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Efficient Schemes for Generating GHZ-Type, W-Type and Cluster ECS of n-Cavity Coherent Fields

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Abstract: We establish a new model for realization of three qubit C-NOT gate operations between two Rydberg atoms and coherent Cavity field. To exploit this C-NOT realization, we enhance many schemes for generation of GHZ-type and W-type entangled coherent state (ECS), to prepare n-cavities GHZ- type (ECS) and n-cavities W- type (ECS). Furthermore, we find the possibility of preparing cluster entangled coherent states using the realized three qubit C-NOT gate. Our model compared with the references schemes generation is more efficient because of simplicity of detecting coherent fields. Also, the rigidity of the model comes from the smaller interaction time needed to realize the three quantum entangled systems due to the proposed interaction.

Keywords: three qubit C-NOT gate, N-cavities GHZ- type ECS, N-cavities W- type ECS, Cluster ECS, Entangled Coherent States.

1 Introduction:

Quantum dense coding [1], quantum teleportation [2], and quantum cryptography [3] are among the important applications used in quantum information processing (QIP). In fact, quantum entanglement is an indispensable and important resource that will be ”consumed” during the QIP. So many types of multipartite entangeled states are investigated in several physical systems [4][5][6][7], specially in the (QIP) field.

On the other hand, the coherent states are a specific kind of quantum states applicable to the electromagnetic field which describe a maximal kind of coherence. That’s promote their usefulness to be used in cryptography and teleportation [8][9][10].

The electromagnetic waves are emitted by many such sources which are in phase, called Fock states.

$$\mid \alpha\rangle = e^{-\frac{|\alpha|^2}{2}} \sum_{n=0}^{\infty} \frac{\alpha^n}{\sqrt{n!}} \mid n\rangle \quad (1)$$

Where $\mid n\rangle$ is the Fock state of the corresponding photon number and α is the corresponding amplitude of the coherent state.

For this aim, based on Cavity (QED), several sechmes have been proposed to prepare specially entangled coherent states. Which include two-qubit entangled states [11][12] ,Greenberger-Horne-Zeilinger (GHZ) states [13][14][15][16][17], W-type states [18], cluster states [19] and Entangled Coherent States [20][21][22]. The generation of entangled coherent states attracs a great deal of attention because of its robustness against single-particule decoherence and its crucial applications in QIP.

To achieve this goal, we use the model of two atoms interacting simultaneously with a coherent field (the two atoms are sent through a cavity submitted to the coherent field). With the help of an effective Hamiltonian, which describe interaction between two three-level atoms simultaneously with a coherent field before eliminating adiabatically the upper level during the interaction process. This is because of the large detuning limit between atomic transition frequency and the cavity field frequency (coherent field frequency). After that, the evolution model allows us to realize a quantum Controlled-NOT (C-NOT) of three gates based on coherent field (two Rydberg atoms - coherent field), as a continuation of a previous work [23] which realize (C-NOT) of two gates (from one rydberg atom to field of coherent field).

In addition, we investigate the established (three qubit C-NOT gate) model to generalize the proposed scheme in

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Ref. [24], in order to generate n-cavities GHZ-type entangled coherent states (ECS) and n-cavities W-type entangled coherent states (ECS). All that are based on separate optical cavities using the approach of entangled continuous-variable states. We also propose a new model for generating of cluster entangled coherent states [25][26][27].

The simplicity of our schemes comes from the use of the C-NOT three gates, the short interaction time which is required, and generation of entanglement on three different quantum systems, which are based on coherent field. We get all these results using a cavities submitted by coherent field which are the major advantages of the present work.

2 The three gates C-NOT Model

We Consider the interaction between two three-level atoms of Λ-type and a coherent optical field Figure 1.

The interaction Hamiltonian of the system is expressed as

$$H = g \sum_{j=1,2} (e^{-i\Delta t} a^{+} S_j^{-} + e^{i\Delta t} a\, S_j^{+}) \quad (2)$$

Where $S_j^- = |\, g_j\rangle\langle e_j \,|$, $S_j^+ = |\, e_j\rangle\langle g_j \,|$, with $|\, g_j\rangle$, $|\, e_j\rangle$ are respectively the ground and the excited state of both rydberg atom.

And $\Delta = \omega_0 - \omega_c$ being the detuning between atomic transition frequency and the frequency of the coherent field, a^+ and a are the creation and annihilation operators for the cavity mode. g is the coupling constant between the cavity mode transitions $|\, e\rangle \rightarrow |\, g\rangle$

The two lower levels of the atoms $|\, g_j\rangle$ and $|\, e_j\rangle$ are degenerated, and the frequency of the coherent optical field ω_c is largely detuned from the atomic transition frequency ω_0 between the degenerated lower levels and the upper level. In this large detuning limit the upper level $|\, i\rangle$ can be adiabatically eliminated during the interaction, if we consider the following initial conditions [23]

$$g = g_1 = g_2 \;\; ; \;\; \lambda = \beta_1 = \beta_2 = \frac{g^2}{\Delta} \quad (3)$$

with g_1 ,g_2 being the coupling constant between the cavity mode transitions $|\, i\rangle \rightarrow |\, e\rangle$, $|\, i\rangle \rightarrow |\, g\rangle$ respectively.

Where $|\, i\rangle$ is the upper energy level of the studied rydberg atom. The effective Hamiltonian [23][28] of the system can then be modified, and written as follow

$$H_{eff} = -\lambda a^+ a \;\{\; |\, e\rangle_{11}\langle g \,| + |\, g\rangle_{11}\langle e \,| + |\, e\rangle_{11}\langle e \,| + |\, g\rangle_{11}\langle g \,| + |\, e\rangle_{22}\langle g \,| + |\, g\rangle_{22}\langle e \,| + |\, e\rangle_{22}\langle e \,| + |\, g\rangle_{22}\langle g \,|\} \quad (4)$$

The effective Hamiltonian of [23] present the interaction between one Rydberg atom and cavity coherent field; y proposed to send one atom throughout cavity QED governed by coherent field. The result of their proposition model is a realization of Quantum two-qubit C-NOT gates. In our work, to achieve realized Quantum three-qubit C-NOT gates, we have to send two Rydberg atoms 1 and 2 simultaneously throughout cavity

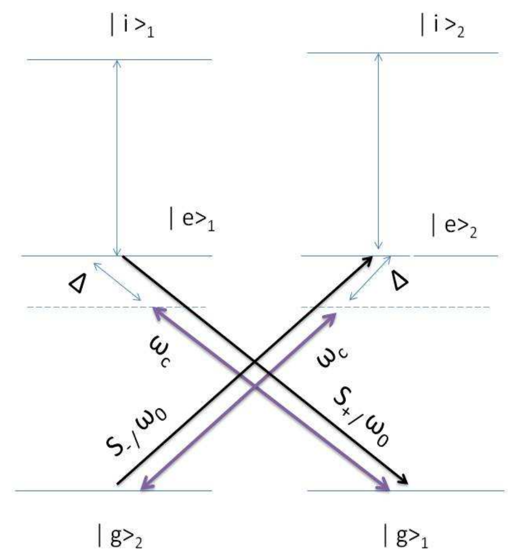

Fig. 1: Two Rydberg atoms interact with a coherent cavity field .

coherent field. So we have to modify the Hamiltonian [23] to be adapted by the new proposed interaction, but by keeping the same initial conditions.

Following the work [23] that establishes a two C-NOT gates, we define the basis states $\{|\, +\rangle, |\, -\rangle\}$ as follows

$$|\, +\rangle_i = \frac{1}{\sqrt{2}}(|\, e\rangle_i + |\, g\rangle_i) \quad (5)$$

$$|\, -\rangle_i = \frac{1}{\sqrt{2}}(|\, e\rangle_i - |\, g\rangle_i) \quad i = 1,2 \quad (6)$$

For the overall state of the two atoms one has the following possible combinations

$$|\, -\rangle_1 |\, -\rangle_2 = \frac{1}{2}\{|\, e\rangle_1 |\, e\rangle_2 - |\, e\rangle_1 |\, g\rangle_2 - |\, g\rangle_1 |\, e\rangle_2 + |\, g\rangle_1 |\, g\rangle_2\} \quad (7)$$

$$|\, -\rangle_1 |\, +\rangle_2 = \frac{1}{2}\{|\, e\rangle_1 |\, e\rangle_2 + |\, e\rangle_1 |\, g\rangle_2 - |\, g\rangle_1 |\, e\rangle_2 - |\, g\rangle_1 |\, g\rangle_2\} \quad (8)$$

$$|\, +\rangle_1 |\, -\rangle_2 = \frac{1}{2}\{|\, e\rangle_1 |\, e\rangle_2 - |\, e\rangle_1 |\, g\rangle_2 + |\, g\rangle_1 |\, e\rangle_2 - |\, g\rangle_1 |\, g\rangle_2\} \quad (9)$$

$$|\, +\rangle_1 |\, +\rangle_2 = \frac{1}{2}\{|\, e\rangle_1 |\, e\rangle_2 + |\, e\rangle_1 |\, g\rangle_2 + |\, g\rangle_1 |\, e\rangle_2 + |\, g\rangle_1 |\, g\rangle_2\} \quad (10)$$

If we suppose that both atoms 1 and 2 are initially prepared in the $|\, -\rangle_1 |\, -\rangle_2$ state and sent simultaneously into a cavity that contains an optical field described by the coherent states $|\, \alpha\rangle$. To have the output state of system after interaction, by applying the Schrodinger equation on (7) as a input state before being sent into the cavity coherent field. The interaction between the atoms and the optical field yields the following state

$$|\, -\rangle_1 |\, -\rangle_2 |\, \alpha\rangle \rightarrow |\, -\rangle_1 |\, -\rangle_2 |\, e^{4i\lambda t}\alpha\rangle. \quad (11)$$

Where i here in (11) means the complex number.

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By adjusting the interaction time $t = \frac{\pi}{2\lambda}$, the previous equation, then, becomes:

$$|-\rangle_1|-\rangle_2|\alpha\rangle \rightarrow |-\rangle_1|-\rangle_2|\alpha\rangle \quad (12)$$

Similarly, the other possible initial states of the system, keeping the same interaction time, will lead to the following final states:

$$|+\rangle_1|+\rangle_2|\alpha\rangle \rightarrow |+\rangle_1|+\rangle_2|\alpha\rangle \quad (13)$$
$$|+\rangle_1|-\rangle_2|\alpha\rangle \rightarrow |+\rangle_1|-\rangle_2|-\alpha\rangle \quad (14)$$
$$|-\rangle_1|+\rangle_2|\alpha\rangle \rightarrow |-\rangle_1|+\rangle_2|-\alpha\rangle \quad (15)$$
$$|-\rangle_1|-\rangle_2|\alpha\rangle \rightarrow |-\rangle_1|-\rangle_2|\alpha\rangle \quad (16)$$

$$|+\rangle_1|+\rangle_2|-\alpha\rangle \rightarrow |+\rangle_1|+\rangle_2|-\alpha\rangle \quad (17)$$
$$|+\rangle_1|-\rangle_2|-\alpha\rangle \rightarrow |+\rangle_1|-\rangle_2|\alpha\rangle \quad (18)$$
$$|-\rangle_1|+\rangle_2|-\alpha\rangle \rightarrow |-\rangle_1|+\rangle_2|\alpha\rangle \quad (19)$$
$$|-\rangle_1|-\rangle_2|-\alpha\rangle \rightarrow |-\rangle_1|-\rangle_2|-\alpha\rangle \quad (20)$$

These results show that the states of the atoms remain unchanged after the interaction and it is the coherent field that changes if the initial atomic states are different. This can be seen as some sort of a three qubit C-NOT gate, where the atomic states serve as control bit and the field state id the target bit.

3 Realization of Cluster Entangled Coherent States

In order to realize tripartite cluster entangled coherent state ($N = 3$), firstly we prepare two atoms in the state $|+\rangle_1|-\rangle_2$, and we use three optical cavities which are governed resepectively by the coherent fields $|-\alpha\rangle_1|\alpha\rangle_2|-\alpha\rangle_3$. After that let these coherent field cross a classical field appropriately so that each field undergoes the following transition

$$|\alpha\rangle_i \rightarrow \frac{1}{\sqrt{2}}(|\alpha\rangle_i + |-\alpha\rangle_i) \quad (21)$$

$$|-\alpha\rangle_i \rightarrow \frac{1}{\sqrt{2}}(|\alpha\rangle_i - |-\alpha\rangle_i) \quad (22)$$

Where the parameter i here means the cavity rank used in the proposed scheme.

So the total state of the system will be

$$\frac{1}{2\sqrt{2}}|+\rangle_1|-\rangle_2(|\alpha\rangle_1 - |-\alpha\rangle_1)(|\alpha\rangle_2 + |-\alpha\rangle_2)(|\alpha\rangle_3 - |-\alpha\rangle_3) \quad (23)$$

Sending simultaneously the two atoms through the three coherent cavities successively from the first cavity to the third one, following the above equations (14) (15) (18) (19) after interaction the global state of the system will be

$$\frac{1}{2\sqrt{2}}|+\rangle_1|-\rangle_2(|-\alpha\rangle_1 - |\alpha\rangle_1)(|\alpha\rangle_2 + |-\alpha\rangle_2)(|-\alpha\rangle_3 - |\alpha\rangle_3) \quad (24)$$

Which can be rewritten as:

$$\frac{1}{2^{\frac{3}{2}}}|+-\rangle_{12}\bigotimes_{i=1}^{3}(-1)^i\Big(|\alpha\rangle_i + (-1)^i|-\alpha\rangle_i\Big) \quad (25)$$

For a multipartite cluster entangled coherent states (dimension N) then we propose to utilize N cavities which contain the coherent field $|(-1)^i\alpha\rangle_i$, it means that the choice of cavity field depends on the parity of the used cavity rank. With the same process used after interaction of the two atoms with the N cavities we get the final form of the cluster entangled coherent state realized

$$\frac{1}{2^{\frac{N}{2}}}|+-\rangle_{12}\bigotimes_{i=1}^{N}(-1)^i\Big(|\alpha\rangle_i + (-1)^i|-\alpha\rangle_i\Big) \quad (26)$$

It's the multipartite cluster entangled coherent state form, generated by coherent cavity, and by investigation of the three gates C-NOT Model.

4 Generation of N-cavities GHZ-type Entangled Coherent States

Let us use this time, two atoms intially prepared in the state: $|++\rangle_{1,2}$, N-cavities governed by the coherent fields $|\alpha,\alpha,...,\alpha\rangle_{1,2,...,N}$, and apply a classical field just on the second atom to have the following transitions

$$|+\rangle_2 \rightarrow (|+\rangle_2 + |-\rangle_2)$$
$$|-\rangle_2 \rightarrow (|+\rangle_2 - |-\rangle_2) \quad (27)$$

We propose to send the two atoms through the N-cavities successively from C_1 to C_N to interact with the previous coherent fields, so the initial state of system will be in this form

$$|++\rangle_{1,2}|\alpha,\alpha,...,\alpha\rangle_{1,2,...,N} + |+-\rangle_{1,2}|\alpha,\alpha,...,\alpha\rangle_{1,2,...,N} \quad (28)$$

After interaction the state of the system is

$$|++\rangle_{1,2}|\alpha,\alpha,...,\alpha\rangle_{1,2,...,N} + |+-\rangle_{1,2}|-\alpha,-\alpha,...,-\alpha\rangle_{1,2,...,N} \quad (29)$$

Applying again the same transitions by classical field on the second atom (27), then

$$N_e\Big[|++\rangle_{1,2}\Big(|\alpha,\alpha,...,\alpha\rangle_{1,2,...,N} + |-\alpha,-\alpha,...,-\alpha\rangle_{1,2,...,N}\Big) + |+-\rangle_{1,2}\Big(|\alpha,\alpha,...,\alpha\rangle_{1,2,...,N} - |-\alpha,-\alpha,...,-\alpha\rangle_{1,2,...,N}\Big)\Big] \quad (30)$$

With N_e the normalization factor, in this case $N_e = \frac{1}{2}$. If the two atoms are selected by detectors after mesurement in the state: $|++\rangle_{1,2}$, then the N-cavities field will be $|\alpha,\alpha,...,\alpha\rangle_{1,2,...,N} + |-\alpha,-\alpha,...,-\alpha\rangle_{1,2,...,N}$, else the state $|\alpha,\alpha,...,\alpha\rangle_{1,2,...,N} - |-\alpha,-\alpha,...,-\alpha\rangle_{1,2,...,N}$ will be realized, which are just the so called N-cavities GHZ-type ECSs.

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5 Generation of n-cavities W-type Entangled Coherent States

We choose to use $2N$ atoms prepared in the initial state

$$N_e\left[\left(\sum_{k=0}^{N-2}\bigotimes_{c=1}^{2k+1}\mid q\rangle_c\bigotimes_{c=2k+2}^{2N}\mid p\rangle_c\right)+\bigotimes_{c=1}^{2N-1}\mid q\rangle_c\mid p\rangle_{2N}\right] \quad (31)$$

With

$$\forall p\in\{+,-\};q\in\{+,-\} \quad (32)$$

$$\mid p\times q\rangle=\mid -\rangle \quad (33)$$

And $N_e=\frac{1}{\sqrt{N}}$ is the normalisation factor

Also we use N optical cavities $C_1,C_2,\ldots,C_N$ that have the coherent fields $\mid\alpha\rangle_1,\mid\alpha\rangle_2,\ldots,\mid\alpha\rangle_N$ respectively, after that we send simultaneously the two first atoms through these N cavities respectively. The initial state of the system before interaction is:

$$\mid\psi\rangle=N_e\left[\left(\sum_{k=0}^{N-2}\otimes_{c=1}^{2k+1}\mid q\rangle_c\otimes_{c=2k+2}^{2N}\mid p\rangle_c\right)+\otimes_{c=1}^{2N-1}\mid q\rangle_c\mid p\rangle_{2N}\right]\otimes_{c=1}^{N}\mid\alpha\rangle_c \quad (34)$$

After interaction the global state becomes:

$$\begin{aligned}\mid\psi(t=\frac{\pi}{2\lambda})\rangle &= N_e\left[\sum_{k=0}^{N-2}\left(\bigotimes_{c=1}^{2k+1}\mid q\rangle_c\bigotimes_{c=2k+2}^{2N}\mid p\rangle_c\right)\left(\bigotimes_{c=1}^{N,c\neq k+1}\mid\alpha\rangle_c\mid-\alpha\rangle_{k+1}\right)\right.\\ &\left.+\left(\bigotimes_{c=1}^{2N-1}\mid q\rangle_c\mid p\rangle_{2N}\right)\left(\bigotimes_{c=1}^{N-1}\mid\alpha\rangle_c\mid-\alpha\rangle_N\right)\right] \quad (35)\end{aligned}$$

Suggest that the $2N$ atoms undergoes a classical field allowing the following transitions

$$\mid p\rangle_c\rightarrow(\mid p\rangle_c+\mid q\rangle_c) \quad (36)$$

$$\mid q\rangle_c\rightarrow(\mid p\rangle_c-\mid q\rangle_c) \quad (37)$$

Then the total state becomes:

$$\begin{aligned}\mid\psi(t=\frac{\pi}{2\lambda})\rangle &= N_e\left[\sum_{k=0}^{N-2}\left(\bigotimes_{c=1}^{2k+1}(\mid p\rangle_c-\mid q\rangle_c)\bigotimes_{c=2k+2}^{2N}(\mid p\rangle_c+\mid q\rangle_c)\right)\right.\\ &\left(\bigotimes_{c=1}^{N,c\neq k+1}\mid\alpha\rangle_c\mid-\alpha\rangle_{k+1}\right)\\ &\left.+\left(\bigotimes_{c=1}^{2N-1}(\mid p\rangle_c-\mid q\rangle_c)(\mid p\rangle_{2N}+\mid q\rangle_{2N})\right)\left(\bigotimes_{c=1}^{N-1}\mid\alpha\rangle_c\mid-\alpha\rangle_N\right)\right] \quad (38)\end{aligned}$$

After that, we apply the transitions (5),(6) on each atom among the $2N$ atoms. This allows to have 2^{2N} different atomic states. A projective measurement on one of these atomic states, yields a W-type ECSs for the coherent field:

$$\pm\sum_{k=0}^{N-2}\left(\bigotimes_{c=1}^{N,c\neq k+1}\mid\alpha\rangle_c\mid-\alpha\rangle_{k+1}\right)\pm\left(\bigotimes_{c=1}^{N-1}\mid\alpha\rangle_c\mid-\alpha\rangle_N\right) \quad (39)$$

A total of 2^N W-type ECSs can be prepared in this way with a probability of $\frac{1}{2}(\frac{1}{\sqrt{2}\times 2N})$.

Example: $N=4$

To give an example of the proposed scheme ,we choose to apply the generalized scheme to generate $4-cavities$ W-type (ECSs), then for this aim we have to use 8 atoms in the initial state

$$\begin{aligned}N_e &\left[\mid-+++++++\rangle_{1,\ldots,8}+\mid---+++++\rangle_{1,\ldots,8}\right.\\ &\left.+\mid-----+++\rangle_{1,\ldots,8}+\mid-------+\rangle_{1,\ldots,8}\right] \quad (40)\end{aligned}$$

In this case, $N_e=\frac{1}{\sqrt{4}}=\frac{1}{2}$

The overall state of system after interaction with 4-cavities which contain the coherent field:$\mid\alpha,\alpha,\alpha,\alpha\rangle_{1,\ldots,4}$ is

$$\begin{aligned}\frac{1}{2}\times &\left[\mid-++++++--\rangle_{1,\ldots,8}\mid-\alpha,\alpha,\alpha,\alpha\rangle_{1,\ldots,4}+\mid---++++--\rangle_{1,\ldots,8}\right.\\ &+\mid\alpha,-\alpha,\alpha,\alpha\rangle_{1,\ldots,4}+\mid-----+--\rangle_{1,\ldots,8}\mid\alpha,\alpha,-\alpha,\alpha\rangle_{1,\ldots,4}\\ &\left.+\mid-------+\rangle_{1,\ldots,8}\mid\alpha,\alpha,\alpha,-\alpha\rangle_{1,\ldots,4}\right] \quad (41)\end{aligned}$$

After that, let us apply the transitions (5),(6) on each atom among the eight atoms. This allows to have 2^8 different atomic states. A projective measurement on one of these atomic states, yields a W-type ECSs for the coherent field:

$$\pm\mid-\alpha,\alpha,\alpha,\alpha\rangle_{1,\ldots,4}\pm\mid\alpha,-\alpha,\alpha,\alpha\rangle_{1,\ldots,4}\pm\mid\alpha,\alpha,-\alpha,\alpha\rangle_{1,\ldots,4}\pm\mid\alpha,\alpha,\alpha,-\alpha\rangle_{1,\ldots,4} \quad (42)$$

A total of 2^4 W-type ECSs can be prepared in this way with a probability of $\frac{1}{2}(\frac{1}{\sqrt{2}\times 8})$.

Conclusion

In summmary, we have proposed a new scheme to prepare cluster coherent states. We have generated n-cavities GHZ-type entangled coherent states (ECS), and n-cavities W-type entangled coherent states (ECS) by exploiting the obtained result of the quantum Controlled-NOT (C-NOT) of three gates. For this goal, we use interaction between two three-level atoms simultaneously and optical cavity exploiting an effective Hamiltonian of interaction between both atoms with coherent field. And we claim that our schemes might be feasible with current technology due to the simplicity of homodyne detection for coherent field, which gives more efficiency to our model compared with the references schemes.

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