

Chain Ratio Type Estimators for Finite Population Mean In Double Sampling for Stratification

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Abstract: In this paper chain ratio type estimator has been proposed in double sampling for stratification with their properties. The bias and mean squared error of the proposed estimator is obtained up to the first degree of approximation. The proposed estimator has been compared with usual unbiased estimator and Ige and Tripathi [3] estimators. In the line of Srivenkataramana and Tracy [15,16] transformation, the improved version of the proposed estimator is also obtained with their properties. Asymptotic optimum estimator (AOE) is also identified. In support of theoretical findings, a numerical illustration is also given.

Keywords: Double sampling for stratification; Bias; Mean squared error

1 Introduction

Use of auxiliary information is often used by researchers in order to improve the efficiencies of estimators in sample surveys. The problem of estimating population parameters have been studied by many researchers including Tailor and Lone[18,19], Lone and Tailor [6,7], Lone and Tailor [4], Lone et al. [5,8], Singh et al. [13] Tailor et al. [20] and others. Double sampling for stratification is good alternative of stratified random sampling when strata weights are not known. The problem of estimating finite population mean in double sampling for stratification has been studied by few researchers including Ige and Tripathi [3],Tripathi and Bahl [21], Singh and Vishwakarma [14],Chouhan [2] ,Sharma [11], Tailor and Lone [17] .Let us consider a finite population $U = \{U_1, U_2, U_3, \dots, U_N\}$ of size N in which strata weight $\frac{N_h}{N}, \{h = 1, 2, 3, \dots, L\}$ are unknown. In double sampling for stratification

(a) A first phase of sample S of size n' using simple random sampling without replacement is drawn and only auxiliary variate x is observed.

(b) the samples is stratified into L strata on the basis of observed variable x . Let n'_h denotes the number of units in h^{th} stratum ($h = 1, 2, 3, \dots, L$) such that $n' = \sum_{h=1}^L n'_h$. (c)

From each n'_h units, a sample of size $n_h = v_h n'_h$ is drawn where $0 < v_h < 1$, $\{h = 1, 2, 3, \dots, L\}$, is the predetermined probability of selecting a sample of size n_h from each strata of size n'_h and it constitutes a sample S' of size $n = \sum_{h=1}^L n_h$

. In S' both study variate y and auxiliary variate x are observed.

Let y be the study variate and x and z are the two auxiliary variate respectively. Let us define

$\bar{x}_{ds} = \sum_{h=1}^L w_h \bar{x}_h$: Unbiased estimator of population mean \bar{X} at second phase or double sampling mean of the auxiliary

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variate x ,

$\bar{y}_{ds} = \sum_{h=1}^L w_h \bar{y}_h$: Unbiased estimator of population mean \bar{Y} at second phase or double sampling mean of the study variate y ,

$\bar{z}_{ds} = \sum_{h=1}^L w_h \bar{z}_h$: Unbiased estimator of population mean \bar{Z} at second phase or double sampling mean of the auxiliary variate z ,

$\bar{x}_h = \frac{1}{n_h} \sum_{i=1}^{n_h} x_{hi}$: Mean of the second phase sample taken from h^{th} stratum for the auxiliary variate x ,

$\bar{y}_h = \frac{1}{n_h} \sum_{i=1}^{n_h} y_{hi}$: Mean of the second phase sample taken from h^{th} stratum for the study variate y ,

$\bar{z}_h = \frac{1}{n_h} \sum_{i=1}^{n_h} z_{hi}$: Mean of the second phase sample taken from h^{th} stratum for the auxiliary variate z ,

$\bar{X} = \frac{1}{N} \sum_{h=1}^L \sum_{i=1}^{N_h} x_{hi}$: Population mean of the auxiliary variate x ,

$\bar{Y} = \frac{1}{N} \sum_{h=1}^L \sum_{i=1}^{N_h} y_{hi}$: Population mean of the study variate y ,

$\bar{Z} = \frac{1}{N} \sum_{h=1}^L \sum_{i=1}^{N_h} z_{hi}$: Population mean of the auxiliary variate z ,

$\bar{X}_h = \frac{1}{N_h} \sum_{i=1}^{N_h} x_{hi}$: h^{th} Stratum mean for the auxiliary variate x ,

$\bar{Y}_h = \frac{1}{N_h} \sum_{i=1}^{N_h} y_{hi}$: h^{th} Stratum mean for the study variate y ,

$\bar{Z}_h = \frac{1}{N_h} \sum_{i=1}^{N_h} z_{hi}$: h^{th} Stratum mean for the auxiliary variate z ,

$S_x^2 = \frac{1}{N-1} \sum_{h=1}^L \sum_{i=1}^{N_h} (x_{hi} - \bar{X}_h)^2$: Population mean square of the auxiliary variate x ,

$S_y^2 = \frac{1}{N-1} \sum_{h=1}^L \sum_{i=1}^{N_h} (y_{hi} - \bar{Y}_h)^2$: Population mean square of the study variate y ,

$S_z^2 = \frac{1}{N-1} \sum_{h=1}^L \sum_{i=1}^{N_h} (z_{hi} - \bar{Z}_h)^2$: Population mean square of the auxiliary variate z ,

$$S_{xh}^2 = \frac{1}{N_h - 1} \sum_{i=1}^{N_h} (x_{hi} - \bar{X}_h)^2 : h^{th} \text{ Stratum population mean square of the auxiliary variate } x ,$$

$$S_{yh}^2 = \frac{1}{N_h - 1} \sum_{i=1}^{N_h} (y_{hi} - \bar{Y}_h)^2 : h^{th} \text{ Stratum population mean of the study variate } y ,$$

$$S_{zh}^2 = \frac{1}{N_h - 1} \sum_{i=1}^{N_h} (z_{hi} - \bar{Z}_h)^2 : h^{th} \text{ Stratum population mean square of the auxiliary variate } z ,$$

$$\rho_{yxh} = \frac{S_{yxh}}{S_{yh} S_{xh}} : \text{Correlation coefficient between } y \text{ and } x \text{ in the stratum } h ,$$

$$\bar{x}'_h = \frac{1}{n'_h} \sum_{h=1}^{n_h} \bar{x}_{hi} : \text{First phase sample mean of the } h^{th} \text{ stratum for the auxiliary variety } x ,$$

$$\bar{z}'_h = \frac{1}{n'_h} \sum_{h=1}^{n_h} \bar{z}_{hi} : \text{First phase sample mean of the } h^{th} \text{ stratum for the auxiliary variety } z ,$$

$$f = \frac{n'}{N} : \text{First phase sampling fraction.}$$

$$n = \sum_{h=1}^L n_h : \text{Size of the sample } S'$$

$$w'_h = \frac{n'_h}{n'} : h^{th} \text{ Stratum weight in the first phase sample,}$$

$$\bar{x}' = \sum_{h=1}^{n_h} w_h \bar{x}'_h : \text{Unbiased estimator of population mean } \bar{X} ,$$

$$\bar{z}' = \sum_{h=1}^{n_h} w_h \bar{z}'_h : \text{Unbiased estimator of population mean } \bar{Z} .$$

Ige and Tripathi [3] defined classical ratio and product estimators in double sampling for stratification as

$$\hat{Y}_{Rd} = \bar{y}_{ds} \left(\frac{\bar{x}'}{\bar{x}_{ds}} \right) \tag{1.1}$$

$$\hat{Y}_{Pd} = \bar{y}_{ds} \left(\frac{\bar{z}_{ds}}{\bar{z}'} \right) . \tag{1.2}$$

Where z is an auxiliary variate which is negatively correlated with the study variate y and notations \bar{z}_{ds} and \bar{z}' have their usual meanings

The biases and mean squared errors of estimators \hat{Y}_{Rd} and \hat{Y}_{Pd} up to the first degree of approximation are obtained as

$$B\left(\hat{Y}_{Rd}\right) = \frac{1}{\bar{X}} \left[\sum_{h=1}^L \frac{W_h}{n'} \left(\frac{1}{v_h} - 1 \right) \left(R_1 S_{xh}^2 - S_{yjh} \right) \right], \quad (1.3)$$

$$B\left(\hat{Y}_{Pd}\right) = \frac{1}{\bar{Z}} \left[\sum_{h=1}^L \frac{W_h}{n'} \left(\frac{1}{v_h} - 1 \right) S_{yzh} \right], \quad (1.4)$$

$$MSE\left(\hat{Y}_{Rd}\right) = S_y^2 \left(\frac{1-f}{n'} \right) + \frac{1}{n'} \sum_{h=1}^L W_h \left(\frac{1}{v_h} - 1 \right) \left[S_{yh}^2 + R_1^2 S_{xh}^2 - 2R_1 S_{yjh} \right] \quad (1.5)$$

And

$$MSE\left(\hat{Y}_{Pd}\right) = S_y^2 \left(\frac{1-f}{n'} \right) + \frac{1}{n'} \sum_{h=1}^L W_h \left(\frac{1}{v_h} - 1 \right) \left[S_{yh}^2 + R_2^2 S_{zh}^2 + 2R_2 S_{yzh} \right]. \quad (1.6)$$

2 The Proposed Chain Ratio-Type estimator

Chand [1] defined a chain ratio-type estimator for \bar{Y} in double sampling as

$$\hat{Y}_{Rd}^{(C)} = \bar{y} \left(\frac{\bar{x}'}{\bar{x}} \right) \left(\frac{\bar{z}}{\bar{z}'} \right). \quad (2.1)$$

Motivated by Chand [1] we propose chain ratio type estimator in double sampling for stratification as

$$\hat{Y}_{Rd}^S = \bar{y}_{ds} \left(\frac{\bar{x}'}{\bar{x}_{ds}} \right) \left(\frac{\bar{z}}{\bar{z}'} \right). \quad (2.2)$$

Where \bar{y}_{ds} and \bar{x}_{ds} are unbiased estimators of population mean \bar{Y} and \bar{X} respectively.

To obtain the bias and mean squared error of the proposed estimator \hat{Y}_{Rd}^S , we write

$$\bar{y}_{ds} = \bar{Y} (1 + e_o), \quad \bar{x}_{ds} = \bar{X} (1 + e_1), \quad \bar{x}' = \bar{X} (1 + e'_1) \quad \text{and} \quad \bar{z}' = \bar{Z} (1 + e'_2)$$

Such that $E(e_o) = E(e_1) = 0 = E(e'_1) = 0$ and

$$E(e_o^2) = \frac{1}{\bar{Y}^2} \left[S_y^2 \left(\frac{1-f}{n'} \right) + \frac{1}{n'} \sum_{h=1}^L W_h S_{yh}^2 \left(\frac{1}{v_h} - 1 \right) \right],$$

$$E(e_1^2) = \frac{1}{\bar{X}^2} \left[S_x^2 \left(\frac{1-f}{n'} \right) + \frac{1}{n'} \sum_{h=1}^L W_h S_{xh}^2 \left(\frac{1}{v_h} - 1 \right) \right],$$

$$E(e'_1{}^2) = \frac{1}{\bar{X}^2} S_x^2 \left(\frac{1-f}{n'} \right),$$

$$E(e_o e_1) = \frac{1}{\bar{Y} \bar{X}} \left[\left(\frac{1-f}{n'} \right) S_{yx} + \frac{1}{n'} \sum_{h=1}^L W_h S_{yjh} \left(\frac{1}{v_h} - 1 \right) \right],$$

$$E(e_0 e_1') = \frac{1}{\bar{Y} \bar{X}} \left(\frac{1-f}{n'} \right) S_{yx} \quad \text{And}$$

$$E(e_1 e_1') = \frac{1}{\bar{X}^2} \left(\frac{1-f}{n'} \right) S_x^2.$$

Adopting the usual procedure for finding the bias and mean squared error, the bias and mean squared error of the proposed estimator \hat{Y}_{Rd}^S up to the first degree of approximation are obtained as

$$B(\hat{Y}_{Rd}^S) = \frac{1}{\bar{Z}} \left(\frac{1-f}{n'} \right) (R_2 S_z^2 - S_{yz}) + \frac{1}{\bar{X}} \frac{1}{n'} \sum_{h=1}^L W_h \left(\frac{1}{v_h} - 1 \right) (R_1 S_{xh}^2 - S_{yxh}) \tag{2.3}$$

And

$$MSE(\hat{Y}_{Rd}^S) = S_y^2 \left(\frac{1-f}{n'} \right) + \frac{1}{n'} \sum_{h=1}^L W_h \left(\frac{1}{v_h} - 1 \right) (S_{yh}^2 + R_1^2 S_{xh}^2 - 2R_1 S_{yxh}) + \left(\frac{1-f}{n'} \right) (R_2^2 S_z^2 - 2R_2 S_{yz}). \tag{2.4}$$

3 Efficiency comparisons

Variance of usual unbiased estimator \bar{y}_{ds} in double sampling for stratification is given as

$$V(\bar{y}_{ds}) = S_y^2 \left(\frac{1-f}{n'} \right) + \frac{1}{n'} \sum_{h=1}^L W_h S_{yh}^2 \left(\frac{1}{v_h} - 1 \right). \tag{3.1}$$

From (1.5), (1.6), (2.4) and (3.1) it is observed that the proposed estimator \hat{Y}_{Rd}^S would be more efficient than

(i) Usual unbiased estimator \bar{y}_{ds} if

$$\rho_{yz} > \frac{1}{2} \left(\frac{C_z}{C_y} + \psi \right), \tag{3.2}$$

(ii) Ige and Tripathi [3] ratio estimator \hat{Y}_{Rd} if

$$\rho_{yz} > \frac{1}{2} \left(\frac{C_z}{C_y} \right), \tag{3.3}$$

(ii) Ige and Tripathi [3] product estimator \hat{Y}_{Pd} if

$$\rho_{yz} > \frac{1}{2} \left(\frac{C_z}{C_y} + \phi \right). \tag{3.4}$$

Let us define

$$M = \sum_{h=1}^L W_h \left(\frac{1}{v_h} - 1 \right) S_{xh}^2, \quad B = \sum_{h=1}^L W_h \left(\frac{1}{v_h} - 1 \right) S_{yxh}, \quad C = \sum_{h=1}^L W_h \left(\frac{1}{v_h} - 1 \right) S_{zh}^2, \quad \beta = 1 - f$$

$$D = \sum_{h=1}^L W_h \left(\frac{1}{v_h} - 1 \right) S_{yzh}$$

Where $\alpha = M R_1^2 - 2 R_1 B$, $\eta = \alpha - (C R_2^2 + 2 D R_2)$, $\psi = \frac{\alpha}{\beta R_2 S_y S_z}$, $\phi = \frac{\eta}{\beta R_2 S_y S_z}$, $C^* = \rho_{yz} \frac{C_y}{C_z}$ and $C = \rho_{yx} \frac{C_y}{C_x}$ have their usual meaning.

4 An Improved Chain Ratio-Type estimator

Using the transformation $U_i = A - Z_i, i = 1, 2, 3, \dots, N$, (A being suitably chosen scalar) proposed by Srivenkataramana and Tracy [15,16] Prasad et al. [10] defined the alternative chain ratio-type estimator for \bar{Y} in double sampling as

$$\hat{Y}_{Rd}^{SB} = \bar{y} \left(\frac{\bar{x}'}{\bar{x}} \right) \left(\frac{\bar{u}'}{\bar{U}} \right) \quad (4.1)$$

1)

Using the same transformation adopted by Srivenkataramana and Tracy [15,16] alternative chain ratio-type estimator for \bar{Y} in double sampling for stratification is proposed as

$$\hat{Y}_{Rd}^{(L)} = \bar{y}_{ds} \left(\frac{\bar{x}'}{\bar{x}_{ds}} \right) \left(\frac{\bar{u}'}{\bar{U}} \right) \quad (4.2)$$

2)

where $\bar{u}' = A - \bar{z}'$ such that $E(\bar{u}') = \bar{U} = A - \bar{Z}$

Now, expressing $\hat{Y}_{Rd}^{(L)}$ in terms of e's we have

$$\hat{Y}_{Rd}^{(L)} = \bar{Y} (1 + e_o) (1 + e_1) (1 + e_1)^{-1} (1 - \theta e_2) \quad (4.3)$$

Where $\theta = \bar{Z} / (A - \bar{Z})$

Adopting the usual procedure for finding the bias and mean squared error, the bias and mean squared errors of the proposed estimator $\hat{Y}_{Rd}^{(L)}$ up to the first degree of approximation are obtained as

$$B \left(\hat{Y}_{Rd}^{(L)} \right) = \bar{Y} \left[\frac{1}{n'} \sum_{h=1}^L W_h \left(\frac{1}{v_h} - 1 \right) C_{xh}^2 (1 - C) - \theta \left(\frac{1-f}{n'} \right) C_z^2 C^* \right], \quad (4.4)$$

$$MSE \left(\hat{Y}_{Rd}^{(L)} \right) = S_y^2 \left(\frac{1-f}{n'} \right) + \frac{1}{n'} \sum_{h=1}^L W_h \left(\frac{1}{v_h} - 1 \right) \left(S_{yh}^2 + R_1^2 S_{xh}^2 - 2 R_1 S_{y x h} \right) + \left(\frac{1-f}{n'} \right) \theta \left(R_2^2 \theta S_z^2 - 2 R_2 S_{yz} \right)$$

(4.5)

$$MSE \left(\hat{Y}_{Rd}^{(L)} \right) = MSE \left(\hat{Y}_{Rd} \right) + \left(\frac{1-f}{n'} \right) \theta C_z^2 \bar{Y}^2 (\theta - 2C^*) \quad (4.6)$$

which is minimized for

$$\theta = C^* = \theta_0 \text{ (say)} \tag{4.7}$$

$$\Rightarrow A = \left(\frac{1+C^*}{C^*} \right) \bar{Z} = A_0 \text{ (say)} \tag{4.8}$$

Using the value of (4.8) in (4.6), we get the minimum MSE of $\hat{Y}_{Rd}^{(L)}$ as

$$\min .MSE\left(\hat{Y}_{Rd}^{(L)}\right) = MSE\left(\hat{Y}_{Rd}\right) - \left(\frac{1-f}{n'}\right) C_z^2 \bar{Y}^2 C^{*2} \tag{4.9}$$

Substitution of (4.8) in (4.2) yields the asymptotic optimum estimator (AOE) of \bar{Y} as

$$\hat{Y}_{Rd}^{(L_0)} = \bar{y}_{ds} \left(\frac{\bar{x}'}{\bar{x}_{ds}} \right) \left[\frac{\bar{Z} + C^* (\bar{Z} - \bar{z}')}{\bar{Z}} \right] \tag{4.10}$$

with same mean square error as given in (4.9)

5 Estimator Based on Estimated optimum

It is to be mentioned that the estimator $\hat{Y}_{Rd}^{(L_0)}$ in (4.10) require the prior knowledge of C^* . In practical sample surveys ,a prior value of C^* can be guessed easily by utilizing appropriate information from the most recent survey taken in the past.

Further if the investigator is unable to guess the value of C^* , the only alternative left to him is to replace C^* in (4.10) by its consistent estimator \hat{C}^* computed from the data at hand. Thus the estimator based on estimated optimum is

$$\hat{Y}_{Rd}^{(\hat{L}_0)} = \bar{y}_{ds} \left(\frac{\bar{x}'}{\bar{x}_{ds}} \right) \left[\frac{\bar{Z} + \hat{C}^* (\bar{Z} - \bar{z}')}{\bar{Z}} \right] \tag{5.1}$$

Where $\hat{C}^* = \left(\frac{\bar{z}}{\bar{y}} \right) b_{yz}$, b_{yz} is the estimate of regression coefficient of y on z

Theorem: - The estimator $\hat{Y}_{Rd}^{(\hat{L}_0)}$ based on estimated optimum has the same MSE to the first degree of approximation as that of AOE $\hat{Y}_{Rd}^{(L_0)}$

Proof: To obtain the MSE of $\hat{Y}_{Rd}^{(\hat{L}_0)}$, we write $\hat{C}^* = C^* (1 + e_3)$

Now, expressing $\hat{Y}_{Rd}^{(\hat{L}_0)}$ in terms of e's we have

$$\hat{Y}_{Rd}^{(\hat{L}_0)} = \bar{Y} (1 + e_o) (1 + e_1)^{-1} (1 + e_1)' \left\{ 1 - C^* (1 + e_3) e_2' \right\} \tag{5.2}$$

To the first degree of approximation the mean squared error of $\hat{Y}_{Rd}^{(\hat{L}_0)}$ upto the first degree of approximation is obtained as

$$MSE\left(\hat{Y}_{Rd}^{(\hat{L}_0)}\right) = MSE(\bar{y}_{Rd}) - \left(\frac{1-f}{n'}\right) C_z^2 \bar{Y}^2 C^{*2} \tag{5.3}$$

Which is same as given in (4.9)

Hence proved the theorem.

6 Efficiency Comparison of $\hat{Y}_{Rd}^{(L)}$

Comparisons of (3.1), (1.5), (1.6), (2.4) and (4.6) shows that the proposed estimator $\hat{Y}_{Rd}^{(L)}$ would be better than

(i) \bar{y}_{ds} if

$$\frac{\beta S_{yz} - \sqrt{\beta^2 S_{yz}^2 - \beta \alpha S_z^2}}{\beta R_2 S_z^2} < \theta < \frac{\beta S_{yz} + \sqrt{\beta^2 S_{yz}^2 - \beta \alpha S_z^2}}{\beta R_2 S_z^2} \quad (6.1)$$

(ii) \hat{Y}_{Rd} if

$$\left. \begin{array}{l} \text{either } 0 < \theta < 2C^* \\ \text{or } 2C^* < \theta < 0 \end{array} \right\} \quad (6.2)$$

(iii) \hat{Y}_{Pd} if

$$\frac{\beta S_{yz} - \sqrt{\beta^2 S_{yz}^2 - \beta \eta S_z^2}}{\beta R_2 S_z^2} < \theta < \frac{\beta S_{yz} + \sqrt{\beta^2 S_{yz}^2 - \beta \eta S_z^2}}{\beta R_2 S_z^2} \quad (6.3)$$

(iv) \hat{Y}_{Rd}^S If

$$\left. \begin{array}{l} \text{either } 2C^* - 1 < \theta < 1 \\ \text{or } 1 < \theta < 2C^* - 1 \end{array} \right\} \quad (6.4)$$

Expressions (6.1) to (6.4) are conditions under which improved proposed estimator $\hat{Y}_{Rd}^{(L)}$ is more efficient than usual unbiased estimator \bar{y}_{ds} , Ige and Tripathi estimators \hat{Y}_{Rd} and \hat{Y}_{Pd} and \hat{Y}_{Rd}^S .

7 Empirical study

To exhibit the performance of the proposed estimator in comparison to other estimator, two population data sets are being considered. The description of population is given below.

Population I [Source: Singh and Choudhary [12], P.177] y : Production (MT/hectare),

x : Production in '000Tons and z Area in '000hectare

$N_{=20}$	$n_{=8}$	$n_{1=4}$	$n_{2=4}$
$n'_{1=6}$	$n'_{2=6}$	$N_{1=10}$	$N_{2=10}$
$\bar{Y}_{1=264}$	$\bar{Y}_{2=214.7}$	$\bar{X}_{1=939}$	$\bar{X}_{2=1121.5}$
$\bar{Z}_{1=263.2}$	$\bar{Z}_{2=202.9}$	$S_{y1}=149.53$	$S_{y2}=192.02$
$S_{x1}=389.67$	$S_{x2}=1165.2$	$S_{z1}=162.85$	$S_{z2}=178.54$
$S_{yx1}=53277$	$S_{yx2}=68650$	$S_{yz1}=23798$	$S_{yz2}=33841$

$S_{xz1} = 58729$	$S_{xz2} = 60376$	$S_y^2 = 31814.87$	$S_z^2 = 31692.05$
$S_{yz} = 29562.58$	$\rho_{yz} = 0.980006$	$C_y = 1.376755$	$C_z = 1.343112$

Population II [source: Murthy [9], p.228]

y : Output, x : Fixed capital and x : Number of workers

$N = 10$	$n = 4$	$n_1 = 2$	$n_2 = 2$
$n'_1 = 4$	$n'_2 = 4$	$N_1 = 5$	$N_2 = 5$
$\bar{Y}_1 = 1925.8$	$\bar{Y}_2 = 3115.6$	$\bar{X}_1 = 214.4$	$\bar{X}_2 = 333.8$
$\bar{Z}_1 = 51.80$	$\bar{Z}_2 = 60.60$	$S_{y1} = 615.92$	$S_{y2} = 340.38$
$S_{x1} = 74.87$	$S_{x2} = 66.35$	$S_{z1} = 0.75$	$S_{z2} = 4.84$
$S_{yx1} = 39361$	$S_{yx2} = 22357$	$S_{yz1} = 411.16$	$S_{yz2} = 1536.24$
$S_{xz1} = 38.08$	$S_{xz2} = 287.92$	$S_y^2 = 668351.00$	$S_z^2 = 34.84$

Table 1: Empirical Study of Theoretical Conditions Obtained in Ssection-6

Theoretical Conditions Obtained in Section 6		
Efficiency Comparisons	Population I	Population II
$MSE(\hat{Y}_{Rd}^{(L)}) < Var(\bar{y}_{ds})$	$\theta \in (0.3075, 1.5089)$	$\theta \in (-1.649, 6.2455)$
$MSE(\hat{Y}_{Rd}^{(L)}) < MSE(\hat{Y}_{Rd})$	$\theta \in (0, 2.009)$	$\theta \in (0, 5.1064)$
$MSE(\hat{Y}_{Rd}^{(L)}) < MSE(\hat{Y}_{Pd})$	$\theta \in (-1.9302, 3.7467)$	$\theta \in (-9.9612, 14.5576)$
$MSE(\hat{Y}_{Rd}^{(L)}) < MSE(\hat{Y}_{Rd}^S)$	$\theta \in (1.00, 1.0091)$	$\theta \in (1.00, 4.1064)$

Table 2: Percent relative Efficiency of \bar{y}_{ds} , \hat{Y}_{Rd} , \hat{Y}_{Pd} , \hat{Y}_{Rd}^S and $\hat{Y}_{Rd}^{(L_0)}$ with respect to \bar{y}_{ds}

Estimators	Percent Relative Efficiencies (PRE's)	
	Population I	Population II
s_y^2	100.00	100.00
\hat{Y}_{Rd}	81.60	159.42
\hat{Y}_{Pd}	37.96	77.20
\hat{Y}_{Rd}^S	120.64	202.31
$\hat{Y}_{Rd}^{(L_0)}$	135.99	260.59

8 Conclusion

Table 2 reveals that the proposed estimators \hat{Y}_{Rd}^S and $\hat{Y}_{Rd}^{(L)}$ have maximum percent relative efficiencies in comparison to other considered estimators. Section 3 provides the conditions under which the proposed estimator \hat{Y}_{Rd}^S has less mean squared error in comparisons to simple mean estimator and Ige and Tripathi [3] estimators. Also section 6 provides the conditions under which the improved proposed estimator $\hat{Y}_{Rd}^{(L)}$ would be more efficient than \bar{y}_{ds} , \hat{Y}_{Rd} , \hat{Y}_{Pd} and \hat{Y}_{Rd}^S . Table 1 establishes those conditions empirically. Thus the proposed estimators are recommended for use in practice for estimating the population mean provided conditions given in section 3 and 6 are satisfied.

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