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The Reduction of the Exceptional Groups of String Theory and the Standard Model

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Abstract: The phenomenology related to the exceptional groups of string theory will be reviewed. A known success of the E_6 model is an accurate prediction of the Weinberg angle. Spontaneous symmetry breaking produces scalar and fermion fields that do not belong to the lepton and quark multiplets, and therefore, a description of the standard model is likely to be derived from a group of less dimension. A comparison with theories that contain compact groups, which are subgroups of the ten-dimensional Lorentz group, is given. Following the reduction of a twelve-dimensional theory, governing the ten-dimensional superstring and heterotic string effective actions in ten dimensions, over the coset manifold $\frac{G_2\times SU(2)\times U(1)}{SU(3)\times U(1)'\times U(1)''}$, or eleven-dimesional supergravity over the compact space M^{klm}, the renormalization group flow of the couplings is found to give the approximate value of $sin^2\theta_W$ only for a certain ratio of the hypercharge to nonabelian gauge couplings, which is found to require modification at supersymmetric scales. The isometry groups of these coset spaces arise from geometrical considerations, while a unique connection with the larger exceptional group is introduced through the intersection form of the manifold in four dimensions.

Keywords: exceptional groups, coset space, particle spectrum, renormalization group flow

1 Introduction

Anomaly cancellation at leading order in the powers of the curvature in heterotic string theory can be achieved initially through the introduction of the groups $SO(32)$ or $E_8\times E_8$. There are other anomalies that arise at the next-to-leading order, and the phenomenology of theories with a residual symmetry may be determined. After the identification of one of the factors of E_8 with gravitational connection, the other factor of E_8 may be reduced to E_6 on compact spaces with SU(3) holonomy, preserving $N=2$ supersymmetry [1]. An E_6 gauge symmetry also would be achieved by the dimensional reduction of the heterotic string effective action over the coset space $G_2/SU(3)$ [2].

Although the dimension of this group is larger than the maximum dimension of groups with an effective pointwise action on a four-dimensional manifold [3], its phenomenological viability is verified in the unique theoretical prediction of the value of the Weinberg angle [4]. This grand unified theory, however, contains many symmetry breaking patterns which yield massive scalar fields. A large number of experiments have been conducted to establish the Higgs boson mass for the electroweak model [5] , and there is almost no evidence of additional massive Higgs scalar fields.

A check of the matter multiplets the reduction of the E_6 supergravity in ten dimensions to four dimensions yields three massless scalar fields [6]. The existence of these massless fields has not been confirmed. The fermions produced also do not match the lepton and quark multiplets of the standard model. There are too many particles arising from the symmetry breaking of this large group. A description of the elementary particle interactions, however, does exist in a model derived from the Clifford algebra with a division algebra module that is a direct sum of a tensor product of $\mathbb{C}$, $\mathbb{H}$ and $\mathbb{O}$. The Clifford algebra corresponding to this tensor product is $\mathbb{R}_{1,9}$ and the space of products of two elements is the Lorentz group $SO(1,9)$ [7].

The problem of determining the symmetries relevant to elementary particle physics then may be considered. The Lorentz group this symmetry of Type II superstrings, where exceptional groups are not necessary. The

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reduction of this symmetry to subgroups coinciding with the standard model gauge groups also exists. By requiring invariance of certain elements and idempotent conditions on the representation of the algebra, a projection to known gauge groups of the standard model is derived [7].

2 Geometry and the Allowed Groups

The location of a particle may be distinguished from the remainder of space-time. However, since it is a point particle, its location may be included in a continuous manifold, and the functional space on the complement can be completed from a dense subset of that on the union, which is necessary for the a consistent limit of a quantum theory. The conditions on a manifold, therefore, will be relevant for a point particle, and the gauge group action could be interpreted as a localization of a transitive group action on a manifold at a point. Specifically, the admission of the effective action of a compact group of dimension less than ten in four dimensions is equivalent to a limit on the dimension of the gauge groups acting at the location of a point particle. The symmetry groups of point-particle limits of quantum theories, therefore, will be required to satisfy this bound.

It has been proven that the allowed effective compact group actions on a four-dimensional manifold have dimensions less than or equal to 10 [3]. The gauge groups of the standard model all satisfy the inequality, whereas the dimensions of the exceptional groups exceed this bound. The potentials of a gauge theory may be identified with the components of a vector field in the vertical subspace of a bundle. A connection in a fibre bundle is given by a choice of the horizontal subspace of the total space of the bundle at each point on the base manifold. The tangent bundle of a four-manifold M that may be approximated in a local neighbourhood by a submanifold of S^4 has a subbundle $T\Sigma \times \mathbb{R}$, where Σ is a hypersurface in M, which can be described by a Pfaffian system of differential equations that have a G_2 symmetry [8]. The maximal dimension of a group acting effectively on the the bundle $T\Sigma \times \mathbb{R}$ is 28.

An analogous bound for the maximal dimension of a pointwise effective compact group on a ten-dimensional manifold would be 55. While G_2, F_4 and $SO(1,9)$ satisfy this inequality, the dimension of E_6 may be considered to be too large for a pointwise effective action to be supported by a ten-dimensional manifold [**?**]. With the inclusion of tangent vectors, the dimension of the natural bundle on the manifold is increased to 190. The point-particle limit of gauge theories in ten dimensions then would allow $E_6 \times E_8$ and not $E_8 \times E_8$.

3 The Phenomenological Gauge Groups and the E_6 Symmetry

Supersymmetric theories with an E_6 gauge symmetry have been investigated with regard to phenomenological viability. Given a choice of the intermediate gauge group in the pattern of symmetry breaking, which may involve more than one scale [10], the number of multiplets of the reduced groups can be evaluated from the initial **27** representations of E_6.

It has been demonstrated that an asymmetric orbifold in heterotic string theory yields a different number of fermion and anti-fermion generations [11] , which is not physically realistic. Different partition functions with an E_6 symmetry can be constructed, although these models are not viable because certain gauge symmetries are not included [12].

It follows that the theories with E_6 symmetry and supersymmetry may not form an adequate basis for phenomenology. The symmetry breaking pattern of E_6 can be described without supersymmetry because the source of the symmetry is not necessarily a supersymmetric field theory. Instead, it may be included in the isometry group of the subbundle of the tangent bundle to a ten-dimensional manifold and could arise from the intersection form of infinite-genus surfaces with nonsmooth boundaries [11].

The preference for a theory which does not have a general E_6 invariance in four dimensions follows from elementary particle phenomenology. Instead, it would be sufficient to have a viable model based on symmetries that can be reduced to the standard model together with a mechanism for introducing the larger exceptional group symmetry for a specific parameter such as $sin^2\theta_W$. The following results have been proven [13]:

1. Infinite-genus surfaces can have nonplanar ends.
2. The metric structure of the infinite-genus surface is not smooth when the capacity of the ideal boundary is nonvanishing.
3. Equality of the integral over the wedge product of the curvature and its dual with the signature of an E_8 homology manifold yields a condition on the measure of the ideal boundary.

Since a physical model in the four-dimensional embedding space must preserve the intersection form in the neighbourhood of the ideal boundary, it would be compatible with an E_8 gauge symmetry. Therefore, this invariance should be present locally for certain scattering parameters including $sin^2\theta_W$. The large-scale gauge group, however, would be determined by the Lagrangian and must be consistent with the elementary particle.

The particle spectrum already can be derived from $M^{k\ell m} = \frac{SU(3)\times SU(2)\times U(1)}{SU(2)\times U(1)'\times U(1)''}$ or $L^{k\ell m}$, a $U(1)$ bundle over $M^{k\ell m}$. This eight-dimensional space may be compared to $\frac{G_2\times SU(2)\times U(1)}{SU(3)\times U(1)'\times U(1)''}$, which gives the particle and antiparticle spectrum of the standard model [13]. The wreath product of the isometry group of the latter coset space and S_3 is isomorphic to the automorphism group of the spinor space of the formulation of the particle spectrum of the standard model with division algebra modules.

Consider the manifold $M^{k\ell m}$, and let the charges generating a $U(1)$ group embedded in the tangent space, $U(1)'$ and $U(1)''$ be

$$\begin{aligned}
Z &= k\left(\frac{i}{2}\sqrt{3}\lambda_8\right) + \ell\left(\frac{i}{2}\sigma_3\right) + m(iY) \\
Z' &= k'\left(\frac{i}{2}\sqrt{3}\lambda_8\right) + \ell'\left(\frac{i}{2}\sigma_3\right) + m'(iY) \\
Z'' &= k''\left(\frac{i}{2}\sqrt{3}\lambda_8\right) + \ell''\left(\frac{i}{2}\sigma_3\right) + m''(iY).
\end{aligned} \tag{1}$$

The quantum numbers of the first generation of quarks and leptons [14] is

$$\begin{array}{lllll}
\begin{pmatrix} u \\ d \end{pmatrix}_L : u_L & Q = \frac{2}{3} & I_3 = \frac{1}{2} & Y = \frac{1}{3} \\
\quad d_L & Q = -\frac{1}{3} & I_3 = -\frac{1}{2} & Y = \frac{1}{3} \\
(u^c)_L : & Q = -\frac{2}{3} & I_3 = 0 & Y = -\frac{4}{3} \\
(d^c)_L : & Q = \frac{1}{3} & I_3 = 0 & Y = \frac{2}{3} \\
\begin{pmatrix} \nu_e \\ e \end{pmatrix}_L : \nu_e & Q = 0 & I_3 = \frac{1}{2} & Y = -1 \\
\quad e_L & Q = -1 & I_3 = -\frac{1}{2} & Y = -1 \\
(e^c)_L : & Q = 1 & I_3 = 0 & Y = 2
\end{array} \tag{2}$$

since $(u^c)_L$ and $(d^c)_L$ transform under the $\bar{\mathbf{3}}$ representation of SU(3) while $(e^c)_L$ belongs to the complex conjugate representation of $U(1)$. Expanding these fermions in terms of a representation of $SU(2)\times U(1)'\times U(1)''$, based on the decomposition $\mathbf{3}\to\mathbf{2}+\mathbf{1}$ [14],

$$\begin{aligned}
\begin{pmatrix} u \\ d \end{pmatrix}_L &= \left|\mathbf{2}; \frac{1}{2}k' + \frac{1}{2}\ell' + \frac{1}{6}m'; \frac{1}{2}k'' + \frac{1}{2}\ell'' + \frac{1}{6}m''\right\rangle \\
&\quad + \left|\mathbf{2}; \frac{1}{2}k' - \frac{1}{2}\ell' + \frac{1}{2}m'; \frac{1}{2}k'' - \frac{1}{2}\ell'' + \frac{1}{6}m''\right\rangle \\
&\quad + \left|\mathbf{1}; -k' + \frac{1}{2}\ell' + \frac{1}{6}m'; -k'' + \frac{1}{2}\ell'' + \frac{1}{6}m''\right\rangle \\
&\quad + \left|\mathbf{1}; -k' - \frac{1}{2}\ell' + \frac{1}{6}m'; -k'' - \frac{1}{2}\ell'' + \frac{1}{6}m''\right\rangle \\
u_R &= \left|2; \frac{1}{2}k' + \frac{2}{3}m'; \frac{1}{2}k'' + \frac{2}{3}m''\right\rangle + \left|\mathbf{1}; -k' + \frac{2}{3}m'; -k'' + \frac{2}{3}m''\right\rangle \\
d_R &= \left|\mathbf{2}; \frac{1}{2}\ell'; \frac{1}{2}\ell''\right\rangle + \left|\mathbf{1}; 0; 0\right\rangle \\
\begin{pmatrix} \nu_e \\ e \end{pmatrix}_L &= \left|\mathbf{2}; \frac{1}{2}\ell' - \frac{1}{2}m'; \frac{1}{2}\ell'' - \frac{1}{2}m''\right\rangle + \left|\mathbf{1}; -\frac{1}{2}\ell' - \frac{1}{2}m'; -\frac{1}{2}\ell'' - \frac{1}{2}m''\right\rangle \\
e_R &= \left|\mathbf{1}; \ell'; \ell''\right\rangle,
\end{aligned} \tag{3}$$

where the sign of the charge of u_R, d_R and e_R is chosen to be identical to that of u_L, d_L and e_L, and therefore, the coefficients of m' and m'' are opposite to that of $\frac{1}{2}Y$ for the $(u^c)_L$, $(d^c)_L$ and $(e^c)_L$.

A matching with quantum numbers of the fermions in the standard model requires the embedding paramters to satisfy $3k' = -m'$ since the quarks have charges that are multiples of $\frac{1}{3}e$. From the relation $I_3 = Q - \frac{1}{2}Y$, $\ell = \mp m'$ reflects contributions of the same magnitude of the isospin and half of the hypercharge for fixed electric charge. The two solutions generating the quantum numbers are

$3k' = \pm\ell' = -m'$, $3k'' = \pm\ell'' = -m''$.

The conditions for supersymmetry in the theory have been proven to be

$$\begin{aligned}
\frac{3k'' - \ell''}{3k' - \ell'} &= \frac{m''}{m'} \\
\frac{3k'' + \ell''}{3k' + \ell'} &= \frac{m''}{m'}
\end{aligned} \tag{4}$$

which will be valid singularly for the two classes of solutions [14].

Since the fermions in the standard model represent the spinor content, there would be scalar fields in the ground state by supersymmetry. However, the masses of these fields could be much larger through supersymmetry breaking. The absence of couplings of these heavy fields to the lighter fields cause the existence of composite particles to be less prevalent. The classical value of the electromagnetic coupling in this model follows from

$$\begin{aligned}
\alpha_i &= \frac{g_i^2}{4\pi} \\
\alpha &= \frac{3}{5}\left[\frac{1}{\alpha_1} + \frac{3}{5}\frac{1}{\alpha_2}\right]^{-1},
\end{aligned} \tag{5}$$

based on a relation between the electromagnetic and hypercharge couplings, and $e^2 = g_2^2 sin^2\theta_W$, where g_1 and g_2 represent the gauge couplings of $U(1)_Y$ and $SU(2)$. At the solution with $N=2$ supersymmetry with the quantum numbers of the standard model, in units with $\kappa^2 = 8\pi G = 1$, $g_1^2 = 18n^2\left(\frac{1}{2}c^{-2}(k'\ell'' - k''\ell')^2\right)^{-1}$ is singular. The breaking of supersymmetry is necessary for a valid formula for g_1. Then the contribution to α from $\frac{1}{\alpha_1}$ would vanish and

$$\alpha \to \alpha_2 = \frac{g_2^2}{4\pi} \tag{6}$$

and $e^2 = g_2^2 sin^2\theta_W$ and $sin^2\theta_W \to 1$.

A renormalization group flow is necessary for a derivation of the couplings and the masses. However, the relation between e and g_2 continues to be valid at low energy scales. With the breaking of supersymmetry and the energy-dependence of the couplings, g_1^2 will be finite and let the contribution of $\frac{1}{\alpha_1}$ equal $k\cdot\frac{3}{5}\frac{1}{\alpha_2}$. Then

$$\alpha = \frac{3}{5}\frac{1}{1+k}\frac{5}{3}\alpha_2 = \frac{1}{1+k}\alpha_2 \tag{7}$$

and

$$sin^2\theta_W = \frac{1}{1+k}. \tag{8}$$

The conventional value of θ_W will be achieved when $k \simeq 3$. The value of the $U(1)_Y$ coupling g_1 satisfies $sin^2\theta_W = \frac{g_1^2}{g_1^2+\frac{5}{3}g_2^2}$.. When $k \simeq 3$, the ratio is $\frac{1}{4}$, and the relation between the hypercharge and nonabelian gauge charge couplings must be modified at supersymmetric scales.

This theory must be distinguished from the G_2 minimal supersymmetric standard models, which require stabilization of the moduli together with a stratification of the particle spectrum [16]. Typically, this space is seven-dimensional with G_2 holonomy [17]. Given the difficulties in verifying the commutation of the E_8 gauge group elements with the holonomy group on this space when the strings are interpreted to be Wilson loops, since the noncovariance under infinitesimal gauge transformations modifies the condition for the centralizer [18][19] and may be restored only with the introduction of a scalar field [20], the phenomenology derived from this formalism is considerably different from the $G_2/SU(3)$ solution to the heterotic string field equations [21][22].

The components of the Ricci tensors of the M^{klm} manifolds are given by

$$R^A{}_B = \frac{3}{4}a^2\left[1-\frac{a^2}{12c^2}(\ell'm''-\ell''m')^2\right]\delta^A{}_B \tag{9}$$

$$R^m{}_n = \frac{b^2}{2}\left[1-\frac{b^2}{2c^2}(k'm''-k''m')^2\right]\delta^m{}_n$$

$$R^3{}_3 = \frac{b^4}{2c^2}(k'm''-k''m')^2+\frac{a^4}{8c^2}(\ell'm''-\ell''m')^2.$$

If $3k' = -m'$ and $3k'' = -m''$, $k'm'' = -\frac{1}{3}k'k'' = k''m'$, and, $\ell' = \mp m'$ and $\ell'' = \mp m''$, $\ell'm'' = \mp m'm'' = \ell''m'$. Then $R^3{}_3 = 0$ and the metric is not a Freund-Rubin solution. Nevertheless, a flat metric can be placed on the parallelizable circle.

The equality of the Z' and Z'' numbers follows from an identification of the $U(1)'$ and $U(1)''$ factors. The resulting manifold, $L^{l\ell m} = \frac{SU(3)\times SU(2)\times U(1)}{SU(2)\times U(1)}$, which can be equivalent to an S^3 bundle over S^5 or an $S^2\times S^1$ bundle over S^5 or have the topologies $\mathbb{CP}^2\times S^3\times S^1$ or $\mathbb{CP}^2\times S^2\times S^1\times S^1$, where the spherical topologies may include squashed spheres. Although the last topology does not admit a metric of the Freund-Rubin form, it is sufficient for the description of a configuration with supersymmetry and a spinor sector consisting of the fermions in the standard model.

It may be noted that, although there is a bosonic sector which can be found by a supersymmetry transformation of the fermionic sector, it is more easily derived by dimensional reduction of the six-dimensional Yang-Mills theory with gauge group G_2 over S^2. Therefore, the other eight-dimensional geometry $\frac{G_2\times SU(2)\times U(1)}{SU(3)\times U(1)'\times U(1)''}$ may be considered with regard to the generation of the quarks, leptons and antiparticles of the standard model. Furthermore, there is a factorization, $G_2/SU(3)\times SU(2)/U(1)'\times U(1)/(U(1)'')$ which is diffeomorphic to $S^6\times S_s^2\times U(1)/U(1)''$, where the squashing parameter may be set equal to zero when $U(1)'$ is identified with the S^1 fibre in the Hopf fibration. By the previous analysis, it would not be usual to identify $U(1)''$ with $U(1)$, for embeddings orthogonal to the tangent space. However, the manifold is well-defined, nevertheless, and a reduction over $S^6\times S^2$ will have $N=1$ supersymmetry. There is no possibility of a coincidence of the topologies, with the exception of $\mathbb{CP}^2\times S^3\times S^1$, because $\frac{G_2}{SU(3)\times U(1)'\times U(1)''}$ is a singular variety, since the rank of G_2 is 2 and the rank of $SU(3)\times U(1)'\times U(1)''$ is 4.

4 Conclusion

The E_6 theories derived from superstring theory tend to have too many fermion generations or the number of particle and antiparticle multiplets are not equal. The dimensionally reduced E_8 super-Yang-Mills theory over $G_2/SU(3)$ yields results in an anomaly-free E_6 theory with $N=1$ supersymmetry. With the scalar potential, it is possible to determine each of the symmetry breaking patterns, which introduce, however, a larger number of extra fields.

The prediction of the Weinberg angle supports an E_6 model without supersymmetry. By contrast, the phenomenological viability of a unified field theory with compactification over the coset space $\frac{G_2\times SU(2)\times U(1)}{SU(3)\times U(1)'\times U(1)''}$ results from the reduction of the fermions in the higher dimensions to the known quarks and leptons and antiparticles and the automorphism group of the spinor space of the standard model. The topology of this compact manifold is different that of the $M^{k\ell m}$ solutions to eleven-dimensional supergravity or the eight-dimensional limits $L^{k\ell m}$. Although both the manifold $L^{k\ell m}$ and the coset space yield the particle spectrum, there is a difference in the value of the couplings, especially since the solution with $N=2$ supersymmetry, necessary to produce the fermion multiplets in the former model, does not generate the value of the Weinberg angle at electroweak scales.

The resolution to the problem of the precise value of $sin^2\theta_W$ and the ratios of the couplings is provided by a new mechanism for the introduction of the exceptional gauge group E_6 without affecting the physical gauge symmetries derived from the compactification. The invariances of a physical theory defined over a four manifold, in which infinite-genus surfaces are embedded, may be enlarged to a larger subgroup of the E_8 homology group of a nonsmooth structure occurring in the neighbourhood of the ideal boundary. The prediction of the E_6 theory without supersymmetry is known for $sin^2\theta_W$ is known to coincide closely with the experimental value. Then, a consistent phenemonological theory may be derived from the compactification of the string model.

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