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A New Laplace Iteration Technique for Solving a Generalize Fractional Nonlinear Schrodinger Equation

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Abstract: In the present paper, we proposed a reliable recipe of iteration method and Laplace transformation namely Laplace iteration method (LIM) to solve a general fractional nonlinear Schrodinger equation. By using an initial value, the analytical solutions of the equation for different cases have been obtained, which accelerate the rapid convergence of the series solution.

Keywords: Schrodinger equations, Laplace decomposition, iterational method

1 Introduction

Over the last 40 years, the nonlinear Schrodinger equation a prototypical dispersive nonlinear partial differential equation (PDE) that has been derived in many areas of physics. It has important applications in various physics such as wave propagation in nonlinear media [1], surface waves on deep waters [2], plasma phenomena and non-uniform dielectric media. Ginzburg and Landau [3] and Ginzburg [4] have been studied of the macroscopic theory of superconductivity. Also, Ginzburg and Pitaevskii [5] have been investigated the theory of super fluidity. Furthermore, the analytical approximation solution for the generalized nonlinear Schr?dinger equation subject to some initial conditions has been obtained [6], by using the Adomian decomposition method. The analytical fractional nonlinear Schrodinger equation in time is governed by the equation

$$\imath\frac{\partial^\alpha u(x,t)}{\partial t^\alpha}+\beta\frac{\partial^2 u(x,t)}{\partial x^2} + V(x)u(x,t)+\gamma|u(x,t)|^2u(x,t)=0 \qquad (1)$$

where $V(x)$ is the trapping potential and β, γ are a real constant, with some initial conditions to find explicit solutions and numerical solutions of this equation rather than the traditional methods. The aim of this paper is to find the approximate solution of the generalized fractional nonlinear Schr?dinger equation with the new proposed analytical technique Laplace iterational method (LIM) that is analytical approach to get series solutions of various types (linear or nonlinear, ordinary and partial differential) equations. The paper is organized as follows: In. Sect. 2, the Laplace iterational method (LIM) for a generalized fractional nonlinear Schrodinger equation is obtained. In. Sect. 3, some examples are given in order to demonstrate the effectiveness of LIM. Finally, conclusions are given in sect.4

2 Analysis of Laplace Iterational Method (LIM)

We consider the generalized NLS equation of the form

$$\imath\frac{\partial^\alpha u(x,t)}{\partial t^\alpha}+\beta\frac{\partial^2 u(x,t)}{\partial x^2} + V(x)u(x,t)+\gamma|u(x,t)|^2u(x,t)=0 \qquad (2)$$

The standard form of the generalized NLS equation in an operator form is

$$D_t^\alpha u(x,t) - i\beta L_x u(x,t) - iV(x)u(x,t) - i\gamma|u(x,t)|^2u(x,t) \qquad (3)$$

where $D_t^\alpha=\partial^\alpha/\partial t^\alpha$ is the Caputo fractional derivatives of order α and $L_x=\partial^2/\partial x^2$. Operating with $D_t^{1-\alpha}$ in both sides of Eq. (3), we find

$$D_t u(x,t) - D_t^{1-\alpha}\{i\beta L_x u(x,t) + iV(x)u(x,t)+i\gamma|u(x,t)|^2u(x,t)\}=0 \qquad (4)$$

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Applying Laplace transform in both sides of Eq. (4), we can rewrite the generalized NLS equation in the form

$$\begin{aligned} NL[u(x,t)] &= \Im_t\,[u(x,t)] - \frac{u(x,0)}{s} \\ &- \frac{1}{s}\Im_t[D_t^{1-\alpha}\{i\beta L_x u(x,t) \\ &+ iV(x)u(x,t) + i\gamma|u(x,t)|^2 u(x,t)\}] = 0 \end{aligned} \tag{5}$$

where $NL[u(x,t)]$ is nonlinear Laplace operator. According to the iterational method and placeLaplace decomposition method, we consider the correction functional in t -direction in the following form

$$u_{n+1}(x,t) = u_n(x,t) - \Im_t^{-1}[NL[u_n(x,t)]] \tag{6}$$

Beginning with an initial approximation $u(x,0) = u_0(x,t)$, we can obtain the analytical approximation solutions

$$\begin{aligned} u_1(x,t) &= u_0(x,t) - \Im_t^{-1}\Bigg[\Im_t\,[u_0(x,t)] - \frac{u(x,0)}{s} \\ &- \frac{1}{s}\Im_t[D_t^{1-\alpha}\{i\beta L_x u_0(x,t) + iV(x)u_0(x,t) \\ &+ i\gamma|u_0(x,t)|^2 u_0(x,t)\}] = 0 \end{aligned} \tag{7}$$

and

$$\begin{aligned} u_2(x,t) &= u_1(x,t) - \Im_t^{-1}\Bigg[\Im_t\,[u_1(x,t)] - \frac{u(x,0)}{s} \\ &- \frac{1}{s}\Im_t[D_t^{1-\alpha}\{i\beta L_x u_1(x,t) + iV(x)u_1(x,t) \\ &+ i\gamma|u_1(x,t)|^2 u_1(x,t)\}] = 0 \end{aligned} \tag{8}$$

and so on. Finally the exact solution is obtained by

$$u(x,t) = \lim_{n\to\infty} u_n(x,t) \tag{9}$$

3 Applications

In the current section, we take three examples to illustrate the pervious method to solve the generalized nonlinear fractional Schrdinger equation.

Example 1: let us consider Eq. (1) $\beta = 1/2$, $V(x) = 0$ and $\gamma = 1$ Subject to the initial condition

$$u(x,0) = \sin(x) \tag{10}$$

By using Eq. (5), we could be able to calculate some of the terms of the series

$$u_1(x,t) = sin(x) - \frac{3it^{\alpha} sin(x)}{2\Gamma(1+\alpha)}$$

$$u_2(x,t) = sin(x) - \frac{3it^{\alpha} sin(x)}{2\Gamma[1+\alpha]} - \frac{9t^{2\alpha} sin(x)}{4\Gamma[1+2\alpha]}$$

$$u_3(x,t) = sin(x) - \frac{3it^{\alpha} sin(x)}{2\Gamma[1+\alpha]} - \frac{9t^{2\alpha} sin(x)}{4\Gamma[1+2\alpha]} + \frac{27it^{3\alpha} sin(x)}{8\Gamma[1+3\alpha]}$$

Thus,

$$\begin{aligned} u_4(x,t) &= sin(x) - \frac{3it^{\alpha} sin(x)}{2\Gamma[1+\alpha]} - \frac{9t^{2\alpha} sin(x)}{4\Gamma[1+2\alpha]} \\ &+ \frac{27it^{3\alpha} sin(x)}{8\Gamma[1+3\alpha]} + \frac{81t^{4\alpha} sin(x)}{16\Gamma[1+4\alpha]} \end{aligned} \tag{11}$$

The exact solution is

$$u_n(x,t) = sin(x)\sum_{n=0}^{\infty}\frac{\left(-\frac{3it^{\alpha}}{2}\right)^n}{\Gamma[1+n\alpha]} \tag{12}$$

This has the exact solution

$$u_n(x,t) = sin(x)E_{\alpha}\left(-\frac{3it^{\alpha}}{2}\right) \tag{13}$$

where E_{α} is the Mittag–Leffler function, it has been studied in [7].

Example 2: let us consider Eq. (1) $\beta = 1/2$, $V(x) = 1$ and $\gamma = 1$ Subject to the initial condition

$$u(x,0) = e^{ix} \tag{14}$$

by using Eq. (6, we could be able to calculate some of the terms of the series

$$u_1(x,t) = e^{ix} + \frac{ie^{ix}t^{\alpha}}{2\Gamma[1+\alpha]}$$

$$u_2(x,t) = e^{ix} + \frac{ie^{ix}t^{\alpha}}{2\Gamma[1+\alpha]} - \frac{e^{ix}t^{2\alpha}}{4\Gamma[1+2\alpha]}$$

$$u_3(x,t) = e^{ix} + \frac{ie^{ix}t^{\alpha}}{2\Gamma[1+\alpha]} - \frac{e^{ix}t^{2\alpha}}{4\Gamma[1+2\alpha]} - \frac{ie^{ix}t^{3\alpha}}{8\Gamma[1+3\alpha]}$$

Therefore,

$$\begin{aligned} u_4(x,t) = e^{ix} + \frac{ie^{ix}t^{\alpha}}{2\Gamma[1+\alpha]} - \frac{e^{ix}t^{2\alpha}}{4\Gamma[1+2\alpha]} - \frac{ie^{ix}t^{3\alpha}}{8\Gamma[1+3\alpha]} \\ + \frac{e^{ix}t^{4\alpha}}{16\Gamma[1+4\alpha]} \end{aligned}$$

The exact solution is

$$u_n(x,t) = e^{ix}\sum_{n=0}^{\infty}\frac{\left(\frac{it^{\alpha}}{2}\right)^n}{\Gamma[1+n\alpha]} \tag{15}$$

The above result is in complete agreement with Ref [6]

Example 3: let us to consider Eq. (1) when the trapping potential and a real constants β, γ are taken into account subject the Subject to the initial condition $u(x,0) = f(x)$, by using Eq. (6), we could be able to calculate some of the terms of the series

$$u_1(x,t) = f_0 + f_1\frac{(it)^{\alpha}}{\Gamma(1+\alpha)}$$

$$u_2(x,t) = f_0 + f_1\frac{(it)^{\alpha}}{\Gamma(1+\alpha)} + f_2\frac{(it)^{2\alpha}}{\Gamma(1+2\alpha)}$$

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$$u_3(x,t) = f_0 + f_1 \frac{(it)^{\alpha}}{\Gamma(1+\alpha)} + f_2 \frac{(it)^{2\alpha}}{\Gamma(1+2\alpha)} + f_3 \frac{(it)^{3\alpha}}{\Gamma(1+3\alpha)}$$

Therefore,

$$u_3(x,t) = f_0 + f_1 \frac{(it)^{\alpha}}{\Gamma(1+\alpha)} + f_2 \frac{(it)^{2\alpha}}{\Gamma(1+2\alpha)} + f_3 \frac{(it)^{3\alpha}}{\Gamma(1+3\alpha)} + f_4 \frac{(it)^{4\alpha}}{\Gamma(1+4\alpha)}$$

where

$$f_1 = \beta f^{''} + fV + \gamma |f|^2 f$$

$$f_2 = \beta^2 f^{(4)} + fV^2 + 2\beta f^{'} V^{'} + 2\beta V f^{''} + \beta f V^{''}$$

.. And so on. The exact solution is given by

$$u_n(x,t) = \sum_{k=0}^{\infty} f_k \frac{(it)^{k\alpha}}{\Gamma(1+k\alpha)} \qquad (16)$$

where

$$f_{k+1} = \beta f_k^{''} + V f_k + \gamma |f_k|^2 f_k \; k = 0, ...\infty$$

4 Conclusions

We used the new technique namely Laplace iterational method for finding the solution of the generalized fractional nonlinear Schr?dinger equation subject to some initial conditions. This technique is very powerful in finding solutions for various physical problems. Also, this method will be very useful for solving many Engineering problems, both analytically and numerically, because it's very fast convergence to the solution.

References

[1] V. E. Zakharov and A. B. Shabat, Sov. Phys. JETP **34** 62 (1972).
[2] V. E. Zakharov, J. Appl. Mech. Tech. Phys. **344** 190 (1968).
[3] Ginzburg VL, Landau LD, Sov Phys JETP **20** 1064 (1950).
[4] Ginzburg VL, Sov Phys JETP **2** 589 (1956).
[5] Ginzburg VL, Pitaevskii LP, Sov Phys JETP **7** 858 (1958).
[6] S.Z. Rida, H.M. El-Sherbiny, A.A.M. Arafa, Phys. Lett. A **372** 553 (2008).
[7] G.M. Mittag-Leffler, Rend. Accad. Lincei **13** 3 (1904).
[8] A. S. Arife; world. Appl. Sci.J. **12** (2011), 2274-2278.
[9] S. J. Liao;. Commun. Non linear; Sci. Numer. Simulat **15** (2010), 2003-2016.
[10] Ahmed Salah and S. S. A. Hassan; world Appl. Sci. J. **21** (2013), 659-664.
[11] Ahmed Salah, Majid Khan and Muhammad Asif Gondol, Neural Coput. Appl. **23** (2013), 269-271.
[12] Muhammad Asif Gondol, Ahmed Salah, Majid Khan and Syeda Iram Battol;; Neural Coput. Appl. (2013).
[13] N. H. Abed-wahab and Ahmed Salah, Eur. Phys. J. Plus. **130** (2015), 1-11.

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