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Soft Regularity and Normality based on Semi Open Soft Sets and Soft Ideals

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Abstract: Shabir and Naz in [27] introduced the notion of soft topological spaces. They defined soft topology on the collection τ of soft sets over X. Consequently, they defined soft separation axioms, soft regular spaces and soft normal spaces and established their several properties. Min in [30] investigate some properties of these soft separation axioms. Kandil et al. [12] introduce the notion of soft semi separation axioms. In particular they study the properties of the soft semi regular spaces and soft semi normal spaces. In the present paper, we introduce the notions of soft regular (normal) spaces based on the notions of semi open soft sets and soft ideals. Also, we discuss some properties of these notions and introduce an alternative descriptions of the notions of soft regular spaces via soft ideals [7], which is more general.

Keywords: Soft topological space, Soft-$\tilde{I}$-regular spaces, Soft-$\tilde{I}$-normal spaces,Irresolute soft functions, Irresolute open soft functions.

1 Introduction

The concept of soft sets was first introduced by Molodtsov [22] in 1999 as a general mathematical tool for dealing with uncertain objects. In [22,23], Molodtsov successfully applied the soft theory in several directions, such as smoothness of functions, game theory, operations research, Riemann integration, Perron integration, probability, theory of measurement, and so on. After presentation of the operations of soft sets [20], the properties and applications of soft set theory have been studied increasingly [4,16,23]. Xiao et al.[29] and Pei and Miao [25] discussed the relationship between soft sets and information systems. They showed that soft sets are a class of special information systems. In recent years, many interesting applications of soft set theory have been expanded by embedding the ideas of fuzzy sets [2,3,5,18, 19,20,21,23,24,31]. To develop soft set theory, the operations of the soft sets are redefined and a uni-int decision making method was constructed by using these new operations [6].
Recently, in 2011, Shabir and Naz [27] initiated the study of soft topological spaces. They defined soft topology on the collection τ of soft sets over X. Consequently, they defined basic notions of soft topological spaces such as open soft and closed soft sets, soft subspace, soft closure, soft nbd of a point, soft separation axioms, soft regular spaces and soft normal spaces and established their several properties. Min in [30] investigate some properties of these soft separation axioms. Kandil et al. [12] introduce the notion of soft semi separation axioms. In particular they studied the properties of the soft semi regular spaces and soft semi normal spaces. Maji et. al. [18] initiated the study involving both fuzzy sets and soft sets. The notion of soft ideal was initiated for the first time by Kandil et al.[14]. They also introduced the concept of soft local function. These concepts are discussed with a view to find new soft topologies from the original one, called soft topological spaces with soft ideal $(X,\tau,E,\tilde{I})$. Applications to various fields were further investigated by Kandil et al. [10,11,13,15].
In the present paper, we introduce the notions of soft-$\tilde{I}$-regular spaces, soft semi-$\tilde{I}$-regular spaces, soft-$\tilde{I}$-normal spaces and soft semi-$\tilde{I}$-normal spaces. Then we investigate some of their properties.

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2 Preliminaries

In this section, we present the basic definitions and results of soft set theory which will be needed in the sequel.

Definition 2.1.[22] Let X be an initial universe and E be a set of parameters. Let $P(X)$ denote the power set of X and A be a non-empty subset of E. A pair (F,A) denoted by F_A is called a soft set over X , where F is a mapping given by $F : A \to P(X)$. In other words, a soft set over X is a parametrized family of subsets of the universe X. For a particular $e \in A$, $F(e)$ may be considered the set of e-approximate elements of the soft set (F,A) and if $e \notin A$, then $F(e) = \phi$ i.e
$F_A = \{F(e) : e \in A \subseteq E,\ F : A \to P(X)\}$. The family of all these soft sets over X denoted by $SS(X)_A$.

Definition 2.2.[20] Let $F_A,\ G_B \in SS(X)_E$. Then F_A is soft subset of G_B, denoted by $F_A \tilde{\subseteq} G_B$, if

(1)$A \subseteq B$, and
(2)$F(e) \subseteq G(e)$, $\forall e \in A$.

In this case, F_A is said to be a soft subset of G_B and G_B is said to be a soft superset of F_A, $G_B \tilde{\supseteq} F_A$.

Definition 2.3.[20] Two soft subset F_A and G_B over a common universe set X are said to be soft equal if F_A is a soft subset of G_B and G_B is a soft subset of F_A.

Definition 2.4.[4] The complement of a soft set (F,A), denoted by $(F,A)'$, is defined by $(F,A)' = (F',A)$, $F' : A \to P(X)$ is a mapping given by $F'(e) = X - F(e)$, $\forall e \in A$ and F' is called the soft complement function of F.
Clearly $(F')'$ is the same as F and $((F,A)')' = (F,A)$.

Definition 2.5.[27] The difference between two soft sets (F,E) and (G,E) over the common universe X, denoted by $(F,E) - (G,E)$ is the soft set (H,E) where for all $e \in E$, $H(e) = F(e) - G(e)$.

Definition 2.6.[27] Let (F,E) be a soft set over X and $x \in X$. We say that $x \in (F,E)$ read as x belongs to the soft set (F,E) whenever $x \in F(e)$ for all $e \in E$.

Definition 2.7.[20] A soft set (F,A) over X is said to be a NULL soft set denoted by $\tilde{\phi}$ or ϕ_A if for all $e \in A$, $F(e) = \phi$ (null set).

Definition 2.8.[20] A soft set (F,A) over X is said to be an absolute soft set denoted by $\tilde{A}$ or X_A if for all $e \in A$, $F(e) = X$. Clearly we have $X'_A = \phi_A$ and $\phi'_A = X_A$.

Definition 2.9.[20] The union of two soft sets (F,A) and (G,B) over the common universe X is the soft set (H,C), where $C = A \cup B$ and for all $e \in C$,

$$H(e) = \begin{cases} F(e),\ e \in A - B, \\ G(e),\ e \in B - A, \\ F(e) \cup G(e),\ e \in A \cap B\,. \end{cases}$$

Definition 2.10.[20] The intersection of two soft sets (F,A) and (G,B) over the common universe X is the soft set (H,C), where $C = A \cap B$ and for all $e \in C$, $H(e) = F(e) \cap G(e)$. Note that, in order to efficiently discuss, we consider only soft sets (F,E) over a universe X in which all the parameter set E are same. We denote the family of these soft sets by $SS(X)_E$.

Definition 2.11.[32] Let I be an arbitrary indexed set and $L = \{(F_i,E), i \in I\}$ be a subfamily of $SS(X)_E$.

(1)The union of L is the soft set (H,E), where $H(e) = \bigcup_{i\in I} F_i(e)$ for each $e \in E$. We write $\tilde{\bigcup}_{i\in I}(F_i,E) = (H,E)$.
(2)The intersection of L is the soft set (M,E), where $M(e) = \bigcap_{i\in I} F_i(e)$ for each $e \in E$. We write $\tilde{\bigcap}_{i\in I}(F_i,E) = (M,E)$.

Definition 2.12.[27] Let τ be a collection of soft sets over a universe X with a fixed set of parameters E, then $\tau \subseteq SS(X)_E$ is called a soft topology on X if

(1)$\tilde{X}, \tilde{\phi} \in \tau$, where $\tilde{\phi}(e) = \phi$ and $\tilde{X}(e) = X$, $\forall e \in E$,
(2)the union of any number of soft sets in τ belongs to τ,
(3)the intersection of any two soft sets in τ belongs to τ.

The triplet (X,τ,E) is called a soft topological space over X.

Definition 2.13.[26] Let (X,τ,E) be a soft topological space. A soft set (F,A) over X is said to be closed soft set in X, if its relative complement $(F,A)'$ is an open soft set.

Definition 2.14.[26] Let(X,τ,E) be a soft topological space. The members of τ are said to be open soft sets in X. We denote the set of all open soft sets over X by $OS(X,\tau,E)$, or when there can be no confusion by $OS(X)$ and the set of all closed soft sets by $CS(X,\tau,E)$, or $CS(X)$.

Definition 2.15.[27] Let (X,τ,E) be a soft topological space and $(F,E) \in SS(X)_E$. The soft closure of (F,E), denoted by $cl(F,E)$ is the intersection of all closed soft super sets of (F,E) i.e
$cl(F,E) = \tilde{\cap}\{(H,E) : (H,E)\ \textit{is closed soft set and}\ (F,E) \tilde{\subseteq} (H,E)\}$).

Definition 2.16.[32] Let (X,τ,E) be a soft topological space and $(F,E) \in SS(X)_E$. The soft interior of (G,E), denoted by $int(G,E)$ is the union of all open soft subsets of (G,E) i.e
$int(G,E) = \tilde{\cup}\{(H,E) : (H,E)\ \textit{is an open soft set and}\ (H,E) \tilde{\subseteq} (G,E)\}$).

Definition 2.17.[32] The soft set $(F,E) \in SS(X)_E$ is called a soft point in X_E if there exist $x \in X$ and $e \in E$ such that $F(e) = \{x\}$ and $F(e') = \phi$ for each $e' \in E - \{e\}$, and the soft point (F,E) is denoted by x_e.

Definition 2.18.[32] The soft point x_e is said to be belonging to the soft set (G,A), denoted by $x_e \tilde{\in} (G,A)$, if for the element $e \in A$, $F(e) \subseteq G(e)$.

Theorem 2.1.[28] Let (X,τ,E) be a soft topological space. A soft point $x_e \tilde{\in} cl(F,E)$ if and only if each soft neighborhood of x_e intersects (F,E).

Definition 2.19.[27] Let (X,τ,E) be a soft topological space and Y be a non null subset of X. Then $\tilde{Y}$ denotes the soft set (Y,E) over X for which $Y(e) = Y\ \forall e \in E$.

Definition 2.20.[27] Let (X,τ,E) be a soft topological space, $(F,E)\in SS(X)_E$ and Y be a non null subset of X. Then the sub soft set of (F,E) over Y denoted by (F_Y,E), is defined as follows:

$$F_Y(e)=Y\cap F(e)\ \forall e\in E.$$

In other words $(F_Y,E)=\tilde{Y}\tilde{\cap}(F,E)$.

Definition 2.21.[27] Let (X,τ,E) be a soft topological space and Y be a non null subset of X. Then

$$\tau_Y=\{(F_Y,E):(F,E)\in\tau\}$$

is said to be the soft relative topology on Y and (Y,τ_Y,E) is called a soft subspace of (X,τ,E).

Theorem 2.2.[27] Let (Y,τ_Y,E) be a soft subspace of a soft topological space (X,τ,E) and $(F,E)\in SS(X)_E$. Then

(1)If (F,E) is an open soft set in Y and $\tilde{Y}\in\tau$, then $(F,E)\in\tau$.
(2)(F,E) is an open soft set in Y if and only if $(F,E)=\tilde{Y}\tilde{\cap}(G,E)$ for some $(G,E)\in\tau$.
(3)(F,E) is a closed soft set in Y if and only if $(F,E)=\tilde{Y}\tilde{\cap}(H,E)$ for some (H,E) is τ-closed soft set.

Definition 2.22.[9] Let (X,τ,E) be a soft topological space and $(F,E)\in SS(X)_E$. Then (F,E) is called semi open soft set if $(F,E)\tilde{\subseteq}cl(int(F,E))$. The set of all semi open soft sets is denoted by $SOS(X,\tau,E)$, or $SOS(X)$ and the set of all semi closed soft sets is denoted by $SCS(X,\tau,E)$, or $SCS(X)$.

Definition 2.23.[9] Let (X,τ,E) be a soft topological space, $(F,E)\in SS(X)_E$ and $x_e\in SS(X)_E$. Then x_e is called semi closure soft point of (F,E) if $(F,E)\tilde{\cap}(H,E)\neq\tilde{\phi}\ \forall\ (H,E)\in SOS(X)$. The set of all semi closure soft points of (F,E) is called semi soft closure of (F,E) and is denoted by $Scl(F,E)$ consequently, $Scl(F,E)=\tilde{\bigcap}\{(H,E):(H,E)\in SCS(X),(F,E)\tilde{\subseteq}(H,E)\}$.

Definition 2.24.[1] Let $SS(X)_A$ and $SS(Y)_B$ be families of soft sets, $u:X\to Y$ and $p:A\to B$ be mappings. Let $f_{pu}:SS(X)_A\to SS(Y)_B$ be a mapping. Then;

(1)If $(F,A)\in SS(X)_A$. Then the image of (F,A) under f_{pu}, written as $f_{pu}(F,A)=(f_{pu}(F),p(A))$, is a soft set in $SS(Y)_B$ such that
$$f_{pu}(F)(b)=\begin{cases}\cup_{x\in p^{-1}(b)\cap A}\ u(F(a)), & p^{-1}(b)\cap A\neq\phi,\\ \phi, & otherwise.\end{cases}$$
for all $b\in B$.
(2)If $(G,B)\in SS(Y)_B$. Then the inverse image of (G,B) under f_{pu}, written as $f_{pu}^{-1}(G,B)=(f_{pu}^{-1}(G),p^{-1}(B))$, is a soft set in $SS(X)_A$ such that
$$f_{pu}^{-1}(G)(a)=\begin{cases}u^{-1}(G(p(a))), & p(a)\in B,\\ \phi, & otherwise.\end{cases}$$
for all $a\in A$.

The soft function f_{pu} is called surjective if p and u are surjective, also is said to be injective if p and u are injective. **Definition 2.25.**[9,17,32] Let (X,τ_1,A) and (Y,τ_2,B) be soft topological spaces and $f_{pu}:SS(X)_A\to SS(Y)_B$ be a function. Then, the function f_{pu} is said to be

(1)Continuous soft if $f_{pu}^{-1}(G,B)\in\tau_1\ \forall\ (G,B)\in\tau_2$.
(2)Open soft if $f_{pu}(G,A)\in\tau_2\forall\ (G,A)\in\tau_1$.
(3)Homeomorphism soft if it is bijective, continuous soft and f_{pu}^{-1} is continuous soft.
(4)Semi open soft if $f_{pu}(G,A)\in SOS(Y)\forall\ (G,A)\in\tau_1$.
(5)Semi continuous soft if $f_{pu}^{-1}(G,B)\in SOS(X)\forall\ (G,B)\in\tau_2$.
(6)Irresolute soft if $f_{pu}^{-1}(G,B)\in SOS(X)\forall\ (G,B)\in SOS(Y)[f_{pu}^{-1}(F,B)\in SCS(X)\forall\ (F,B)\in SCS(Y)]$.
(7)Irresolute open soft (resp. irresolute closed soft) if $f_{pu}(G,A)\in SOS(Y)\forall\ (G,A)\in SOS(X)(resp.f_{pu}(F,A)\in SCS(Y)\forall\ (F,A)\in SCS(Y))$.

Theorem 2.3.[1] Let $SS(X)_A$ and $SS(Y)_B$ be families of soft sets. For the soft function $f_{pu}:SS(X)_A\to SS(Y)_B$, the following statements hold,

(a)$f_{pu}^{-1}((G,B)')=(f_{pu}^{-1}(G,B))'\forall\ (G,B)\in SS(Y)_B$.
(b)$f_{pu}(f_{pu}^{-1}((G,B)))\tilde{\subseteq}(G,B)\forall\ (G,B)\in SS(Y)_B$. If f_{pu} is surjective, then the equality holds.
(c)$(F,A)\tilde{\subseteq}f_{pu}^{-1}(f_{pu}((F,A)))\forall\ (F,A)\in SS(X)_A$. If f_{pu} is injective, then the equality holds.
(d)$f_{pu}(\tilde{X})\tilde{\subseteq}\tilde{Y}$. If f_{pu} is surjective, then the equality holds.
(e)$f_{pu}^{-1}(\tilde{Y})=\tilde{X}$ and $f_{pu}(\tilde{\phi}_A)=\tilde{\phi}_B$.
(f)If $(F,A)\tilde{\subseteq}(G,A)$, then $f_{pu}(F,A)\tilde{\subseteq}f_{pu}(G,A)$.
(g)If $(F,B)\tilde{\subseteq}(G,B)$, then $f_{pu}^{-1}(F,B)\tilde{\subseteq}f_{pu}^{-1}(G,B)\ \forall\ (F,B),(G,B)\in SS(Y)_B$.
(h)$f_{pu}^{-1}[(F,B)\tilde{\cup}(G,B)]=f_{pu}^{-1}(F,B)\tilde{\cup}f_{pu}^{-1}(G,B)$ and $f_{pu}^{-1}[(F,B)\tilde{\cap}(G,B)]=f_{pu}^{-1}(F,B)\tilde{\cap}f_{pu}^{-1}(G,B)$ $\forall\ (F,B),(G,B)\in SS(Y)_B$.
(I)$f_{pu}[(F,A)\tilde{\cup}(G,A)]=f_{pu}(F,A)\tilde{\cup}f_{pu}(G,A)$ and $f_{pu}[(F,A)\tilde{\cap}(G,A)]\tilde{\subseteq}f_{pu}(F,A)\tilde{\cap}f_{pu}(G,A)$ $\forall\ (F,A),(G,A)\in SS(X)_A$. If f_{pu} is injective, then the equality holds.

Proposition 2.1.[12] Let (X,τ,E) be a soft topological space, $(F,E)\in SS(X)_E$ and $x\in X$. Then:

(1)$x\in(F,E)$ if and only if $x_E\tilde{\subseteq}(F,E)$.
(2)If $x_E\tilde{\cap}(F,E)=\tilde{\phi}$, then $x\notin(F,E)$.

Definition 2.26.[8]. A non-empty collection I of subsets of a set X is called an ideal on X, if it satisfies the following conditions

(1)$A\in I$ and $B\in I\Rightarrow A\cup B\in I$,
(2)$A\in I$ and $B\subseteq A\Rightarrow B\in I$,
i.e. I is closed under finite unions and subsets.

Definition 2.27.[14] Let $\tilde{I}$ be a non-null collection of soft sets over a universe X with a fixed set of parameters E, then $\tilde{I}\subseteq SS(X)_E$ is called a soft ideal on X with a fixed set E if

(1)$(F,E)\in\tilde{I}$ and $(G,E)\in\tilde{I}\Rightarrow(F,E)\tilde{\cup}(G,E)\in\tilde{I}$,

(2)$(F,E) \in \tilde{I}$ and $(G,E)\tilde{\subseteq}(F,E) \Rightarrow (G,E) \in \tilde{I}$,
i.e. $\tilde{I}$ is closed under finite soft unions and soft subsets.

Theorem 2.4.[14] Let (X,τ,E) be a soft topological space, $\tilde{I}$ be a soft ideal over X with the same set of parameters E and $cl^* : SS(X)_E \to SS(X)_E$ be the soft closure operator. Then there exists a unique soft topology over X with the same set of parameters E, finer than τ, called the $*$-soft topology, denoted by $\tau^*(\tilde{I})$ or τ^*, given by

$$\tau^*(\tilde{I}) = \{(F,E) \in SS(X)_E : \ cl^*(F,E)' = (F,E)'\}. \qquad (1)$$

Theorem 2.5.[11] Let $(X_1,\tau_1,A,\tilde{I})$ be a soft topological space with soft ideal, (X_2,τ_2,B) be a soft topological space and $f_{pu} : (X_1,\tau_1,A,\tilde{I}) \to (X_2,\tau_2,B)$ be a soft function. Then $f_{pu}(\tilde{I}) = \{f_{pu}((F,A)) : (F,A) \in \tilde{I}\}$ is a soft ideal on X_2.

3 Soft regular spaces via soft ideals

In the present section, we introduce the notions of soft regular spaces based on the notions of semi open soft sets and soft ideals. Also, we discuss some properties of these notions and introduce an alternative descriptions of the notions of soft regular spaces via soft ideals [7], which is more general.

Definition 3.1. Let $(X,\tau,E,\tilde{I})$ be a soft topological space with soft ideal and (H,E) be a closed soft set over X such that $x \notin (H,E)$ for $x \in X$. Then $(X,\tau,E,\tilde{I})$ is called a soft-$\tilde{I}$-regular space if there exist disjoint open soft sets (F,E) and (G,E) such that $x \in (F,E)$ and $(H,E)-(G,E) \in \tilde{I}$.

Definition 3.2. Let $(X,\tau,E,\tilde{I})$ be a soft topological space with soft ideal and (H,E) be a closed soft set over X such that $x \notin (H,E)$ for $x \in X$. Then $(X,\tau,E,\tilde{I})$ is called a soft-$\tilde{I}$-regular space if there exist disjoint open soft sets (F,E) and (G,E) such that $x \in (F,E)$ and $(H,E)-(G,E) \in \tilde{I}$.

Examples 3.1.

(1)Let $X = \{h_1,h_2\}$, $E = \{e_1,e_2\}$ and $\tau = \{\tilde{X},\tilde{\phi},(F_1,E),(F_2,E),(F_3,E),(F_4,E)\}$ where $(F_1,E),(F_2,E),(F_3,E),(F_4,E)$ are soft sets over X defined as follows:
$F_1(e_1) = \{h_1\}, \quad F_1(e_2) = \{h_2\}$,
$F_2(e_1) = \{h_2\}, \quad F_2(e_2) = \{h_1\}$,
$F_3(e_1) = \phi, \quad F_3(e_2) = \{h_1\}$,
$F_4(e_1) = \{h_1\}, \quad F_4(e_2) = X$.
Then τ defines a soft topology on X. Let $\tilde{I} = \{\tilde{\phi},(I_1,E)\}$ be a soft ideal over X where (I_1,E) is soft set over X defined by $I_1(e_1) = \{h_2\}$, $I_1(e_2) = \phi$. Then $(X,\tau,E,\tilde{I})$ is a soft-$\tilde{I}$-regular space.

(2)Let $X = \{h_1,h_2,h_3\}$, $E = \{e_1,e_2\}$ and $\tau = \{\tilde{X},\tilde{\phi},(F_1,E),(F_2,E),(F_3,E)\}$ where $(F_1,E),(F_2,E),(F_3,E)$ are soft sets over X defined as follows:
$F_1(e_1) = \{h_1\}, \quad F_1(e_2) = \{h_2\}$,
$F_2(e_1) = \{h_1,h_3\}, \quad F_2(e_2) = \{h_2,h_3\}$,
$F_3(e_1) = \{h_1,h_2\}, \quad F_3(e_2) = \{h_1,h_2\}$.
Then τ defines a soft topology on X. Let $\tilde{I} = \{\tilde{\phi},(I_1,E),(I_2,E),(I_3,E)\}$ be a soft ideal over X where $(I_1,E),(I_2,E),(I_3,E)$ are soft sets over X defined by;
$I_1(e_1) = \{h_2\}, I_1(e_2) = \phi$,
$I_2(e_1) = \phi, I_2(e_2) = \{h_1\}$,
$I_3(e_1) = \{h_2\}, I_3(e_2) = \{h_1\}$.
Then $(X,\tau,E,\tilde{I})$ is not soft-$\tilde{I}$-regular space. Because, for the closed soft set (F_1',E) such that $h_1 \notin (F_1',E)$, there are no two disjoint open soft sets (A,E) and (B,E) such that $h_1 \in (A,E)$ and $(F_1',E)-(B,E) \in \tilde{I}$.

Lemma 3.1. Let (X,τ,E) be a soft topological space and $x \in X$. If $(X,\tau,E,\tilde{I})$ is soft-$\tilde{I}$-regular space. Then, for every closed soft set (G,E) such that $x_E\tilde{\cap}(G,E) = \tilde{\phi}$, there exist disjoint open soft sets (F_1,E) and (F_2,E) such that $x_E - (F,E) \in \tilde{I}$ and $(G,E)-(F_2,E) \in \tilde{I}$.
Proof. Obvious.

Proposition 3.1.

(1)Every soft regular space is a soft-$\tilde{I}$-regular.
(2)If $\tilde{I} = \tilde{\phi}$, then $(X,\tau,E,\tilde{I})$ is soft regular space if and only if it is soft-$\tilde{I}$-regular space.

Proof. Obvious.

Theorem 3.1. (X,τ^*,E) is a soft-$\tilde{I}$-regular space if and only if (X,τ,E) is soft-$\tilde{I}$-regular space.
Proof. ($:\Rightarrow$) Let (X,τ^*,E) be a soft-$\tilde{I}$-regular space and (H,E) be a τ-closed soft set such that $x \notin (H,E)$. Since $\tau \subseteq \tau^*$. It follows that, (H,E) is τ^*-closed soft set such that $x \notin (H,E)$. Since (X,τ^*,E) is a soft-$\tilde{I}$-regular space. Hence, there exist disjoint τ^*-open soft sets (F,E) and (G,E) such that $x_E-(F,E) \in \tilde{I}$ and $(H,E)-(G,E) \in \tilde{I}$ from Lemma 3.1. Since (F,E) and (G,E) are τ^*-open soft sets. Then, $(F,E) = (A,E)-(I_1,E)$ and $(G,E) = (B,E)-(I_2,E)$ where $(A,E),(B,E) \in \tau$ and $(I_1,E),(I_2,E) \in \tilde{I}$. Thus, $x_E-(A,E) \in \tilde{I}$ and $(H,E)-(G,E) = (H,E)-((B,E)-(I_2,E)) \in \tilde{I}$. From Definition 2.27, $(H,E)-(B,E) \in \tilde{I}$. Therefore, (X,τ,E) is soft-$\tilde{I}$-regular space.
($\Leftarrow$:) Let (X,τ,E) be a soft-$\tilde{I}$-regular space and (H,E) be τ^*-closed soft set such that $x \notin (H,E)$. Since $(H,E)'$ is τ^*-open soft sets. Then, $(H,E)' = (G,E)-(I,E)$ where $(G,E) \in \tau$ and $(I,E) \in \tilde{I}$. Hence, $(G,E)'$ is τ-closed soft set such that $x \notin (G,E)'$. Since (X,τ,E) is soft-$\tilde{I}$-regular space, then there exist disjoint τ-open soft sets (A,E) and (B,E) such that $x_E-(A,E) \in \tilde{I}$ and $(G,E)'-(B,E) \in \tilde{I}$ from Lemma 3.1. So, $(G,E)'-(B,E) = [(H,E)-(I,E)]-(B,E) \in \tilde{I}$. It follows that, $(H,E)-(B,E) \in \tilde{I}$. Therefore, (X,τ^*,E) is a soft-$\tilde{I}$-regular space.

Theorem 3.2. A soft subspace $(Y,\tau_Y,E,\tilde{I}_Y)$ of a soft-$\tilde{I}$-regular space $(X,\tau,E,\tilde{I})$ is soft-$\tilde{I}_Y$-regular space.
Proof. Let $y \in Y$ and (G,E) be a closed soft set in Y such that $y \notin (G,E)$. Then $(G,E) = (Y,E)\tilde{\cap}(H,E)$ for some closed soft set (H,E) in X from Theorem 2.2. Hence, $y \notin (Y,E)\tilde{\cap}(H,E)$. But $y \in (Y,E)$, so $y \notin (H,E)$. Since

$(X,\tau,E,\tilde{I})$ is soft-$\tilde{I}$-regular space. So, there exist disjoint open soft sets (F_1,E) and (F_2,E) in X such that $y_E-(F_1,E)\in\tilde{I}$ and $(H,E)-(F_2,E)\in\tilde{I}$ from Lemma 3.1. Since $(Y,E)\tilde{\cap}(F_1,E)$ and $(Y,E)\tilde{\cap}(F_2,E)$ are disjoint open soft sets in Y such that $y_E-[(Y,E)\tilde{\cap}(F_1,E)]\in\tilde{I}_Y$ and $(Y,E)\tilde{\cap}[(G,E)-(F_2,E)]=(Y,E)\tilde{\cap}[(H,E)-(F_2,E)]\in\tilde{I}_Y$. It follows that, $(Y,\tau_Y,E,\tilde{I}_Y)$ is soft-$\tilde{I}_Y$-regular spaces.

Theorem 3.3. Let (X,τ,E) be a soft topological space and $x\in X$. Then the following are equivalent:

(1)$(X,\tau,E,\tilde{I})$ is a soft-$\tilde{I}$-regular space.
(2)For every open soft set (A,E) such that $x\in(A,E)$, there exists an open soft set (F,E) such that $x_E-(F,E)\in\tilde{I}$ and $cl(F,E)-(A,E)\in\tilde{I}$.
(3)For every closed soft set (B,E) such that $x\notin(B,E)$, there exists an open soft set (F,E) such that $x_E-(F,E)\in\tilde{I}$ and $cl(F,E)\tilde{\cap}(B,E)\in\tilde{I}$.

Proof.

(1) $\Rightarrow$ (2) Let (A,E) be an open soft set such that $x\in(A,E)$. Then, $(A,E)'$ is closed soft set such that $x\notin(A,E)'$. It follows by (1), there exist disjoint open soft sets (G,E) and (H,E) such that $x_E-(G,E)\in\tilde{I}$ and $(A,E)'-(H,E)=(H,E)'-(A,E)\in\tilde{I}$. Since $(G,E)\tilde{\cap}(H,E)=\tilde{\phi}$. Then, $(G,E)\tilde{\subseteq}(H,E)'$. So, $cl(G,E)\tilde{\subseteq}(H,E)'$. Hence, $cl(G,E)-(A,E)\tilde{\subseteq}(H,E)'-(A,E)\in\tilde{I}$.
(2) $\Rightarrow$ (3) Let (B,E) be a closed soft set such that $x\notin(B,E)$. Then, $(B,E)'$ is open soft set such that $x\in(B,E)'$. From (2), there exists an open soft set (F,E) such that $x_E-(F,E)\in\tilde{I}$ and $cl(F,E)-(B,E)'=cl(F,E)\tilde{\cap}(B,E)\in\tilde{I}$.
(3) $\Rightarrow$ (1)Let (H,E) be a closed soft set such that $x\notin(H,E)$. From (3), there exists an open soft sets (F,E) such that $x_E-(F,E)\in\tilde{I}$ and $cl(F,E)\tilde{\cap}(H,E)=(H,E)-[cl(F,E)]'\in\tilde{I}$, where (F,E) and $[cl(F,E)]'$ are disjoint open soft sets. Therefore, $(X,\tau,E,\tilde{I})$ is soft-$\tilde{I}$-regular space.

Theorem 3.4. Let $f_{pu}:SS(X)_A\to SS(Y)_B$ be an injective soft function. Then, $f_{pu}[F_A-G_A]=f_{pu}(F_A)-f_{pu}(G_A)$.
Proof. Let $y\in f_{pu}[F_A-G_A]$. Then,

$$y\in f_{pu}[F_A-G_A](b)=\begin{cases}\cup_{a\in p^{-1}(b)\cap A}\ u[(F_A-G_A)(a)], & p^{-1}(b)\cap A\neq\phi,\\ \phi, & otherwise.\end{cases}$$

for all $b\in B$. Since f_{pu} is injective, then u is injective and $y\in u[F(a)]-[G(a)]$. It follows that, $y\in u[F(a)]$ and $y\notin u[G(a)]$. Hence, $y\in f_{pu}[F_A(b)]$ and $y\notin f_{pu}[G_A(b)]$. Therefore,
$y\in f_{pu}[F_A(b)]-f_{pu}[G_A(b)]=f_{pu}[(F_A-G_A)(b)]$. So, $y\in f_{pu}(F_A)-f_{pu}(G_A)$. Thus, $f_{pu}[F_A-G_A]=f_{pu}(F_A)-f_{pu}(G_A)$.

Theorem 3.5. Let $f_{pu}:SS(X)_A\to SS(Y)_B$ be a homeomorphism soft function. If (X,τ_1,A) is a soft-$\tilde{I}$-regular space, then (Y,τ_2,B) is soft-$f_{pu}(\tilde{I})$-regular space.

Proof. Let (G,B) be a closed soft set in Y and $y\in Y$ such that $y\notin(G,B)$. Since f_{pu} is homeomorphism soft function, then $\exists x\in X$ such that $u(x)=y$ and $(F,A)=f_{pu}^{-1}(G,B)$ is a closed soft set in X such that $x\notin(F,A)$. By hypothesis, there exist disjoint open soft sets (H,A) and (Z,A) such that $x\in(H,A)$ and $(Z,A)-(F,A)\in\tilde{I}$. It follows that, $x\in H_A(e)$ for all $e\in A$ and $f_{pu}[(Z,A)-(F,A)]\in f_{pu}(\tilde{I})$ from Theorem 2.5. Hence, $u(x)=y\in u[H_A(e)]$ for all $e\in A$ and $f_{pu}(Z,A)-f_{pu}(F,A)\in f_{pu}(\tilde{I})$ from Theorem 3.4. Therefore, $y\in f_{pu}(H,A)=(W,B)$ and $(U,B)-(V,B)\in f_{pu}(\tilde{I}$, where (W,B) is an open soft set in Y. Therefore, (Y,τ_2,B) is soft-$f_{pu}(\tilde{I})$-regular space.

4 Soft semi regular spaces via soft ideals

Definition 4.1. Let $(X,\tau,E,\tilde{I})$ be a soft topological space with soft ideal and (H,E) be a semi closed soft set over X such that $x\notin(H,E)$ for $x\in X$. Then $(X,\tau,E,\tilde{I})$ is called a soft semi-$\tilde{I}$-regular space if there exist disjoint semi open soft sets (F,E) and (G,E) such that $x\in(F,E)$ and $(H,E)-(G,E)\in\tilde{I}$.

Examples 4.1.

(1)Let $X=\{h_1,h_2,h_3\}$, $E=\{e_1,e_2\}$ and $\tau=\{\tilde{X},\tilde{\phi},(F_1,E),(F_2,E),(F_3,E)\}$ where $(F_1,E),(F_2,E)$,
(F_3,E) are soft sets over X defined as follows:
$F_1(e_1)=\{h_1\}$, $F_1(e_2)=\{h_1\}$.
$F_2(e_1)=\{h_2\}$, $F_2(e_2)=\{h_2\}$.
$F_3(e_1)=\{h_1,h_2\}$, $F_3(e_2)=\{h_1,h_2\}$.
Then τ defines a soft topology on X. Let $\tilde{I}=\{\tilde{\phi},(I_1,E),(I_2,E),(I_3,E)\}$ be a soft ideal over X where $(I_1,E),(I_2,E),(I_3,E)$ are soft sets over X defined by:
$I_1(e_1)=\{h_3\}$, $I_1(e_2)=\phi$,
$I_2(e_1)=\phi$, $I_2(e_2)=\{h_3\}$,
$I_3(e_1)=\{h_3\}$, $I_3(e_2)=\{h_3\}$,
Then, $(X,\tau,E,\tilde{I})$ is a soft semi-$\tilde{I}$-regular space.
(2)Let $X=\{h_1,h_2\}$, $E=\{e_1,e_2\}$ and $\tau=\{\tilde{X},\tilde{\phi},(F_1,E),(F_2,E),(F_3,E),$
$(F_4,E)\}$ where $(F_1,E),(F_2,E),(F_3,E),(F_4,E)$ are soft sets over X defined as follows:
$F_1(e_1)=\{h_1\}$, $F_1(e_2)=\{h_2\}$,
$F_2(e_1)=\{h_2\}$, $F_2(e_2)=\{h_1\}$,
$F_3(e_1)=\phi$, $F_3(e_2)=\{h_1\}$,
$F_4(e_1)=\{h_1\}$, $F_4(e_2)=X$.
Then τ defines a soft topology on X. Let $\tilde{I}=\{\tilde{\phi},(I_1,E)\}$ be a soft ideal over X where (I_1,E) is soft set over X defined by $I_1(e_1)=\{h_1\}$, $I_1(e_2)=\phi$. Then $(X,\tau,E,\tilde{I})$ is not soft semi-$\tilde{I}$-regular space. Because, for the closed soft set (F_1',E) such that $h_1\notin(F_1',E)$, there are no two disjoint semi open soft sets (A,E) and (B,E) such that $h_1\in(A,E)$ and $(F_1',E)-(B,E)\in\tilde{I}$.

Lemma 4.1 Let (X,τ,E) be a soft topological space and $x \in X$. If $(X,\tau,E,\tilde{I})$ is soft semi-$\tilde{I}$-regular space. Then, for every semi closed soft set (G,E) such that $x_E \tilde{\cap} (G,E) = \tilde{\phi}$, there exist disjoint semi open soft sets (F_1,E) and (F_2,E) such that $x_E - (F,E) \in \tilde{I}$ and $(G,E) - (F_2,E) \in \tilde{I}$.
Proof. Immediate.

Proposition 4.1

(1)Every soft semi regular space is a soft semi-$\tilde{I}$-regular.
(2)If $\tilde{I} = \tilde{\phi}$, then $(X,\tau,E,\tilde{I})$ is soft semi regular space if and only if it is soft semi-$\tilde{I}$-regular space.

Proof. Immediate.

Theorem 4.1. (X,τ^*,E) is a soft semi-$\tilde{I}$-regular space if and only if (X,τ,E) is soft semi-$\tilde{I}$-regular space.
Proof. The proof is similar to the proof of Theorem 4.1.

Theorem 4.2. A soft subspace $(Y,\tau_Y,E,\tilde{I}_Y)$ of a soft semi-$\tilde{I}$-regular space $(X,\tau,E,\tilde{I})$ is soft semi-$\tilde{I}_Y$-regular space.
Proof.

Let $y \in Y$ and (G,E) be a semi closed soft set in Y such that $y \notin (G,E)$. Then $(G,E) = (Y,E)\tilde{\cap}(H,E)$ for some semi closed soft set (H,E) in X from Theorem 2.2. Hence, $y \notin (Y,E)\tilde{\cap}(H,E)$. But $y \in (Y,E)$, so $y \notin (H,E)$. Since $(X,\tau,E,\tilde{I})$ is soft semi-$\tilde{I}$-regular space. So, there exist disjoint semi open soft sets (F_1,E) and (F_2,E) in X such that $y \in (F_1,E)$, $(H,E) - (F_2,E) \in \tilde{I}$. Since $(Y,E)\tilde{\cap}(F_1,E)$ and $(Y,E)\tilde{\cap}(F_2,E)$ are disjoint semi open soft sets in Y such that $y \in (Y,E)\tilde{\cap}(F_1,E)$ and $(Y,E)\tilde{\cap}[(G,E) - (F_2,E)] = (Y,E)\tilde{\cap}[(H,E) - (F_2,E)] \in \tilde{I}_Y$. It follows that, $(Y,\tau_Y,E,\tilde{I}_Y)$ is soft semi-$\tilde{I}_Y$-regular spaces.

Theorem 4.3. Let (X,τ,E) be a soft topological space and $x \in X$. Then the following are equivalent:

(1)$(X,\tau,E,\tilde{I})$ is soft semi-$\tilde{I}$-regular space.
(2)For every semi open soft set (A,E) such that $x \in (A,E)$, there exists an semi open soft set (F,E) such that $x \in (F,E)$ and $SScl(F,E) - (A,E) \in \tilde{I}$.
(3)For every semi closed soft set (B,E) such that $x \notin (B,E)$, there exists an semi open soft set (F,E) such that $x \in (F,E)$, $SScl(F,E)\tilde{\cap}$ $(B,E) \in \tilde{I}$.

Proof. The proof is similar to the proof of Theorem 3.3.

Theorem 4.4. Let $f_{pu} : SS(X)_A \rightarrow SS(Y)_B$be a soft function which is bijective, irresolute soft and irresolute open soft. If (X,τ_1,A) is soft semi-$\tilde{I}$-regular space, then (Y,τ_2,B) is soft semi-$f_{pu}(\tilde{I})$-regular space.
Proof. The proof is similar to the proof of Theorem 3.5.

5 Soft semi normal spaces via soft ideals

Definition 5.1.

Let $(X,\tau,E,\tilde{I})$ be a soft topological space with soft ideal and $(H,E),(K,E)$ be disjoint semi closed soft sets over X. Then $(X,\tau,E,\tilde{I})$ is called a soft semi-$\tilde{I}$-normal space if there exist disjoint semi open soft sets (F,E) and (G,E) such that $(H,E) - (G,E) \in \tilde{I}$ and $(K,E) - (F,E) \in \tilde{I}$.

Proposition 5.1

(1)Every soft semi normal space is a soft semi-$\tilde{I}$-normal.
(2)If $\tilde{I} = \tilde{\phi}$, then $(X,\tau,E,\tilde{I})$ is soft semi normal space if and only if it is soft semi-$\tilde{I}$-normal space.

Proof. Immediate.

Theorem 5.1. (X,τ^*,E) is a soft semi-$\tilde{I}$-normal space if and only if (X,τ,E) is soft semi-$\tilde{I}$-normal space.
Proof. $(:\Rightarrow)$ Let (X,τ^*,E) be a soft semi-$\tilde{I}$-normal space and $(H,E),(K,E)$ be disjoint τ-semi closed soft sets. Since $\tau \subseteq \tau^*$. It follows that, $(H,E),(K,E)$ are disjoint τ^*-semi closed soft sets. Since (X,τ^*,E) is soft semi-$\tilde{I}$-normal space. Hence, there exist disjoint τ^*-semi open soft sets (F,E) and (G,E) such that $(K,E) - (F,E) \in \tilde{I}$ and $(H,E) - (G,E) \in \tilde{I}$. Since (F,E) and (G,E) are τ^*-open soft sets. Then, $(F,E) = (A,E) - (I_1,E)$ and $(G,E) = (B,E) - (I_2,E)$ where $(A,E),(B,E) \in \tau$ and $(I_1,E),(I_2,E) \in \tilde{I}$. Thus, $(K,E) - (F,E) = (K,E) - ((A,E) - (I_1,E)) \in \tilde{I}$ and $(H,E) - (G,E) = (H,E) - ((B,E) - (I_2,E)) \in \tilde{I}$. From Definition 2.27, $(K,E) - (A,E) \in \tilde{I}$ and $(H,E) - (B,E) \in \tilde{I}$. Therefore, (X,τ,E) is soft semi-$\tilde{I}$-normal space.
$(\Leftarrow:)$ Let (X,τ,E) be a soft semi-$\tilde{I}$-normal space and $(H,E),(K,E)$ be disjoint τ^*-semi closed soft sets. Since $(H,E)',(K,E)'$ are τ^*-semi open soft sets. Then, $(H,E)' = (G,E) - (I_1,E)$ and $(K,E)' = (F,E) - (I_1,E)$, where $(G,E),(F,E) \in \tau$ and $(I_1,E),(I_2,E) \in \tilde{I}$. Hence, $(G,E)',(F,E)'$ are disjoint τ-closed soft set. Since (X,τ,E) is soft semi-$\tilde{I}$-normal space, then there exist disjoint τ-semi open soft sets (A,E) and (B,E) such that $(F,E)' - (A,E) \in \tilde{I}$ and $(G,E)' - (B,E) \in \tilde{I}$. So, $(F,E)' - (A,E) = [(K,E) - (I_1,E)] - (A,E) \in \tilde{I}$ and $(G,E)' - (B,E) = [(H,E) - (I_2,E)] - (B,E) \in \tilde{I}$. It follows that, $(K,E) - (A,E) \in \tilde{I}$ and $(H,E) - (B,E) \in \tilde{I}$. Therefore, (X,τ^*,E) is a soft semi-$\tilde{I}$-normal space.

Theorem 5.2. A closed soft semi subspace $(Y,\tau_Y,E,\tilde{I}_Y)$ of a soft semi-$\tilde{I}$-normal space $(X,\tau,E,\tilde{I})$ is soft semi-$\tilde{I}_Y$-normal.
Proof. Let $(H,E),(K,E)$ be disjoint semi closed soft sets over Y. Since $\tilde{Y}$ is a closed soft set over X. Then, $(H,E),(K,E)$ are disjoint semi closed soft sets over X. By hypothesis, there exist disjoint semi open soft sets (F,E) and (G,E) over X such that $(H,E) - (G,E) \in \tilde{I}$ and $(K,E) - (F,E) \in \tilde{I}$. It follows that, $\tilde{Y}\tilde{\cap}[(H,E) - (G,E)] \in \tilde{I}_Y$ and $\tilde{Y}\tilde{\cap}[(K,E) - (F,E)] \in \tilde{I}_Y$. Hence, $(H,E) - [\tilde{Y}\tilde{\cap}(G,E)] \in \tilde{I}_Y$ and $(K,E) - [\tilde{Y}\tilde{\cap}(F,E)] \in \tilde{I}_Y$, where $\tilde{Y}\tilde{\cap}(G,E)$ and $\tilde{Y}\tilde{\cap}(F,E)$ are disjoint semi open soft sets over Y. Therefore, $(Y,\tau_Y,E,\tilde{I}_Y)$ is a soft-$\tilde{I}_Y$-normal space.

Theorem 5.3. Let (X,τ,E) be a soft topological space and $x \in X$. Then the following are equivalent:

(1)$(X,\tau,E,\tilde{I})$ is a soft semi-$\tilde{I}$-normal space.
(2)For every semi open soft set (A,E) containing a semi closed soft set (H,E), there exists a semi open soft set (G,E) such that $(H,E)-(G,E)\in\tilde{I}$ and $SScl(G,E)-(A,E)\in\tilde{I}$.
(3)For every disjoint semi closed soft sets (H,E) and (K,E), there exists a semi open soft set (G,E) such that $(H,E)-(G,E)\in\tilde{I}$ and $SScl(G,E)\tilde{\cap}(K,E)\in\tilde{I}$.

Proof.

(1) $\Rightarrow$ (2) Let (A,E) be a semi open soft set containing a semi closed soft set (H,E). Then, $(A,E)'$ and (H,E) are disjoint semi semi closed soft sets over X. It follows by (1), there exist disjoint semi open soft sets (F_1,E) and (F_2,E) such that $(H,E)-(F_1),E)\in\tilde{I}$ and $(A,E)'-(F_2,E)\in\tilde{I}$. Since $(F_1,E)\tilde{\cap}(F_2,E)=\tilde{\phi}$. Then, $SScl(F_1,E)\tilde{\subseteq}(F_2,E)'$. It follows that, $SScl(F_1,E)-(A,E)=(A,E)'\tilde{\cap}SScl(F_1,E)\tilde{\subseteq}(A,E)'\tilde{\cap}(F_2,E)'=(A,E)'-(F_2,E)\in\tilde{I}$. From Definition 2.27, $SScl(F_1,E)-(A,E)\in\tilde{I}$.

(2) $\Rightarrow$ (3) Let (H,E) and (K,E) be disjoint semi semi closed soft sets. Then, $(K,E)\tilde{\subseteq}(H,E)'$. This means that, $(H,E)'$ is a semi open soft set containing a semi closed soft set (K,E). From (2), there exists a semi open soft set (G,E) such that $(K,E)-(G,E)\in\tilde{I}$ and $SScl(G,E)\tilde{\cap}(H,E)=SScl(G,E)-(H,E)'\in\tilde{I}$.

(3) $\Rightarrow$ (1)Let (H,E) and (K,E) be disjoint semi semi closed soft sets. From (3), there exists a semi open soft set (G,E) such that $(H,E)-(G,E)\in\tilde{I}$ and $(K,E)-[SScl(G,E)]'=SScl(G,E)\tilde{\cap}(K,E)\in\tilde{I}$, where (G,E) and $[SScl(G,E)]'$ are disjoint semi open soft sets. Therefore, $(X,\tau,E,\tilde{I})$ is soft semi-$\tilde{I}$-normal space.

Theorem 5.4. Let $f_{pu}:SS(X)_A\rightarrow SS(Y)_B$ be a soft homeomorphism function. If (X,τ_1,A) is a soft semi-$\tilde{I}$-normal space, then (Y,τ_2,B) is a soft semi-$f_{pu}(\tilde{I})$-normal space.

Proof. Let $(F,B),(G,B)$ be disjoint closed soft sets in Y. Since f_{pu} is soft homeomorphism, then $(S,A)=f_{pu}^{-1}(F,B)$ and $(V,A)=f_{pu}^{-1}(G,B)$ are closed soft sets in X such that $(S,A)\tilde{\cap}(V,A)=f_{pu}^{-1}[(F,B)\tilde{\cap}(G,B)]=f_{pu}^{-1}[\tilde{\phi}_B]=\tilde{\phi}_A$ from Theorem 2.3. By hypothesis, there exist open soft sets (K,A) and (H,A) in X such that $(S,A)-(K,A)\in\tilde{I}$ and $(V,A)-(H,A)\in\tilde{I}$. It follows that, $f_{pu}[(S,A)-(K,A)]=f_{pu}(S,A)-f_{pu}(K,A)\in f_{pu}(\tilde{I})$ and $f_{pu}[(V,A)-(H,A)]=f_{pu}(V,A)-f_{pu}(H,A)\in f_{pu}(\tilde{I})$ from Theorem 2.5 and Theorem 3.4. Since f_{pu} is a soft homeomorphism. Then, $f_{pu}(K,A)=(K,B)$ and $f_{pu}(H,A)=(H,B)$ are disjoint open soft sets in Y. Therefore, (Y,τ_2,B) is a soft semi-$f_{pu}(\tilde{I})$-normal space.

6 Conclusion

Recently, many scientists have studied and improved the soft set theory, which is initiated by Molodtsov [22] and easily applied to many problems having uncertainties from social life.Shabir and Naz in [27] introduced and studied the notion of soft topological spaces. They defined soft topology on the collection τ of soft sets over X. Consequently, they defined soft separation axioms, soft regular spaces and soft normal spaces and established their several properties. Won in [30] investigate some properties of these soft separation axioms. The notion of soft ideal is initiated for the first time by Kandil et al.[14]. They also introduced the concept of soft local function. These concepts are discussed with a view to find new soft topologies from the original one, called soft topological spaces with soft ideal $(X,\tau,E,\tilde{I})$. Applications to various fields were further investigated by Kandil et al. [10,11,13,15] introduce the notion of soft semi separation axioms. In particular, they study the properties of the soft semi regular spaces and soft semi normal spaces. In the present paper,we introduce the notions of soft-$\tilde{I}$-regular spaces and soft-$\tilde{I}$-normal spaces. Also, the notions of soft semi regular spaces and soft semi normal spaces via soft ideals, which is weaker than soft semi regularity and soft semi normality mentioned in [12]. Also, we discuss their properties in detail. We hope that the results in this paper will help researcher enhance and promote the further study on soft topology to carry out a general framework for their applications in practical life.

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