Journal of Statistics Applications & Probability
An International Journal

http://dx.doi.org/10.12785/jsap/040215

On The Estimation of Population Mean in Current Occasion in Two- Occasion Rotation Patterns

Housila P. Singh and *Surya K. Pal* *.

School of Studies in Statistics, Vikram University Ujjain, M.P., 456010 India.

Received: 19 Feb. 2015, Revised: 10 Apr. 2015, Accepted: 12 Apr. 2015.
Published online: 1 Jul. 2015.

Abstract: The present work is an effort to develop some estimators of current population mean in two-occasion successive sampling utilizing the known population mean $\bar{Z}$ alongwith known correlation coefficient ρ_{yz} and coefficient of kurtosis $\beta_2(z)$ of the auxiliary variable *z* on both occasions in successive sampling. Optimum replacement policy relevant to the proposed estimators has been discussed. Numerical illustration is carried out and appropriate recommendations are made.

Keywords: Auxiliary variable, Study variable, Bias, Mean square error, Efficiency Comparison.

1 Introduction

The problem of sampling on two successive occasions was first considered by Jessen [1] who introduced the idea of sampling on two occasions by using the information gathered on the previous occasions to improve the precision of the current estimate. Later several authors including Patterson [2], Narain [3], Eckler [4], Rao and Graham [5], Gordon [6], Arnab and Okafor [7] and Singh et al. [8], among others have developed the theory of successive sampling. Sen [9] applied this theory in designing the strategies for estimating the population mean on the current occasion using information on two auxiliary variables. Sen [10, 11] extended his work for multiple auxiliary variables. Fen and Zou [12] and Biradar and Singh [13] used the auxiliary information on both occasions for estimating the current population mean in the successive sampling. Singh [14], Singh and Vishwakarma [15, 16, 17] have used the auxiliary information on both occasions and envisaged several estimators for estimating the population mean on current (second) occasion in two - occasion successive (rotation) sampling.

Recently Singh and Pal [18, 19] have suggested some estimators utilizing the known population mean $\bar{Z}$ alongwith known coefficient of variation C_z and Standard deviation S_z of the auxiliary variable *z* on both occasions in successive sampling for estimating the current (second) population mean in two occasion successive sampling. Motivated by Singh and Pal [18, 19], Singh and Tailor [20] and Singh et al. [21] we have suggested some estimators utilizing the known population mean $\bar{Z}$ alongwith known correlation coefficient ρ_{yz} and coefficient of kurtosis $\beta_2(z)$ of the auxiliary variable *z* on both occasions in successive sampling for estimating the current (second) population mean in two occasion successive sampling.

2 The Suggested Class of Estimators

Let $U=(U_1, U_2 \ldots U_N)$ be the finite population of size N units, which has been sampled over two occasions. The character under study be denoted by *x* (*y*) on the first (second) occasion respectively. It is assumed that information on population coefficient of Kurtosis $\beta_2(z)$ of an auxiliary variable z along with known population mean $\bar{Z}$ is available for the both the occasions. As pointed by Singh and Tailor [20] and Sahai and Sahai [22] the population correlation coefficient ρ_{yz} between the study variable *y* and auxiliary variable z can also be made known from the post data or experience gathered in due to course of time. So we have assumed that ρ_{yz} is also known along with the population mean $\bar{Z}$ of an auxiliary

*Corresponding author e-mail: suryakantpal6676@gmail.com

variable z on both occasions. In such situation, we have to proposed two combined classes of estimators of the population mean $\bar{Y}$ on the current (second) occasion, one is based on (ρ_{yz} , $\bar{Z}$) and second on ($\beta_2(z)$, $\bar{Z}$).

A simple random sample of n units is drawn without replacement (*WOR*) on the first occasion. A random sub sample of m (= $n\lambda$) units is retained (matched) from the sample drawn on the first occasion for its use on the current (second) occasion, while a fresh sample of size u= (n-m) = $n\mu$ units is drawn on the current (second) occasion, from the entire population by simple random sampling without replacement (*SRSWOR*) method so that the sample size on the current (second) is also n. The fractions of the matched and fresh samples are respectively designated by λ and μ such that $\lambda+\mu=1$.

The following notations have been used throughout the paper.

$\bar{X}$, $\bar{Y}$, $\bar{Z}$: The population means of the variables x, y and z respectively.

$\bar{x}_m$, $\bar{x}_n$, $\bar{y}_u$, $\bar{y}_m$, $\bar{z}_u$, $\bar{z}_n$: The sample means of the respective variables based on the sample sizes indicated in suffices.

C_x, C_y, C_z : The coefficients of variation of the variables x, y and z respectively,

ρ_{yx}, ρ_{yz}, ρ_{xz} : The correlation coefficients between the variables shown in suffices .

$S_x^2=(N-1)^{-1}\sum_{i=1}^{N}(x_i-\bar{X})^2$, $S_y^2=(N-1)^{-1}\sum_{i=1}^{N}(y_i-\bar{Y})^2$, $S_z^2=(N-1)^{-1}\sum_{i=1}^{N}(z_i-\bar{Z})^2$ are

The population mean squares of x, y and z respectively,

$f=n/N$: The sampling fraction.

For estimating the population mean $\bar{Y}$ on the second (current) occasion two different sets of estimators are proposed. One set of estimators $\tau_u=\{\tau_{1u},\tau_{2u}\}$ based on sample size $u(=n\mu)$ drawn afresh on the second occasion and the second set of estimators $\tau_m=\{\tau_{1m},\tau_{2m}\}$ based on the sample size $m(=n\lambda)$ common with both occasions. Estimators of sets τ_u and τ_m are given below:

$$\tau_{1u}=\bar{y}_u\left(\frac{\bar{Z}+\rho_{yz}}{\bar{z}_u+\rho_{yz}}\right) \tag{2.1}$$

$$\tau_{2u}=\bar{y}_u\left(\frac{\bar{Z}+\beta_2(z)}{\bar{z}_u+\beta_2(z)}\right), \tag{2.2}$$

And

$$\tau_{1m}=\bar{y}_m\left(\frac{\bar{x}_n}{\bar{x}_m}\right)\left(\frac{\bar{Z}+\rho_{yz}}{\bar{z}_n+\rho_{yz}}\right), \tag{2.3}$$

$$\tau_{2m}=\bar{y}_m\left(\frac{\bar{x}_n}{\bar{x}_m}\right)\left(\frac{\bar{Z}+\beta_2(z)}{\bar{z}_n+\beta_2(z)}\right). \tag{2.4}$$

Where (ρ_{yz} , $\beta_2(z)$) are known correlation coefficient between y and x and coefficient of kurtosis.

Combining the estimators of sets τ_u and τ_m , we have the following estimators of the population mean $\bar{Y}$ at the second (current) occasion:

(i) when ρ_{yz} is known along with population mean $\bar{Z}$:

$$\tau_1=\varpi_1\tau_{1u}+(1-\varpi_1)\tau_{1m} \ , \tag{2.5}$$

(ii) when $\beta_2(z)$ is known along with population mean $\bar{Z}$:

$$\tau_2 = \varpi_2 \tau_{2u} + (1-\varpi_2)\tau_{2m}. \tag{2.6}$$

In short, we can define the estimators (2.5) and (2.6) as:

$$\tau_i = \varpi_i \tau_{iu} + (1-\varpi_i)\tau_{im}, \ (i=1,2); \tag{2.7}$$

Where ϖ_i (i=1, 2) are unknown constants to be determined under certain criterion.

3 Bias and Mean Squared Errors of Suggested Estimators τ_i (i=1, 2)

The biases and mean squared errors of the class of estimators τ_i (i =1, 2) are given in the following Theorems 3.1 and 3.2.

Theorem 3.1 Biases of the proposed estimators τ_i (i =1, 2) to the first order of approximation are given by

$$B(\tau_i) = \varpi_i B(\tau_{iu}) + (1-\varpi_i)B(\tau_{im}), \tag{3.1}$$

Where

$$B(\tau_{iu}) = \bar{Y}((1/u)-(1/N))\delta_i(\delta_i C_z^2 - \rho_{yz}C_yC_z), (i=1,2), \tag{3.2}$$

$$B(\tau_{im}) = \bar{Y}[((1/m)-(1/N))(C_x^2 - \rho_{yx}C_yC_x) + ((1/n)-(1/N))\delta_i(\delta_i C_z^2 - \rho_{yz}C_yC_z)], \tag{3.3}$$

$$\delta_1 = \bar{Z}/(\bar{Z}+\rho_{yz}) \text{ And } \delta_2 = \bar{Z}/(\bar{Z}+\beta_2(z)).$$

Proof is simple so omitted.

Theorem 3.2 The mean squared errors of the suggested combined class of estimators τ_i (i, =1, 2) to first degree of approximation are given by

$$M(\tau_i) = [\varpi_i^2 M(\tau_{iu}) + (1-\varpi_i)^2 M(\tau_{im}) + 2\varpi_i(1-\varpi_i)Cov(\tau_{iu},\tau_{im})], (i=1,2), \tag{3.4}$$

Where

$$M(\tau_{iu}) = \bar{Y}^2((1/u)-(1/N))[C_y^2 + \delta_i(\delta_i C_z^2 - 2\rho_{yz}C_yC_z)],$$

$$= \bar{Y}^2((1/u)-(1/N))(C_y^2 + \omega_i) \tag{3.5}$$

$$M(\tau_{im}) = \bar{Y}^2[(1/m)\omega + (1/n)(C_y^2 - \omega + \omega_i) - 1/N(C_y^2 + \omega_i)],$$

$$= \bar{Y}^2[(1/m)\omega + (1/n)\omega_i^* - 1/N(C_y^2 + \omega_i)], \tag{3.6}$$

$$Cov(\tau_{iu},\tau_{im}) = -(\bar{Y}^2/N)(C_y^2 + \omega_i), (i=1,2), \tag{3.7}$$

$$\omega = (C_y^2 + C_x^2 - 2\rho_{yx}C_yC_x), \tag{3.8}$$

$$\omega_i = \delta_i(\delta_i C_z^2 - 2\rho_{yz}C_yC_z), \tag{3.9}$$

$$\omega_i^* = (C_y^2 - \omega + \omega_i) \tag{3.10}$$

$$= \{2\rho_{yx}C_yC_x - C_x^2 + \delta_i(\delta_i C_z^2 - 2\rho_{yz}C_yC_z)\}; (i=1, 2).$$

4 Minimum Mean Squared Error of Proposed Estimators τ_i (i =1, 2)

Differentiating $M(\tau_i)$ at (3.4) with respect to ϖ_i and equating to zero we get the optimum value of ϖ_i as

$$\varpi_{iopt} = \frac{[M(\tau_{im}) - Cov(\tau_{iu}, \tau_{im})]}{[M(\tau_{iu}) + M(\tau_{im}) - 2Cov(\tau_{iu}, \tau_{im})]},$$

$$= \frac{\mu_i(A_i - \mu_i B_i)}{(A_i - \mu_i^2 B_i)}, \tag{4.1}$$

Where $A_i = \{1 + \delta_i(\delta_i - 2\rho_{yz})\}$ and $B_i = \{2\rho_{yx} - 1 + \delta_i(\delta_i - 2\rho_{yz})\}$.

Thus the resulting minimum *MSE* of τ_i (i =1, 2) is given by

$$\min .M(\tau_i) = \frac{[M(\tau_{iu})M(\tau_{im}) - \{Cov(\tau_{iu}, \tau_{im})\}^2]}{[M(\tau_{iu}) + M(\tau_{im}) - 2Cov(\tau_{iu}, \tau_{im})]},$$

$$= \frac{S_y^2}{n}\frac{A_i[(1-f)A_i - \mu_i B_i + f\mu_i^2 B_i]}{(A_i - \mu_i^2 B_i)}. \tag{4.2}$$

5 Optimum Replacement Policy

To obtain the optimum value of μ_i (fraction of a sample to be drawn afresh on the second occasion) so that the population mean $\bar{Y}$ may be estimated with maximum precision. Differentiating $\min .M(\tau_i)$ at (4.2) with respect to μ_i and equating it to zero we get

$$B_i \mu^2 - 2A_i \mu + A_i = 0, \ (i=1, 2) \tag{5.1}$$

Solution of the above equation is given by

$$\mu_{io} = \frac{A_i \pm \sqrt{2(1-\rho_{yx})A_i}}{B_i}, \ (i=1, 2). \tag{5.2}$$

From (5.2) two values of μ_{i0} are possible, hence to choose a value of μ_{i0}, it should be recommended that $0 \le \mu_{i0} \le 1$, all other values of μ_{i0} are inadmissible. Substituting the value of μ_{i0} from equation (5.2) into equation (4.2), we have

$$\min .M(\tau_i)_{opt} = \frac{S_y^2}{n}\frac{A_i[(1-f)A_i - \mu_{i0} B_i + f\mu_{i0}^2 B_i]}{(A_i - \mu_{i0}^2 B_i)}, \ (i=1, 2) \tag{5.3}$$

6 Efficiency Comparison

The percent relative efficiencies(*PREs*) of the suggested estimators 'τ_i' with respect to usual unbiased estimator $\bar{y}_n$, when there is no matching and the estimator $\hat{\hat{Y}} = \psi\, \bar{y}_u + (1-\psi)\bar{y}'_m$, when no auxiliary information is used at any occasion, where $\bar{y}'_m = \bar{y}_m + \beta_{yx}(\bar{x}_n - \bar{x}_m)$ and β_{yx} is the known population regression coefficient; have been obtained for various choices ρ_{yx}, ρ_{yz} and δ_i.

Following Sukhatme et al. [23], the variance of usual unbiased estimator $\bar{y}_n$ and optimum variance of $\hat{\hat{Y}}$ are respectively given by

$$V(\bar{y}_n) = (1-f)\frac{S_y^2}{n}, \tag{6.1}$$

And

$$V(\hat{\hat{Y}})=[(1/2)\left(1+\sqrt{(1-\rho_{yx}^2)}\right)-f]\frac{S_y^2}{n}, \tag{6.2}$$

For N=1000, n=100 and the various choices ρ_{yx}, ρ_{yz}, $\bar{Z}$ and $\beta_2(z)$, Table 6.1 and Table 6.2 depicts the optimum values μ_{i0} of μ_i (i =1, 2) and the percent relative efficiencies (*PREs*) $E_i^{(1)}$ and $E_i^{(2)}$ of the suggested estimator τ_i (i =1, 2) with respect to $\bar{y}_n$ and $\hat{\hat{Y}}$ respectively by using the following formulae:

$$\begin{aligned} E_i^{(1)} &= \frac{V(\bar{y}_n)}{\min.M(\tau_i)_{opt}}\times 100 \\ &= \frac{(1-f)(A_i-\mu_{i0}^2B_i)}{A_i[(1-f)A_i-\mu_{i0}B_i+f\mu_{i0}^2B_i]}\times 100, \end{aligned} \tag{6.3}$$

And

$$\begin{aligned} E_i^{(2)} &= \frac{V(\hat{\hat{Y}})_{opt}}{\min.M(\tau_i)_{opt}}\times 100 \\ &= \frac{\left[\left(\frac{1}{2}\right)\left(1+\sqrt{(1-\rho_{yx}^2)}\right)-f\right](A_i-\mu_{i0}^2B_i)}{A_i[(1-f)A_i-\mu_{i0}B_i+f\mu_{i0}^2B_i]}\times 100, \end{aligned} \tag{6.4}$$

Findings are shown in Table 6.1 and 6.2. To make the numerical values obtained for (*PREs*).

In Table 6.1 and 6.2 more comprehensible to the readers we have represented these through figures 6.1 and 6.2 respectively.

Table 6.1: The percent relative efficiencies (*PREs*) of τ_i (i =1, 2) with respect to $\bar{y}_n$ and $\hat{\hat{Y}}$ for different values of ρ_{yx}, ρ_{yz} and $\bar{Z}$.

ρ_{yx}	ρ_{yz}	$\bar{Z}$	μ_{10}	$E_1^{(1)}$	$E_1^{(2)}$	$\bar{Z}$	μ_{10}	$E_1^{(1)}$	$E_1^{(2)}$	$\bar{Z}$	μ_{10}	$E_1^{(1)}$	$E_1^{(2)}$
0.50	0.60	10.0	0.4654	121.87	112.80	20.0	0.4686	119.69	110.79	30.0	0.4697	118.92	110.07
0.50	0.70	10.0	0.4291	149.54	138.41	20.0	0.4325	146.72	135.80	30.0	0.4338	145.69	134.84
0.50	0.80	10.0	0.3801	196.99	182.33	20.0	0.3832	193.50	179.10	30.0	0.3845	192.11	177.81
0.50	**0.90**	**10.0**	**0.3037**	**305.87**	**283.11**	**20.0**	**0.3054**	**302.94**	**280.39**	**30.0**	**0.3063**	**301.19**	**278.77**
0.60	0.60	10.0	0.4932	129.96	115.52	20.0	0.4964	127.59	113.42	30.0	0.4976	126.75	112.67
0.60	0.70	10.0	0.4566	160.09	142.30	20.0	0.4601	157.02	139.57	30.0	0.4613	155.89	138.57
0.60	0.80	10.0	0.4067	212.01	188.45	20.0	0.4099	208.18	185.05	30.0	0.4112	206.66	183.70
0.60	0.90	10.0	0.3278	331.84	294.97	20.0	0.3295	328.60	292.09	30.0	0.3305	326.67	290.38
0.70	0.60	10.0	0.5292	140.55	118.23	20.0	0.5324	137.92	116.02	30.0	0.5335	136.99	115.24
0.70	0.70	10.0	0.4925	174.04	146.40	20.0	0.4959	170.61	143.52	30.0	0.4972	169.36	142.46
0.70	0.80	10.0	0.4418	232.09	195.23	20.0	0.4451	227.80	191.62	30.0	0.4464	226.09	190.19
0.70	0.90	10.0	0.3603	367.23	308.91	20.0	0.3620	363.56	305.83	30.0	0.3631	361.38	303.99
0.80	0.60	10.0	0.5792	155.58	121.01	20.0	0.5823	152.58	118.67	30.0	0.5834	151.51	117.84
0.80	0.70	10.0	0.5431	194.09	150.96	20.0	0.5465	190.13	147.88	30.0	0.5478	188.69	146.76
0.80	0.80	10.0	0.4922	261.47	203.36	20.0	0.4956	256.46	199.47	30.0	0.4969	254.47	197.92

0.80	0.90	10.0	0.4082	420.42	326.99	20.0	0.4100	416.07	323.61	30.0	0.4112	413.49	321.60
0.90	0.60	10.0	0.6606	180.78	124.13	20.0	0.6635	177.10	121.60	30.0	0.6645	175.80	120.70
0.90	0.70	10.0	0.6270	228.37	156.80	20.0	0.6302	223.46	153.43	30.0	0.6314	221.66	152.19
0.90	0.80	10.0	0.5782	313.12	214.99	20.0	0.5815	306.77	210.63	30.0	0.5828	304.25	208.90
0.90	0.90	10.0	0.4938	518.23	355.82	20.0	0.4957	512.55	351.92	30.0	0.4969	509.17	349.60

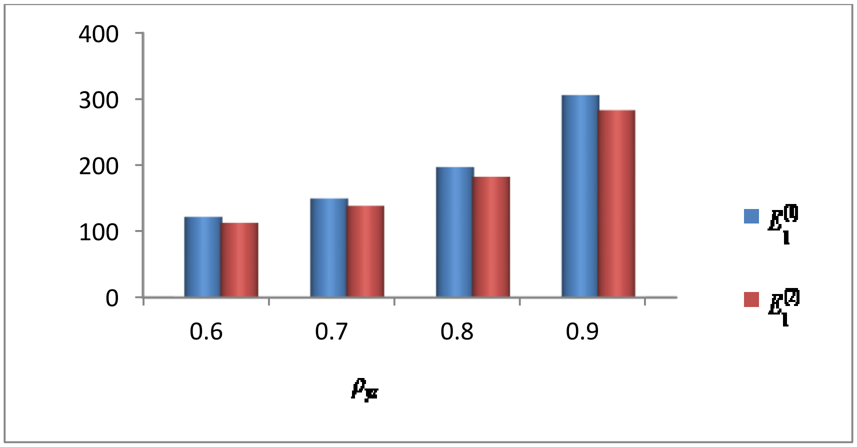

Figure 6.1: The percent relative efficiencies (*PREs*) of τ_i (i =1, 2) with respect to $\bar{y}_n$ and $\hat{\bar{Y}}$ for fixed values of $\rho_{yx} = 0.5$, $\bar{Z} = 10$ and different values of ρ_{yz}.

Table 6.2: The percent relative efficiencies of (*PREs*) τ_i (i =1, 2) with respect to $\bar{y}_n$ and $\hat{\bar{Y}}$ for different values of ρ_{yx}, ρ_{yz}, $\bar{Z}$ and $\beta_2(z) = 5.0$.

ρ_{yx}	ρ_{yz}	$\beta_2(z)$	$\bar{Z}$	μ_{20}	$E_2^{(1)}$	$E_2^{(2)}$	$\bar{Z}$	μ_{20}	$E_2^{(1)}$	$E_2^{(2)}$	$\bar{Z}$	μ_{20}	$E_2^{(1)}$	$E_2^{(2)}$
0.50	0.50	5.0	10.0	0.4686	119.67	110.76	20.0	0.4782	113.31	104.88	30.0	0.4837	109.84	101.66
0.50	0.60	5.0	10.0	0.4453	136.54	126.37	20.0	0.4519	131.52	121.73	30.0	0.4566	128.09	118.56
0.50	0.70	5.0	10.0	0.4169	160.17	148.25	20.0	0.4190	158.30	146.51	30.0	0.4224	155.31	143.75
0.50	0.80	5.0	10.0	0.3807	196.32	181.71	20.0	0.3750	202.70	187.62	30.0	0.3761	201.49	186.50
0.50	**0.90**	**5.0**	**10.0**	**0.3308**	**260.88**	**241.46**	**20.0**	**0.3090**	**296.44**	**274.37**	**30.0**	**0.3046**	**304.33**	**281.68**
0.60	0.50	5.0	10.0	0.4965	127.57	113.39	20.0	0.5061	120.66	107.26	30.0	0.5116	116.89	103.90
0.60	0.60	5.0	10.0	0.4730	145.92	129.70	20.0	0.4797	140.45	124.85	30.0	0.4844	136.73	121.53
0.60	0.70	5.0	10.0	0.4442	171.70	152.62	20.0	0.4464	169.65	150.80	30.0	0.4498	166.39	147.90
0.60	0.80	5.0	10.0	0.4073	211.28	187.80	20.0	0.4015	218.27	194.02	30.0	0.4026	216.95	192.84
0.60	0.90	5.0	10.0	0.3560	282.23	250.87	20.0	0.3333	321.43	285.71	30.0	0.3287	330.14	293.46
0.70	0.50	5.0	10.0	0.5324	137.89	115.99	20.0	0.5420	130.25	109.57	30.0	0.5474	126.08	106.06
0.70	0.60	5.0	10.0	0.5089	158.26	133.13	20.0	0.5156	152.19	128.02	30.0	0.5203	148.05	124.54
0.70	0.70	5.0	10.0	0.4800	186.98	157.29	20.0	0.4821	184.70	155.37	30.0	0.4856	181.06	152.30
0.70	0.80	5.0	10.0	0.4424	231.27	194.54	20.0	0.4365	239.12	201.15	30.0	0.4376	237.63	199.89
0.70	0.90	5.0	10.0	0.3896	311.14	261.72	20.0	0.3660	355.44	299.00	30.0	0.3612	365.31	307.29
0.80	0.50	5.0	10.0	0.5824	152.54	118.64	20.0	0.5917	143.81	111.85	30.0	0.5970	139.05	108.15
0.80	0.60	5.0	10.0	0.5593	175.91	136.82	20.0	0.5659	168.93	131.39	30.0	0.5706	164.18	127.70
0.80	0.70	5.0	10.0	0.5306	209.05	162.59	20.0	0.5327	206.41	160.54	30.0	0.5362	202.20	157.26
0.80	0.80	5.0	10.0	0.4929	260.51	202.62	20.0	0.4868	269.67	209.75	30.0	0.4880	267.94	208.39
0.80	0.90	5.0	10.0	0.4387	354.16	275.46	20.0	0.4142	406.46	316.14	30.0	0.4092	418.14	325.22

0.90	0.50	5.0	10.0	0.6635	177.06	121.57	20.0	0.6721	166.38	114.24	30.0	0.6769	160.57	110.25
0.90	0.60	5.0	10.0	0.6422	205.82	141.31	20.0	0.6484	197.20	135.40	30.0	0.6527	191.35	131.38
0.90	0.70	5.0	10.0	0.6152	247.05	169.62	20.0	0.6172	243.74	167.35	30.0	0.6205	238.48	163.74
0.90	0.80	5.0	10.0	0.5788	311.91	214.16	20.0	0.5729	323.55	222.15	30.0	0.5741	321.34	220.63
0.90	0.90	5.0	10.0	0.5251	432.00	296.62	20.0	*	-	-	30.0	0.4948	515.25	353.77

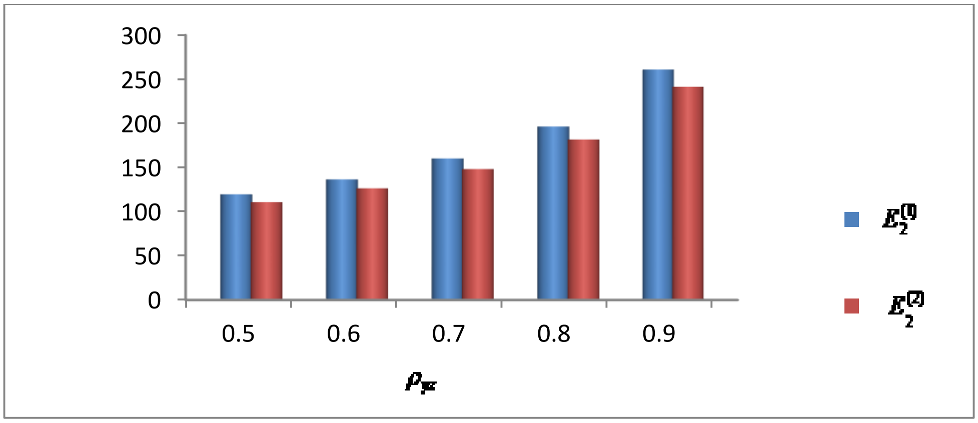

Figure 6.2: The percent relative efficiencies (*PREs*) of τ_i (i =1, 2) with respect to $\bar{y}_n$ and $\hat{\hat{\bar{Y}}}$ for fixed values of $\rho_{yx}=0.5$, $\beta_2(z)=5.0$, $\bar{Z}=10$ and different values of ρ_{yz}.

From Table 6.1 it is observed that:

(i) For fixed values of (ρ_{yx}, ρ_{yz}), the values of μ_{i0} increases as the value of $\bar{Z}$ increases while the values of $E_i^{(1)}$ and $E_i^{(2)}$ decrease.

(ii) for fixed values of (ρ_{yx}, $\bar{Z}$), the values of μ_{i0} decreases as the value of ρ_{yz} increases

While the values of $E_i^{(1)}$ and $E_i^{(2)}$ increase.

(iii) For fixed values of ($\bar{Z}$, ρ_{yz}), the values of μ_{i0}, $E_i^{(1)}$ and $E_i^{(2)}$ increase with increasing value of ρ_{yx}. This behavior is in agreement with Sukhatme et al. [23], results which explained that more the value of ρ_{yx}, more the fractions of fresh sample required at the current occasion.

(iv) minimum value of μ_{10} is 0.3037($\cong 0.30$) which shows that the fraction to be replaced at the current occasion is as low as about 30 percent of the total sample size leading to a reduction of considerable amount in the cost of the survey.

Similar trend and conclusions can be drawn from Table 6.2.

In addition to these it is observed from Tables 6.1 and 6.2 that there is appreciable gain in efficiencies by using the proposed estimators τ_i (i =1, 2) over usual unbiased estimator $\bar{y}_n$ and the estimator $\hat{\hat{\bar{Y}}}$. Thus we infer that the use of auxiliary information at the estimation stage is highly rewarding in terms of the proposed estimator τ_i (i =1, 2).

We also note from Table 6.1 (and Figure 6.1) and Table 6.2 (and Figure 6.2) that the proposed estimators τ_1 and τ_2 yield more gain over $\bar{y}_n$ as compared to $\hat{\hat{\bar{Y}}}$.

7 Conclusion

In this article, an efficient estimation procedure has been developed utilizing the known population mean $\bar{Z}$ alongwith known correlation coefficient ρ_{yz} and coefficient of kurtosis $\beta_2(z)$ of the auxiliary variable *z* on both the occasions for estimating the current (second) population mean in two occasion successive sampling. From numerical illustrations, it may be concluded that the suggested classes of estimators is more useful in estimation of the population mean of the study variable at the current occasion in two-occasion successive sampling. Finally looking on the good performance of the envisaged estimator, our suggestion is to use the proposed classes of estimators in practice.

8 Acknowledgements

The authors are highly grateful to the Editor-in-Chief and learned referees for their excellent comments which helped us to improve the previous draft of the paper.

References

[1] R. J. Jessen, Statistical investigation of a sample survey for obtaining form facts. Iowa Agricultural Experiment Station Road Bulletin no. 304: Ames, USA, (1942).

[2] H.D. Patterson, Sampling on successive occasions with partial replacement of units. *Jour. Roy. Statist. Assoc*, B, **12**, 241- 255, (1950).

[3] R. D. Narain, On the recurrence formula in sampling on successive occasions. *Journal of the Indian society of agriculture statistics*, **5**, 96-99, (1953).

[4] A.R. Eckler, Rotation sampling. *Ann. Math. Statist.* **26**, 664-685, (1955).

[5] J.N.K. Rao, and J.E. Graham, Rotation design for sampling on repeated occasions. *Jour. Amer. Statist. Assoc.*, **59**, 492-509, (1964).

[6] L. Gordon, Successive sampling in finite population. *The Annals of statistics*, **11(2)**, 702-706, (1983).

[7] R. Arnab, Okafor, F.C., A note on double sampling over two occasions. *Pakistan Journal of Statistics*, **8**, 9-18, (1992).

[8] H.P. Singh, H.P. Singh and V.P. Singh, A generalized efficient class of estimators of population mean in two-phase and successive Sampling. *Int. Jour. Manage. Systems*, **8(2)**, 173-183, (1992).

[9] A .R. Sen, Successive sampling with two auxiliary variables. *Sankhya*, B, **33**, 371- 378, (1971).

[10] A .R. Sen, Successive sampling with p (p≥1) auxiliary variables, *Ann. Math.*, **43**, 2031-2034, (1972).

[11] A .R. Sen, Theory and application of sampling on repeated occasions with several auxiliary variables. *Biometrics*, 29: 381-385, (1973).

[12] S. Feng and G. Zou, Sampling rotation method with auxiliary variable. *Commun. Statist. Theo. Meth.*, 26(6): 1497-1509, (1997).

[13] R. S. Biradar and H.P. Singh, Successive sampling using auxiliary information on both the occasions. *Calcutta Statist. Assoc. Bull.*, **51**, 243-251, (2001).

[14] G.N. Singh, on the use of chain type ratio estimator in successive Sampling Statistics in Transition - new series, **7(1)**, 21-26, (2005).

[15] H. P. Singh, and G.K. Vishwakarma, Modified exponential ratio product estimators for finite population mean in double sampling. *Austrian Jour. Statist.*, **36(3)**, 217 – 225, (2007a).

[16] H.P. Singh, and G.K. Vishwakarma, A general class of estimators in successive sampling. *Metron*, **65(2),** 201-227, (2007b).

[17] H.P. Singh, and G.K. Vishwakarma, A general procedure for estimating population mean in successive sampling. *Commun. Statist. Theo. Meth.*, **38**, 293-308, (2009).

[18] H. P. Singh and S. K. Pal, on the estimation of population mean in successive sampling. *Int. Jour. Math. Sci. Applica.*, **5(1),** 179-185, (2015a).

[19] H. P. Singh and S. K. Pal, on the estimation of population mean in rotation sampling. *Jour. Statist. Applica. Pro. Lett.* **2(2)**, 131-136, (2015b).

[20] H.P. Singh and R. Tailor, Use of known correlation coefficient in estimating the finite population mean. *Statistics in Transition*, **6 (4)**, 555-560, (2003).

[21] H.P. Singh, Tailor, Rajesh, Tailor, Ritesh and M. S. Kakran, An improved estimator of population mean using power transformation. *Journal of the Indian Society of Agricultural Statistics*, **58 (2)**, 223-230, (2004).

[22] A. Sahai and A. Sahai. An efficient use of auxiliary information. *Jour. Statist. Plan.Inf.***12**, 203-212, (1985).

[23] P. V. Sukhatme, B. V. Sukhatme, and C. Ashok. Sampling theory of surveys with applications, 3rd ed. Ames, IA, Iowa State University Press, (1984).