Estimation of Mass Diffusion Relaxation Time in the Binary Mixture between Two-Phase Bubbly Flow

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Abstract: The mass diffusion relaxation time constant is derived on the basis of relaxed model with unequal phase-mass diffusion for a binary mixture. The mass diffusion and state equations are solved analytically for two finite boundaries. The relaxation time is affected by void fraction and vaporized mass diffusion fraction values. Mass diffusion relaxation time obtained in this work has a larger values than that obtained by Mohammadein [9] and Moby Dick experiment [3]; which satisfied for some values of the physical parameters.

Keywords: Mass diffusion relaxation time. Binary mixture. State equation. Moby Dick experiment. Void fraction.

1 Introduction

Two-phase flow may occur in many technical devices, which are present in heat exchangers, refrigerators, and installations for cooling nuclear reactors. The problem of heat transfer in a pure or binary superheated liquid and growing vapor bubbles was widely discussed by many authors [1]-[15]. Scriven [13] was the first one who formulated the heat exchange problem in heat conduction equation between two-phase densities. Moreover, the temperature distribution in an infinite volume of mixture is obtained by [8,10]. The formulation of relaxation time "θ" is discussed by empirical, calculation, and derivation methods. This appear through the references [2,3,4,7,9, 12]. The empirical correlation of relaxation time obtained by Bauer et al [2] can be written as

$$\theta = 660 P^{-0.505} w^{-1.89} \phi^{-0.954} \qquad (1)$$

One of the model including phenomenon of non-equilibrium is the homogeneous relaxation model HRM [3,12]. The interpretation of the MOBY DICK experiment has been done by Reocreux [12]. Within this framework it is assumed that vapour and liquid phases are moving with equal velocities, and the structure of the flow is homogeneous mixture of the two phases. Furthermore, it is considered that the flow is in one- dimensional. For a steady flow through a nozzle of varying cross section $A = A(z)$, without the influence of mass, forces and without heat exchange with the surroundings, the conservation laws for mass, momentum and energy of homogeneous two-phase mixture can be written in the form

$$\frac{1}{\rho}\frac{d\rho}{dz} + \frac{1}{w}\frac{dw}{dz} = -\frac{dA}{dz}, \qquad (2)$$

$$\rho w \frac{dw}{dz} + \frac{dP}{dz} = -\tau \frac{C}{A}, \qquad (3)$$

$$\frac{dh}{dz} + w \frac{dw}{dz} = 0. \qquad (4)$$

The enthalpy h can be written as

$$h = h(P, \rho, x) = x h^{"}(P) + (1-x) h_1(P, T_1(P, \rho, x)), \qquad (5)$$

where $h^{"}(P)$ denotes vapour enthalpy on the vapour-pressure line which corresponds to the local pressure p; T_1 is the non-equilibrium temperature of the superheated liquid, and h_1 denotes liquid enthalpy in a metastable state, as well as with the equation defining shearing stress τ at the nozzle wall as follows

$$\tau = \frac{1}{2} f w^2 \rho, \qquad (6)$$

Where f is the empirical friction coefficient.
The reinterpretation of the MOBY DICK experiment has been done by Bilicki et al. [3]. Moby Dick experiment represent the flow in a conical tube as given in Fig. 1. The experimental data of Moby Dick were reinterpreted by

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using the simplest relaxation homogeneous model (HRM). The relation between pressure, void fraction and relaxation time is shown by Fig.2. It is observed that the pressure, relaxation time and void fraction are relaxed to the equilibrium state when void fraction φalong the conical tube lies in the interval $(0 < \varphi < 0.3)$.

The thermodynamic non-equilibrium in the HRM can be described by the evaluation equation in terms of dryness fraction x

$$\frac{dx}{dz} = -\frac{x - \bar{x}}{\theta_x w}. \qquad (7)$$

The experimental data of Moby Dick were reinterpreted by using the simplest relaxation homogeneous model. The system (2-4) is augmented by a state equation (5), closure equation (6), and relaxation equation(7). This system is solved numerically by using Runge-Kutta method for the given parameters (inlet temperature, pressure distribution, void fraction distribution, mass flow rate) as obtained by Bilicki et al. [4]. Then θ_x can be obtained in a pure mixture as follows

$$\theta_x = \frac{\bar{x} - \varphi(z)\frac{\rho_v}{\rho_l}}{w(z)} \left(\frac{\rho_v}{\rho_l}\frac{d\varphi}{dz} - \varphi(z)\frac{\rho_v}{\rho_l^2}\frac{d\rho}{dz} + \frac{\varphi(z)}{\rho_l}\frac{d\rho_v}{dp}\frac{dp}{dz} \right)^{-1}. \qquad (8)$$

To find thermal relaxation time θ_T, the system of equations (2-6) are solved by using the relaxation equation in terms of temperature as follows

$$\frac{d\bar{T}(t)}{dt} = -\frac{\bar{T}(t) - T_s}{\theta_T}, \qquad (9)$$

and then thermal relaxation time θ_T in terms of some thermodynamic parameters in a pure mixture has the following form

$$\theta_T = -\frac{T_l - T_{vs}}{w(z)}\left(\frac{\partial T_l}{\partial h_l}\right)_p^{-1}$$
$$\times \left\{ \frac{1}{1-x}\frac{dh}{dz} - \frac{h'' + h}{(1-x)^2}\frac{dx}{dz} - \left[\frac{x}{1-x}\frac{dh''}{dP} + \left(\frac{\partial h_l}{\partial P}\right)_{T_l} \right]\frac{dP}{dz} \right\}^{-1}, \qquad (10)$$

Moreover, in a pure mixture, the derivation of thermal relaxation time θ_T is obtained by Mohammadein [9] on the basis of the relaxation equation and the average of temperature distribution surrounded a growing vapour bubbles can be written in the following form

$$\theta_T = \frac{R_0^2}{2C}\left(\left\{ \varphi_0 \left(1 + \frac{1 + [(C_{PL} - C_{PV})\Delta\theta_0/L]}{J_a[1 - \exp(-1)]}\right) \right\}^{-\frac{2}{3}} - 1 \right),$$

$$C = \frac{6a_l}{\pi}\left[\frac{J_a}{1 + ((C_{PL} - C_{PV})\Delta\theta_0/L)} \right]^2. \qquad (11)$$

where θ_T represents the derived thermal relaxation time in terms of void fraction, superheating, Jacob number, and other physical parameters.

The above survey of evaluation of θ includes the empirical correlation, Moby Dick experiment (see refs.
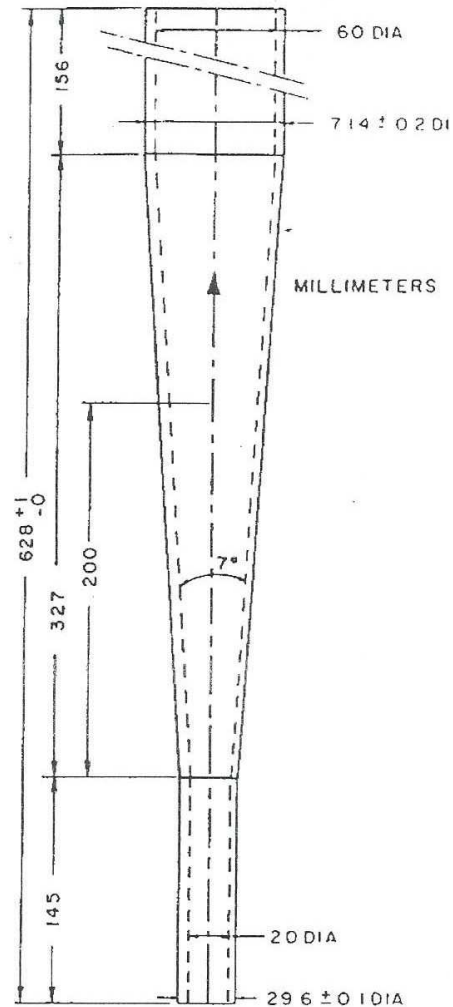

Fig. 1: The Channel used in Moby Dick experiment.

[2,3,12]) and analytical formula [9] in a pure mixture of vapour and superheated liquids.

In the binary mixtures, heat diffusion is linked with mass diffusion of the more volatile component which; is rapidly exhausted in the liquid (see refs. [5,14,15]). A low concentration or mass diffusivity of the more component results in the slowing down of bubble growth as the mass diffusivity is an order of magnitude smaller than the thermal diffusivity.

The volatile parameters in a binary mixture are analogous to those of pure mixture and obtained by replacing T, a, $\Delta\theta_0$, L/c and β byχ, D, $(\chi_0 - \chi)$, $(y - \chi)$and $\mu = \left(\frac{a}{D}\right)^{\frac{1}{2}}$ respectively.

Mohammadein et al [11] reformulated Scriven equation in terms of two-phase mass diffusion in a binary mixture using spherical co-ordinates to derive mass diffusion distribution $\chi(r,t)$ around a growing vapour bubbles as
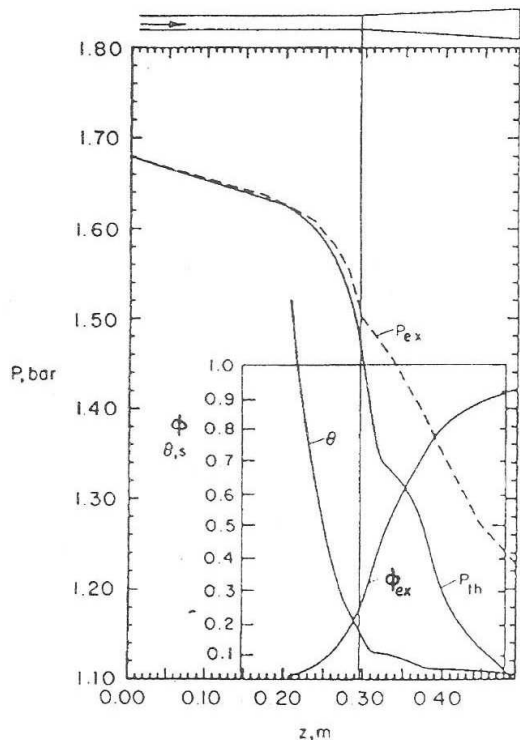
Fig. 2: Experimental results of Moby Dick for run 400.

follows

$$\chi(r,t) = \chi_0 - \frac{\rho_2 A(y-\chi)}{\rho_1 D} R(t_i)$$
$$\times \int_r^{R_m} \frac{1}{r^2} \exp\left\{-\frac{A}{D}\left(\frac{r^2}{2R^2(t_i)} + \frac{\varepsilon R(t_i)}{r} - \left(\varepsilon + \frac{1}{2}\right)\right)\right\} dr'$$
$$(12)$$

and the growth of vapour bubble radius in terms of mass diffusivity D and vaporized mass diffraction fraction G_d for a binary mixture can be written in the form

$$R(t) = \sqrt{2A(t-t_0) + R_0^2}. \qquad (13)$$

The average of mass diffusion distribution surrounded a growing vapour bubbles can be written in the following form

$$\bar{\chi}(\theta_m) = \frac{1}{V_l} \int_{R(\theta_m)}^{R_m} (4\pi r^2)\, \chi(r,\theta_m)\, dr, \qquad (14)$$

where

$$V_l = \frac{4}{3}\pi\left(R_m^3 - R^3(\theta_m)\right).$$

The previous works for evaluation of relaxation times in a pure mixture are introduced empirically, numerically, analytically as in references [2,4,9].

In section 2, the relaxation equation is solved by analytical technique to estimate the mass diffusion relaxation time. In section 3, the procedure was carried

out to the numerical calculations as shown from the figures and discussion of results. In section 4, the concluded remarks are indicated to importance of study for mass diffusion relaxation time in a complicated properties of fluid and flow.

The present work is devoted to derive the analytical formula for the mass diffusion relaxation time θ_m for a non-equilibrium system of a binary mixture between two-phase flow. The relaxation time is evaluated under conditions different than that studied before. Results are compared with previous theoretical and experimental results obtained by [9] and [3].

2 Analysis

In a binary mixture, the non-steady mass diffusion distribution surrounding the growing vapour bubble is obtained by Mohammadein and Elgammal [11]. The relaxation equation of state is written in terms of average mass diffusion distribution. Mass diffusion relaxation equation is solved analytically to find mass diffusion relaxation time θ_m. Within this framework it is assumed that the more and less volatile components in the mixture are moving with equal velocities, Moreover, the structure of the flow is homogeneous mixture of the two phases.

The non-equilibrium of binary mixtures introduces relaxation to reach the equilibrium state. Mass diffusion relaxation time θ_m is considered as constant time according to the following relaxation equation

$$\frac{d\bar{\chi}(t)}{dt} = -\frac{\bar{\chi}(t) - \chi_s}{\theta_m}, \qquad (15)$$

with the initial condition

$$At\ t = t_0 \qquad \bar{\chi}(t_\circ) = \chi_\circ, \qquad (16)$$

by integrating Eq.(15) w. r. to t, then

$$\bar{\chi}(\theta_m) = \chi_s + (\chi_\circ - \chi_s)\exp\left(\frac{t_\circ}{\theta_m} - 1\right). \qquad (17)$$

Substituting from Eq.(12) in Eq., (14), then

$$\bar{\chi}(\theta_m) = \frac{1}{R_m^3 - R^3(\theta_m)}$$
$$\times \left[R_m^3 \chi_\circ - R^3(\theta_m)\chi\{R(\theta_m),\theta_m\}\int_{R(\theta_m)}^{R_m} r^3 \frac{\partial\chi(r,\theta_m)}{\partial r}dr\right],$$
$$(18)$$

Equating Eqs, (17) and (18), then

$$\chi_s + (\chi_\circ - \chi_s)\exp\left(\frac{t_\circ}{\theta_m} - 1\right)$$

$$= \frac{1}{R_m^3 - R^3(\theta_m)}$$
$$\times \left[R_m^3 \chi_\circ - R^3(\theta_m)\chi\{R(\theta_m),\theta_m\}\int_{R(\theta_m)}^{R_m} r^3 \frac{\partial\chi(r,\theta_m)}{\partial r}dr\right].$$
$$(19)$$

The integral in Eq. (19) can be written in the form

$$\int_{R(\theta_m)}^{R_m} r^3 \frac{\partial \chi(r,\theta_m)}{\partial r} dr = MR^3(\theta_m) \left[I_1 + \frac{\varepsilon A R(\theta_m)}{D} I_2 \right],$$ (20)

where

$$M = \frac{\rho_v}{\rho_l}(y - \chi_s),$$ (21)

$$A = \frac{6D}{\pi}\left(\frac{\rho_l}{\rho_v}G_d\right)^2,$$ (22)

$$G_d = \frac{\chi_0 - \chi_s}{y - x_s},$$ (23)

$$I_1 = \int_{R(\theta_m)}^{R_m} \frac{A}{D}\left(\frac{r}{R^2(\theta_m)} - \frac{\varepsilon R(\theta_m)}{r^2}\right)$$
$$\times \exp\left\{-\frac{A}{D}\left(\frac{r^2}{2R^2(\theta_m)} + \frac{\varepsilon R(\theta_m)}{r} - \left(\frac{1}{2} + \varepsilon\right)\right)\right\}dr,$$ (24)

and

$$I_2 = \int_{R(\theta_m)}^{R_m} \frac{1}{r^2}\exp\left\{-\frac{A}{D}\left(\frac{r^2}{2R^2(\theta_m)} + \frac{\varepsilon R(\theta_m)}{r} - \left(\frac{1}{2} + \varepsilon\right)\right)\right\}dr.$$ (25)

In addition, when $\rho_v \ll \rho_l$, then $\varepsilon \approx 1$ and Eq. (19) can be written as follows

$$(\chi_\circ - \chi_s)\left[1 - \exp\left(\frac{t_\circ}{\theta_m} - 1\right)\right] = \frac{M}{\frac{R_m^3}{R^3(\theta_m)} - 1} I_1|_{\varepsilon=1}$$ (26)

where

$$I_1|_{\varepsilon=1} = 1 - \exp\left\{-\frac{A}{D}\left(\frac{r^2}{2R(\theta_m)} + \frac{R(\theta_m)}{r} - \frac{3}{2}\right)\right\}.$$ (27)

When $D \ll 1$, then $I_1 \to 1$, and equation (26) becomes

$$R(\theta_m) = R_m\left\{1 + \frac{M}{(\chi_\circ - \chi_s)\left[1 - \exp\left(\frac{t_\circ}{\theta_m} - 1\right)\right]}\right\}^{\frac{-1}{3}}.$$ (28)

Substituting from equation (28) in Eq.(13) with consideration $t_0 \ll \theta_m$, then the mass diffusion relaxation time θ_m in a binary mixture has the following form

$$\theta_m(\varphi_\circ) = \frac{R_\circ^2}{2A}\left(\left[\varphi_\circ\left\{1 + \frac{\rho_v}{\rho_l G_d\left[1 - \exp(-1)\right]}\right\}\right]^{\frac{-2}{3}} - 1\right).$$ (29)

The mass diffusion relaxation time. θ_m is obtained in terms of void fraction $\phi_\circ$ and vaporized mass diffusion fraction G_d for binary mixture.

3 Discussion of results

In a binary mixture, the relaxation mass diffusion problem (15) is solved analytically in terms of average

Table 1: rrr

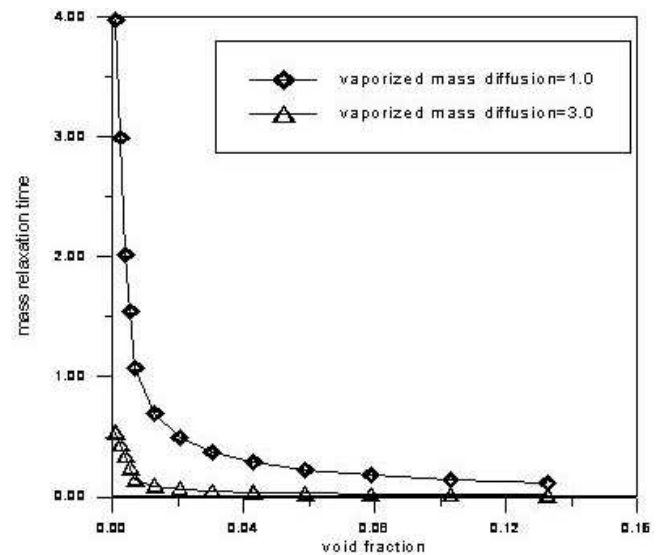|	Value		Value
D	$9.9x10^{-10} m^2/s$	L	2256700 J/kg
ρ_l	$958.3\ Kg m^{-3}$	k_l	$0.679 W/(mK)$
ρ_v	$0.597\ Kg m^{-3}$	R_0	$0.0001\ m$
C_{pl}	$4220\ J/(kgK)$	R_m	$0.001\ m$
C_{pv}	$2030\ J/(kgK^0)$	G_d	0.006
$\Delta\chi$	1	Δy	3
μ	13.1		

Fig. 3: Mass diffusion relaxation time in terms of void fraction for two different values of vaporized mass diffusion fraction G_d.

mass diffusion. Relaxation time is calculated from the distribution of mass diffusion surrounded a growing vapour bubbles. Mass diffusion relaxation time θ_m is given by relation (29). The values of physical parameters considered in this study is taken from the ref. [6] at saturation pressure (Pl=101.3 k Pa) in a steam water-1-butanol are given in the Table 1

Mass diffusion relaxation time θ_m in terms of void fraction for two different values of vaporized mass diffusion fraction G_d is shown in Fig.3. It is observed that, the mass diffusion relaxation time θ_m is decreasing with increasing of void fraction $\phi_\circ$ and vaporized mass diffusion fraction G_d for original mixture. Comparison of present relaxation time with thermal relaxation in terms of void fraction when $\Delta\theta_0 = 1.0 K$ is shown in Fig.4. It is observed that, the mass diffusion relaxation time in a binary mixture is larger than thermal relaxation time in a pure mixture for all different values of superheating. The mass diffusion relaxation time θ_m is compared with the thermal relaxation time θ_T obtained by [9] and [3] as shown in Fig.5. It is observed that, the
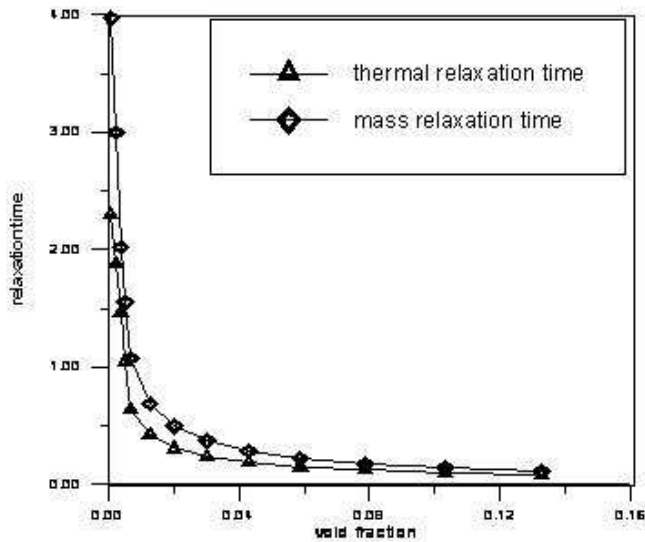
Fig. 4: Comparison of mass diffusion and thermal relaxation times in terms of void fraction when $\Delta\theta_0 = 1.0K$..
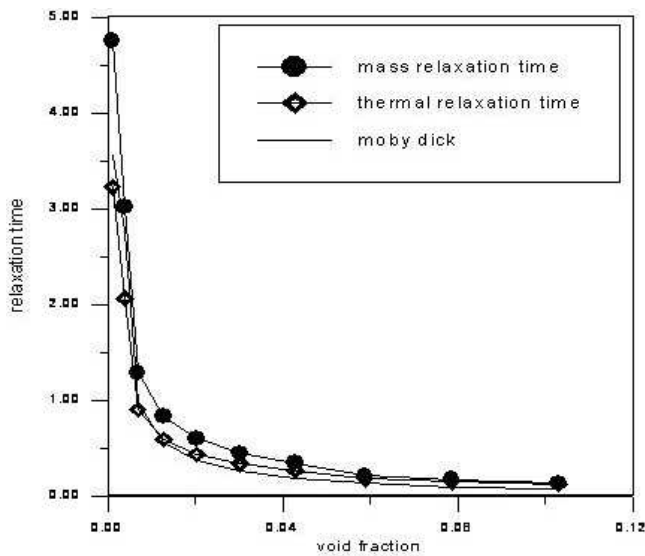

Fig. 5: Comparison of mass diffusion relaxation time θ_m with thermal relaxation time θ_T [9] and Moby Dick experiment [3].

calculated mass diffusion relaxation time θ_m performs higher values than thermal relaxation time θ_T; which obtained by authors [9] and [3]. Moreover, the mass diffusion relaxation time θ_m is compared with pressure and thermal relaxation times θ_p and θ_T as shown in Fig.6. It is observed that, the mass diffusion relaxation time θ_m performs higher values than pressure and thermal relaxation times obtained before (see refs, [7] and [9]).
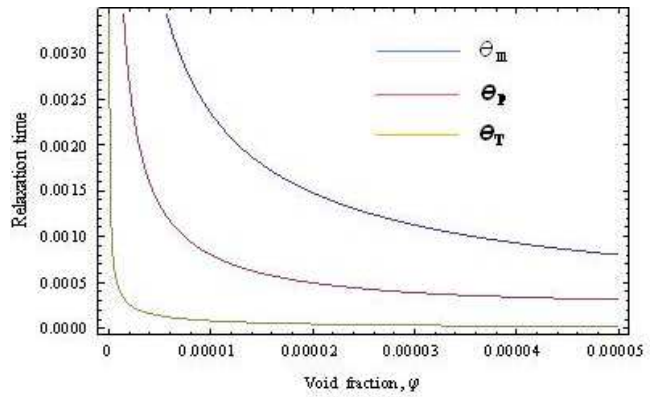

Fig. 6: Comparison of mass diffusion relaxation time with pressure and thermal relaxation times by Mohammadein and Omran [7, 9] in terms of void fraction values.

4 Conclusion

The relaxation equation of state (15) is solved analytically in terms of average mass diffusion of a binary mixture surrounded a growing vapour bubbles. The approximate formula of mass diffusion relaxation time θ_m is obtained directly from the relaxed model with unequal phase-mass diffusions.

The following important remarks can be written:

1. The mass diffusion relaxation time is proportional inversely with void fraction and vaporized mass diffusion fraction.

2. Mass diffusion relaxation time θ_m performed higher values than the values of the thermal relaxation time θ_T for different values of superheating and void fractions.

3. The mass diffusion relaxation time θ_m in a binary mixture performed higher values than thermal relaxation time θ_T in a pure mixture obtained by Mohammadein [9] and Moby Dick experiment [3].

4. The mass diffusion relaxation time in a binary mixture performs a higher values than pressure and thermal relaxation times θ_P and θ_T for some values of void fraction.

5. Mass diffusion relaxation time θ_m in a binary mixture performed the same order of magnitude of relaxation time in the pure mixtures.

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