in Example 3.1.

Definition 3.5. Let $(X, \mathfrak{T}_1, E)$, $(Y, \mathfrak{T}_2, K)$ be fuzzy soft topological spaces and $f_{pu} : FSS(X)_E \to FSS(Y)_K$ be a fuzzy soft function. Then, the function f_{pu} is called fuzzy α-irresolute soft if $f_{pu}^{-1}(g_B) \in F\alpha OS(X) \forall\, g_B \in F\alpha OS(Y)$.

Theorem 3.4. Let $(X_1, \mathfrak{T}_1, E)$ and $(X_2, \mathfrak{T}_2, K)$ be fuzzy soft topological spaces and $f_{pu} : (X_1, \mathfrak{T}_1, E) \to (X_2, \mathfrak{T}_2, K)$ be a fuzzy α-irresolute surjective soft function. If $(X_1, \mathfrak{T}_1, E)$ is fuzzy soft α-connected, then $(X_2, \mathfrak{T}_2, K)$ is also a fuzzy soft α-connected.

Proof. Let $(X_2, \mathfrak{T}_2, K)$ be a fuzzy soft α-disconnected space. Then, there exist f_A, g_B pair of non null proper fuzzy α-open soft subsets of $\tilde{1}_K$ such that $f_A \sqcap g_B = \tilde{0}_K$ and $\tilde{1}_K = f_A \sqcup g_B$. Since f_{pu} is fuzzy α-irresolute soft function, then $f_{pu}^{-1}(f_A), f_{pu}^{-1}(g_B)$ are pair of non null proper fuzzy α-open soft subsets of $\tilde{1}_E$ such that $f_{pu}^{-1}(f_A) \sqcap f_{pu}^{-1}(g_B) = f_{pu}^{-1}(f_A \sqcap g_B) = f_{pu}^{-1}(\tilde{0}_K) = \tilde{0}_E$ and $f_{pu}^{-1}(f_A) \sqcup f_{pu}^{-1}(g_B) = f_{pu}^{-1}(f_A \sqcup g_B) = f_{pu}^{-1}(\tilde{1}_K) = \tilde{1}_E$ from Theorem 2.1. This means that, $f_{pu}^{-1}(f_A), f_{pu}^{-1}(g_B)$ forms a fuzzy soft α-separation of $\tilde{1}_E$, which is a contradiction with the fuzzy soft α-connectedness of $(X_1, \mathfrak{T}_1, E)$. Therefore, $(X_2, \mathfrak{T}_2, K)$ is fuzzy soft α-connected.

4 Fuzzy soft α-s-connectedness

In this section, we introduce the notions of fuzzy soft α-separated sets and use it to introduce the notions of fuzzy α-s-connectedness in fuzzy soft topological spaces and study its basic properties.

Definition 4.1. A non null fuzzy soft subsets f_A, g_B of fuzzy soft topological space $(X, \mathfrak{T}, E)$ are said to be fuzzy soft α-separated sets if $F\alpha cl(f_A) \sqcap g_B = F\alpha cl(g_B) \sqcap f_A = \tilde{0}_E$.

Theorem 4.1. Let $f_A \sqsubseteq g_B$, $h_C \sqsubseteq k_D$ and g_B, k_D are soft fuzzy soft α-separated subsets of fuzzy soft topological space $(X, \mathfrak{T}, E)$. Then, f_A, h_C are fuzzy soft α-separated sets.

Proof. Let $f_A \sqsubseteq g_B$, then $F\alpha cl(f_A) \sqsubseteq F\alpha cl(g_B)$. It follows that, $F\alpha cl(f_A) \sqcap h_C \sqsubseteq F\alpha cl(f_A) \sqcap k_D \sqsubseteq F\alpha cl(g_B) \sqcap k_D = \tilde{0}_E$. Also, since $h_C \sqsubseteq k_D$. Then, $F\alpha cl(h_C) \sqsubseteq F\alpha cl(k_D)$. Hence, $f_A \sqcap F\alpha cl(h_C) \sqsubseteq F\alpha cl(k_D) \sqcap g_B = \tilde{0}_E$. Thus, f_A, h_C are fuzzy soft α-separated sets.

Theorem 4.2. Two fuzzy α-closed soft subsets of fuzzy soft topological space $(X, \mathfrak{T}, E)$ are fuzzy soft α-separated sets if and only if they are disjoint.

Proof. Let f_A, g_B are fuzzy soft α-separated sets. Then, $F\alpha cl(g_B) \sqcap f_A = g_B \sqcap F\alpha cl(f_A) = \tilde{0}_E$. Since f_A, g_B are fuzzy α-closed soft sets. Then, $f_A \sqcap g_B = \tilde{0}_E$. Conversely, let f_A, g_B are disjoint fuzzy α-closed soft sets. Then, $g_B \sqcap F\alpha cl(f_A) = f_A \sqcap g_B = \tilde{0}_E$ and $F\alpha cl(g_B) \sqcap f_A = f_A \sqcap g_B = \tilde{0}_E$. It follows that, f_A, g_B are fuzzy soft α-separated sets.

Definition 4.2. A fuzzy soft topological space $(X, \mathfrak{T}, E)$ is said to be fuzzy soft α-s-connected if and only if $\tilde{1}_E$ can not expressed as the fuzzy soft union of two fuzzy soft α-separated sets in $(X, \mathfrak{T}, E)$.

Theorem 4.3. Let $(Z,\mathfrak{T}_Z,E)$ be a fuzzy soft subspace of fuzzy soft topological space $(X,\mathfrak{T},E)$ and f_A, $g_B \sqsubseteq z_E \sqsubseteq \tilde{1}_E$. Then, f_A and g_B are fuzzy soft α-separated on $\mathfrak{T}_Z$ if and only if f_A and g_B are fuzzy soft α-separated on $\mathfrak{T}$, where $\mathfrak{T}_Z$ is the fuzzy soft subspace for z_E.

Proof. Assume that f_A and g_B are fuzzy soft α-separated on $\mathfrak{T}_Z \Leftrightarrow F\alpha cl_{\mathfrak{T}_Z}(f_A) \sqcap g_B = \tilde{\phi}$ and $f_A \sqcap F\alpha cl_{\mathfrak{T}_Z}(g_B) = \tilde{0}_E \Leftrightarrow [F\alpha cl_{\mathfrak{T}}(f_A) \sqcap z_E] \sqcap g_B = F\alpha cl_{\mathfrak{T}}(f_A) \sqcap g_B = \tilde{0}_E$ and $[F\alpha cl_{\mathfrak{T}}(g_B) \sqcap z_E] \sqcap f_A = F\alpha cl_{\mathfrak{T}}(g_B) \sqcap f_A = \tilde{0}_E \Leftrightarrow f_A$ and g_B are fuzzy soft α-separated sets on $\mathfrak{T}$.

Theorem 4.4. Let z_E be a fuzzy soft subset of fuzzy soft topological space $(X,\mathfrak{T},E)$. Then, z_E is fuzzy soft α-s-connected w.r.t $(X,\mathfrak{T},E)$ if and only if z_E is fuzzy soft α-s-connected w.r.t $(Z,\mathfrak{T}_Z,E)$.

Proof. Suppose that z_E is not fuzzy soft α-s-connected w.r.t $(Z,\mathfrak{T}_Z,E)$. Then, $z_E = f_{1A} \sqcup f_{2B}$, where f_{1A} and f_{2B} are fuzzy soft α-separated sets on $\mathfrak{T}_Z \Leftrightarrow z_E = f_{1A} \sqcup f_{2B}$, where f_{1A} and f_{2B} are fuzzy soft α-separated on $\mathfrak{T}$ from Theorem 4.3 $\Leftrightarrow z_E$ is not fuzzy soft α-s-connected w.r.t $(X,\mathfrak{T},E)$.

Theorem 4.5. Let $(Z,\mathfrak{T}_Z,E)$ be a fuzzy soft α-s-connected subspace of fuzzy soft topological space $(X,\mathfrak{T},E)$ and f_A, g_B be fuzzy soft α-separated of $\tilde{1}_E$ with $z_E \sqsubseteq f_A \sqcup g_B$, then either $z_E \sqsubseteq f_A$, or $z_E \sqsubseteq g_B$.

Proof. Let $z_E \sqsubseteq f_A \sqcup g_B$ for some fuzzy soft α-separated subsets f_A, g_B of $\tilde{1}_E$. Since $z_E = (z_E \sqcap f_A) \sqcup (z_E \sqcap g_B)$. Then,
$(z_E \sqcap f_A) \sqcap F\alpha cl_{\mathfrak{T}}(z_E \sqcap g_B) \sqsubseteq (f_A \sqcap F\alpha cl_{\mathfrak{T}} g_B) = \tilde{0}_E$. Also,
$F\alpha cl_{\mathfrak{T}}(z_E \sqcap f_A) \sqcap (z_E \sqcap g_B) \sqsubseteq F\alpha cl_{\mathfrak{T}}(f_A) \sqcap g_B = \tilde{0}_E$. Since $(Z,\mathfrak{T}_Z,E)$ is fuzzy soft α-s-connected. Thus, either $z_E \sqcap f_A = \tilde{0}_E$ or $z_E \sqcap g_B = \tilde{0}_E$. It follows that, $z_E = z_E \sqcap f_A$ or $z_E = z_E \sqcap g_B$. This implies that, $z_E \sqsubseteq f_A$ or $z_E \sqsubseteq g_B$.

Theorem 4.6. Let $(Z,\mathfrak{T}_Z,N)$ and $(Y,\mathfrak{T}_Y,M)$ be fuzzy soft α-s-connected subspaces of fuzzy soft topological space $(X,\mathfrak{T},E)$ such that none of them is fuzzy soft α-separated. Then, $z_N \sqcup y_M$ is fuzzy soft α-s-connected.

Proof. Let $(Z,\mathfrak{T}_Z,N)$ and $(Y,\mathfrak{T}_Y,M)$ be fuzzy soft α-s-connected subspaces of $\tilde{1}_E$ such that $z_N \sqcup y_M$ is not fuzzy soft α-s-connected. Then, there exist two non null fuzzy soft α-separated sets k_D and h_C of $\tilde{1}_E$ such that $z_N \sqcup y_M = k_D \sqcup h_C$. Since z_N, y_M are fuzzy soft α-s-connected, $z_N, y_M \sqsubseteq z_N \sqcup f_A = k_D \sqcup h_C$. By Theorem 4.5, either $z_N \sqsubseteq k_D$ or $z_N \sqsubseteq h_C$, also, either $y_M \sqsubseteq k_D$ or $y_M \sqsubseteq h_C$. If $z_N \sqsubseteq k_D$ or $z_N \sqsubseteq h_C$. Then, $z_N \sqcap h_C \sqsubseteq k_D \sqcap h_C = \tilde{0}_E$ or $z_N \sqcap k_D \sqsubseteq z_N \sqcap k_D = \tilde{0}_E$. Therefore, $[z_N \sqcup y_M] \sqcap k_D = [z_N \sqcap k_D] \sqcup [y_M \sqcup k_D] = [y_M \sqcap k_D] \sqcup \tilde{0}_E = y_M \sqcap k_D = y_M$ since $y_M \sqsubseteq k_D$. Similarly, if $y_M \sqsubseteq k_D$ or $y_M \sqsubseteq h_C$. we get $[z_N \sqcup y_M] \sqcap h_C = z_N$.

Now, $[(z_N \sqcup y_M) \sqcap h_C] \sqcap F\alpha cl[(z_N \sqcup y_M) \sqcap k_D] \sqsubseteq [(z_N \sqcup y_M) \sqcap h_C] \sqcap [F\alpha cl(z_N \sqcup y_M) \sqcap F\alpha cl(k_D)] = [z_N \sqcup y_M] \sqcap [h_C \sqcap F\alpha cl(k_D)] = \tilde{0}_E$ and $F\alpha cl[(z_N \sqcup y_M) \sqcap h_C] \sqcap [(z_N \sqcup y_M) \sqcap k_D] \sqsubseteq [F\alpha cl(z_N \sqcup y_M) \sqcap F\alpha cl(h_C)] \sqcap [(z_N \sqcup y_M) \sqcap k_D] = [z_N \sqcup y_M] \sqcap [F\alpha cl(h_C) \sqcap k_D] = \tilde{0}_E$. It follows that, $[z_N \sqcup y_M] \sqcap k_D = z_N$ and $[z_N \sqcup y_M] \sqcap h_C = y_M$ are fuzzy soft α-separated, which is a contradiction. Hence, $z_N \sqcup y_M$ is fuzzy soft α-s-connected.

Theorem 4.7. Let $(Z,\mathfrak{T}_Z,N)$ be a fuzzy soft α-s-connected subspace of fuzzy soft topological space $(X,\mathfrak{T},E)$ and $S_M \in FSS(X)_A$. If $z_N \sqsubseteq S_M \sqsubseteq F\alpha cl(z_N)$. Then, $(S,\mathfrak{T}_S,M)$ is fuzzy soft α-s-connected subspace of $(X,\mathfrak{T},E)$.

Proof. Assume that $(S,\mathfrak{T}_S,M)$ is not fuzzy soft α-s-connected subspace of $(X,\mathfrak{T},E)$. Then, there exist fuzzy soft α-separated sets f_A and g_B on $\mathfrak{T}$ such that $S_M = f_A \sqcup g_B$. So, we have z_N is fuzzy soft α-s-connected subset of fuzzy soft α-s-disconnected space. By Theorem 4.5, either $z_N \sqsubseteq f_A$ or $z_N \sqsubseteq g_B$. If $z_N \sqsubseteq f_A$. Then, $F\alpha cl(z_N) \sqsubseteq F\alpha cl(f_A)$. It follows $F\alpha cl(z_N) \sqcap g_B \sqsubseteq F\alpha cl(f_A) \sqcap g_B = \tilde{0}_E$. Hence, $g_B = F\alpha cl(z_N) \sqcap g_B = \tilde{0}_E$ which is a contradiction. If $z_N \sqsubseteq g_B$. By a similar way, we can get $f_A = \tilde{0}_E$, which is a contradiction. Hence, $(S,\mathfrak{T}_S,M)$ is fuzzy soft α-s-connected subspace of $(X,\mathfrak{T},E)$.

Corollary 4.1. If $(Z,\mathfrak{T}_Z,N)$ is fuzzy soft α-s-connected subspace of fuzzy soft topological space $(X,\mathfrak{T},E)$. Then, $F\alpha cl(z_N)$ is fuzzy soft α-s-connected.

Proof. It obvious from Theorem 4.7.

Theorem 4.8. If for all pair of distinct fuzzy soft point f_e, g_e, there exists a fuzzy soft α-s-connected set $z_N \sqsubseteq \tilde{1}_E$ with $f_e, g_e \tilde{\in} z_N$, then $\tilde{1}_E$ is fuzzy soft α-s-connected.

Proof. Suppose that $\tilde{1}$ is fuzzy soft α-s-disconnected. Then, $\tilde{1}_E = f_A \sqcup g_B$, where f_A, g_B are fuzzy soft α-separated sets. It follows $f_A \sqcap g_B = \tilde{0}_E$. So, $\exists\ f_e \tilde{\in} f_A$ and $g_e \tilde{\in} g_B$. Since $f_A \sqcap g_B = \tilde{0}_E$. Then, f_e, g_e are distinct fuzzy soft point in $\tilde{1}_E$. By hypothesis, there exists a fuzzy soft α-s-connected set z_N such that $f_e, g_e \tilde{\in} z_N \sqsubseteq \tilde{1}_E$ and $f_e, g_e \tilde{\in} z_N$. Moreover, we have z_N is fuzzy soft α-s-connected subset of a a fuzzy soft α-s-disconnected space. It follows by Theorem 4.5, either $z_N \sqsubseteq f_A$ or $z_N \sqsubseteq g_B$ and both cases is a contradiction with the hypothesis. Therefore, $\tilde{1}_E$ is fuzzy soft α-s-connected.

Theorem 4.9. Let $\{(Z_j,\mathfrak{T}_{Z_j},N) : j \in J\}$ be a non null family of fuzzy soft α-s-connected subspaces of fuzzy soft topological space $(X,\mathfrak{T},E)$. If $\sqcap_{j\in J}(z_j,N) \neq \tilde{0}_E$, then $(\sqcup_{j\in J}Z_j, \mathfrak{T}_{\sqcup_{j\in J}Z_j}, N)$ is also a fuzzy soft α-s-connected fuzzy subspace of $(X,\mathfrak{T},E)$.

Proof. Assume that $(Z,\mathfrak{T}_Z,N) = (\sqcup_{j\in J}Z_j, \mathfrak{T}_{\sqcup_{j\in J}Z_j}, N)$ is fuzzy soft α-s-disconnected. Then, $z_N = f_A \sqcup g_B$ for some fuzzy soft α-separated subsets f_A, g_B of $\tilde{1}_E$. Since $\sqcap_{j\in J}(z_j,N) \neq \tilde{0}_E$. Then, $\exists\ f_e \tilde{\in} \sqcap_{j\in J}(z,N)_j$. It follows that, $f_e \tilde{\in} z_N$. So, either $f_e \tilde{\in} f_A$ or $f_e \tilde{\in} g_B$. Suppose that $f_e \tilde{\in} f_A$. Since $f_e \tilde{\in} (z_j,N)\ \forall j \in J$ and $(z_j,N) \sqsubseteq z_N$. So, we have (z_j,N) is fuzzy soft α-s-connected subset of fuzzy soft α-s-disconnected set z_N. Then, by Theorem 4.5, either $(z_j,N) \sqsubseteq f_A$ or $(z_j,N) \sqsubseteq g_B\ \forall j \in J$. If $(z_j,N) \sqsubseteq f_A\ \forall j \in J$. Then, $z_N \sqsubseteq f_A$. This implies that, $g_B = \tilde{0}_E$, which is a contradiction. Also, if $(z_j,N) \sqsubseteq g_B\ \forall j \in J$. Also, if $f_e \tilde{\in} g_B$, by a similar way, we get $f_A = \tilde{0}_E$, which is a contradiction. Therefore,

$(Z,\mathfrak{T}_Z,N) = (\sqcup_{j\in J}Z_j,\mathfrak{T}_{\sqcup_{j\in J}Z_j},N)$ is fuzzy soft α-*s*-connected.

Theorem 4.10. Let $\{(Z_j,\mathfrak{T}_{Z_j},N) : j\in J\}$ be a family of fuzzy soft α-*s*-connected subspaces of fuzzy soft topological space $(X,\mathfrak{T},E)$ such that one of the members of the family intersects every other members, then $(\sqcup_{j\in J}Z_j,\mathfrak{T}_{\sqcup_{j\in J}Z_j},N)$ is fuzzy subspace of $(X,\mathfrak{T},E)$.

Proof. Let $(Z,\mathfrak{T}_Z,N) = (\sqcup_{j\in J}Z_j,\mathfrak{T}_{\sqcup_{j\in J}Z_j},N)$ and $(z,N)_{jo} \in \{(z_j,N) : j\in J\}$ such that $(z,N)_{jo}\sqcap(z_j,N)\neq\tilde{0}_E\ \ \forall j\in J$. Then, $(z,N)_{jo}\sqcup(z_j,N)$ is fuzzy soft α-*s*-connected $\forall j\in J$ by Theorem 4.6. Hence, the collection $\{(z,N)_{jo}\sqcup(z_j,N) : j\in J\}$ is a collection of fuzzy soft α-*s*-connected subsets of $\tilde{1}$, which having a non null fuzzy soft intersection. Therefore, $(Z,\mathfrak{T}_Z,N) = (\sqcup_{j\in J}Z_j,\mathfrak{T}_{\sqcup_{j\in J}Z_j},N)$ is fuzzy soft α-*s*-connected subspace of $(X,\mathfrak{T},E)$ by Theorem 4.6.

Theorem 4.11. Let $(X_1,\mathfrak{T}_1,E)$ and $(X_2,\mathfrak{T}_2,K)$ be fuzzy soft topological spaces and $f_{pu}:(X_1,\mathfrak{T}_1,E)\to(X_2,\mathfrak{T}_2,K)$ be a fuzzy α-irresolute surjective soft function. If $(X_1,\mathfrak{T}_1,E)$ is fuzzy soft α-*s*-connected, then $(X_2,\mathfrak{T}_2,K)$ is also a fuzzy soft α-*s*-connected.

Proof. Let $(X_2,\mathfrak{T}_2,K)$ be fuzzy soft α-disconnected space. Then, there exist f_A,g_B pair of non null proper fuzzy soft α-separated sets such that $\tilde{1}_K = f_A\sqcup g_B$, $F\alpha cl(f_A)\sqcap g_B = F\alpha cl(g_B)\sqcap f_A = \tilde{0}_E$. Since f_{pu} is fuzzy α-irresolute soft function, then $f_{pu}^{-1}(f_A), f_{pu}^{-1}(g_B)$ are pair of non null proper fuzzy α-open soft subsets of $\tilde{1}_E$ such that $F\alpha cl(f_{pu}^{-1}(f_A))\sqcap f_{pu}^{-1}(g_B)\sqsubseteq f_{pu}^{-1}(F\alpha cl(f_A))\sqcap f_{pu}^{-1}(g_B) = f_{pu}^{-1}(f_A\sqcap g_B) = f_{pu}^{-1}(\tilde{0}_K) = \tilde{0}_E$, $f_{pu}^{-1}(f_A)\sqcap F\alpha cl(f_{pu}^{-1}(g_B))\sqsubseteq f_{pu}^{-1}(f_A)\sqcap f_{pu}^{-1}(F\alpha cl(g_B)) = f_{pu}^{-1}(f_A\sqcap g_B) = f_{pu}^{-1}(\tilde{0}_K) = \tilde{0}_E$ and $f_{pu}^{-1}(f_A)\sqcup f_{pu}^{-1}(g_B)) = f_{pu}^{-1}(f_A\sqcup g_B) = f_{pu}^{-1}(\tilde{1}_K) = \tilde{1}_E$ from Theorem 2.1 and [[25], Theorem 4.2]. This means that, $f_{pu}^{-1}(f_A), f_{pu}^{-1}(g_B)$ are pair of non null proper fuzzy soft α-separated sets of $\tilde{1}_E$, which is a contradiction of the fuzzy soft α-*s*-connectedness of $(X_1,\mathfrak{T}_1,E)$. Therefore, $(X_2,\mathfrak{T}_2,K)$ is fuzzy soft α-*s*-connected.

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5 Conclusion

Since the authors introduced topological structures on fuzzy soft sets [8,14,42], so the fuzzy soft topological properties, which introduced by Mahanta et al.[29], is generalized here to the fuzzy α-soft sets which will be useful in the fuzzy systems. Because there exists compact connections between soft sets and information systems [38,44], we may use the results deducted from the studies on fuzzy soft topological space to improve these kinds of connections. We hope that the findings in this paper will help researcher enhance and promote the further study on fuzzy soft topology to carry out a general framework for their applications in practical life.

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References

[1] A. M. Abd El-latif, Characterizations of fuzzy soft pre separation axioms, Journal of New Theory, **7**, 47-63 (2015).
[2] A. M. Abd El-latif, fuzzy soft separation axioms based on fuzzy β-open soft sets, Ann. Fuzzy Math. Inform, vol(x) (2015).
[3] A. M. Abd El-latif and Rodyna A. Hosny, Fuzzy soft pre-connected properties in fuzzy soft topological spaces, South Asian J. Math., **5 (5)**, 202-213 (2015).
[4] B. Ahmad and A. Kharal, Mappings of fuzzy soft classes, Adv. Fuzzy Syst. (2009), Art. ID 407890, 6 pp.
[5] B. Ahmad and A. Kharal, On fuzzy soft sets, Adv. Fuzzy Syst. 2009, Art. ID 586507, 6 pp.
[6] H. Aktas and N. Cagman, Soft sets and soft groups, Information Sciences, **1 (77)**, 2726-2735 (2007).
[7] M. I. Ali, F. Feng, X. Liu, W. K. Min and M. Shabir, On some new operations in soft set theory, Computer and Math. with appl., **57**, 1547-1553 (2009).
[8] Bakir Tanay and M. Burcl Kandemir, Topological structure of fuzzy soft sets, Computer and mathematics with applications **(61)**, 2952-2957 (2011).
[9] Bin Chen, Some Local Properties of Soft Semi-Open Sets, Hindawi Publishing Corporation, (1-6) 2013.
[10] N. agman, F.itak and S. Enginoglu, Fuzzy parameterized fuzzy soft set theory and its applications, Turkish Journal of Fuzzy Systems, **1 (1)**, 21-35 (2010).
[11] N.agman and S. Enginoglu, Soft set theory and uni-Fint decision making, European Journal of Operational Research **207**, 848-855 (2010).
[12] C. L. Change, Fuzzy topological spaces, J. Math. Anal. Appl., **24**, 182-190 (1968).
[13] S. A. El-Sheikh and A. M. Abd El-latif, Characterization of b-open soft sets in soft topological spaces, Journal of New Theory, **2**, 8-18 (2015) .
[14] Jianyu Xiao, Minming Tong, Qi Fan and Su Xiao, Generalization of Belief and Plausibility Functions to Fuzzy Sets, Applied Mathematics Information Sciences, **6**, 697-70 (2012).
[15] A. Kandil, O. A. E. Tantawy, S. A. El-Sheikh, A. M. Abd El-latif and S. El-Sayed, Fuzzy soft connectedness based on fuzzy β-open soft sets, Journal of Mathematics and Computer Applications Research, **2 (1)**, 37-46 (2015).
[16] A.Kandil, O. A. E. Tantawy, S. A. El-Sheikh and A. M. Abd El-latif, Fuzzy soft semi connected properties in fuzzy soft topological spaces, Math. Sci. Lett., **4 (2)**, 171-179 (2015).

[17] A.Kandil, O. A. E. Tantawy, S. A. El-Sheikh and A. M. Abd El-latif, γ-operation and decompositions of some forms of soft continuity in soft topological spaces, Ann. Fuzzy Math. Inform., **7**, 181-196 (2014).
[18] A.Kandil, O. A. E. Tantawy, S. A. El-Sheikh and A. M. Abd El-latif, γ-operation and decompositions of some forms of soft continuity of soft topological spaces via soft ideal, Ann. Fuzzy Math. Inform., **9 (3)**, 385-402 (2015).
[19] A.Kandil, O. A. E. Tantawy, S. A. El-Sheikh and A. M. Abd El-latif, Soft connectedness via soft ideals, Journal of New Results in Science, **4**, 90-108 (2014).
[20] A. Kandil, O. A. E. Tantawy, S. A. El-Sheikh and A. M. Abd El-latif, Soft ideal theory, Soft local function and generated soft topological spaces, Appl. Math. Inf. Sci., **8 (4)**, 1595-1603. (2014).
[21] A.Kandil, O. A. E. Tantawy, S. A. El-Sheikh and A. M. Abd El-latif, Soft regularity and normality based on semi open soft sets and soft ideals, Appl. Math. Inf. Sci. Lett., **3 (2)**, 47-55 (2015).
[22] A. Kandil, O. A. E. Tantawy, S. A. El-Sheikh and A. M. Abd El-latif, Soft semi compactness via soft ideals, Appl. Math. Inf. Sci., **8 (5)**, 2297-2306 (2014).
[23] A.Kandil, O. A. E. Tantawy, S. A. El-Sheikh and A. M. Abd El-latif, Soft semi (quasi) Hausdorff spaces via soft ideals, South Asian J. Math., **4** (6), 265-284 (2014).
[24] A.Kandil, O. A. E. Tantawy, S. A. El-Sheikh and A. M. Abd El-latif, Soft semi separation axioms and irresolute soft functions, Ann. Fuzzy Math. Inform., **8 (2)**, 305-318 (2014).
[25] A.Kandil, O. A. E. Tantawy, S. A. El-Sheikh and A. M. Abd El-latif, Some fuzzy soft topological properties based on fuzzy semi open soft sets, South Asian J. Math., **4 (4)**, 154-169 (2014).
[26] A. Kandil, O. A. E. Tantawy, S. A. El-Sheikh and A. M. Abd El-latif, Supra generalized closed soft sets with respect to an soft ideal in supra soft topological spaces, Appl. Math. Inf. Sci., **8 (4)**, 1731-1740 (2014).
[27] D. V. Kovkov, V. M. Kolbanov and D. A. Molodtsov, Soft sets theory-based optimization, Journal of Computer and Systems Sciences Finternational, **46 (6)**, 872-880 (2007).
[28] F. Lin, Soft connected spaces and soft paracompact spaces, International Journal of Mathematical Science and Engineering, **7 (2)**, 1-7 (2013).
[29] J. Mahanta and P.K. Das, Results on fuzzy soft topological spaces, arXiv:1203.0634v1,2012.
[30] P. K. Maji, R. Biswas and A. R. Roy, Fuzzy soft sets, Journal of Fuzzy Mathematics, **9 (3)**, 589-602 (2001).
[31] P. K. Maji, R. Biswas and A. R. Roy, intuitionistic fuzzy soft sets, Journal of Fuzzy Mathematics, **9 (3)**, 677-691 (2001).
[32] P. K. Maji,R. Biswas and A. R. Roy, Soft set theory, Computer and Math. with appl., **45**, 555-562 (2003).
[33] P. Majumdar and S. K. Samanta, Generalised fuzzy soft sets, Computer and Math. with appl., **59**, 1425-1432 (2010).
[34] D. A. Molodtsov, Soft set theory-firs tresults, Computer and Math. with appl., **37**, 19-31 (1999).
[35] D.Molodtsov, V. Y. Leonov and D. V. Kovkov, Soft sets technique and its application, Nechetkie Sistemy i Myagkie Vychisleniya, **1 (1)**, 8-39 (2006).
[36] A. Mukherjee and S. B. Chakraborty, On Fintuitionistic fuzzy soft relations, Bulletin of Kerala Mathematics Association, **5 (1)**, 35-42 (2008).
[37] B. Pazar Varol and H. Aygun, Fuzzy soft topology, Hacettepe Journal of Mathematics and Statistics, **41 (3)**, 407-419 (2012).
[38] D. Pei and D. Miao, From soft sets to information systems, in: X. Hu, Q. Liu, A. Skowron, T. Y. Lin, R. R. Yager, B. Zhang (Eds.), Proceedings of Granular Computing, in: IEEE, vol.2, (2005), pp. 617-621.
[39] S. Roy and T. K. Samanta, A note on fuzzy soft topological spaces, Ann. Fuzzy Math. Inform., **3 (2)**, 305-311 (2012).
[40] S. Roy and T. K. Samanta, An introduction to open and closed sets on fuzzy topological spaces, Ann. Fuzzy Math. Inform., **6 (2)**, 425-431 (2012).
[41] M. Shabir and M. Naz, On soft topological spaces, Comput. Math. Appl., **61**, 1786-1799 (2011).
[42] B. Tanay and M. B. Kandemir, Topological structure of fuzzy soft sets, Computer and Math. with appl., **61**, 412-418 (2011).
[43] Won Keun Min, A note on soft topological spaces, Computer and Math. with appl., **62**, 3524-3528 (2011).
[44] Z. Xiao, L. Chen, B. Zhong and S. Ye, Recognition for soft information based on the theory of soft sets, in: J. Chen (Ed.), Proceedings of ICSSSM-05, vol. 2, IEEE, (2005), pp. 1104-1106.
[45] Yong Chan Kim and Jung Mi Ko, Fuzzy G-closure operators, commun Korean Math. Soc., **18 (2)**, 325-340 (2008).
[46] L. A. Zadeh, Fuzzy sets, Information and Control, **8**, 338-353 (1965).
[47] Y. Zou and Z. Xiao, Data analysis approaches of soft sets under incomplete information, Knowledge-Based Systems, **21**, 941-945 (2008).

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