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Enhancement of Qubit-Qubit Correlation via Non-Resonance Two-Mode Raman Coupled Model by Applying Bang-Bang Pulses

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Abstract: The influence of a pulsed control on nonclassical correlation and entanglement is studied. To illustrate, a scheme of initially entangled noninteracting two qubits coupled independently to a couple of quantized field modes via a nonresonant Raman interaction has been proposed. Mainly, two types of measures are considered, namely, quantum discord and concurrence. We discuss how concurrence can play efficiently the same role as discord in absence of entanglement. Dynamics of states, initially, having "X"-structure density matrix are studied to help in shedding light on this important role. We mainly focus on extended Werner-like states (EWL). We show that, for a specific EWL state, interaction induces a loss of the initial entanglement of the two qubits and it causes "entanglement sudden death (ESD)", while for other EWL state, the behavior of a nonclassical correlation can be easily determined from that of entanglement and vice versa. Also, regardless of EWL states type, by adjusting efficient values of specific parameters, a pulsed control of both qubits causes steady behavior of nonclassical correlation as well as entanglement. In that case, the amount of correlation between both qubits can be increased where correlation maxima can be almost reached. Our observations may have important implications in exploiting these correlations in quantum information processing and transmission.

Keywords: Two qubits; Entanglement; Bang-bang pulse; Nonresonance Two-mode Raman coupled model.

1 Introduction

Correlations are ubiquitous in nature. The concept of correlation, i.e., information of one system about another, is a key element in many-body physics. Correlations can be both from classical and quantum sources[1]. Quantum entanglement is a special quantum correlation and has been recognized not only as a vital concept in physics but also a prime resource for quantum information processing (QIP)[2,3,4,5]. However, the entanglement [6,7,8,9,10,11,12,13,14] is not the only type of quantum correlation and there exist quantum tasks that display quantum advantage without entanglement [15,18,16,17]. The discovery that separable quantum states can exhibit non-classical correlation other than entanglement has led to a new understanding of the quantum aspects of a physical system. It has been demonstrated both theoretically [19,20,21,18,55] and experimentally[56,56,57] that other nonclassical correlation can be responsible for the computational speedup for certain quantum tasks. These correlations are more general and more fundamental than entanglement. In the field of quantum information it is important to distinguish between quantum and classical aspects of correlation in a composite quantum state. Such nonclassical correlations are characterized as quantum discord[32]. Quantum discord, which quantifies nonclassical correlations of more general and more fundamental type than entanglement, was introduced by Ollivier and Zurek [32] and is defined as the difference between the quantum mutual information and the classical correlation. It is largly accepted that quantum mutual information is the information-theoretic measure of the total correlation in a bipartite quantum state. Another interesting fact is that quantum discord is present in typical instances of a mixed-state quantum computation[33], even when entanglement is absent [21,22,56]. Moreover, it has been proposed as the key resource present in certain quantum

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communication tasks and quantum computational models without containing much entanglement [21,58,59]. In addition to its conceptual role, some recent results[21] suggest that quantum discord and not entanglement may be responsible for the efficiency of a mixed state based quantum computer. For this purpose, enhancement of quantum discord is important for some quantum algorithms that could work well in absence of entanglement. For implementations of those algorithms in the case that entanglement approaches zero, quantum discord should be larger than zero[42]. Therefore, it is important to study, characterize and quantify quantum and classical correlations.

About entanglement, dense work has been performed for bipartite systems, because it plays a crucial role in many applications of QIP theory, such as teleportation[29] and quantum key distribution[30]. However, the interaction between the environment and quantum system of interest can destroy quantum coherence and lead to decoherence, as a result, entanglement degradation. The possibility of preventing or avoiding entanglement degradation is of significant importance for any technological use of quantum systems, aimed at processing, communicating or storing information. To this end, one must understand and model all of the relevant features characterizing the environment of the physical system to be protected. It is therefore of great importance to prevent or minimize the influence of decoherence in the practical realization of quantum information processing. Several different protocols have been designed to protect quantum information[49,50,43,44,45,46]. One of the protocols to prevent the quantum decoherence is dynamical decoupling strategies [26- 28] by means of a train of instantaneous pulses(”bang-bang” pulses)[43,44,45,54]. Several applications for this strategy of bang-bang control were proposed[23,47,48]. Recently, experimental realizations of the quantum ”bang-bang” control have been reported [51,52,53], where experimentally suppression of polarization decoherence in a ring cavity using bang-bang decoupling technique has been performed[51].

A problem of quantum discord dynamics of two effective two-level atoms independently interacting with two quantized field modes through a Raman interaction in the presence of phase decoherence has been already studied in Ref.[54]. However, the schemes doesn't take advantage of bang-bang control of the two qubits during interaction. In addition, a scheme for investigating quantum correlations for the cavity quantum electrodynamics system consisting of two noninteracting two-level atoms each locally interacting with its own quantized field mode by bang-bang pulses has been also presented[23]. Again, the scheme doesn't take advantage of nonresonance two field modes Raman interaction. In this paper we are motivated by the aim of investigation of quantum correlations of two qubits from a different point of view.

Our analysis is focused on the dynamics of a class of states having an ”X”-structure density matrix, namely the extended Werner-like states (EWL). This class of states plays a crucial role in many applications of quantum information theory, such as teleportation[29] and quantum key distribution[30]. Moreover, such a choice will give us the chance to study how non-classical correlations and entanglement dynamics and their revivals are related to the purity and the amount of entanglement of the initial state. We also focus on the interplay between entanglement and correlations for the qubits states when they are pulse-controlled. We investigate the connection between these two quantities, comparing concurrence, mutual information and quantum discord dynamics for initial EWL states, and finding clear correlations between them when various shapes of field inside a nonresonance cavity are considered. In other words, the influence bang-bang control, detuning and type of photon distribution inside the cavity on dynamics of the above mentioned quantities when the two qubits coupled independently to two quantized field modes through a nonresonance Raman interaction. The cavity fields are initially in either Fock or thermal states.

The paper is organized as follows: In section 2, the effective Hamiltonian of two-mode Raman coupled model, which represents the main starting point of our going calculations, is derived, followed by Reduced density operator which is calculated for general two-qubits X states in Sec. 3. In Sec. 4, a short survey about quantum meseure we are going to use is presented. Sepcial cases of generally derived formula that are presented in Sec. 5. While, discussion of numerical calculations is presented in Sec.6. Finally, our conclusion is showed in Sec.7.

2 Effective Hamiltonian of two-mode Raman coupled model

In this section we derive the effective Hamiltonian we use to study the dynamics of two qubits interacting with quantized two-mode Raman coupled model [24,25,26,27,54]. The Hamiltonian of the system, in the rotating wave approximation in presence of bang-bang pulse, can be written as

$$H = H_1 + H_P, \tag{1}$$

where,

$$H_1 = \omega_1 a_1^{\Sigma} a_1 + \omega_2 a_2^{\dagger} a_2 + \frac{\omega}{2}\sigma_z + g(a_1 a_2^{\dagger}\sigma_+ + a_1^{\dagger} a_2 \sigma_-)$$

$$= H_0 + H_{int}, \tag{2}$$

is the Hamiltonian of one atom interacting with its own quantized two-mode Raman coupled model[27,54] in absence of bang-bang pulse, with

$$H_0 = \omega_1 a_1^{\dagger} a_1 + \omega_2 a_2^{\dagger} a_2 + \frac{\omega}{2}\sigma_z, \quad H_{int} = g(a_1 a_2^{\dagger}\sigma_+ + a_1^{\dagger} a_2 \sigma_-), \tag{3}$$

The operators σ_z and $\sigma_\pm$ represent, respectively, the atomic spin flip, raising and lowring operators characterizing the two-level atom with transition frequency ω. The parameter g is the atom-field coupling constant and a_i ($a_i^\dagger$) are annihilation (creation) operators of the i^{th} mode light field of frequencies ω_i. While[23],

$$H_P = V\sigma_z \sum_{n=0}^{\infty} \theta(t-T-n(T+\tau))\theta((n+1)(T+\tau)-t), \tag{4}$$

is the Hamiltonian for a train of identical pulses of duration τ, where T is the time interval between two consecutive pulses and the amplitude V of the control field is specified to be $\frac{2\pi}{\tau}$, which means that we consider the π-pulse only.

In absence of bang-bang pulse, following the same procedure of Ref. [27], in terms of $SU(2)$ generators, we can write the Hamiltonian Eq. (2) in the form

$$H_1 = \omega_1 K_1 + \omega_2 K_2 + \Delta S_0 + g\sqrt{K_1 K_2}(S_+ + S_-), \tag{5}$$

where, $K_1 = a_1^\dagger a_1 + \frac{1+\sigma_z}{2}$, $K_2 = a_2^\dagger a_2 + \frac{1-\sigma_z}{2}$ are constants of motion in the Hamiltonian, and $\Delta = (\omega + \omega_2 - \omega_1)$. Note that the three operators $S_0 = \frac{\sigma_z}{2}$, $S_+ = \frac{a_1 a_2^\dagger \sigma_+}{\sqrt{K_1 K_2}}$ and $S_- = \frac{a_1^\dagger a_2 \sigma_-}{\sqrt{K_1 K_2}}$ are introduced. Again, with the help of $SU(2)$ dynamical algebraic structure, we can diagonalize the Hamiltonian Eq. (5) by the unitary transformation[27]

$$U = \exp\left[\frac{\theta(K_1,K_2)}{2}(S_+ - S_-)\right], \tag{6}$$

where

$$\theta(k_1,K_2) = \tan^{-1}\frac{2g\sqrt{K_1K_2}}{\Delta}, \tag{7}$$

and get transformed Hamiltonian

$$H_1' = (\omega_1 K_1 + \omega_2 K_2) + 2\Omega(K_1,K_2)S_0, \tag{8}$$

where

$$\Omega(K_1,K_2) = \sqrt{\frac{\Delta^2}{4} + g^2 K_1 K_2}. \tag{9}$$

In terms of Eq.(8), it is not difficult, with the help of an $SU(2)$ dynamical algebraic structure, to write down the time evolution operator in the absence of control pulses field directly as $U_0(t) = e^{-iH_1 t}$ as

$$U_0(t) = \exp(-i(\omega_1 K_1 + \omega_2 K_2)t)\Bigg[\cos[\Omega(K_1,K_2)t] - \frac{i}{2}(\Delta\sigma_z + 2H_{int})\frac{\sin[\Omega(K_1,K_2)t]}{\Omega(K_1,K_2)}\Bigg]. \tag{10}$$

In presence of control pulses field, with strong enough pulses, i.e. the duration $\tau \to 0$, the time evolution operator can be approximated as[23]

$$U_p(\tau) = \exp\big[-iH\tau\big] \approx \exp\big[-i\frac{\pi}{2}\sigma_z\big], \tag{11}$$

which leads to

$$U_p U_0(T) U_p = -e^{-i(H_0 - H_{int})T}, \tag{12}$$

this condition is valid for the stroboscopic dynamics of the system. Focusing on the stroboscopic evolution at times t_{2N}[28], the evolution is driven by an effective average Hamiltonian[23,43]

$$U(t_{2N}) = [U_c]^N = \exp[-iH_{eff}t_{2N}]. \tag{13}$$

For sufficiently short T[28], the effective Hamiltonian is accurately represented by the following Hamiltonian

$$H_{eff} = H_0 - ig_{eff}(a_1 a_2^\dagger \sigma_+ - a_1^\dagger a_2 \sigma_-). \tag{14}$$

The coupling parameter $g_{eff} = \frac{g\Delta T}{2!}$ is proportional to the detuning Δ and the time interval T between two successive pulses. Obviously, the interaction between the atom and field is averaged to zero by the bang-bang pulses when $T \to 0$. With the help of the $SU(2)$ algebraic structure as before, the evolution operator at times t_{2N} can be expressed as

$$U(t_{2N}) = \exp(-i(\omega_1 K_1 + \omega_2 K_2)t_{2N})\Bigg[\cos[\Omega_{eff}(K_1,K_2)t_{2N}] - i\Big(\Delta S_0 + H_{eff} - H_0\Big)\frac{\sin[\Omega_{eff}(K_1,K_2)t_{2N}]}{\Omega_{eff}(K_1,K_2)}\Bigg], \tag{15}$$

with

$$\Omega_{eff}(K_1,K_2) = \sqrt{\frac{\Delta^2}{4} + g_{eff}^2 K_1 K_2}. \tag{16}$$

In general, at a certain time $t = t_{2N} + \tilde{t}$, the evolution operator is given by[23,47]

$$U(t) = \begin{cases} U_0(\tilde{t})[U_c]^N, & 0 \le \tilde{t} < T \\ U(\tilde{t}-T)U_P U_0(T)[U_c]^N & T \le \tilde{t} < 2T \end{cases}, \tag{17}$$

where $N = [t/2T]$, $[\cdot]$ denotes the integer part, and $\tilde{t} = t - 2NT$ is the residual time after N cycles. It is now easy to calculate the time-dependent reduced density operator of the atomic subsystem which will be starting point of our future calculations.

3 Reduced density operator

Dynamics of correlations and entanglement of two qubits prepared in EWL states have attracted much attention. Here, we concerned with the reduced density operator of atomic sub-system prepared initially in generalized EWL states of non-maximally entangled states part, as

$$\rho^{\xi}_{EWL}(0) = r|\xi\rangle\langle\xi| + \frac{1-r}{4}I, \tag{18}$$

where we focus on $|\xi\rangle$ to be either of Bell-like states $|\Phi\rangle$ (two excitations) or $|\Psi\rangle$ (one excitation) as

$$|\Phi\rangle = \mu|ee\rangle + \nu|gg\rangle, \tag{19}$$

$$|\Psi\rangle = \mu|ge\rangle + \nu|eg\rangle. \tag{20}$$

For $r = 0$, the EWL states become totally mixed, while they reduce to the Bell-like pure state $|\Phi\rangle$ or $|\Psi\rangle$ in the case of $r = 1$. I is a 4×4 identity matrix. The density operator of the four cavity fields are initially expressed as

$$\rho_f(0) = \sum_{n=0}^{\infty} p_{nn}|nn\rangle\langle nn| \otimes \sum_{k=0}^{\infty} p_{kk}|kk\rangle\langle kk|, \tag{21}$$

where, for thermal field state, we express p_{mm} as

$$p_{mm} = \Pi_{j=1}^{2} \frac{(\bar{m}_j)^{m_j}}{(1+\bar{m}_j)^{(m_j+1)}}, \qquad \bar{m}_j = (e^{\omega\beta_j} - 1)^{-1}, \tag{22}$$

and $\bar{m}_j$ is the mean photon number at the inverse temperature β_j. For Fock number state, p_{mm} can be expressed as

$$p_{mm} = \Pi_{j=1}^{2} \delta_{m_j l_j}. \tag{23}$$

In terms of initial states Eqs.(18, 21), the full initial state operator (atom-field) is

$$\rho(0) = \rho_f(0) \otimes \rho_{EWL}^{\xi}(0). \tag{24}$$

At any time t, the reduced density operator of the two atoms can be derived by tracing out the cavity fields,

$$\rho_{aa}(t) = \mathrm{Tr}_f\Bigg(U^{(1)}(t)U^{(2)}(t)\rho_a^{\Phi}(0) \times \sum_{n=0}^{\infty}\sum_{k=0}^{\infty} p_n p_k |nnkk\rangle\langle nnkk|\, U^{(1)\dagger}(t)U^{(2)\dagger}(t)\Bigg), \tag{25}$$

where $U^{(i)}(t)(i = 1,2)$ is the evolution operator acting on the i^{th} atom and cavity fields. Matrix representation of the reduced density operator, Eq. (25) spanned by an arbitrary two-atom product states, becomes

$$\rho_{aa}(t) = \begin{pmatrix} \rho_{11} & 0 & 0 & \rho_{14} \\ 0 & \rho_{22} & \rho_{23} & 0 \\ 0 & \rho_{32} & \rho_{33} & 0 \\ \rho_{41} & 0 & 0 & \rho_{44} \end{pmatrix}. \tag{26}$$

Apparently, the matrix representation shows two-qubit X states[34], with the eigenvalues,

$$\lambda_{1,2} = \frac{1}{2}\left[(\rho_{11}+\rho_{44}) \pm \sqrt{(\rho_{11}-\rho_{44})^2 + 4|\rho_{14}|^2}\right], \tag{27}$$

$$\lambda_{2,3} = \frac{1}{2}\left[(\rho_{22}+\rho_{33}) \pm \sqrt{(\rho_{22}-\rho_{33})^2 + 4|\rho_{23}|^2}\right], \tag{28}$$

where, expressions of $\rho_{ij}(i, j = 1,2,3,4)$ will depend on the product states we intend to use.
Performing the reduced density operator enables us to calculate any related phenomena of interest. In the next section we will give a brief survey about quantum measures we are going to use through our future analysis, namely, quantum discord, mutual information and concurrence.

4 Quantum measure

As already mentioned above, the definition of quantum discord is based on quantum mutual information, which contains both classical and quantum correlations. For a bipartite system ρ_{AB}, its total correlations can be measured by their quantum mutual information[32]

$$I(\rho^{AB}) = S(\rho^A) + S(\rho^B) - S(\rho^{AB}), \tag{29}$$

where $\rho^A(\rho^B)$ denotes the density operator of part $A(B)$ and $S(\rho_{A(B)}) = -\mathrm{Tr}_{\mathrm{B(A)}}(\rho_{AB}\log\rho_{AB})$ is the von Neumann entropy[3]. The classical correlation, which depends on the maximal information obtained with measurement on one of the subsystems, is given by[34,35]

$$C(\rho^{AB}) = S(\rho^A) - \sup_{\{B_k\}}[S(\rho^{AB}|\{B_k\})], \tag{30}$$

where $\{B_k\}$ describes a set of one-dimensional projectors for subsystem B, $S(\rho^{AB}|\{B_k\}) = \sum_k p_k S(\rho_k)$ is the based-on-measurement quantum conditional entropy, and $\rho_k = \frac{1}{p_k}(I\otimes B_k)\rho^{AB}(I\otimes B_k)$ is the conditional density operator with the probability $p_k = \mathrm{Tr}[(I\otimes B_k)\rho(I\otimes B_k)]$. Then, the quantum discord is defined as the difference of the total correlation and the maximum classical correlation,[32]

$$\mathscr{Q}(\rho^{AB}) = I(\rho^{AB}) - C(\rho^{AB}). \tag{31}$$

Quantum discord includes quantum correlations that can be present in states that are not entangled[32], revealing that all the entanglement measurements such as concurrence, entanglement of formation, etc, do not capture the whole of quantum correlation between two mixed separate systems. To this end, as a comparison with an entanglement measure will be fruitful and may shed light on some properties of these mentioned measures. We employ Wootter's concurrence[31] representing the entanglement[40,41] as a comparison. Concurrence[31] is considered as an important quantifier of entanglement dynamics of quantum system. Concurrence attains its maximum value of 1 for maximally entangled states and vanishes for separable states. Concurrence is defined by

$$\mathscr{C}_E(t) = \max(\sqrt{\lambda_1} - \sqrt{\lambda_2} - \sqrt{\lambda_3} - \sqrt{\lambda_4}), \tag{32}$$

where $\{\lambda_i\}$ are the eigenvalues of the matrix

$$R = \rho(\sigma_y\otimes\sigma_y)\rho^*(\sigma_y\otimes\sigma_y), \tag{33}$$

with ρ^* denoting the complex conjugate of ρ and $\rho^{A(B)}$ are the Pauli matrices of the atoms A and B.

5 Special cases

5.1 Case 1: EWL state includes two excitations Bell-like state

In this case, in the product basis $|ee\rangle = |1\rangle$, $|eg\rangle = |2\rangle$, $|ge\rangle = |3\rangle$, and $|gg\rangle = |4\rangle$, the X

structure is preserved during the evolution, but with $\rho_{14} = \rho_{41} = 0$, while

$$\rho_{11} = \sum_{n_1,n_2=0}^{\infty} \sum_{k_1,k_2=0}^{\infty} p_{nn} p_{kk} \Big\{ |\mathscr{A}(k_1+1,k_2)|^2 ||\mathscr{B}(n_1,n_2+1)|^2 \times \rho_{11}(0) + |\mathscr{A}(k_1+1,k_2)|^2 |\mathscr{A}(n_1+1,n_2)|^2 \rho_{22}(0) + |\mathscr{B}(k_1,k_2+1)|^2 |\mathscr{B}(n_1,n_2+1)|^2 \rho_{33}(0) + |\mathscr{B}(k_1,k_2+1)|^2 |\mathscr{A}(n_1+1,n_2)|^2 \rho_{44}(0) \Big\}, \tag{34}$$

$$\rho_{22} = \sum_{n_1,n_2=0}^{\infty} \sum_{k_1,k_2=0}^{\infty} p_{nn} p_{kk} \Big\{ |\mathscr{B}(k_1+1,k_2)|^2 |\mathscr{B}(n_1,n_2+1)|^2 \times \rho_{11}(0) + |\mathscr{B}(k_1+1,k_2)|^2 |\mathscr{A}(n_1+1,n_2)|^2 \rho_{22}(0) + |\mathscr{A}(k_1,k_2+1)|^2 |\mathscr{B}(n_1,n_2+1)|^2 \rho_{33}(0) + |\mathscr{A}(k_1,k_2+1)|^2 |\mathscr{A}(n_1+1,n_2)|^2 \rho_{44}(0) \Big\}, \tag{35}$$

$$\rho_{33} = \sum_{n_1,n_2=0}^{\infty} \sum_{k_1,k_2=0}^{\infty} p_{nn} p_{kk} \Big\{ |\mathscr{A}(k_1+1,k_2)|^2 |\mathscr{A}(n_1,n_2+1)|^2 \times \rho_{11}(0) + |\mathscr{A}(k_1+1,k_2)|^2 |\mathscr{B}(n_1+1,n_2)|^2 \rho_{22}(0) + |\mathscr{B}(k_1,k_2+1)|^2 |\mathscr{A}(n_1,n_2+1)|^2 \rho_{33}(0) + |\mathscr{B}(k_1,k_2+1)|^2 |\mathscr{B}(n_1+1,n_2)|^2 \rho_{44}(0) \Big\}, \tag{36}$$

$$\rho_{44} = \sum_{n_1,n_2=0}^{\infty} \sum_{k_1,k_2=0}^{\infty} p_{nn} p_{kk} \Big\{ |\mathscr{B}(k_1+1,k_2)|^2 |\mathscr{A}(n_1,n_2+1)|^2 \times \rho_{11}(0) + |\mathscr{B}(k_1+1,k_2)|^2 |\mathscr{B}(n_1+1,n_2)|^2 \rho_{22}(0) + |\mathscr{A}(k_1,k_2+1)|^2 |\mathscr{A}(n_1,n_2+1)|^2 \rho_{33}(0) + |\mathscr{A}(k_1,k_2+1)|^2 |\mathscr{B}(n_1+1,n_2)|^2 \rho_{44}(0) \Big\}, \tag{37}$$

$$\rho_{23} = \sum_{n_1,n_2=0}^{\infty} \sum_{k_1,k_2=0}^{\infty} p_{nn} p_{kk} \mathscr{A}^*(k_1+1,k_2) \mathscr{A}(n_1,n_2+1) \mathscr{A}^*(k_1,k_2+1) \mathscr{A}(n_1+1,n_2), \tag{38}$$

where

$$\mathscr{A}(m,n) = \langle e, m-1, n | U(t) | e, m-1, n \rangle; \quad m \geq 1, \tag{39}$$

$$\mathscr{B}(m,n) = \langle g, m, n-1 | U(t) | g, m, n-1 \rangle; \quad n \geq 1. \tag{40}$$

Here $\rho_{ij}(0)(i,j = 1,2,3,4)$ are given by Eq.(24) with $\rho_{11}(0) = r|\mu|^2 + \frac{1-r}{4}$, $\rho_{44}(0) = r|\nu|^2 + \frac{1-r}{4}$, $\rho_{22}(0) = \frac{1-r}{4} = \rho_{33}(0)$, and $\rho_{23}(0) = r\mu\nu^* = \rho_{32}^*(0)$. Now, the quantum mutual information $I(\rho)$ can be expressed as

$$I(\rho) = S(\rho^{(1)}) + S(\rho^{(2)}) + \sum_{i=1}^{4} \lambda_i \log_2 \lambda_i, \tag{41}$$

where the quantities $S(\rho^{(1)})$ and $S(\rho^{(2)})$ are the marginal entropies of the density operator $\rho(t)$ with $\rho^{(i)}(i = 1,2)$ are the reduced density matrices of the ith atom where

$$S(\rho^{(1)}) = -(\rho_{11} + \rho_{22}) \log_2 (\rho_{11} + \rho_{22}) - (\rho_{33} + \rho_{44}) \log_2 (\rho_{33} + \rho_{44}), \tag{42}$$

$$S(\rho^{(2)}) = -(\rho_{11} + \rho_{33}) \log_2 (\rho_{11} + \rho_{33}) - (\rho_{22} + \rho_{44}) \log_2 (\rho_{22} + \rho_{44}). \tag{43}$$

For classical correlation $C(\rho)$, following procedures of Ref.[34], an explicit and simple expression can be calculated by

$$C(\rho) = S(\rho^A) - \min\{S_1, S_2\}, \tag{44}$$

where

$$S_1 = -2\left[\rho_{44} \log_2 \frac{\rho_{44}}{\rho_{22} + \rho_{44}} + \rho_{22} \log_2 \frac{\rho_{22}}{\rho_{22} + \rho_{44}}\right], \tag{45}$$

$$S_2 = -\sum_{i=1}^{2} \left(\frac{\rho_{11} + \rho_{33} + (-1)^i |\rho_{23}|^2}{2(\rho_{11} + \rho_{33})} \times \log_2 \frac{\rho_{11} + \rho_{33} + (-1)^i |\rho_{23}|^2}{2(\rho_{11} + \rho_{33})} \right). \tag{46}$$

For concurrence, the following simple analytic expression can be easily obtained as

$$C_E(t) = \max(0, 2\sqrt{\rho_{11}\rho_{44}} - 2\sqrt{\rho_{22}\rho_{33}}), \tag{47}$$

5.2 Case 2: EWL state includes one excitation Bell-like state

For this initial state, in the product basis $|ge\rangle = |1\rangle$, $|ee\rangle = |2\rangle$, $|gg\rangle = |3\rangle$, and $|eg\rangle = |4\rangle$, the X structure is also preserved during the evolution, but with $\rho_{23} = \rho_{32} = 0$, while ρ_{11}, ρ_{22}, ρ_{33} and ρ_{44} can be obtained, respectively, by replacing $\rho_{11}(0), \rho_{22}(0), \rho_{33}(0), \rho_{44}(0)$ by $\rho_{22}(0), \rho_{44}(0), \rho_{11}(0), \rho_{33}(0)$ in Eqs. (34-37). The element $\rho_{14}(0)$ can be obtained by replacing $\mathscr{A}(k_1+1, k_2)$ and $\mathscr{A}(k_1, k_2+1)$ by their conjugates in ρ_{23}, Eq.(38). Recalling Eq.(44), an expression for classical correlation $C(\rho)$ can be obtained, where S_1 can be obtained from Eq.(45) by replacing ρ_{44} by ρ_{11} and ρ_{22} by ρ_{33} and vice versa for S_2 in addition to replacing ρ_{23} by ρ_{14}. In this case, the concurrence can be expressed as

$$C_E(t) = \max(0, 2|\rho_{23}| - 2\sqrt{\rho_{22}\rho_{33}}), \qquad (48)$$

6 Discussion of the numerical results

To analyze, figures representation of mutual information I, quantum discord $\mathscr{Q}$ and then entanglement C_E dynamics with considered specific initial atomic EWL states given by Eqs. (18-20) are depicted when the cavity fields are initially in either Fock or thermal states. Our analysis is focusing on the effect of the variation of detuning parameter δ, time interval T between two consecutive pulses, and purity parameter r, while change of photon number inside the cavity is also considered.

At first, it is worth mentioning that I, C and $\mathscr{Q}$ behave in a similar manner regardless of any particular type of EWL with $r = 1.0, \mu = \nu = \frac{1}{\sqrt{2}}$ (Bell-like state) of interest. For this special case, $\rho_{11}(0) = \rho_{44}(0) = \frac{1}{\sqrt{2}}$ and $\rho_{22}(0) = \rho_{33}(0) = 0$ for Bell-like state $|\Phi\rangle$, Eqs.(19), while for Bell-like state $|\Psi\rangle$, Eqs.(20), $\rho_{22}(0) = \rho_{33}(0) = \frac{1}{\sqrt{2}}$, and $\rho_{11}(0) = \rho_{44}(0) = 0$. This resulted in same expressions of I, C and $\mathscr{Q}$, Eqs.(29-31). This can be clear on recalling previous indicated details of obtaining $\rho_{ij} (i, j = 1, 2, 3, 4)$ in case of EWL state of second case Eqs.(18, 20), Subsec. 5.2.

For fields of a product of Fock states, the results were shown in mesh graphs Figs.(1,4). To put our results in perspective, we took a snapshot when $r = 1$, Figs.(2,3,5,6). It can be seen from the figures that I and $\mathscr{Q}$ vanish and revive periodically with time as $r > 1/3$ and are always zero for $r \leq 1/3$[23,34]. Periodicity is preserved according to the relation $\frac{10\pi}{n}$ (n is the number state value) regardless of change in any other parameters. Interesting is the effect of δ, where, regardless of EWL state type, adjusting a kind of slight difference between field modes and atomic levels frequencies has clearly positive effect on correlation between two atoms. As a result, correlations appear earlier where dark period of time, at which I and $\mathscr{Q}$ vanish, becomes clearly shorter, see Figs. (1b,d). When $r = 1$, atoms are initially maximally entangled, bang-bang control diminishes atomic correlation, specially when detuning is high, while atoms return maximally correlated as soon as detuning returns considerably small. In this case, both I and $\mathscr{Q}$ fall down to the possible minimum, while applying condition of small detuninig they reach their maxima, respectively, on time sacle periods of $\approx 2\pi$. This can be understood for

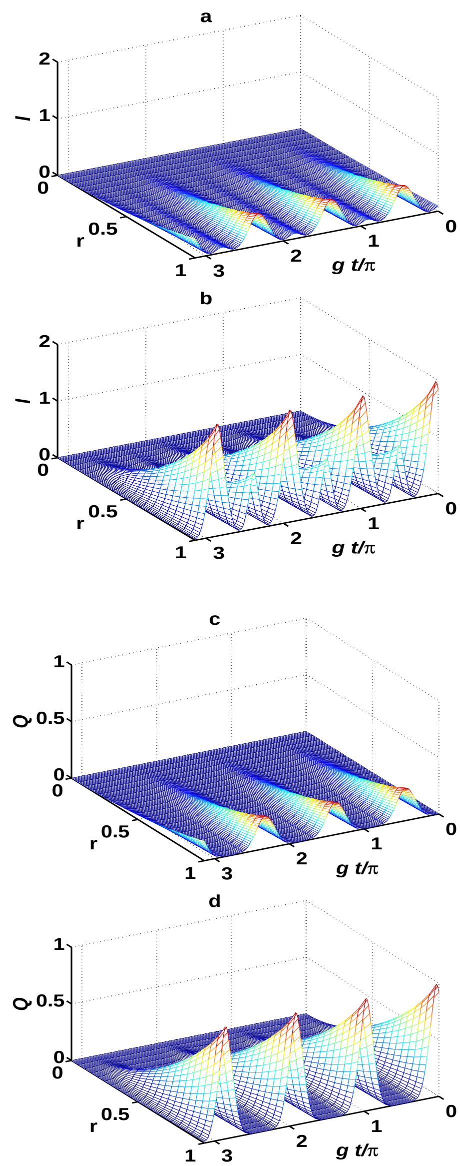

Fig. 1: Fock state field $I(t,r)$ and $\mathscr{Q}(t,r)$ for $k_1 = k_2 = n_1 = n_2 = 10.0$, $T = 0.6$, where (a,c) $\delta = 0.6$, and (b,d) $\delta = 0.001$.

both Ω_{eff} and g_{eff} in Eq.(16) are proportional to δ, where the contribution in $U(t_{2N})$ will come only from $\cos[\Omega_{eff}(K_1, K_2)t_{2N}]$ which causes amplification of correlations amplitudes. Moreover, as both Ω_{eff} and g_{eff}

are proportional to δ and $\Omega_{eff}(k)$ is not sensitive to k when T is small[23], amplification of correlations amplitudes occurs only on long time intervals between two consecutive pulses. Taking into consideration these details, we can conclude that, on on-resonance interaction bang-bang pulse control loses its efficiency. This can be understood if we notice that $\lim_{\delta\to 0}|U(t_{2N})| = 1$.
The effect of bang-bang pulses can be noticed clearly when the two atoms are initially prepared in the maximally entangled pure state, see Figs (2 and 3). We can see clearly that the quantum discord between the two atoms can be enhanced by the pulses because the increased amount of quantum mutual information is always larger than the classical correlation and the quantum discord recovers to its initial value at the points t_{2N}. The increased amount of the quantum correlations is larger for longer time intervals T of the control pulses. It is worth noting that, correlations strength changes to maximun more rapidly on considerably small detuning, to the extent that variation in time interval T between pulses loses completely its efficiency except for the role of acting as a shift tool of periods, see Fig. (2b,d). This result conflicts with that found in Ref.[23], where each atom interacts locally with single field mode. For higher number state inside the cavity, $k_i = 10$, we can approximate Ω_{eff} as $\Omega_{eff} = g_{eff}\sqrt{K_1K_2}$. In this case, correlations exhibit rapid Rabi oscillations almost similar to the case when bang-bang control is absent, see Fig. (3b,d). Worthy note that such effect of increasing the number state contained in the cavity is in contrary to similar studies in literature without bang-bang control, see for example[6,7]. Moreover, it is worth pointing out that different choices of μ and ν do not give dynamics of quantum correlations qualitatively different from the case treated here. Focusing on entanglement, preserving parameters' values as previously indicated, concurrence C_E was depicted in Figs.(4-6). It is well known that for any Bell stats $C_E(\rho(0)) = 1$, which can be seen clearly from our graphical results. Interestingly and surprising is the various orders of entanglement as a function of t and r that depend on EWL state type. This can be easily noticed by comparing dark periods of time, where C_E is zero, of our two considered cases. For case 1 where EWL state includes two excitations Bell-like state, given in 5.1, figures show that, dark period of time is extended to larger values of r; $r \leq 0.54$ on absence of detuning δ while it diminishes on presence of detuning effect. Moreover, in contrary to concurrence C_E, mutual information I, and quantum discord $\mathscr{Q}$ are different from zero even if a quantum systems is separable[37,32]. Comparing Figs. (1c,d) and (4a,b), we notice C_E behaves periodically similar to $\mathscr{Q}$ with only one difference, such that, in periods where $\mathscr{Q}$ dead and born asymptotically, C_E dead and born suddenly, which means entanglement sudden death and sudden birth. Hence, I, and $\mathscr{Q}$, show the existence of quantum correlation where concurrence shows that the two atoms are separable. As seen from the figures, although concurrence shows no entanglement,

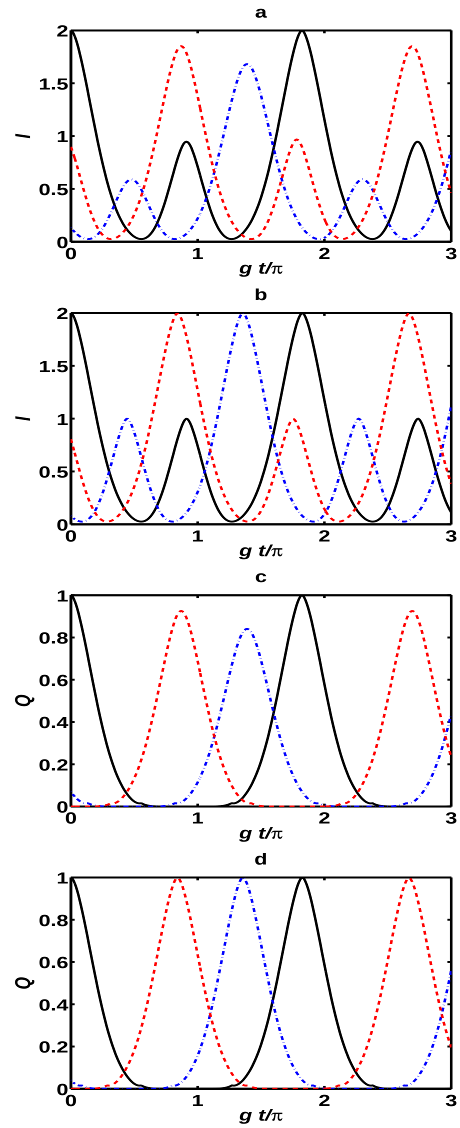

Fig. 2: Fock state field $I(t)$ and $\mathscr{Q}(t)$ for $k_1 = k_2 = n_1 = n_2 = 5.0$, and $r = 1.0$ where (a,c) $\delta = 0.6$, and (b,d) $\delta = 0.001$ for different T: $T = 0.0$ (black curve), $T = 0.1$ (dashed-dotted blue curve), $T = 0.6$ (dashed red curve).

other kinds of nonclassical correlations appear. The appearence of ESD and ESB can be deduced from Eqs. (47) where entanglement dynamics is a function of the population of the super-radiant state, $2\sqrt{\rho_{22}(t)\rho_{33}(t)}$ for the particular initial state of Eq. (19). Therefore whenever the population $2\sqrt{\rho_{22}(t)\rho_{33}(t)}$ reaches its relative maxima, the state attains a maximum value of mixedness, the time-dependent part of the concurrence $2\sqrt{\rho_{11}(t)\rho_{44}(t)} - 2\sqrt{\rho_{22}(t)\rho_{33}(t)}$ becomes negative and entanglement disappears. On the other hand, whenever the population of the super-radiant state reaches a minimum, the population of the $|11\rangle$ excited state and the

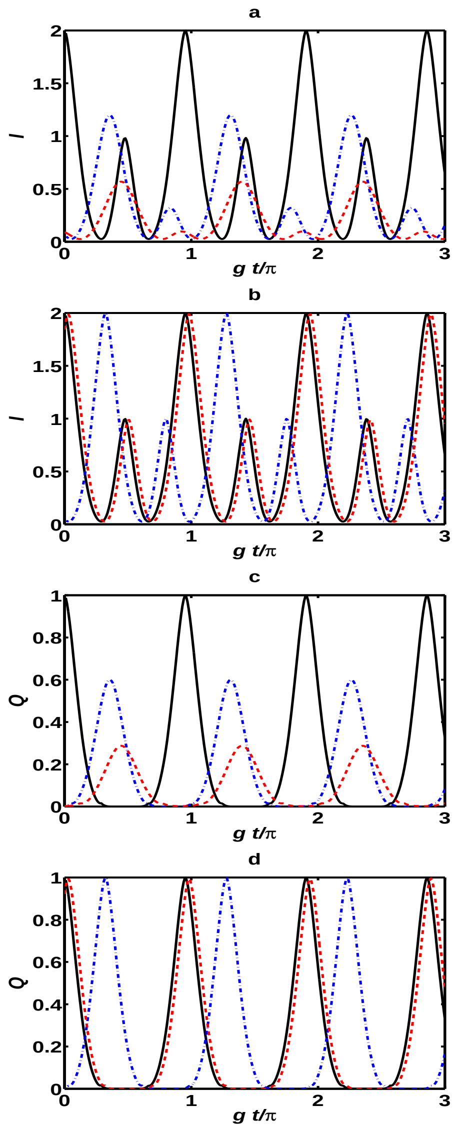

Fig. 3: The same as Fig.(2) but for $k_1 = k_2 = n_1 = n_2 = 10.0$.

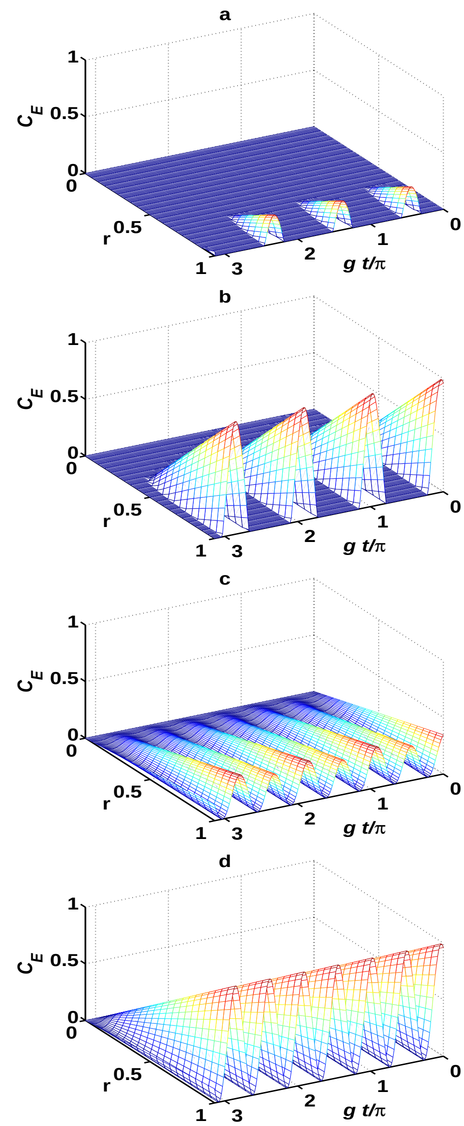

Fig. 4: Fock state field $\mathscr{C}_E(t,r)$ for $k_1 = k_2 = n_1 = n_2 = 10.0$, $T = 0.6$, where (a,b) $\delta = 0.6$, and (c,d) $\delta = 0.001$. up: EWL state Eqs. (18,19); down: EWL state Eqs. (18,20).

$|00\rangle$ ground state have their maxima, and the system goes toward a Bell- like state Eq. (19). As a consequence the system becomes purer and entanglement is again recovered. When the system is prepared in a Bell-like state as the one of Eq. (20) there is no ESD, however, it is still of interest to analyze the asymptoticall-dead and -birth entanglement related to such type of EWL state. For the Bell state $|\Psi\rangle$, Eq. (20), entanglement has exactly the same dynamics of the population of the super-radiant state, see Eq. (48). Moreover the zeroes of entanglement and super-radiant $2\sqrt{\rho_{22}(t)\rho_{33}(t)}$ coincide. For those times, in fact, the system goes into the ground state which is pure and factorized. When some population returns in the super-radiant state entanglement is recovered, and the state is again mixed. This behavior can be notices clearly on considering case 2, where EWL state includes one excitation Bell-like state. Comparing Figs (1a,b) and (4c,d), we notice similar periodical behaviors with asymptotically death and birth in both C_E and I. Here concurrence is directly given by the coherence between the $|10\rangle$ and $|01\rangle$ states. Since the coherence vanishes in asymptotic way, entanglement can be smoothly generated but it cannot suddenly appear. This result actually depends on the type of EWL state. It is worth noting that, it has been shown that, ESD does not occur for the same Bell-like state (with one excitation) as in Eq. (20) even if the two qubits are coupled to structured reservoir[65] or if the dipolar interaction between the qubits is included[66].

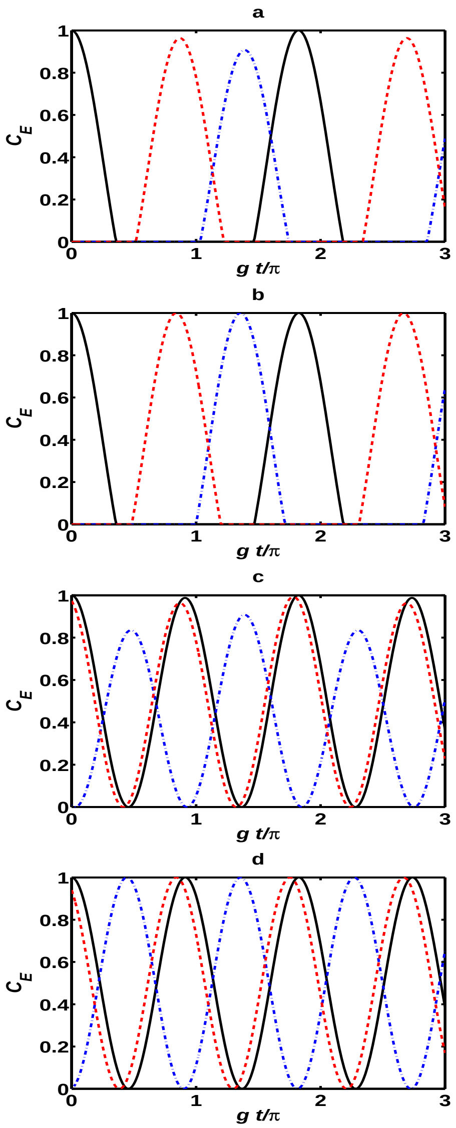

Fig. 5: Fock state field $\mathscr{C}_E(t)$ for $k_1 = k_2 = n_1 = n_2 = 5.0$, and $r = 1.0$ where (a,b) $\delta = 0.6$, and (c,d) $\delta = 0.001$ for different T: $T = 0.0$ (black curve), $T = 0.1$ (dashed-dotted blue curve), $T = 0.6$ (dashed red curve). up: EWL state Eqs. (18,19); down: EWL state Eqs. (18,20).

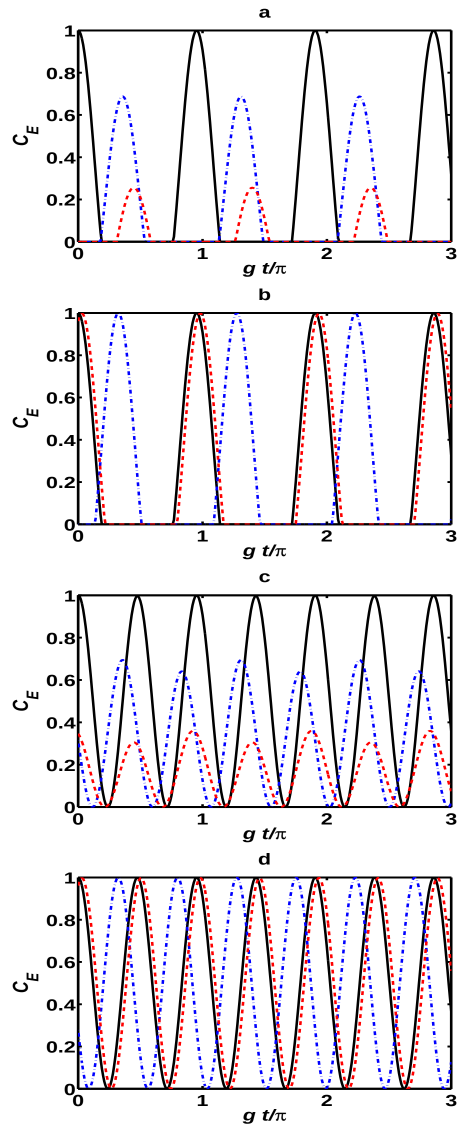

Fig. 6: The same as Fig.(5) but for $k_1 = k_2 = n_1 = n_2 = 10.0$.

Strictly speaking, in this case, concurrence is directly given by the coherence between the $|10\rangle$ and $|01\rangle$ states as in Eq.(48) and since the coherence vanishes in asymptotic way, there cannot be ESD for any generic state with maximum one excitation[67]. Analogously, entanglement can be smoothly generated but it cannot suddenly appear. This result does not depend on the degree of purity of the state. Here, we realize that concurrence collapses and revives asymptotically with periods of half of case 1. Hence, for this type of EWL state, concurrence can be more efficient for quantifying nonclassical correlation. A general shared property between both cases is that the revivals of correlations appear roughly when the state becomes purer.

A clearer insight can be visualized from behavior snapshot when $r = 1$, Figs. (3a,b, 6c,d) and Figs. (3c,d, 6a,b). From the figures we see clearly the effect of long time interval T between pulses and considerably small detuning on creation of maximally entangled atoms, where entanglement attain its maxima and minima at same times as the case when bang-bang pulses are absent. As the number state increases, where all other parameters kept as above, sensitivity of $\Omega_{eff}(k)$ to number state k become higher with considerable large time period T between two consecutive pulses, we notice rapid increase

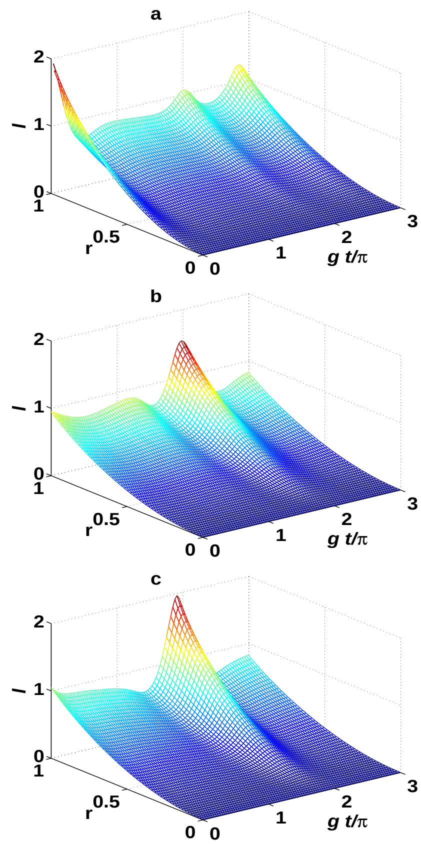

Fig. 7: Thermal sate field $I(t,r)$ (left colmn) and $\mathscr{Q}(t,r)$ (right colmn) for $\bar{k}_1 = \bar{k}_2 = \bar{n}_1 = \bar{n}_2 = 0.1$, for ($\delta = 0.6\ T = 0.0$), ($\delta = 0.6\ T = 0.6$), and ($\delta = 0.01\ T = 0.6$), respectively.

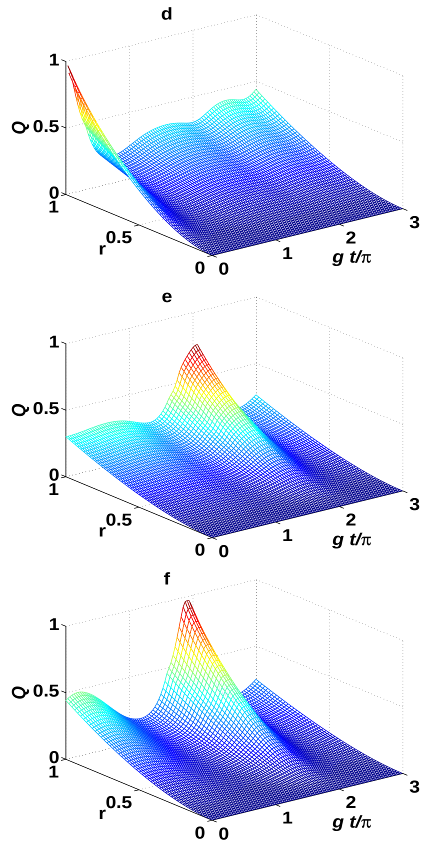

Fig. (7) continued

to correlation maxima for the same change in parameters under consideration.

Turning our attention to case of interaction with initially thermal field. Bearing in mind figures order details as in Fock state, graphical results are showed in figs. (7-10). Figures shape displays interesting and surprising results. For mesh graphs, we keep time interval between two consecutive pulses fixed such as $T = 0.6$, while cases of $T = 0.0$ are depicted as a reference case. We have assumed a thermal filed with various mean photon numbers in the cavity is applied. In absence of bang-bang pulses but detuning is high, I and $\mathscr{Q}$ begin maximum when interaction is switching on, then they fluctuate around half of this value see Fig(7). As soon as driven bang-bang pulse are switched on, I and $\mathscr{Q}$ start live from half of their maximum while fluctuate and peak at about $\approx 2\pi$ which means shifted peak to the right by 2π of time scale. If detuning considerably decreased, keeping small mean photon number $\bar{k}_i$ such as $\bar{k}_i = 0.1$, fluctuations preserved with peaks localized at same position for both I and $\mathscr{Q}$ but with higher amplitudes reach their maximum values when bang-bang pulses were switched on. The interesting here is the effect of change of the mean photon number $\bar{k}_i$. Situation now is completely different. Keeping the time interval T long and fixed such that $T = 0.6$, effect of detuning is kept with no change in shape, while correlated atoms (almost fixed correlation amplitude) can be obtained when mean photon number decreased near vacuum, see Fig. (8c,f). In case where $r = 1$ the two atoms remain maximally correlated as time developes.

For concurrence, results are more interesting and surprising. Bearing in mind the settings as indicated above, for EWL state of two excitations Bell-like state, Eqs. (18, 19), the dark period of time, during which the concurrence is zero, is shorter for smaller values of mean photon numbers $\bar{k}_i$, in contrary to Fock state. In this case, for bigger mean photon numbers $\bar{k}_i$, probability of interference between modes waves is highly occurred and interaction between subsystems is invocked in wider areas. This actually means that entanglement sudden death(ESD) phenomenon can be diminished by proper setting up of mean photon number inside the cavity, see Figs. (9c, 10b). Moreover, with same setting of efficient parameters, ESD may disappear absolutely if interaction begins while atoms prepared initially in EWL state of one excitation Bell-like state, Eqs. (18, 20), see Figs. (9, 10). Furthermore, on considering samll mean photon number

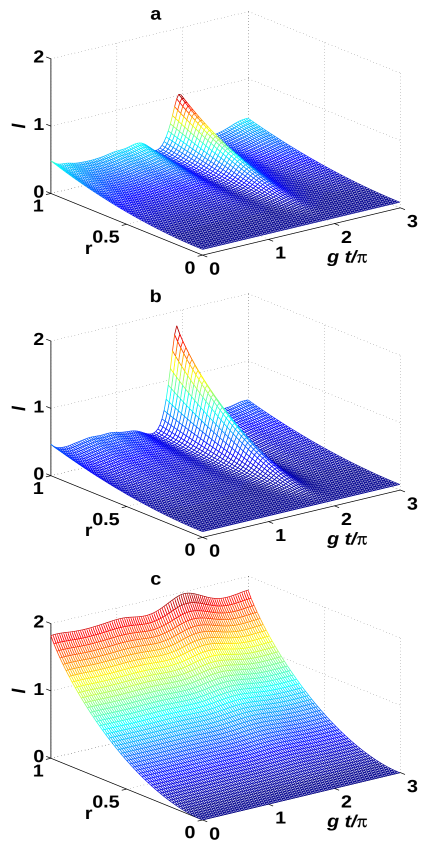

Fig. 8: The same as Fig.(7) but for ($\bar{k}_1 = \bar{k}_2 = \bar{n}_1 = \bar{n}_2 = 0.5$, $\delta = 0.6$), ($\bar{k}_1 = \bar{k}_2 = \bar{n}_1 = \bar{n}_2 = 0.5$, $\delta = 0.01$) and ($\bar{k}_1 = \bar{k}_2 = \bar{n}_1 = \bar{n}_2 = 0.01$, $\delta = 0.6$).

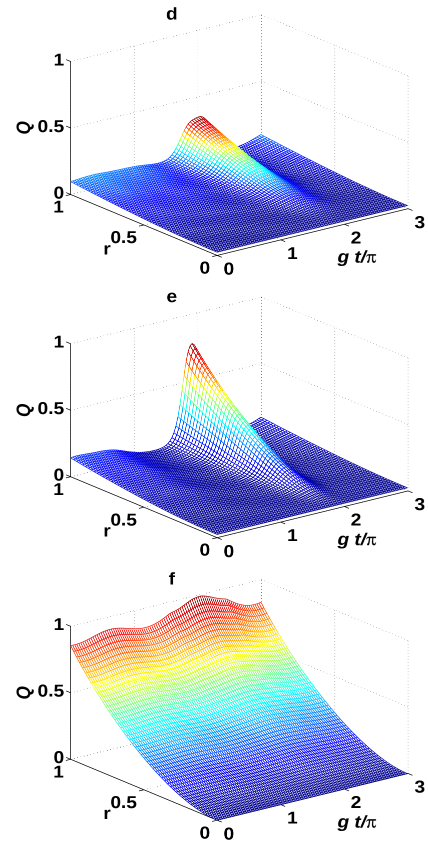

Fig. (8) continued

near vacuum inside thermal cavity, entanglement grows faster towards maximum with change in purity parameter r towards pure entangle states of the atoms, see Figs. (9e,f, 10d,e). Note that entanglement also fluctuates with initially appeared peak shifted towards right as indicated above in case of I and $\mathscr{Q}$. We emphesize that the optimized steady entagled atomic state with $\mathscr{C}_E \approx 1$ can be obtained on starting interaction with atomic system initially in EWL state Eq.(18, 20). It is apparent that, for high mean photon numbers, entanglement fluctuates and the amplitudes depend on EWL state type.

7 Conclusion

In conclusion, both nonclassical correlation and entanglement of initially entangled noninteracting two qubits coupled independently to a couple of quantized field modes via a nonresonant Raman interaction, can be enhanced if we set up interaction to be barely detuned while qubits are driven by bang-bang pulses with long time intervals between two consecutive pulses. Moreover, for two qubits prepared initially in EWL states includes one excitation Bell-like state, ESD disappears where entanglement between them vanishes asymptotically. In adddition, when the cavity contains thermal field distribution, as possible as preserving the mean photon number small near vacuum, maximally correlated qubits can be obtained where almost linear correlation extended over all time scale can be shown regardless of change in any other parameters.

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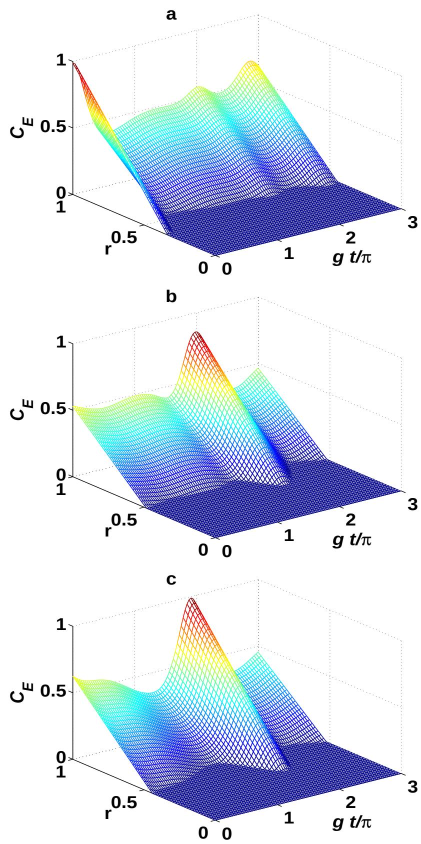

Fig. 9: Thermal sate field $\mathscr{C}_E(t,r)$ when $\bar{k}_1 = \bar{k}_2 = \bar{n}_1 = \bar{n}_2 = 0.1$ wher (left colmn, for EWL Eqs.(18,19), (right colmn for EWL Eqs.(18,20) for ($\delta = 0.6$ $T = 0.0$), ($\delta = 0.6$ $T = 0.6$), and ($\delta = 0.001$ $T = 0.6$), respectively.

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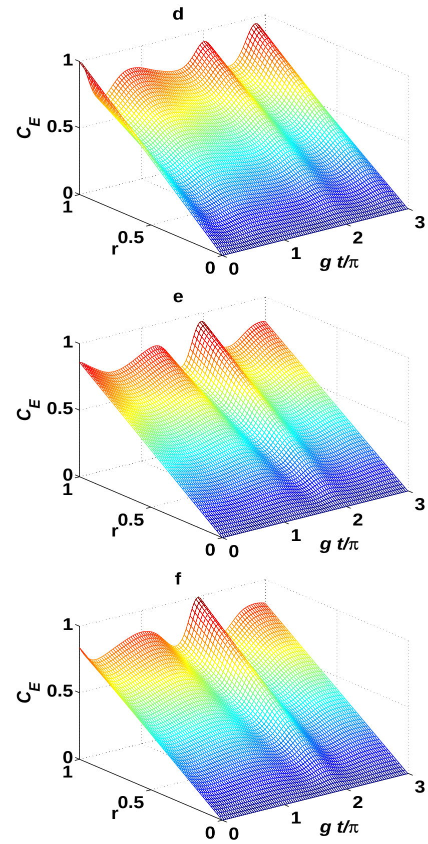

Fig. (9) continued

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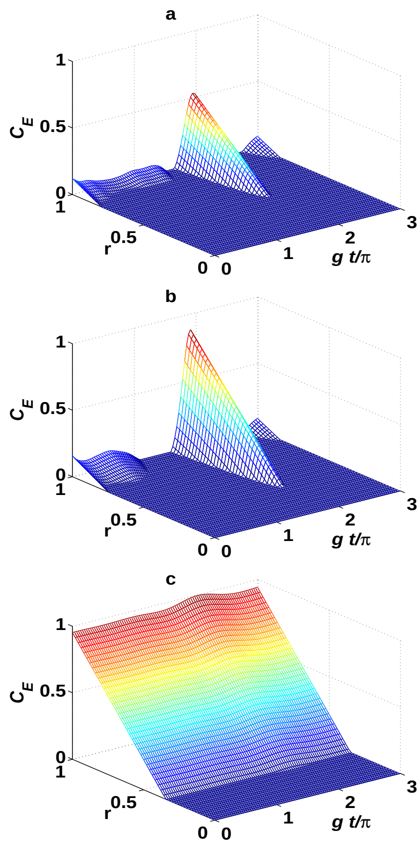

Fig. 10: The same as Fig.(9) but for ($\bar{k}_1 = \bar{k}_2 = \bar{n}_1 = \bar{n}_2 = 0.5$, $\delta = 0.6$), ($\bar{k}_1 = \bar{k}_2 = \bar{n}_1 = \bar{n}_2 = 0.5$, $\delta = 0.001$), and ($\bar{k}_1 = \bar{k}_2 = \bar{n}_1 = \bar{n}_2 = 0.01$, $\delta = 0.6$), respectively.

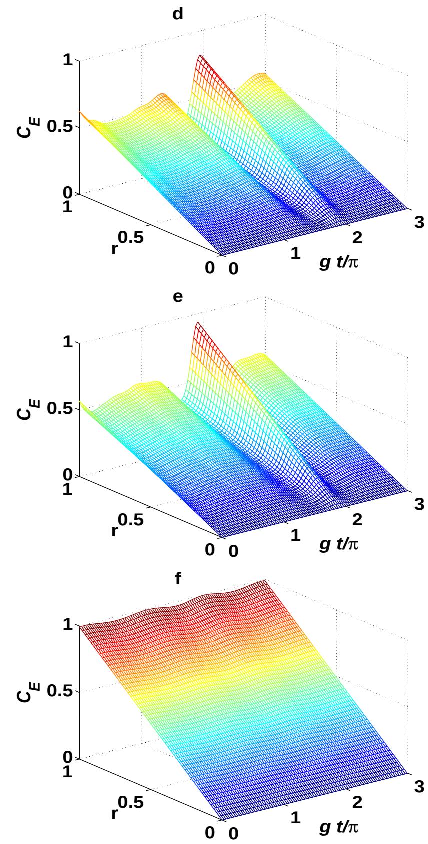

Fig. (10) continued

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