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Two Generalized Soft Intersection Filters in BL-algebras

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Abstract: In this paper, we introduce the concepts of two generalized soft intersection filters of BL-algebras. Some characterizations of these soft intersection filters are discussed. In particular, the relationships among these soft intersection filters are established. This provides a new soft algebraic tool in many uncertainties problems.

Keywords: Soft set; BL-algebras; filter; (M,N)-SI filter; (M,N)-SI implicative ((M,N)-SI-PI, (M,N)-SI fantastic) filter.

1 Introduction

Fuzzy logic has been appeared as a new discipline from the necessity to deal with some kinds of data and imprecise information caused by indistinguishability of objects in certain experimental environments. A BL-algebra has been introduced by Hájek as the algebraic structures for his Basic Logic [8]. A well known example of a BL-algebra is the interval [0,1] endowed with the structure induced by a continuous t-norm. In fact, the MV-algebras, Gödel algebras and product algebras are the most known classes of BL-algebras. BL-algebras are further discussed by many researchers. Turunen [23,24,25] studied some types of filters and proved that the implicative filters are equivalent to the Boolean filters in BL-algebras. Liu [12] applied fuzzy set theory to BL-algebras and introduced the concepts of fuzzy implicative and positive implicative filters, and discussed their related properties. Furthermore, Ma and Zhan investigated some kinds of generalized fuzzy filters in BL-algebras and obtained some important results, see [14,15,28,30]. Zhang [32] described the relations between pseudo-BL and pseudo-effect algebras. The other related results can be found in [10,20,26,31].

It is known well that the complexities of modeling uncertain data in economics, engineering, environmental science, sociology, information sciences and many other fields can not be successfully dealt with by classical methods. To overcome these difficulties, Molodtsov [18] proposed a completely new approach for modeling vagueness and uncertainty, which is called soft set theory. Since then, especially soft set operations have been studied intensively. See [1,2,7,16,19]. Note that soft set theory emphasizes a balanced coverage of both theory and practice. Nowadays, it has promoted a breath of the discipline of information sciences, intelligent systems, expert and decision support systems, expert and decision support systems, knowledge systems and decision making, and so on. For examples, see[4,5,6,17].

At the same time, the soft set theory has a wide range of applications in the algebraic structures such as [9,22,29]. In particular, Çağman, Sezgin and Jun applied the soft intersection theory to groups [3], near-rings [21] and BL-algebras [11]. Recently, Ma et al. [13,27] put forward the new concept: (M,N)-soft intersection set as a generalization of soft intersection sets. They introduced the concepts of (M,N)-soft intersection filters and (M,N)-soft intersection implicative filters in BL-algebras respectively.

As a continuation of [13,27], we organize the present paper as follows. In section 2, we recall some concepts and results of both BL-algebras and soft sets. In section 3, we investigate some characterizations of (M,N)-SI-PI filters of BL-algebras. In section 4, we investigate some characterizations of (M,N)-SI fantastic filters of BL-algebras. Finally, we prove that a soft set in BL-algebras is an (M,N)-SI implicative filter if and only if it is both an (M,N)-SI-PI filter and an (M,N)-SI fantastic filter.

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2 Preliminaries

Recall that an algebra $L=(L,\leq,\wedge,\vee,\odot,\rightarrow,0,1)$ is called a *BL-algebra* [8] if it is a bounded lattice satisfying the following conditions:

(i) $(L,\odot,1)$ is a commutative monoid,

(ii) $\odot$ and $\rightarrow$ form an adjoin pair, i.e., $z\leq x\rightarrow y$ if and only if $x\odot z\leq y$ for all $x,y,z\in L$,

(iii) $x\wedge y=x\odot(x\rightarrow y)$,

(iv) $(x\rightarrow y)\vee(y\rightarrow x)=1$.

In what follows, L is a BL-algebra unless otherwise specified.

In any BL-algebra L, the following statements are true (see [8,23,24]):

(a_1) $x\leq y\Leftrightarrow x\rightarrow y=1$,

(a_2) $x\rightarrow(y\rightarrow z)=(x\odot y)\rightarrow z=y\rightarrow(x\rightarrow z)$,

(a_1) $x\odot y\leq x\wedge y$,

(a_4) $x\rightarrow y\leq(z\rightarrow x)\rightarrow(z\rightarrow y), x\rightarrow y\leq(y\rightarrow z)\rightarrow(x\rightarrow z)$,

(a_5) $x\rightarrow x'=x''\rightarrow x$,

(a_6) $x\vee x'=1\Rightarrow x\wedge x'=0$,

(a_7) $(x\rightarrow y)\odot(y\rightarrow z)\leq x\rightarrow z$,

(a_8) $x\leq y\Rightarrow x\rightarrow z\geq y\rightarrow z$,

(a_9) $x\leq y\Rightarrow z\rightarrow x\leq z\rightarrow y$,

(a_{10}) $x\vee y=((x\rightarrow y)\rightarrow y)\wedge((y\rightarrow x)\rightarrow x)$,

where $x'=x\rightarrow 0$.

A non-empty subset A of L is called a *filter* of L if it satisfies (I1) $1\in A$, (I2) $\forall x\in A,\forall y\in L, x\rightarrow y\in A\Rightarrow y\in A$. It is easy to check that a non-empty subset A of L is a filter of L if and only if it satisfies: (I3) $\forall x,y\in L, x\odot y\in A$, (I4) $\forall x\in A,\forall y\in L, x\leq y\Rightarrow y\in A$ (see [24]).

Now, we call a non-empty subset A of L an *implicative filter* if it satisfies (I1)and

(I5) $x\rightarrow(z'\rightarrow y)\in A, y\rightarrow z\in A\Rightarrow x\rightarrow z\in A$.

A non-empty subset A of L is said to be a *positive implicative filter* of L if it satisfies (I1) and

(I6) $x\rightarrow(y\rightarrow z)\in A, x\rightarrow y\in A\Rightarrow x\rightarrow z\in A$.

A non-empty subset A of L is called a *fantastic filter* of L if it satisfies (I1) and

(I7) $z\rightarrow(y\rightarrow x)\in A, z\in A\Rightarrow((x\rightarrow y)\rightarrow y)\rightarrow x\in A$.

A non-empty subset A of L is said to be a *Boolean filter* of L if it satisfies $x\vee x'\in A$, for all $x\in A$. (see [24, 12,14,15])

From now on, we let L be an BL-algebra and let U be an initial universe. We let E be a set of parameters and let $P(U)$ be the power set of U and $A,B,C\subseteq E$. We let $\emptyset\subseteq M\subset N\subseteq U$.

Definition 2.1. [18,4] A soft set f_A over U is a set defined by $f_A:E\rightarrow P(U)$ such that $f_A(x)=\emptyset$ if $x\notin A$. Here f_A is also called an *approximate function*. A soft set over U can be represented by the set of ordered pairs $f_A=\{(x,f_A(x))|x\in E, f_A(x)\in P(U)\}$. It is clear to see that a soft set is a parameterized family of subsets of U. Note that the set of all soft sets over U will be denoted by $S(U)$.

Definition 2.2. [4] Let $f_A,f_B\in S(U)$.

(1) f_A is said to be a *soft subset* of f_B and denoted by $f_A\tilde{\subseteq}f_B$ if $f_A(x)\subseteq f_B(x)$, for all $x\in E$. f_A and f_B are said to be *soft equal*, denoted by $f_A=f_B$, if $f_A\tilde{\subseteq}f_B$ and $f_A\tilde{\supseteq}f_B$.

(2) The union of f_A and f_B, denoted by $f_A\tilde{\cup}f_B$, is defined as $f_A\tilde{\cup}f_B=f_{A\cup B}$, where $f_{A\cup B}(x)=f_A(x)\cup f_B(x)$ for all $x\in E$.

(3) The intersection of f_A and f_B, denoted by $f_A\tilde{\cap}f_B$, is defined as $f_A\tilde{\cap}f_B=f_{A\cap B}$, where $f_{A\cap B}(x)=f_A(x)\cap f_B(x)$ for all $x\in E$.

Definition 2.3. [11](1) A soft set f_L over U is called an *SI-filter* of L over U if it satisfies:

(S_1) $f_L(x)\subseteq f_L(1)$ for any $x\in L$,

(S_2) $f_L(x\rightarrow y)\cap f_L(x)\subseteq f_L(y)$ for all $x,y\in L$,

(2) A soft set f_L over U is called an *SI-implicative filter* of L over U if it satisfies (S_1) and

(S_3) $f_L(y\rightarrow z)\cap f_L(x\rightarrow(z'\rightarrow y))\subseteq f_L(x\rightarrow z)$ for all $x,y,z\in L$,

(3) A soft set f_L over U is called an *SI-positive implicative filter* of L over U if it satisfies (S_1) and

(S_4) $f_L(x\rightarrow y)\cap f_L(x\rightarrow(y\rightarrow z))\subseteq f_L(x\rightarrow z)$ for all $x,y,z\in L$.

Definition 2.4. A soft set f_L over U is called an *SI-fantastic filter* of L over U if it satisfies (S_1) and

(S_5) $f_L(z)\cap f_L(z\rightarrow(y\rightarrow x))\subseteq f_L(((x\rightarrow y)\rightarrow y)\rightarrow x)$ for all $x,y,z\in L$.

In [13,27], Ma et al. introduced the concepts of (M,N)-*SI* filters and (M,N)-*SI* implicative filters in BL-algebras, respectively.

Definition 2.5. (1) A soft set f_L over U is called an (M,N)-*soft intersection filter* (briefly, (M,N)-*SI filter*) of L over U if it satisfies:

(SI_1) $f_L(x)\cap N\subseteq f_L(1)\cup M$ for all $x\in L$,

(SI_2) $f_L(x\rightarrow y)\cap f_L(x)\cap N\subseteq f_L(y)\cup M$ for all $x,y\in L$.

(2) A soft set f_L over U is called an (M,N)-*soft intersection implicative filter* (briefly, (M,N)-*SI implicative filter*) of L over U if it satisfies (SI_1) and

(SI_3) $f_L(y\rightarrow z)\cap f_L(x\rightarrow(z'\rightarrow y))\cap N\subseteq f_L(x\rightarrow z)\cup M$ for all $x,y,z\in L$.

Define an ordered relation "$\tilde{\subseteq}_{(M,N)}$" on $S(U)$ as follows: For any $f_L,g_L\in S(U),\emptyset\subseteq M\subset N\subseteq U$, we define $f_L\tilde{\subseteq}_{(M,N)}g_L\Leftrightarrow f_L\cap N\tilde{\subseteq}g_L\cup M$. And we define a relation "$=_{(M,N)}$" as follows: $f_L=_{(M,N)}g_L\Leftrightarrow f_L\tilde{\subseteq}_{(M,N)}g_L$ and $g_L\tilde{\subseteq}_{(M,N)}f_L$.

Definition 2.6. [13,27] (1) A soft set f_L over U is called an (M,N)-*SI filter* of L over U if it satisfies:

(SI_1') $f_L(x)\tilde{\subseteq}_{(M,N)}f_L(1)$ for all $x\in L$,

(SI_2') $f_L(x\rightarrow y)\cap f_L(x)\tilde{\subseteq}_{(M,N)}f_L(y)$ for all $x,y\in L$.

(2) A soft set f_L over U is called an (M,N)-*SI implicative filter* of L over U if it satisfies (SI_1') and

(SI_3') $f_L(y\rightarrow z)\cap f_L(x\rightarrow(z'\rightarrow y))\tilde{\subseteq}_{(M,N)}f_L(x\rightarrow z)$ for all $x,y,z\in L$.

3 (M,N)-*SI-PI* filters

In this section, we introduce the concept of (M,N)-*SI-PI* filters in *BL*-algebras, and investigate their properties.

Definition 3.1. A soft set f_S over U is called an (M,N)-*soft intersection positive implicative filter* (briefly, (M,N)-*SI-PI filter*) of L over U if it satisfies (SI_1) and

(SI_4) $f_L(x\to(y\to z))\cap f_L(x\to y)\cap N\subseteq f_L(x\to z)\cup M$

for all $x,y,z\in L$.

Remark 3.2. If f_L is an (M,N)-*SI-PI* filter of L over U, then f_L is an $(\emptyset,U)$-*SI-PI* filter of L over U. Hence every *SI*-positive implicative filter of L is an (M,N)-*SI-PI* filter of L, but the converse need not be true in general. See the following example.

Example 3.3. Assume that $U=S_3$, the symmetric 3-group, is the universal set and let $L=\{0,a,b,1\}$, where $0<a<b<1$. We define $x\wedge y:=\min\{x,y\}, x\vee y:=\max\{x,y\}$, and $\odot$ and $\to$ as follows:

$\odot$	0	a	b	1
0	0	0	0	0
a	0	a	a	a
b	0	a	b	b
1	0	a	b	1

$\to$	0	a	b	1
0	1	1	1	1
a	0	1	1	1
b	0	a	1	1
1	0	a	b	1

It is clear that $(L,\wedge,\vee,\odot,\to,0,1)$ is a *BL*-algebra. Let $M=\{(13),(123)\}$ and $N=\{(1),(12),(13),(123)\}$. Define a soft set f_L over U by $f_L(1)=\{(1),(12),(123)\}$, $f_L(a)=f_L(b)=\{(1),(12),(13),(123)\}$ and $f_L(0)=\{(1),(12)\}$. Then we can easily check that f_L is an (M,N)-*SI-PI* filter of L over U, but it is not an *SI*-positive implicative filter of L over U since $f_L(a)\not\subseteq f_L(1)$.

By means of "$\widetilde{\subseteq}_{(M,N)}$", we obtain the following equivalent concept.

Definition 3.4. A soft set f_S over U is called an (M,N)-*SI-PI filter* of L over U if it satisfies (SI_1') and

(SI_4') $f_L(x\to(y\to z))\cap f_L(x\to y)\widetilde{\subseteq}_{(M,N)}\subseteq f_L(x\to z)$

for all $x,y,z\in L$.

Theorem 3.5. *Every (M,N)-SI-PI filter of L over U is an (M,N)-SI filter.*

Proof. If we let $x:=1$ in (SI_4'), then $f_L(1\to z)\widetilde{\supseteq}_{(M,N)} f_L(1\to(y\to z))\cap f_L(1\to y)$, that is, $f_L(z)\widetilde{\supseteq}_{(M,N)} f_L(y\to z)\cap f_L(y)$. This shows that (SI_2') holds. Thus, f_L is an (M,N)-*SI* filter of L. □

Remark 3.6. The converse of Theorem 3.5 may not be true as shown in the following example.

Example 3.7. Assume that $U=\{0,t_1,t_2,t_3,t_4,1\}$, where $0<t_1<t_2<t_3<t_4<1$, is the universal set and let $L=\{0,a,b,1\}$ be a chain with the following Cayley tables:

$\odot$	0	a	b	1
0	0	0	0	0
a	0	a	a	a
b	0	a	a	b
1	0	a	b	1

$\to$	0	a	b	1
0	1	1	1	1
a	0	1	1	1
b	0	b	1	1
1	0	a	b	1

Define the operations $\wedge$ and $\vee$ on L as "min" and "max", respectively. Then $(L,\wedge,\vee,\odot,\to,0,1)$ is a *BL*-algebra. Let $M=\{t_2\}$ and $N=\{t_2,t_3\}$. Define a soft set f_L over U by $f_L(1)=\{t_3,t_4\}$, $f_L(a)=f_L(b)=\{t_2\}$ and $f_L(0)=\{0,t_1\}$. One can easily check that f_L is an (M,N)-*SI* filter of L over U, since $f_L(b\to a)\cup M=f_L(b)\cup M=\{t_2\}\not\supseteq f_L(b\to(b\to a))\cap f_L(b\to b)\cap N=f_L(1)\cap N=\{t_3,t_4\}\cap\{t_2,t_3\}=\{t_3\}$.

Lemma 3.8. [13] *Let f_L be an (M,N)-SI-filter of L over U. Then for any $x,y,z\in L$, we have*

(1) $x\le y\Rightarrow f_L(x)\widetilde{\subseteq}_{(M,N)} f_L(y)$,

(2) $f_L(x\to y)=f_L(1)\Rightarrow f_L(x)\widetilde{\subseteq}_{(M,N)} f_L(y)$,

(3) $f_L(x\odot y)=_{(M,N)} f_L(x)\cap f_L(y)=_{(M,N)} f_L(x\wedge y)$,

(4) $f_L(0)=_{(M,N)} f_L(x)\cap f_L(x')$,

(5) $f_L(x\to y)\cap f_L(y\to z)\widetilde{\subseteq}_{(M,N)} f_L(x\to z)$,

(6) $f_L(x)\cap f_L(y)\widetilde{\subseteq}_{(M,N)} f_L(x\odot z\to y\odot z)$,

(7) $f_L(x\to y)\widetilde{\subseteq}_{(M,N)} f_L((y\to z)\to(x\to z))$,

(8) $f_L(x\to y)\widetilde{\subseteq}_{(M,N)} f_L((z\to x)\to(z\to y))$.

Now, we characterize (M,N)-*SI-PI* filters of *BL*-algebras as follows.

Theorem 3.9. *Let f_L be an (M,N)-SI filter of L over U. Then the following are equivalent:*

(1) *f_L is an (M,N)-SI-PI filter of L,*

(2) *$f_L(x\to y)\widetilde{\supseteq}_{(M,N)} f_L((x\to(x\to y))$ for all $x,y\in L$,*

(3) *$f_L(x\to y)=_{(M,N)} f_L(x\to(x\to y))$ for all $x,y\in L$.*

Proof. (1) ⇒ (2). Assume that f_L is an (M,N)-*SI-PI* filter of L over U. If we let $z:=y$ and $y:=x$ in (SI_4'), then

$$\begin{aligned} f_L(x\to y) &\widetilde{\supseteq}_{(M,N)} f_L(x\to(x\to y))\cap f_L(x\to x)\\ &= f_L(x\to(x\to y))\cap f_L(1)\\ &\widetilde{\supseteq}_{(M,N)} f_L(x\to(x\to y)). \end{aligned}$$

Thus (2) holds.

(2) ⇒ (3). For any $x,y\in L$, we have

$$\begin{aligned} &f_L(x\to(x\to y))\\ \widetilde{\supseteq}_{(M,N)}\ & f_L(x\to y)\cap f_L((x\to y)\to(x\to(x\to y)))\\ =\ & f_L(x\to y)\cap f_L(x\to((x\to y)\to(x\to y)))\\ =\ & f_L(x\to y)\cap f_L(x\to 1)\\ =\ & f_L(x\to y)\cap f_L(1)\\ \widetilde{\supseteq}_{(M,N)}\ & f_L(x\to y), \end{aligned}$$

which implies $f_L(x\to(x\to y))\widetilde{\supseteq}_{(M,N)} f_L(x\to y)$. Combining (2), we have $f_L(x\to(x\to y))=_{(M,N)} f_L(x\to y)$.

(3) ⇒ (1). Using (3) and Lemma 3.8 (5), we have

$$\begin{aligned} f_L(x\to z) &=_{(M,N)} f_L(x\to(x\to z))\\ &\widetilde{\supseteq}_{(M,N)} f_L(x\to y)\cap f_L(y\to(x\to z))\\ &= f_L(x\to y)\cap f_L(x\to(y\to z)). \end{aligned}$$

Thus, by Definition 3.4, f_L is an (M,N)-*SI-PI* filter of L over U. □

Theorem 3.10. *If f_L is an (M,N)-SI filter of L over U, then the following are equivalent:*
(1) *f_L is an (M,N)-SI-PI filter of L,*
(2) *$f_L(x\to(y\to z))\widetilde{\subseteq}_{(M,N)} f_L((x\to y)\to(x\to z))$ for all $x,y,z\in L$,*
(3) *$f_L(x\to(y\to z)) =_{(M,N)} f_L((x\to y)\to(x\to z))$ for all $x,y,z\in L$,*
(4) *$f_L((x\odot y)\to z) =_{(M,N)} f_L((x\wedge y)\to z)$ for all $x,y,z\in L$.*

Proof. $(1)\Rightarrow(2)$. Assume f_L is an (M,N)-*SI-PI* filter of L over U. If we let $y:=y\to z$ and $z:=(x\to y)\to z$ in (SI_4') and Lemma 3.8, we obtain

$$\begin{aligned}
&f_L((x\to y)\to(x\to z))\\
&= f_L(x\to((x\to y)\to z))\\
&\widetilde{\supseteq}_{(M,N)} f_L(x\to((y\to z)\to((x\to y)\to z)))\\
&\qquad \cap f_L(x\to(y\to z))\\
&= f_L((y\to z)\to((x\to y)\to(x\to z)))\\
&\qquad \cap f_L(x\to(y\to z))\\
&= f_L(1)\cap f_L(x\to(y\to z))\\
&\widetilde{\supseteq}_{(M,N)} f_L(x\to(y\to z)).
\end{aligned}$$

Thus (2) holds.

$(2)\Rightarrow(3)$. Obvious.

$(3)\Rightarrow(4)$. By (a_2), (iii) and (3), we have $(x\odot y)\to z = x\to(y\to z)$ and $(x\wedge y)\to z=(x\odot(x\to y))\to z=(x\to y)\to(x\to z)$, and so $f_L((x\odot y\to z)=_{(M,N)} f_L((x\wedge y)\to z))$.

$(4)\Rightarrow(1)$. By Lemma 3.8, we have $f_L(x\to(y\to z))\widetilde{\supseteq}_{(M,N)} f_L(x\to(y\to z))\cap f_L(x\to y)$. By (a_2) and (4), we have $f_L(x\to(x\to z)) = f_L((x\odot x)\to z) =_{(M,N)} f_L((x\wedge x)\to z) = f_L(x\to z)$. Hence $f_L(x\to z)\widetilde{\supseteq}_{(M,N)} f_L(x\to(y\to z))\cap f_L(x\to y)$. Hence (SI_4') holds. Therefore, f_L is an (M,N)-*SI-PI* filter of L. □

The relationship between (M,N)-*SI* Boolean filters and (M,N)-*SI* implicative filters of *BL*-algebras is shown as follows:

Lemma 3.11. [27] *A soft set f_L of L over U is an (M,N)-SI Boolean filter of L if and only if it is an (M,N)-SI implicative filter.*

Finally, we give a relationship between (M,N)-*SI* implicative (Boolean) filters and (M,N)-*SI-PI* filters of *BL*-algebras.

Theorem 3.12. *Every (M,N)-SI implicative (Boolean) filter of L over U is an (M,N)-SI-PI filter.*

Proof. Assume that f_L is an (M,N)-*SI* implicative (Boolean)filter of L over U. Then, for any $x,y,z\in L$, we have

$$\begin{aligned}
f_L(x\to z) &\widetilde{\supseteq}_{(M,N)} f_L((x\vee x')\to(x\to z))\cap f_L(x\vee x')\\
&=_{(M,N)} f_L((x\wedge x')\to(x\to z))\cap f_L(1)\\
&\widetilde{\supseteq}_{(M,N)} f_L((x\vee x')\to(x\to z)).
\end{aligned}$$

Since $(x\vee x')\to(x\to z)=(x\to(x\to z))\wedge(x'\to(x\to z))=x\to(x\to z)$, we have $f_L((x\wedge x')\to(x\to z))=_{(M,N)} f_L(x\to(x\to z))$. Hence $f_L(x\to z)\widetilde{\supseteq}_{(M,N)} f_L(x\to(x\to z))$. It follows from Theorem 3.9 that f_L is an (M,N)-*SI-PI* filter of L over U. □

Remark 3.13. The converse of Theorem 3.12 may not be true as shown in the following example:

Example 3.14. Consider the *BL*-algebra $L=\{0,a,b,1\}$ as in Example 3.3. Let $U=\{0,t_1,t_2,1\}$, where $0<t_1<t_2<1$ and let $M=\{t_1\}$ and $N=\{t_1,t_2\}$. Define a soft set f_L over U by $f_L(1)=\{t_2,1\}$, $f_L(a)=f_L(b)=\{t_1\}$ and $f_L(0)=\{0,t_1\}$. Then one can easily check that f_L is an (M,N)-*SI-PI* filter of L over U, but it is not an (M,N)-*SI* Boolean filter of L, since $f_L(a\vee a')\cap N=f_L(a)\cap N=\{t_1\}\neq f_L(1)\cup M=\{t_2,1,t_1\}$.

4 (M,N)-*SI* fantastic filters

In this section, we investigate some characterizations of (M,N)-*SI* fantastic filters of *BL*-algebras. Finally, we prove that a soft set in *BL*-algebras is an (M,N)-*SI* implicative filter if and only if it is both an (M,N)-*SI-PI* filter and an (M,N)-*SI* fantastic filter.

Definition 4.1. A soft set f_L over U is called an (M,N)-*soft intersection fantastic filter* (briefly, (M,N)-*SI-fantastic filter*) of L over U if it satisfies (SI_1) and

(SI_5) $f_L(z\to(y\to x))\cap f_L(z)\cap N\subseteq f_L(((x\to y)\to y)\to x)\cup M$ for all $x,y,z\in L$.

Remark 4.2. If f_L is an (M,N)-*SI* fantastic filter of L over U, then f_L is an $(\emptyset,U)$-*SI* fantastic filter of L. Hence every *SI*-fantastic filter of L is an (M,N)-*SI* fantastic filter of L, but the converse may not be true.

Example 4.3. Assume that $U=D_2=\{<x,y>|x^2=y^2=e,xy=yx\}=\{e,x,y,yx\}$, the Dihedral 2-group, is the universe set. Let $L=\{0,a,b,1\}$, where $0<a<b<1$. We define $x\wedge y:=\min\{x,y\}$, $x\vee y:=\max\{x,y\}$, and $\odot$ and $\to$ as follows:

$\odot$	0	a	b	1
0	0	0	0	0
a	0	0	0	a
b	0	0	a	b
1	0	a	b	1

$\to$	0	a	b	1
0	1	1	1	1
a	b	1	1	1
b	a	b	1	1
1	0	a	b	1

Then $(L,\wedge,\vee,\odot,\rightarrow,0,1)$ is a BL-algebra. Let $M=\{e,y\}$ and $N=\{e,x,y\}$. Define a soft set f_L over U by $f_L(1)=\{e,x\}$ and $f_L(a)=f_L(b)=f_L(0)=\{e,x,y\}$. Then f_L is an (M,N)-*SI* fantastic filter of L over U, but it is not an *SI*-fantastic filter of L over U, since $f_L(a)\nsubseteq f_L(1)$.

By using the relation " $\subseteq_{(M,N)}$ ", we can obtain an equivalent relation of (M,N)-*SI* fantastic filters.

Definition 4.4. A soft set f_L over U is called an (M,N)-*SI fantastic filter* of L over U if it satisfies (SI_1') and (SI_5')
$f_L(z\rightarrow(y\rightarrow x))\cap f_L(z)\widetilde{\subseteq}_{(M,N)} f_L(((x\rightarrow y)\rightarrow y)\rightarrow x)$, for all $x,y,z\in L$.

Theorem 4.5. *Every (M,N)-SI fantastic filter of L over U is an (M,N)-SI filter.*

Proof. If we let $y:=1$ in (SI_5'), then $f_L(((x\rightarrow 1)\rightarrow 1)\rightarrow x)\widetilde{\supseteq}_{(M,N)} f_L(z\rightarrow(1\rightarrow x))\cap f_L(z)$, that is, $f_L(x)\widetilde{\supseteq}_{(M,N)} f_L(z\rightarrow x)\cap f_L(z)$. This proves (SI_2') holds. Thus f_L is an (M,N)-*SI* filter of L. □

Remark 4.6. The converse of Theorem 4.5 may not be true as shown in the following:

Example 4.7. In Example 3.14, we know that f_L is an (M,N)-*SI* filter of L over U, but it is not an (M,N)-*SI* fantastic filter of L over U, since $f_L(((b\rightarrow a)\rightarrow a)\rightarrow b)\cup M=f_L(b)\cup M=\{t_1\}$, $f_L(1\rightarrow(a\rightarrow b))\cap f_L(1)\cap N=\{t_2\}$. This implies that $f_L(((b\rightarrow a)\rightarrow a)\rightarrow b)\cup M\nsupseteq f_L(1\rightarrow(a\rightarrow b))\cap f_L(1)\cap N$.

Now we investigate some properties of (M,N)-*SI* fantastic filter of BL-algebras.

Theorem 4.8. *Let f_L be an (M,N)-SI filter of of L over U. Then f_L is an (M,N)-SI fantastic filter of L if and only if it satisfies the following condition:*

(SI_6') $f_L(((x\rightarrow y)\rightarrow y)\rightarrow x)\widetilde{\supseteq}_{(M,N)} f_L(y\rightarrow x)$ *for all* $x,y,z\in L$.

Proof. Assume that f_L is an (M,N)-*SI* filter of L over U. If we let $z:=1$ in (SI_5'), then $f_L(((x\rightarrow y)\rightarrow y)\rightarrow x)\widetilde{\supseteq}_{(M,N)} f_L(1\rightarrow(y\rightarrow x))\cap f_L(1)\widetilde{\supseteq}_{(M,N)} f_L(y\rightarrow x)$. Thus (SI_6') holds.

Conversely, for any $x,y,z\in L$, since f_L is an (M,N)-*SI* filter of of L over U, we have $f_L(y\rightarrow x)\widetilde{\supseteq}_{(M,N)} f_L(z\rightarrow(y\rightarrow x))\cap f_L(z)$. By applying (SI_6'), we obtain $f_L(((x\rightarrow y)\rightarrow y)\rightarrow x)\widetilde{\supseteq}_{(M,N)} f_L(y\rightarrow x)\widetilde{\supseteq}_{(M,N)} f_L(z\rightarrow(y\rightarrow x))\cap f_L(z)$. Thus (SI_5')holds. This shows that f_L is an (M,N)-*SI* fantastic filter of L. □

Now we investigate some relationships between (M,N)-*SI* implicative (Boolean) filters and (M,N)-*SI* fantastic filters of BL-algebras.

Lemma 4.9. [27] *Let f_L be an (M,N)-SI filter of L over U. Then the following are equivalent:*

(1) *f_L is an (M,N)-SI implicative (Boolean) filter of L,*
(2) $f_L(x)=_{(M,N)} f_L(x'\rightarrow x)$, *for all* $x\in L$,
(3) $f_L(x)\widetilde{\supseteq}_{(M,N)} f_L(x'\rightarrow x)$, *for all* $x\in L$,
(4) $f_L(x)\widetilde{\supseteq}_{(M,N)} f_L((x\rightarrow y)\rightarrow x)$, *for all* $x,y\in L$,
(5) $f_L(x)\widetilde{\supseteq}_{(M,N)} f_L(z\rightarrow((x\rightarrow y)\rightarrow x))\cap f_L(z)$, *for all* $x,y,z\in L$.

Theorem 4.10. *Every (M,N)-SI implicative (Boolean) filter of L over U is an (M,N)-SI fantastic filter of L.*

Proof. For any $x,y\in L, 1=((x\rightarrow y)\rightarrow y)\rightarrow(x\rightarrow x)=x\rightarrow(((x\rightarrow y)\rightarrow y)\rightarrow x)$, that is, $x\leq(((x\rightarrow y)\rightarrow y)\rightarrow x)$, and hence $(((x\rightarrow y)\rightarrow y)\rightarrow x)\rightarrow y\leq x\rightarrow y$, which implies

$$\begin{aligned}&((((x\rightarrow y)\rightarrow y)\rightarrow x)\rightarrow y)\rightarrow(((x\rightarrow y)\rightarrow y)\rightarrow x)\\&\geq(x\rightarrow y)\rightarrow(((x\rightarrow y)\rightarrow y)\rightarrow x)\\&=((x\rightarrow y)\rightarrow y)\rightarrow((x\rightarrow y)\rightarrow x)\\&\geq y\rightarrow x\end{aligned}$$

By Lemma 3.8, $f_L((((x\rightarrow y)\rightarrow y)\rightarrow x)\rightarrow y)\rightarrow(((x\rightarrow y)\rightarrow y)\rightarrow x)\widetilde{\supseteq}_{(M,N)} f_L(y\rightarrow x)$. It follows from Lemma 4.9 that $f_L(((x\rightarrow y)\rightarrow y)\rightarrow x)\widetilde{\supseteq}_{(M,N)} f_L((((x\rightarrow y)\rightarrow y)\rightarrow x)\rightarrow y)\rightarrow(((x\rightarrow y)\rightarrow y)\rightarrow x)\widetilde{\supseteq} f_L(y\rightarrow x)$. This proves that (SI_6') holds. Therefore, by Theorem 4.8, f_L is an (M,N)-*SI* fantastic filter of L. □

Remark 4.11. The converse of Theorem 4.10 may not be true as shown in the following.

Example 4.12. Consider the universe set U and BL-algebra L as in Example 4.3. Let $M=\{y\}$ and $N=\{x,y\}$. Define a soft set f_L over U by $f_L(1)=\{e,x,y\}$ and $f_L(a)=f_L(b)=f_L(0)=\{e,x\}$. Then we can check that f_L is an (M,N)-*SI* fantastic filter of L over U, but it is not an (M,N)-*SI* implicative (Boolean) filter of L over U, since $f_L(a\vee a')\cap N=f_L(b)\cap N=\{x\}\neq\{e,x,y\}=f_L(1)\cup M$.

Finally, we discuss the relationships among these kinds of (M,N)-*SI* filters of BL-algebras.

Theorem 4.13. *A soft set f_L over U is an (M,N)-SI implicative (Boolean) filter of L and only if it is both an (M,N)-SI-PI filter and an (M,N)-SI fantastic filter of L.*

Proof. Necessity: It follows from Theorems 3.12 and 4.10 immediately.
Sufficiency: Suppose that f_L is both an (M,N)-*SI-PI* filter and an (M,N)-fantastic filter of L over U. Since $(x\rightarrow y)\rightarrow x\leq(x\rightarrow y)\rightarrow((x\rightarrow y)\rightarrow y)$, by Lemma 3.8, we obtain $f_L((x\rightarrow y)\rightarrow((x\rightarrow y)\rightarrow y))\widetilde{\supseteq}_{(M,N)} f_L((x\rightarrow y)\rightarrow x)$. Since f_L an (M,N)-*SI-PI* filter of L, by Theorem 3.9 (2), we have $f_L((x\rightarrow y)\rightarrow y)\widetilde{\supseteq}_{(M,N)} f_L((x\rightarrow y)\rightarrow((x\rightarrow y)\rightarrow y))$. Thus

$$(*)\qquad f_L((x\rightarrow y)\rightarrow y)\widetilde{\supseteq}_{(M,N)} f_L((x\rightarrow y)\rightarrow x)$$

On the other hand, since f_L is an (M,N)-*SI* fantastic filter of L, by Theorem 4.8, we have

$f_L(((x \to y) \to y) \to x) \widetilde{\supseteq}_{(M,N)} f_L(y \to x)$. Since $(x \to y) \to x \leq y \to x$, by Lemma 3.8, $f_L(y \to x) \widetilde{\supseteq}_{(M,N)} f_L((x \to y) \to x)$. Thus

$$(**) \quad f_L(((x \to y) \to y) \to x) \widetilde{\supseteq}_{(M,N)} f_L((x \to y) \to x)$$

Now, by using $(*)$ and $(**)$, we obtain

$$\begin{aligned} & f_L(x) \\ \widetilde{\supseteq}_{(M,N)} \ & f_L((x \to y) \to y) \cap f_L(((x \to y) \to y) \to x) \\ \widetilde{\supseteq} \ & f_L((x \to y) \to x) \end{aligned}$$

It follows from Lemma 4.9 that f_L is an (M,N)-*SI* implicative (Boolean) filter of L. □

A *BL*-algebra L is called an *MV-algebra* if $x'' = x$, or equivalently, $(x \to y) \to y = (y \to x) \to x$, for all $x, y \in L$.

Summarizing the above theorems, we can obtain the following corollaries:

Corollary 4.14. *Let f_L be a soft set of an MV-algebra L over U. Then the following are equivalent:*
(1) *f_L is an (M,N)-SI filter,*
(2) *f_L is an (M,N)-SI fantastic filter.*

Corollary 4.15. *Let f_L be an (M,N)-SI filter of an MV-algebra L over U. Then the following are equivalent:*
(1) *f_L is an (M,N)-SI implicative filter,*
(2) *f_L is an (M,N)-SI Boolean filter,*
(3) *f_L is an (M,N)-SI-PI filter.*

5 Conclusions

As a continuation of papers [13,27], we introduce the concepts of (M,N)-*SI-PI* and (M,N)-*SI* fantastic filters in *BL*-algebras based on a new soft intersection set. Some good examples are explored. Moreover, some characterizations of soft intersection filters are discussed. Finally the relationships among these kinds of soft intersection filters are established. We hope it can lay a foundation for providing a new soft algebraic tool in many uncertainties problems.

For further investigation in this field, one can apply this theory to other fields such as algebras, topologies and other mathematical branches. Maybe one can apply this new idea to decision making, data analysis and knowledge based systems.

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