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Uni-Soft Bi-Ideals and Uni-Soft Interior Ideals of AG-groupoids

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Abstract: In this paper we introduce uni-soft subAG-groupoid, uni-soft bi-ideals and uni-soft interior ideals of AG-groupoids. We investigate some interesting characterizations of uni-soft bi-ideals and uni-soft interior ideals of AG-groupoids. We also provide some related results to uni-soft quasi ideals of AG-groupoids.

Keywords: Uni-soft AG-groupoid, Uni-soft subAG-groupoid, Uni-soft bi-ideal, Uni-soft interior ideal, Uni-soft quasi ideal

1 Introduction

In 1999, Molodtsov [16] initiated the theory of soft sets as a new mathematical tool for dealing with uncertainties. He applied this particular theory to many directions like; smoothness of function, Perron and Riemann integrations, game theory, measurement theory, probability theory etc. After Molodtsov [16], Maji et al. [17] presented the theoretical study of soft set theory. They defined several operations in soft set theory and also verified De Morgan's laws for the soft sets. Yang [32] and Ali et al. [4] pointed out some errors in the work of Maji et al. [17]. Therefore Ali et al. [4] defined some new operations in soft set theory to improve some new results. In similar way Cagman et al. [5] also redefined some operations in soft set theory and designed a uni-int (union-intersection) soft decision making method which was used in the solutions of many complicated problems. Further, Jun [10] initiated the idea of soft BCK/BCI-algebras and soft sub-algebras. Also, Jun et al. [12] extended the idea of soft set theory and discussed some more concepts like; soft Hilbert algebra, soft Hilbert deductive algebra and soft Hilbert abysmal algebra and studied their related properties.

The theoretical structures of soft set theory have been studied increasingly in recent years. Aktas and Cagman [3] were the first, who presented the idea of soft groups and investigated their related properties. Also, Feng et al. [8] introduced the idea of soft semiring and discussed some related properties to soft smearings. Similarly, many other authors (Acar et al. [1], Shabir et al. [29], Ali et al. [30] etc.) discussed some theoretical structures of soft set theory and played a significant role in this area.

In 2011, Shah et al. [31] injected the idea of soft set theory in the field of AG-groupoids and initiated the idea of soft ordered AG-groupoids, soft ordered subAG-groupoid and soft ideals of soft ordered AG-groupoids. A groupoid S is called an Abel-Grassman's groupoid (AG-groupoid) if the identity: $(ab)c = (cb)a$ holds for all $a,b,c \in S$ [24]. The algebraic structure of AG-gropupoid was first introduced by Kazim and Nasseruddin [14] in 1972. They called it left almost semigroup (or simply LA-semigroup). Some other names have also been used in literature for AG-groupoid. For example, Cho et al. [7] studied this structure under the name of right modular groupoid, Holgate [9] studied it as left invertive groupoid and similarly Stevanovic and Protic called this structure an Abel-Grassmann groupoid (or simply AG-groupoid), which is the primary name under which this structure is known nowadays. Throughout this study we use the term AG-groupoid. Generally an AG-groupoid is a non associative algebraic structure but it has a closed link with semigroups. It provides the generalization of commutative semigroups. There are many important applications of AG-groupoids in the theory of flocks [22].

Recently, In 2013 Kim et al. [13] initiated the notions of uni-soft semigroups, uni-soft left(right) ideals and uni-soft quasi ideals of semigroups. In relation to the work of Kim et al. [13], the concept of uni-soft

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AG-groupoids, uni-soft left(right) ideals and uni-soft quasi ideals of AG-groupoids have been introduced in [26]. The main purpose of this research paper is to extend the concept of uni-soft ideals initiated in [26], by introducing some new ideals namely; uni-soft bi-ideals and uni-soft interior ideals of AG-groupoids and also discuss some related results.

2 Preliminaries

A groupoid $(S,.)$ is called an AG-groupoid if it satisfy the left invertive law: $(ab)c = (cb)a, \forall a,b,c \in S$. A non-empty subset H of an AG-groupoid S is called a subAG-groupoid if $ab \in H$ for all $a,b \in H$. An AG-groupoid may or may not contain left identity and if it contains a left identity then it is unique. An AG-groupoid with right identity becomes commutayive semigroup. In an AG-groupoid S, the medial law: $(ab)(cd) = (ac)(bd)$ holds $\forall a,b,c,d \in S$ [7]. In an AG-groupoid S with left identity, the paramedial law: $(ab)(cd) = (db)(ca)$ holds $\forall a,b,c,d \in S$ [7]. An AG-groupoid S is called weak associative or AG^*-groupoid if it satisfies the identity $(ab)c = b(ac)$, for all $a,b,c \in S$ [21]. Also an AG-groupoid is called AG^{**}-groupoid if $a(bc) = b(ac)$ for all $a,b,c \in S$ [23]. An AG-groupoid S is called locally associative if it satisfies $(aa)a = a(aa)$ for all $a \in S$. An AG-groupoid S is called an AG-group if there exist a left identity $e \in S$ (i.e $ea = a$, $\forall a \in S$), for all $a \in S$ there exist $a^{-1} \in S$ such that $a^{-1}a = aa^{-1} = e$ [15]. An element $a \in S$ of an AG-groupoid S is called idempotent if $a^2 = a$ and S is called AG-2-band or simply AG-band if all the elements of S are idempotent [27]. Also an AG-groupoid S is called AG-3-band, if $(aa)a = a(aa) = a$ for all $a \in S$ [27]. An element $a \in S$ is called regular, if there exist some $x \in S$ such that $a = (ax)a, \forall x \in S$. An AG-groupoid S is called regular if every element of S is regular. Similarly some new subclasses of AG-groupoids such as: $T^1, T^2, T^3_l, T^3_r, T^3, T^4_f, T^4_b, T^4$, left alternative, right alternative, alternative, and transitively commutative AG-groupoigs were also defined in [28].

A non-empty subset A of an AG-groupoid S is called a left(right) ideal of S if $SA \subseteq A(AS \subseteq A)$. A nonempty subset A of S is called an ideal(or a two sided ideal) of S if it is both left and right ideal of S. A non-empty subset Q of an AG-groupoid S is called a quasi ideal of S if $SQ \cap QS \subseteq Q$. It is easy to see that every one sided ideal of S is a quasi ideal of S. A non-empty subset B of an AG-groupoid S is called a generalized bi-ideal of S, if $(BS)B \subseteq B$. A subAG-groupoid B of an AG-groupoid S is called a bi-ideal of S, if $(BS)B \subseteq B$. A non-empty subset A of an AG-groupoid S is called an interior ideal of S if, $(SA)S \subseteq A$. If S is an AG-groupoid and A and B are any two subset of S, then the multiplication of A and B is defined by

$$AB = \{ab \in S | a \in A \text{ and } b \in B\}.$$

Definition 2.1.[16]. Let U be an initial universe and E be a set of parameters. Let A be a non-empty subset of E and $P(U)$ denotes the power set of U. A fair (F,A) is called a soft set over U, where F is a mapping given by $F : A \to P(U)$. In other words we can say that a soft set is a parameterized family of subset of the universe set U. The function F is called approximate function of the soft set (F,A).

Definition 2.2.[13]. Let (F,S) and (G,S) be any two soft sets over a common universe U, then (F,S) is called the soft subset of (G,S) denoted by $(F,S) \subseteq (G,S)$, if $F(x) \subseteq G(x), \forall x \in S$.
Two soft sets (F,S) and (G,S) are said to be soft equal if $F(x) \subseteq G(x)$ and $F(x) \supseteq G(x)$.

Definition 2.3.[13]. Let (F,S) and (G,S) be two soft sets over a common universe U, then the soft union of (F,S) and (G,S) is given by the soft set $(F \cup G,S)$, where $F \cup G$ is define by

$$(F \cup G)(x) = F(x) \cup G(x), \forall x \in S.$$

Definition 2.4.[13]. Let (F,S) and (G,S) be two soft sets over a common universe U, then the soft intersection of (F,S) and (G,S) is given by the soft set $(F \cap G,S)$, where $F \cap G$ is define by

$$(F \cap G)(x) = F(x) \cap G(x), \forall x \in S.$$

Definition 2.5.[13] Let (F,A) be a soft set over a universe U and u be any subset of U, then the u-exclusive set of (F,A), denoted by $e_A(F;u)$ is defined to be the set:

$$e_A(F;u) = \{x \in A / F(x) \subseteq u\}.$$

Definition 2.6.[26] For a nonempty subset A of S, the soft set $(\mathbb{X}_A,S)$ is called the uni-characteristic soft set where $\mathbb{X}_A$ is defined by:

$$\mathbb{X}_A : S \longrightarrow P(U), x \longmapsto \begin{cases} U & \text{if } x \notin A, \\ \emptyset & \text{if } x \in A. \end{cases}$$

The soft set $(\mathbb{X}_S,S)$ is called the uni-empty soft set over U.

3 Uni-soft subAG-groupoid

In what follows, we take $E = S$, where E is the set of parameters and S is a non-associative AG-groupoid, unless otherwise specified. To define uni-soft subAG-groupoid, we recall the definition of uni-soft AG-groupoid as follows.

Definition 3.1.[26] Let (F,S) be a soft set over U, then (F,S) is called a uni-soft AG-groupoid over U, if for all $x,y \in S$ we have

$$F(xy) \subseteq F(x) \cup F(y).$$

Definition 3.2. Let (F,S) be a uni-soft AG-groupoid over U and (H,S) be a soft subset of (F,S) over U. Then we say that (H,S) is a uni-soft subAG-groupoid over U, if for all $x,y \in S$ we have

$$H(xy) \subseteq H(x) \cup H(y).$$

Example 3.1. Consider a universal set $U = \{a,b,c,d,e,f\}$ and an AG-groupoid $S = \{1,2,3,4,5\}$, whose Cayley table is given by:

$\cdot$	1	2	3	4	5
1	1	2	3	4	5
2	5	3	3	5	5
3	3	3	3	3	3
4	4	2	3	1	5
5	2	2	3	2	3

Since $(2.4).1 = 2 \neq 5 = 2.(4.1)$, implies that S is a non-associative AG-groupoid. Now define a soft set (F,S) over U by:

$$F : S \to P(U), x \mapsto \begin{cases} \{a,b\} & \text{if } x = 3, \\ \{a,b,d\} & \text{if } x = \{2,5\}, \\ \{a,b,d,f\} & \text{if } x = \{1,4\}. \end{cases}$$

Then we can easily check that (F,S) is a uni-soft AG-groupoid over U. Also, consider the soft set (H,S) over U defined by:

$$H : S \to P(U), x \mapsto \begin{cases} \{a\} & \text{if } x = 3, \\ \{a,d\} & \text{if } x = \{2,5\}, \\ \{a,d,f\} & \text{if } x = \{1,4\}. \end{cases}$$

Then we see that $(H,S) \subseteq (F,S)$ and for all $x,y \in S$, we have $H(xy) \subseteq H(x) \cup H(y)$. Hence (H,S) is a uni-soft subAG-groupoid of (F,S).

To prove some related results to uni-soft quasi ideals of AG-groupoids, first we recall the definition of uni-soft quasi ideal as given below.

Definition 3.3.[26]. A soft set (H,S) over U is called a uni-soft quasi ideal over U if

$$(H \circ \mathbb{X}_S, S) \cup (\mathbb{X}_S \circ H, S) \supseteq (H,S).$$

Theorem 3.1. For a nonempty subset A of S the following are equivalent:

(1) The uni-characteristic soft set $(\mathbb{X}_A, S)$ is a uni-soft quasi ideal over U.

(2) A is a quasi ideal of S.

Proof. Suppose that the uni-characteristic soft set $(\mathbb{X}_A, S)$ is a uni-soft quasi ideal over U and let $a \in AS \cap SA$. Then we can write $a = bx = yc$ for some $b,c \in A$ and $x,y \in S$. Since $(\mathbb{X}_A, S)$ is a uni-soft quasi ideal over U, so by Definition (3.3) we have

$$\mathbb{X}_A(a) \subseteq \big((\mathbb{X}_A \circ \mathbb{X}_S) \cup (\mathbb{X}_S \circ \mathbb{X}_A)\big)(a)$$

$$= (\mathbb{X}_A \circ \mathbb{X}_S)(a) \cup (\mathbb{X}_S \circ \mathbb{X}_A)(a)$$

$$= \Big(\bigcap_{a=uv} \{\mathbb{X}_A(u) \cup \mathbb{X}_S(v)\}\Big) \cup \Big(\bigcap_{a=uv} \{\mathbb{X}_S(u) \cup \mathbb{X}_A(v)\}\Big)$$

$$= \Big(\bigcap_{a=uv} \{\mathbb{X}_A(u)\}\Big) \cup \Big(\bigcap_{a=uv} \{\mathbb{X}_A(v)\}\Big) = \emptyset \cup \emptyset = \emptyset,$$

implies that $\mathbb{X}_A(a) = \emptyset$.
Therefore $a \in A$ and thus $AS \cap SA \subseteq A$. Hence A is a quasi ideal of S.

Conversely, let A be a quasi ideal of S and $a \in S$. If $a \in A$, then

$$\mathbb{X}_A(a) = \emptyset \subseteq \big((\mathbb{X}_A \circ \mathbb{X}_S) \cup (\mathbb{X}_S \circ \mathbb{X}_A)\big)(a).$$

Now if $a \notin A$, then $\mathbb{X}_A(a) = U$. Also let $\big((\mathbb{X}_A \circ \mathbb{X}_S) \cup (\mathbb{X}_S \circ \mathbb{X}_A)\big)(a) = \emptyset$, then $(\mathbb{X}_A \circ \mathbb{X}_S)(a) = \emptyset$ and $(\mathbb{X}_S \circ \mathbb{X}_A)(a) = \emptyset$. Therefore

$$(\mathbb{X}_A \circ \mathbb{X}_S)(a) = \bigcap_{a=xy} \{\mathbb{X}_A(x) \cup \mathbb{X}_S(y)\} = \emptyset$$

and

$$(\mathbb{X}_S \circ \mathbb{X}_A)(a) = \bigcap_{a=xy} \{\mathbb{X}_S(x) \cup \mathbb{X}_A(y)\} = \emptyset.$$

Which shows that there exist some $b,c,d,e \in S$ with $a = bc = de$ such that $\mathbb{X}_A(b) = \emptyset$ and $\mathbb{X}_A(e) = \emptyset$, which implies that $b,e \in A$. Thus $a = bc = de \in AS \cap SA$, but $AS \cap SA \subseteq A$ as A is a quasi ideal of S, this implies $a \in A$, which is a contradiction to the fact that $a \notin A$. Hence $(\mathbb{X}_A \circ \mathbb{X}_S, S) \cup (\mathbb{X}_S \circ \mathbb{X}_A, S) \supseteq (\mathbb{X}_A, S)$ and therefore $(\mathbb{X}_A, S)$ is a uni-soft quasi ideal over U.

Theorem 3.2. For an AG-groupoid S, the following are equivalent:

(1) S is regular.

(2) $(F,S) = (F \circ \mathbb{X}_S \circ F, S)$ for every uni-soft quasi ideal (F,S) over U.

Proof. Let S be a regular AG-groupoid and let (F,S) be a uni-soft quasi ideal over U, Then for any $a \in S$, there exist some $x \in S$ such that $a = (ax)a$. Now,

$$(F \circ \mathbb{X}_S \circ F)(a) = \bigcap_{a=yz} \{(F \circ \mathbb{X}_S)(y) \cup F(z)\}$$

$$\subseteq (F \circ \mathbb{X}_S)(ax) \cup F(a) = \bigcap_{ax=bc} \{(F(b) \cup \mathbb{X}_S)(c)\} \cup F(a)$$

$$= \bigcap_{ax=bc} \{(F(b)\} \cup F(a) = F(a),$$

shows that $(F \circ \mathbb{X}_S \circ F, S) \subseteq (F,S)$. Also since (F,S) is a uni-soft quasi ideal over U, so

$$(F,S) \subseteq (\mathbb{X}_S \circ F, s) \cup (F \circ \mathbb{X}_S, S) \subseteq (F \circ \mathbb{X}_S \circ F, S),$$

which shows that $(F,S) \subseteq (F \circ \mathbb{X}_S \circ F, S)$. Hence $(F,S) = (F \circ \mathbb{X}_S \circ F, S)$.

Conversely, let $(F,S) = (F \circ \mathbb{X}_S \circ F, S)$ holds and A be any quasi ideal of S, then $(AS)A \subseteq SA \cap AS \subseteq A$. Also by Theorem (3.1), $(\mathbb{X}_A, S)$ is a uni-soft quasi ideal over U. Now for any $a \in A$, with $a = xy$ we have,

$$\emptyset = \mathbb{X}_A(a) = (\mathbb{X}_A \circ \mathbb{X}_S \circ \mathbb{X}_A)(a) = \bigcap_{a=xy} \{(\mathbb{X}_A \circ \mathbb{X}_S)(x) \cup \mathbb{X}_A(y)\}$$

$$\Rightarrow \bigcap_{a=xy} \{(\mathbb{X}_A \circ \mathbb{X}_S)(x) \cup \mathbb{X}_A(y)\} = \emptyset,$$

shows that there exist some $c,d \in S$ such that $a = cd$, $(\mathbb{X}_A \circ \mathbb{X}_S)(c) = \emptyset$ and $\mathbb{X}_A(d) = \emptyset$. Also

$$\emptyset = (\mathbb{X}_A \circ \mathbb{X}_S)(c) = \bigcap_{c=mn} \{(\mathbb{X}_A(m) \cup \mathbb{X}_S)(n))\},$$

implies that there exist some $r,s \in S$ such that $c = rs$, $\mathbb{X}_A(r) = \emptyset$ and $\mathbb{X}_S(s) = \emptyset$. Thus $d,r \in A$ and $s \in S$ which follows that $a = cd = (rs)d \in (AS)A$. Therefore $A \subseteq (AS)A$ and thus $A = (AS)A$. Hence S is regular.

4 Uni-soft Bi-ideals and uni-soft interior ideals

Definition 4.1. Let (F,S) be a soft set over U, then (F,S) is called a uni-soft bi-ideal over U, if for all $x \in S$ we have

$$F\big((xy)z\big) \subseteq F(x) \cup F(z).$$

Definition 4.2. Let (F,S) be a soft set over U, then (F,S) is called a uni-soft interior ideal over U, if for all $x \in S$ we have

$$F\big((xy)z\big) \subseteq F(y).$$

It should be noted that every uni-soft bi-ideal and uni-soft interior ideal over U is a uni-soft AG-groupoid over U.

Example 4.1. Let $S = \{1,2,3,4\}$ be an AG-groupoid with following Cayley table:

·	1	2	3	4
1	2	2	4	4
2	2	2	2	2
3	1	2	3	4
4	1	2	1	2

Since $(1.3).1 = 1 \neq 2 = 1.(3.1)$, implies that S is a non-associative AG-groupoid. Now let (F,S) be a soft set over U defined by:

$$F : S \to P(U),\, x \mapsto \begin{cases} u_1 \text{ if x=2}, \\ u_2 \text{ if x=}\{1,4\}, \\ u_3 \text{ if x=3}, \end{cases}$$

where $u_1,u_2,u_3, \in P(U)$ with $u_1 \subset u_2 \subset u_3$. Then (F,S) is a uni-soft bi-ideal, uni-soft interior ideal and hence a uni-soft AG-groupoid over U.

In the following example we will see that not every uni-soft AG-groupoid over U is a uni-soft bi-ideal and a uni-soft interior ideal over U, respectively.

Example 4.2. Let $S = \{0,1,2,3,4\}$ be an AG-groupoid with following Cayley table:

·	0	1	2	3	4
0	0	1	2	3	4
1	4	0	1	2	3
2	3	4	0	1	2
3	2	3	4	0	1
4	1	2	3	4	0

Since $(1.3).0 = 3 \neq 1 = 1.(3.0)$, implies that S is a non-associative AG-groupoid. Now let (F,S) be a soft set over U defined by:

$$F : S \to P(U),\, x \mapsto \begin{cases} u_1 \text{ if x=0}, \\ u_2 \text{ if x=}\{1,2,3,4\}, \end{cases}$$

where $u_1,u_2 \in P(U)$ with $u_1 \subset u_2$. Then (F,S) is a uni-soft AG-groupoid over U but it is not a uni-soft bi-ideal and a uni-soft interior ideal over U. Since

$$F\big(0.4)0\big) = F(3) = u_2 \nsubseteq u_1 = F(0) = F(0) \cup F(0),$$

shows that (F,S) is not a uni-soft bi-ideal over U. Similarly,

$$F\big(2.0)1\big) = F(3) = u_2 \nsubseteq u_1 = F(0),$$

implies that (F,S) is not a uni-soft interior ideal over U.

Theorem 4.1. If (F,S) and (G,S) are any two uni-soft bi-ideals over U, then their soft union $(F \cup G,S)$ is also a uni-soft bi-ideal over U.

Proof. Since (F,S) and (G,S) are uni-soft bi-ideals over U, so we can write $F\big((xy)z\big) \subseteq F(x) \cup F(z)$ and $G\big((xy)z\big) \subseteq G(x) \cup G(z); \forall x,y,z \in S$. Now,

$$\begin{aligned}(F \cup G)\big((xy)z\big) &= F\big((xy)z\big) \cup G\big((xy)z\big) \\ &\subseteq \big(F(x) \cup F(z)\big) \cup \big(G(x) \cup G(z)\big) \\ &= \big(F(x) \cup G(x)\big) \cup \big(F(z) \cup G(z)\big) \\ &= (F \cup G)(x) \cup (F \cup G)(z),\end{aligned}$$

implies that $(F \cup G)\big((xy)z\big) \subseteq (F \cup G)(x) \cup (F \cup G)(z)$. Hence $(F \cup G,S)$ is a uni-soft bi-ideal over U.

Theorem 4.2. If (F,S) and (G,S) are two uni-soft interior ideals over U, then the soft union $(F \cup G,S)$ is also a uni-soft interior ideal over U.

Proof. Let (F,S) and (G,S) are any two uni-soft interior ideals over U, then can write $F\big((xy)z\big) \subseteq F(y)$ and $G\big((xy)z\big) \subseteq G(y); \forall x,y,z \in S$. Now,

$$\begin{aligned}(F \cup G)\big((xy)z\big) &= F\big((xy)z\big) \cup G\big((xy)z\big) \\ &\subseteq F(y) \cup G(y) = (F \cup G)(y),\end{aligned}$$

shows that $(F \cup G)\big((xy)z\big) \subseteq (F \cup G)(y)$. Hence $(F \cup G,S)$ is a uni-soft interior ideal over U.

Theorem 4.3. For any soft set (F,S) over U, the following are equivalent:

(1) (F,S) is a uni-soft bi-ideal over U.

(2) For all $u \in P(U)$ with $e_S(F;u) \neq \emptyset$, $e_S(F;u)$ is a bi-ideal of S.

Proof. Suppose (F,S) is a uni-soft bi-ideal over U and $u \subseteq U$ such that $e_S(F;u) \neq \emptyset$. Let $x,y,z \in S$ such that $x,z \in e_S(F;u)$, then by Definition (2.5) we have $F(x) \subseteq u$ and $F(z) \subseteq u$. Since (F,S) is a uni-soft bi-ideal over U, it follows that

$$F\big((xy)z\big) \subseteq F(x) \cup F(z) \subseteq u \cup u = u,$$

$$\Rightarrow F\big((xy)z\big) \subseteq u,$$

which shows that $(xy)z \in e_S(F;u)$. Thus $e_S(F;u)$ is a bi-ideal of S.

Conversely, suppose that $\forall\, u \in P(U)$ with $e_S(F;u) \neq \emptyset$, $e_S(F;u)$ is a bi-ideal of S. Also let $x,y,z \in S$ such that $F(x) \subseteq u^1$ and $F(z) \subseteq u^2$. Take $u = u^1 \cup u^2$, then $F(x) \subseteq u$ and $F(z) \subseteq u$, implies that $x,z \in e_S(F;u)$. But $e_S(F;u)$ is a bi-ideal of S which implies that $(xy)z \in e_S(F;u)$ for all $x,y,z \in S$. Now

$$F\big((xy)z\big) \subseteq u = u^1 \cup u^2 = F(x) \cup F(z).$$

Hence (F,S) is a uni-soft bi-ideal over U.

Theorem 4.4. For any soft set (F,S) over U, the following are equivalent:

(1) (F,S) is a uni-soft interior ideal over U.

(2) For all $u \in P(U)$ with $e_S(F;u) \neq \emptyset$, $e_S(F;u)$ is an interior ideal of S.

Proof. Let(F,S) is a uni-soft interior ideal over U and $u \subseteq U$ such that $e_S(F;u) \neq \emptyset$. Let $x,y,z \in S$ such that $y \in e_S(F;u)$, then by Definition (2.5) we have $F(y) \subseteq u$. Since (F,S) is a uni-soft interior ideal over U, it follows that

$$F\big((xy)z\big) \subseteq F(y) \subseteq u,$$
$$\Rightarrow F\big((xy)z\big) \subseteq u,$$

which shows that $(xy)z \in e_S(F;u)$. Thus $e_S(F;u)$ is an interior ideal of S.

Conversely, suppose that $\forall\, u \in P(U)$ with $e_S(F;u) \neq \emptyset$, $e_S(F;u)$ is an interior ideal of S. Also let $x,y,z \in S$ such that $F(y) = u$. Then $F(y) = u$, implies that $y \in e_S(F;u)$. But $e_S(F;u)$ is an interior ideal of S which shows that $(xy)z \in e_S(F;u)$ for all $x,y,z \in S$. Now

$$F\big((xy)z\big) \subseteq u = F(y).$$

Hence (F,S) is a uni-soft interior ideal over U.

Definition 4.3.[26] Let (F,S) be a soft set over U and u be any subset of U with $e_S(F;u) \neq \emptyset$. Define a soft set (F^*,S) over U as:

$$F^*: S \to P(U),\ x \mapsto \begin{cases} F(x) & \text{if } x \in e_S(F;u), \\ V & \text{otherwise.} \end{cases}$$

Where V is a subset of U with $F(x) \subset V$.

Theorem 4.5. If (F,S) is a uni-soft bi-ideal over U, then $(F^*$,S) is also a uni-soft bi-ideal over U.

Proof. Let (F,S) is a uni-soft bi-ideal over U, and $x,y,z \in S$. If $x,z \in e_S(F;u)$, then $(xy)z \in e_S(F;u)$ as $e_S(F;u)$ is a bi-ideal of S (by Theorem (4.3)). So

$$F^*\big((xy)z\big) = F\big((xy)z\big) \subseteq F(x) \cup F(z) = F^*(x) \cup F^*(z).$$

Now, if x or $z \notin e_S(F;u)$, then $F^*(x) = V$ or $F^*(z) = V$. Thus

$$F^*\big((xy)z\big) \subseteq V = V \cup V = F^*(x) \cup F^*(z),$$

which implies that $F^*\big((xy)z\big) \subseteq F^*(x) \cup F^*(z)$. Hence (F^*,S) is a uni-soft bi-ideal over U.

Theorem 4.6. If (F,S) is a uni-soft interior ideal over U, then $(F^*$,S) is also a uni-soft interior ideal over U.

Proof. Suppose (F,S) is a uni-soft interior ideal over U, and $x,y,z \in S$. Let $y \in e_S(F;u)$, then $(xy)z \in e_S(F;u)$ as $e_S(F;u)$ is an interior ideal of S (by Theorem (4.4)). So

$$F^*\big((xy)z\big) = F\big((xy)z\big) \subseteq F(y) = F^*(y).$$

Now, if $y \notin e_S(F;u)$, then $F^*(y) = V$. Thus

$$F^*\big((xy)z\big) \subseteq V = F^*(y)$$

which shows that $F^*\big((xy)z\big) \subseteq F^*(y)$. Hence (F^*,S) is a uni-soft interior ideal over U.

Theorem 4.7. For a nonempty subset B of S, the following are equivalent:

(1) B is a bi-ideal ideal of S.

(2) The uni-characteristic soft set $(\mathbb{X}_B,S)$ over U is the uni-soft bi-ideal ideal over U.

proof. Assume that B is a bi-ideal ideal of S and $x,y,z \in S$. If $x,z \in B$, then $(xy)z \in B$ as B is a bi-ideal ideal of S. Therefore $\mathbb{X}_B\big((xy)z\big) = \emptyset = \mathbb{X}_B(x) \cup \mathbb{X}_B(z)$. Now, if one of the x or $z \notin B$, then

$$\mathbb{X}_B\big((xy)z\big) \subseteq U = \mathbb{X}_B(x) \cup \mathbb{X}_B(z).$$

Hence $(\mathbb{X}_B,S)$ is a uni-soft bi-ideal over U.

Conversely, suppose that $(\mathbb{X}_B,S)$ is uni-soft bi-ideal over U, we have to show that B is a bi-ideal of S. For this let $x,y,z \in S$ such that $x,z \in B$. Since $(\mathbb{X}_B,S)$ is uni-soft bi-ideal over U, so we can write $\mathbb{X}_B\big((xy)z\big) \subseteq \mathbb{X}_B(x) \cup \mathbb{X}_B(z)$. Since $\mathbb{X}_B(x) = \emptyset$ and $\mathbb{X}_B(z) = \emptyset$ as $x,z \in B$. Therefore

$$\mathbb{X}_B\big((xy)z\big) \subseteq \mathbb{X}_B(x) \cup \mathbb{X}_B(z) = \emptyset \cup \emptyset = \emptyset,$$

which shows that $(xy)z \in B$ and hence B is a bi-ideal of S.

Theorem 4.8. For a nonempty subset A of S, the following are equivalent:

(1) A is an interior ideal of S.

(2) The uni-characteristic soft set $(\mathbb{X}_A,S)$ over U is the uni-soft interior ideal over U.

proof. Let A is an interior ideal of S and $x,y,z \in S$. If $y \in A$, then $(xy)z \in A$ as A is an interior ideal of S. Therefore $\mathbb{X}_A\big((xy)z\big) = \emptyset = \mathbb{X}_A(y))$. Now, if $y \notin A$, then $\mathbb{X}_A\big((xy)z\big) \subseteq U = \mathbb{X}_A(y)$. Thus $(\mathbb{X}_A,S)$ is a uni-soft interior ideal over U.

Conversely, suppose that $(\mathbb{X}_A,S)$ is uni-soft interior ideal over U, we will show that A is an interior ideal of S. Let $x,y,z \in S$ with $y \in A$. Since $(\mathbb{X}_A,S)$ is uni-soft interior ideal over U, so we can write $\mathbb{X}_A\big((xy)z\big) \subseteq \mathbb{X}_A(y)$. Since $\mathbb{X}_A(y) = \emptyset$, as $y \in A$. Therefore

$$\mathbb{X}_A\big((xy)z\big) \subseteq \mathbb{X}_A(y) = \emptyset,$$

which shows that $(xy)z \in A$ and hence A is an interior ideal of S.

5 Conclusion

The present work is an extension to the concept of uni-soft ideals in the field of AG-groupoids, initiated in [26], by introducing some new ideals namely; uni-soft bi-ideals and uni-soft interior ideals of AG-groupoids. Similarly we can extend these notions to the emerging algebraic structure of AG-gorupoids and semigroups such as rings, semirings, Γ-semigroups and Γ-AG-groupoids etc..

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