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Transmitted Torque Analysis of a Magnetic Gear Mechanism with Rectangular Magnets

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Abstract: Magnetic gears transmit torque by noncontact magnetic coupling rather than meshed mechanical gear teeth. In contrast to traditional mechanical gear mechanisms, non-contact magnetic gear mechanisms possess unique features of low mechanical energy loss, overload protection ability, no need for lubricants, tolerance of misalignment and vibration isolation between adjacent mechanical parts. The analysis of transmitted torque is a prerequisite task when evaluating the performance of a magnetic gear mechanism. In this study, a feasible method for predicting transmitted torque generated by external-type magnetic gear sets with rectangular magnets is demonstrated by employing the current sheet approach. Based on the current sheet mathematical model presented, the effects of geometrical and material parameters, including the number of magnet pole pairs, the width of the air gap and the remanence of permanent magnets, on transmitted torques of external-type magnetic gear sets can be analytically evaluated and discussed. The two-dimensional finite-element analysis (FEA) is further employed in computing the magnetic fields and transmitted torques of external-type magnetic gear sets. The results of this paper are beneficial for further design purposes and optimization of magnetic gear mechanisms.

Keywords: magnetic gear mechanism, transmitted torque, magnetic field, finite-element analysis

1 Introduction

A gear is a power-transmitting element that transmits rotary motion from one shaft to another by means of successively engaging teeth. A gear mechanism or a gear train is any collection of two or more meshing gears, which is widely used in mechanical devices and machines nowadays. However, there are inherent problems such as vibration, noise, abrasion and the necessity of lubrication due to contact force and friction. Recently, a noncontact magnetic gear that utilizes permanent magnets in order to solve these problems has been developed. It transmits torque by noncontact magnetic coupling rather than meshed mechanical gear teeth. Compared with conventional mechanical gear mechanisms, noncontact magnetic gear mechanisms possess unique attributes, including: no need for lubricants, low mechanical energy losses, overload protection, and tolerance of misalignment between the input and output shafts [1]. Beginning in 1941, Faus [2] presented a magnetic gear, with a geometric configuration analogous to ancient mechanical pin gears, with magnetic pins inserted in the base of the magnetic gear. A magnetic worm and worm gear set was presented in the same U. S. patent. Originating in 1995, Yao et al. [3,4] proposed a bi-axial external magnetic gear set with sector-shaped magnets. They presented the computer-aided simulation results and experimental analyses of the magnetostatic field generated by two external-type magnetic gears with parallel axes. Besides, they found that the transmitted torque, which is an important performance index of magnetic gear mechanisms, is highly related to the number of magnet pole pairs, the magnet's materials and the distance between two central axes of magnetic gears. In 1997, Furlani [5] proposed a magnetic charge model to predict the transmitted torque between two external magnetic gears. Nagrial et al. [6,7,8] made use of NdFeB magnets to allow an external-type magnetic gear set to generate high magnetic flux densities within the air gap. They also compared the predicted and experimental performance of such a magnetic gear set. However, these studies mentioned above only discussed magnetic gear mechanisms with radially polarized and sector-shaped magnets.

The aim of this study is to analyze the transmitted torque of an external-type magnetic gear mechanism with rectangular magnet blocks. A two-dimensional (2-D)

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analytical approach employing the current sheet model is applied to evaluate the magnetic flux densities within the air gap, and then, to calculate the transmitted torque of the proposed magnetic gear mechanism. These two significant items can be derived in terms of the magnet's material properties and the geometric parameters of the magnetic gear mechanism. The analytical results are also compared to those of a 2-D finite-element analysis (FEA) by employing a commercial package, ANSOFT/ Maxwell.

2 An external-type magnetic gear mechanism

A magnetic gear consists of alternate magnet poles affixed to the inner or outer surface of a ring-shaped iron yoke. Permanent magnets with even numbers are required for a magnetic gear. The simplest magnetic gear mechanism consists of two separated magnetic gears constrained to rotate about their respective axes. Figure 1 shows an external magnetic gear set with rectangular magnets. The space between the magnetic gears is the air gap. All permanent magnets are equipped with identical width and height. Compared with the sector-shaped magnets, rectangular magnets have the merit of low cost

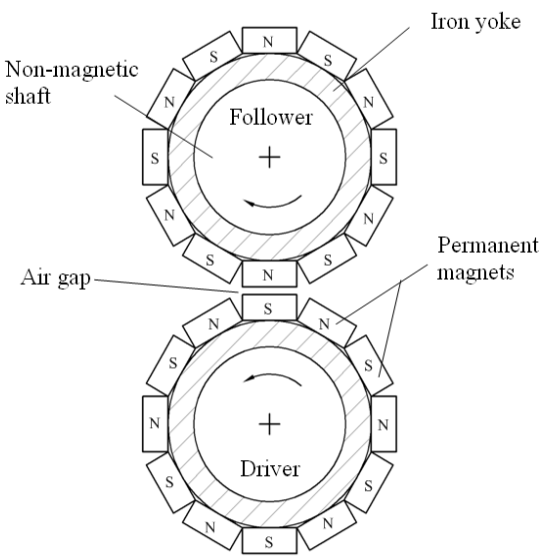

Figure 1: An external magnetic gear set with rectangular magnets.

due to available dimensional materials. The ring-shaped iron yoke provides the base for mounting the magnets and efficiently conducts the magnetic flux to form closed flux loops due to its high permeability. The rotations of the driver and the follower are in opposite directions for the external-type magnetic gear set. When the driver rotates, it induces a magnetic torque, i.e. the transmitted torque, to cause the follower to rotate simultaneously. It is a periodic torque that varies with respect to the angular displacement of the driver. The maximum transmitted torque occurs when the angular displacement is half of the pitch angle of a magnet pole. Once the driving torque from the driver exceeds the critical value of the transmitted torque, then, slipping occurs. Therefore, a magnetic gear mechanism is not only a transmission device but also an overload protection device.

3 Transmitted torque analysis

Because the transmitted torque is an important performance index of a magnetic gear mechanism, the prediction of this torque is an essential task to the complete design of magnetic gears. However, magnetic field distribution governs the transmitted torque of a magnetic gear mechanism. There are several approaches to evaluate the magnetic field of magnetic devices including analytical methods and numerical techniques. The analytical method, based on reasonable hypotheses and simplified mathematical models, gives sufficient insight into the effects of geometric and material parameters on the magnetic properties. In this study, the approach of the current sheet model is employed to calculate the magnetic field of an external-type magnetic gear mechanism, especially for the magnetic flux densities within the air gap. The magnetomotive force (MMF) of each permanent magnet is equivalent to a series of infinitesimal current sheets [9] located along the direction of magnetization of the permanent magnet, i.e. the direction of magnet height. For rectangular magnetic poles, shown in Figure 1, a two-dimensional model with orthogonal coordinates is utilized. Some hypotheses for simplifying the analysis of the magnetic modeling are listed as follows:

1. The field is only two-dimensional.
2. The ring-shaped iron yoke has a high permeability, and there is no magnetic saturation in the iron yoke.
3. The relative permeability of the magnet is 1.
4. The slot effect is neglected, and the air gap is regarded as a uniform region.
5. One third of the permanent magnets of the driver are magnetically coupled to those of the follower for this magnetic gear set at any instant.

The quasi-static magnetic field $\overrightarrow{B}$ is expressed as $\nabla \times \overrightarrow{A} = \overrightarrow{B}$, where $\overrightarrow{A}$ is the magnetic vector potential. Such a magnetic field problem can be simplified as the solution of Laplace's Equation $\nabla \cdot \nabla \overrightarrow{A} = 0$. It is generally expressed in Cartesian coordinates as:

$$\frac{\partial^2 A}{\partial x^2} + \frac{\partial^2 A}{\partial y^2} = 0 \tag{1}$$

The solution for magnetic vector potential $\overrightarrow{A}$ for equation (1) is [10] :

$$A_n = [P_n sinh(my) + Q_n cosh(my)] \cdot cos(mx) \tag{2}$$

where the symbol n denotes the regions a', a'', and d, as illustrated in Figure 2. The parameter m is equal to π/τ, and τ is the magnet pole pitch of the magnetic gear. The magnetic flux densities can be separated into the x component and y component, i.e., $B_{nx} = \frac{\partial A_n}{\partial y}$ and $B_{ny} = -\frac{\partial A_n}{\partial x}$, can be derived as:

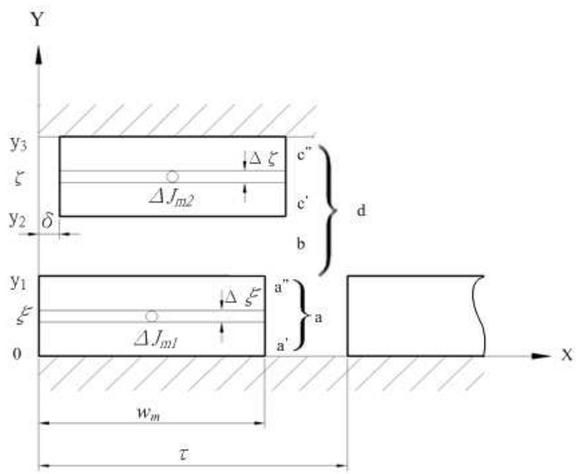

Figure 2: A two-dimensional model of two coupled rectangular magnets [10].

$$B_{nx} = m[P_n \cosh(my) + Q_n \sinh(my)] \cdot \cos(mx) \quad (3)$$

$$B_{ny} = m[P_n \sinh(my) + Q_n \cosh(my)] \cdot \sin(mx) \quad (4)$$

The above coefficients P_n and Q_n can be obtained according to the boundary conditions at the interfaces of different materials. They must satisfy a continuous normal component of magnetic field $\overrightarrow{B}$. In addition, a continuous tangential component of the magnetic field of intensity $\overrightarrow{H}$ should be followed. Based on the results of Huang and Sung [10], the y component, i.e. the radial component, of the magnetic flux density within the air gap $B_{y-airgap}$ is:

$$B_{y-airgap} = (\frac{\mu_0 \hat{J}}{m}) \cdot \sinh(my_1) \cdot \operatorname{csch}(my_3) \cdot \cosh(my - my_3) \cdot \sin(mx) \quad (5)$$

where u_o is the permeability of air, $\hat{J} = (\frac{4B_r}{\mu_0 \tau}) \cdot \sin(\frac{\pi}{2} \cdot \alpha_e)$, B_r is the residual flux density of the permanent magnet, α_e is eqal to w_m/τ, and w_m is the width of a permanent magnet. From Lorentz's Force Equation, the transmitted torque of this magnetic gear set, in which one third of the permanent magnets contribute to torque transmission based on the assumptions mentioned above, can be written as:

$$T = \frac{nL}{3} \int_{y_2}^{y_3} \int_0^{\tau} \hat{J} \cdot (y + R_2) \cdot \cos(mx + m\delta) \cdot B_{y-airgap}\, dx\, dy \quad (6)$$

where n is the number of magnet pole pairs, L is the length of the magnetic gear in the axial direction, R_2 is the outer radius of the iron yoke, y_2 and y_3 are shown in Table 1 and δ is the relative shift between the driver and the follower, respectively. Because the above derivation is based on a two-dimensional model, there exists a shift angle α between the driver and the follower. Figure 3 illustrates that the width of the air-gap should be corrected as: $g' = \Delta + g$. According to the geometric relationship shown in Figure 3, length Δ is equal to $(R_3 - R_3 \cos\alpha)$. Besides, angle α is equal to $(\pi/2n)$ which is half of the pitch angle of a magnetic pole. The modified air-gap width is further applied to evaluate the maximum transmitted torque of a magnetic gear mechanism [10]. Due to the derivation of the width of the air-gap for an external-type magnetic gear mechanism, the radiuses y_2 and y_3 should be further corrected, as listed in Table 1. For numerical calculations, a magnetic gear mechanism with two identical magnetic gears is taken to illustrate the proposed mathematical model. Figure 4 shows the related cross-sectional view and design parameters of this magnetic gear mechanism. Related values of the magnet's material properties and geometric dimensions are given in Table 2. Figure 5 illustrates the transmitted torque of the external-type magnetic gear mechanism against the angular displacement of the driver. As expected, the maximum transmitted torque occurs when the angular displacement is 180°/(2*6)=15°. Figure 6 depicts relationships between the maximum transmitted torque and the number of magnet pole pairs, which is varied between 5 and 10. The total volume of permanent magnets is identical for each magnetic gear set, so that the MMF of each gear set is the same. It is interesting to find that the transmitted torque increases with the number of magnet pole pairs and diminishes after it researches six pole pairs for this external-type of magnetic gear topology. Therefore, the optimal design for selecting the number of magnet pole pairs is six, so that the magnetic gear mechanism generates the largest transmitted torque. Figure 7 shows the relationships between the maximum transmitted torque and the width of the air-gap which is changed from 1.5 mm to 3.0 mm. It is clear that the maximum transmitted torque decreases as the width of the air gap increases. Besides, Figure 8 demonstrates the maximum transmitted torque plotted against the remanence of the permanent magnet changed from 0.6 T to 1.3 T. The maximum transmitted torque increases in accordance with the magnet remanence when no flux saturation occurs in the iron yoke. Therefore, the maximum transmitted torque of an external-type magnetic gear mechanism is sensitive to the number of magnet poles, the width of the air-gap and the remanence of the permanent magnet.

Table 1: Modification of dimensions [10].

Items	Original dimensions	Modified dimensions
Air-gap width	g	$g' = g + (R_3 - R_3 \cos\alpha)$
y_2	$h+g$	$h+g'$
y_3	$2h+g$	$2h+g'$

* h is the height of the permanent magnet.

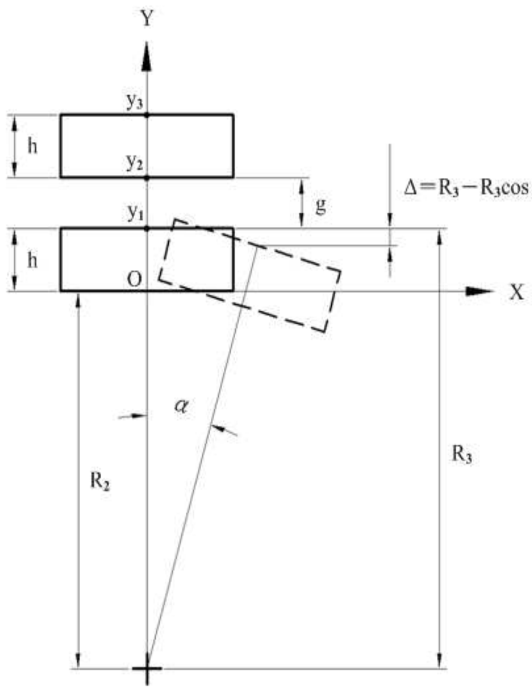

Figure 3: Planar model of two coupled rectangular magnets [10].

Table 2: Specifications of a magnetic gear set shown in Figure 4.

NdFeB (N-35) magnet		
Items	Symbol	Values
Remanence (T)	B_r	1.15
Relative permeability	u_r	1.09
Magnet width (mm)	w_m	10.70
Magnet height (mm)	h	5.00
Axial length (mm)	L	40.00
Number of pole pairs	n	6
Geometric dimensions of the gear set		
Items	Symbol	Values
Inner radius of the iron yoke (mm)	R_1	15.00
Outer radius of the iron yoke (mm)	R_2	20.00
Air-gap length (mm)	g	1.50
Length of pole pitch (mm)	τ	11.78
Ratio of pole width to pole pitch	α_e	0.91

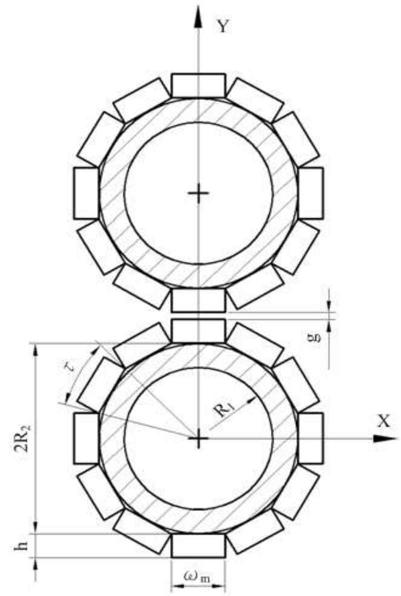

Figure 4: Geometric parameters of the proposed magnetic gear set.

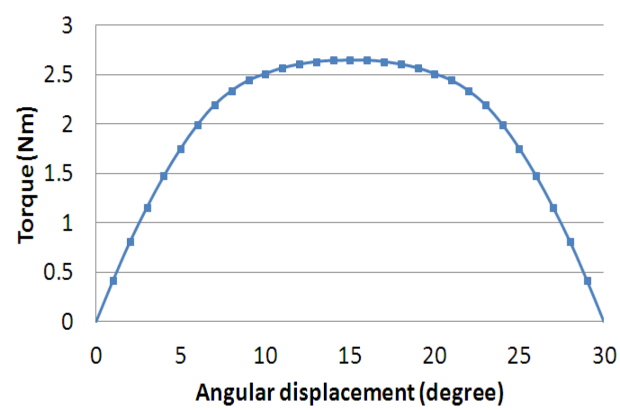

Figure 5: Transmitted torque against the angular displacement of the driver.

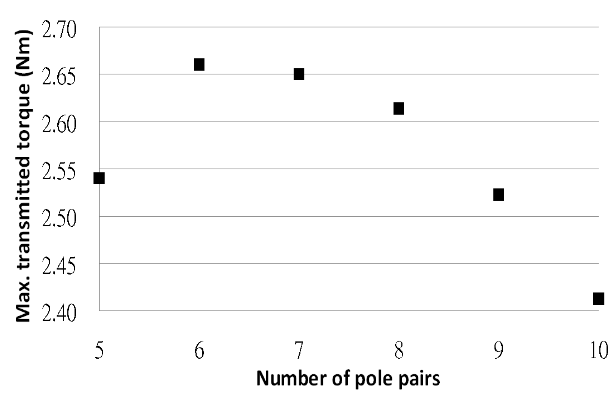

Figure 6: Relationships between the maximum transmitted torque and the number of magnet pole pairs.

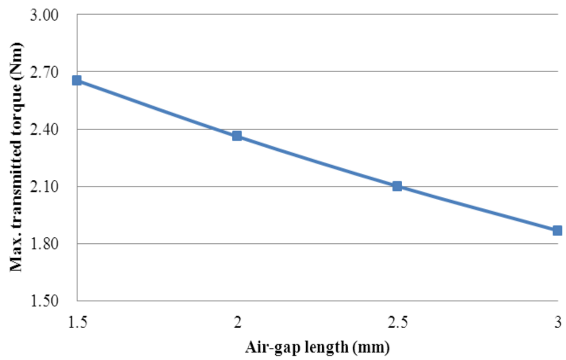

Figure 7: Relationships between the maximum transmitted torque and the width of the air-gap.

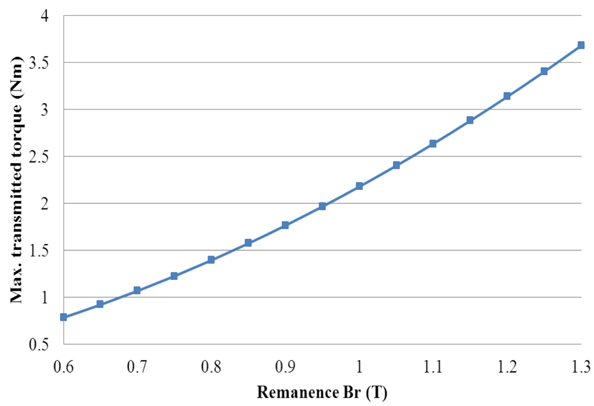

Figure 8: Relationships between the maximum transmitted torque and the remanence (Br) of the magnet.

4 FEA verification

In this study, the finite-element method is further employed to calculate the transmitted torque of the proposed magnetic gear mechanism, and also, to verify the validity of the above mentioned analytical model. Commercial FEA software Ansoft/Maxwell is used on the magnetic field and transmitted torque analyses. Figure 9 shows the finite element meshing of the proposed magnetic gear set. Figure 10 demonstrates the flux line distribution. We can find some leakage flux failures to magnetically couple with other flux. So, the leakage flux can not effectively contribute to the transmitted torque and should be carefully detected when designing magnetic gears. Figure 11 presents the flux density distribution of the magnetic gear set when the driver rotates 15°. This is the position where the maximum magnetic flux density and maximum transmitted torque occur. The maximum magnetic flux density is about 1.50 T that occurs at the narrow space between the permanent magnet and the iron yoke due to the magnetic flux concentration. Besides, we can find that the magnetic flux density is under 1.2 T at the iron yoke. Since it is less than the saturated magnetic flux density of 1.39 T for the iron yoke, no saturation occurs on the main body of this magnetic gear set. Table 3 shows a comparison of the analytical computations and the FEA results for the maximum transmitted torque of the proposed magnetic gear mechanism when the air-gap width is equal to 1.5 mm. The errors between these two methods are within 10%. The errors mainly arise from the assumption that one third of the permanent magnets of the driver are coupled to those of the follower, since it is difficult to predict exactly how many magnet poles are coupled to one another. Besides, the analytical model does not take the leakage flux into consideration. These two approaches both depict that the maximum transmitted torque occurs when the number of magnet pole pairs is equal to six for the proposed external-type magnetic gear mechanism with rectangular magnets.

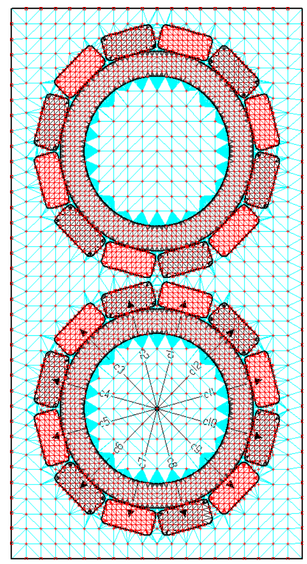

Figure 9: Finite element meshing of the magnetic gear mechanism.

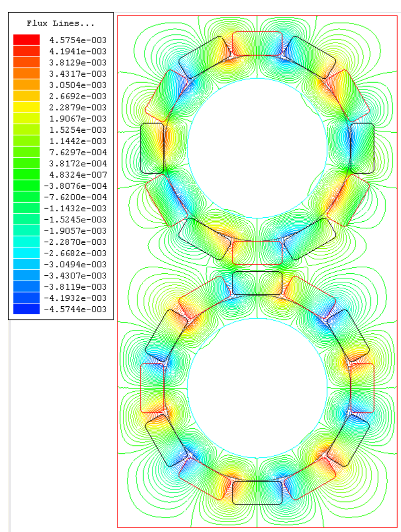

Figure 10: Distribution of the magnetic flux lines of the magnetic gear mechanism by FEA.

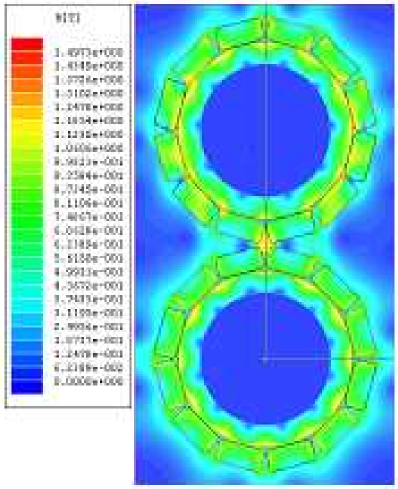

Figure 11: Distribution of magnetic flux densities for the proposed magnetic gear mechanism when the driver rotates 15°.

Table 3: Comparison of the maximum transmitted torque for FEA and analytical calculation.

Number of pole pairs	Analytical results	FEA results	Error (%)
5	2.54	2.81	-9.61
6	2.66	2.87	-7.32
7	2.65	2.76	-3.99
8	2.61	2.59	0.77
9	2.52	2.40	5.00
10	2.41	2.21	9.05

* Unit: Nm
**Error=
(Analytical results-FEA results)/FEA results*100%

5 Application fields

Magnetic gear mechanisms possess the features of low cost, simple geometries, compact size and overload protection and are highly related to green technology. They can be appropriately employed in some mobile and handheld information devices to replace the traditional gearbox with a counter weight on the output shaft to form a vibration geared motor within these devices. The magnetic gear may make the gearbox more reliable and does not need lubricants and maintenance.

6 Conclusions

The magnetic field and the transmitted torque of an external magnetic gear set with rectangular magnets have been investigated by using the current sheet model and the FEA. Although the FEA is widely considered to provide the most accurate estimations when the geometric discretization is fine enough, it is often time-consuming and does not provide useful information for further optimal design. In contrast, the major advantage of the current sheet model is the analytical expression of the transmitted torque which depicts clear information and relationships between the transmitted torque, the magnet's material properties and geometric dimensions of a magnetic gear mechanism. This is beneficial to further design purposes and optimization of magnetic gear mechanisms. The results of this work can be extended to external or internal magnetic gear sets with rectangular or sector-shaped permanent magnets.

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