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An Alternative Solution Technique of the JIT Lot-Splitting Model for Supply Chain Management

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Abstract: Recently, Kim and Ha [S.-L. Kim and D. Ha, A JIT lot-splitting model for supply chain management: Enhancing buyer-supplier linkage, *Internat. J. Prod. Econ.* **86** (2003), 1–10] proposed a model to determine the optimal order quantity, the number of shipments and size delivered over a finite planning horizon in a JIT single-buyer and single-supplier scenario. Kim and Ha's model is interesting. However, their solution procedure and some theoretical results may not be generally true. In this paper, we propose an analytical solution procedure free from using convexity to correct and improve on Kim and Ha's model. Some flaws shown in Kim and Ha's paper are also corrected. This paper further presents sufficient conditions to illustrate when the single-setup-multiple-delivery (SSMD) policy is more beneficial over the single-delivery policy. Furthermore, this paper finds the minimum order quantity $Q_{\min}$ that makes the SSMD policy favorable over the single-del!ivery policy. Numerical examples are provided to illustrate the above results.

Keywords: JIT (Just-in-Time) manufacturing; JIT lot-splitting strategy; Buyer-supplier linkage; Integrated inventory model; Supply chain model; Optimal solution.
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1 Introduction

Numerous studies have revealed that many manufacturing processes have greater improvements in performance due to the implementation of the Just-in-Time (JIT) policy. One of the most important means to assure that JIT is successfully implemented is the integrated inventory policy. Goyal [11] was a pioneer in the study of the integrated joint optimization inventory models consisting of a single supplier and a single buyer. Subsequently, many excellent researchers (for example, Joglekar [17], Yang and Wee [28] and Lin and Yeh [21]) employed their ideas into such different scenarios as (for example) deteriorating items, imperfect items, *et cetera*. An up-to-date review of the integrated inventory model for a lot with equal- and/or unequal-sized shipments has been provided by Zavanella and Zanoni [30], Hoque [13], and Glock [10], respectively. Ben-Daya *et al.* [2] dealt with a three-layer supply chain model in which the system consisted of a single supplier, a single manufacturer and multiple retailers. They employed a derivative-free solution procedure to derive a near optimal solution to the model at hand. Jaber *et al.* [16] investigated a three-layer supply chain (supplier-manufacturer-retailer) where the manufacturing operations undergo a learning-based consideration improvement process. In their work, mathematical models achieving chain-wide lot-sizing integration were developed allowing the manufacturer to justify a policy based on more frequent, smaller lot size production. Hoque [12] developed a generalized single-vendor multi-buyer integrated supply chain model considering production flow synchronization. They employed general differentiation and Lagrange multiplier techniques to obtain minimal cost solutions and showed single-vendor single-buyer as well as the single-vendor multiple-buyer models as their special cases. Many other closely-related recent investigations on the subject of this paper can be found in (for example) [4] to [8] (and also in

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many of the references to earlier works cited in each of these recent publications).

Recently, Kim and Ha [18] developed a JIT lot-splitting model that dealt with single-buyer single-supplier coordination. They explored the effects of a JIT lot-splitting strategy on the joint total relevant costs by examining the optimal order quantity, the number of shipments and the delivery size over a finite planning horizon. Some recent works demonstrated Kim and Ha's idea still received researcher's attention. Huang [14] expanded Kim and Ha's model to the imperfect nature of items by employing the concept of Salameh and Jaber [25]. Rau and OuYang [24] presented an integrated production-inventory policy under a finite planning horizon and a linear trend in demand in which the vendor supplies a single product to a buyer with a non-periodic and the JIT (Just-in-Time) replenishment policy in a supply chain environment. Huang *et al.* [15] and Chen and Kang [3] take the issue of trade credit into JIT implement account. Yan *et al.* [27] developed an integrated single-supplier and single-buyer inventory model for a deteriorating item in a JIT environment. Cost functions for the supplier, the buyer and the integrated supply chain are derived. Lin [20] integrated overlapped delivery and imperfect items into the production-distribution model and observed that Kim and Ha's work is a special case of his model. Omar *et al.* [22] considered a three-stage production-distribution model, under a JIT manufacturing environment, where the manufacturer must deliver the products in small quantity to minimize the suppliers as well as the buyers holding cost. Lee and Kim [19] mentioned that JIT still has a certain level of dominance in spite of the Toyota recall shock in 2009 and 2010, due to the effect of its contributions and improvement on the global economics ever since its emergence. Moreover, Deloof [9], Ramachandran and Jankiraman [23], Yıldız and Ustaoğlu [29] focusing on Belgian, Indian, and Turkish companies, respectively, have demonstrated that JIT had a positive influence on business performance. The above discussions illustrate the fact that Kim and Ha's model still received many attentions in recent years. In Kim and Ha's work the integrated total relevant cost $TC(Q,N)$ is treated as a function of two decision variables N (the number of deliveries per batch cycle from the supplier to the buyer) and Q (the order quantity for the buyer). Their model is correct and interesting. However, as we have pointed out in our present investigation, their solution procedures and theoretical results may not be generally true. From the academic viewpoint, we do need to remove Kim and Ha's flaws in [18] and thus to help managers making his decision correctly. Therefore, the purpose of this paper is six-fold as indicated below:

(A) Kim and Ha [18, p. 5] indicated that the integrated total relevant cost $TC(Q,N)$ is convex. However, this paper reveals that $TC(Q,N)$ is generally not necessarily convex.

(B) This paper gives some sufficient conditions to illustrate when the single-setup-multiple-delivery (SSMD) policy is more beneficial over the single-delivery policy.

(C) This paper finds the minimum order quantity $Q_{\min}$ that makes the SSMD policy favorable over the single-delivery policy.

(D) This paper shows that Fact 1(b) in Kim and Ha [18, p. 6] is not necessarily true.

(E) This paper reveals that Theorem 1 and Corollary 1 in Kim and Ha [18, p. 7] are not necessarily true.

(F) This paper develops an analytical solution procedure free of using the convexity to correct and improve Kim and Ha [18].

Numerical examples are also provided to illustrate the above results.

2 Formulation of the Mathematical Model

The notations and assumptions adopted by this paper are the same as those in Kim and Ha [18].

Notations:

A = Ordering cost for buyer

D = Annual demand rate for buyer

F = Fixed transportation cost per trip

C = Supplier's hourly setup cost

H_B = Holding cost/unit/year for buyer

H_S = Holding cost/unit/year for supplier

N = Number of deliveries per batch cycle (integer value)

P = Annual production rate for supplier, $P > D$

Q = Order quantity for buyer

q = Delivery size per trip, $q = Q/N$

S = Setup time/set up for supplier

V = Unit variable cost for order handling and receiving

Assumptions:

(1) Supply chain system consists of a single supplier and a single buyer
(2) Demand for the item is constant over time
(3) Production rate is uniform and finite
(4) Delivery times is constant
(5) Transportation and order handling costs are paid by the buyer in order to facilitate frequent deliveries

(6) The supplier splits the order quantity into small lot sizes and deliveries them over multiple periods
(7) No quantity discounts
(8) Shortages are not allowed

Kim and Ha [18, p. 3] assume that $H_B > H_S$. However, in this paper, we do not need this assumption to generalize and improve the work of Kim and Ha [18]. Based upon the above notations and assumptions, Kim and Ha [18, p. 5] obtain the integrated total relevant cost function $TC(Q,N)$ for buyer and supplier as follows:

$$\begin{aligned} TC(Q,N) = {} & \frac{D}{Q}(A+CS) \\ & + \frac{Q}{2N}\left[H_B + H_S\left(\frac{(2-N)D}{P} + N - 1\right)\right] \\ & + \frac{DNF}{Q} + DV. \end{aligned} \tag{1}$$

3 The Convexity of $TC(Q,N)$

Equation (1) yields the first-order and second-order partial derivatives with respect to Q and N as follows (see, for example, [26]):

$$\frac{\partial TC(Q,N)}{\partial N} = \frac{Q[-H_B + H_S(1-2D/P)]}{2N^2} + \frac{DF}{Q}, \tag{2}$$

$$\begin{aligned} \frac{\partial TC(Q,N)}{\partial Q} = {} & \frac{-D(A+CS+NF)}{Q^2} \\ & + \frac{[H_B - H_S(1-2D/P)]/N + (1-D/P)H_S}{2}, \end{aligned} \tag{3}$$

$$\frac{\partial^2 TC(Q,N)}{\partial N^2} = \frac{Q[H_B - H_S(1-2D/P)]}{N^3}, \tag{4}$$

$$\frac{\partial^2 TC(Q,N)}{\partial Q^2} = \frac{2D(A+CS+NF)}{Q^3}, \tag{5}$$

$$\frac{\partial TC(Q,N)}{\partial N \partial Q} = \frac{[-H_B + H_S(1-2D/P)]}{2N^2} - \frac{DF}{Q^2} \tag{6}$$

and

$$\begin{aligned} & \left(\frac{\partial^2 TC(Q,N)}{\partial N^2}\right)\left(\frac{\partial^2 TC(Q,N)}{\partial Q^2}\right) - \left(\frac{\partial^2 TC(Q,N)}{\partial N \partial Q}\right)^2 \\ & = \frac{1}{4N^4Q^4}\Big[8DNQ^2(A+CS+NF)[H_B - H_S(1-2D/P)] \\ & - \left\{-Q^2[H_B - H_S(1-2D/P)] - 2N^2DF\right\}^2\Big]. \end{aligned} \tag{7}$$

Theorem 4.30 of Avriel [1, p. 91] demonstrates that $TC(Q,N)$ is convex if and only if

$$\frac{\partial^2 TC(Q,N)}{\partial Q^2} > 0, \tag{8}$$

$$\frac{\partial^2 TC(Q,N)}{\partial N^2} > 0 \tag{9}$$

and

$$\left(\frac{\partial^2 TC(Q,N)}{\partial N^2}\right)\left(\frac{\partial^2 TC(Q,N)}{\partial Q^2}\right) - \left(\frac{\partial^2 TC(Q,N)}{\partial N \partial Q}\right)^2 > 0 \tag{10}$$

for all $N \geqq 1$ and $Q > 0$.

Kim and Ha [18, p. 5] indicate that the Hessian matrix of the equation (1) is positive definite and ensures that the total cost function in the equation (1) is jointly convex. The following example shows that the declaration of Kim and Ha [18] may not necessarily be true.

Example 1. Let $A = 10$, $C = 10$, $S = 5$, $N = 5$, $Q = 1000$, $H_B = 5$, $H_S = 3$, $P = 10000$, $D = 5000$ and $F = 100$. Both the equations (8) and (9) then hold true. However,

$$\left(\frac{\partial^2 TC(Q,N)}{\partial N^2}\right)\left(\frac{\partial^2 TC(Q,N)}{\partial Q^2}\right) - \left(\frac{\partial^2 TC(Q,N)}{\partial N \partial Q}\right)^2 = -0.188224 < 0. \tag{11}$$

Equation (11) illustrates the following two things:

(A) The Hessian matrix of the equation (1) is not positive definite, and
(B) The total cost function in the equation (1) is not jointly convex.

However, the solution procedure presented in Kim and Ha [18] is based on the convexity of $TC(Q,N)$. Both (A) and (B) reveal that the validity of the solution procedure in Kim and Ha [18] is questionable. Naturally, therefore, we try to develop an alternative solution procedure free from using convexity to overcome the shortcomings occurring in Kim and Ha's solution procedure [18].

4 The Solution Procedure

Consider both of the following equations:

$$\frac{\partial TC(Q,N)}{\partial N} = 0 \tag{12}$$

and

$$\frac{\partial TC(Q,N)}{\partial Q} = 0. \tag{13}$$

Solving the equations (12) and (13) simultaneously, we obtain their solution $(\overline{N}^*, \overline{Q}^*)$ as follows:

$$\overline{N}^* = \sqrt{\frac{(A+CS)\{P(H_B - H_S) + 2DH_S\}}{F(P-D)H_S}} \tag{14}$$

and

$$\overline{Q}^* = \overline{N}^* = \sqrt{\frac{2DFP}{(H_B - H_S)P + 2DH_S}}. \tag{15}$$

Substituting the equation (14) into the equation (15), we get

$$\overline{Q}^* = \sqrt{\frac{2D(A+CS)}{H_S(1-D/P)}}. \tag{16}$$

If $\overline{N}^*$ in equation (14) is an integer, Kim and Ha [18] take $(Q^*,N^*) = (\overline{Q}^*,\overline{N}^*)$ and the equations (12) and (13) are satisfied at (Q^*,N^*). However, if $\overline{N}^*$ in the equation (14) is not an integer, Kim and Ha [18, p. 5] choose (Q^*,N^*) such that

$$TC(Q^*,N^*) = \min\left\{TC(\overline{Q}^*,N^+), TC(\overline{Q}^*,N^-)\right\}, \tag{17}$$

where N^+ and N^- represent the nearest integers larger and smaller than the optimal N^*. Equation (17) implies that

$$(Q^*,N^*) = (\overline{Q}^*,N^+) \qquad \text{or} \qquad (\overline{Q}^*,N^-).$$

Under these circumstances, both $(\overline{Q}^*,N^+)$ and $(\overline{Q}^*,N^-)$ do not satisfy the equations (12) and (13) simultaneously. Therefore, it is not appropriate that solving the equations (12) and (13) simultaneously is in order to locate the optimal solution (Q^*,N^*) of $TC(Q,N)$. To overcome the shortcoming in Kim and Ha [18], let us solve the equation (13) when N is fixed. We have

$$Q^*(N) = \sqrt{\frac{2DN(A+CS+NF)}{[N(1-D/P)+(2D/P-1)]H_S+H_B}}, \tag{18}$$

Substituting the equation (18) into the equation (1), we can obtain

$$\begin{aligned} &TC(Q^*(N),N) \\ &= \sqrt{\frac{2D(A+CS+NF)\{H_B+H_S[2D/P-1+N(1-D/P)]\}}{N}} \\ &+DV. \end{aligned} \tag{19}$$

Ignoring the constant terms in the equation (19), from the number under the radical, we find that minimizing $TC(Q^*(N),N)$ in the equation (19) is equivalent to minimizing the following expression:

$$Z(N) = FH_SN(1-D/P) + \frac{(A+CS)[H_S(2D/P-1)+H_B]}{N}. \tag{20}$$

Then the equation (20) yields the first-order derivative with respect to N as follows:

$$\frac{dZ(N)}{dN} = FH_S(1-D/P) - \frac{(A+CS)[H_S(2D/P-1)+H_B]}{N^2}. \tag{21}$$

There are the following two cases that occur:

Case 1. $H_B + H_S\left(\frac{2D}{P}-1\right) > 0$.
Let

$$\Omega = \sqrt{\frac{(A+CS)[H_B+H_S(2D/P-1)]}{FH_S(1-D/P)}}. \tag{22}$$

Then, clearly, Case 1 implies that

$$\frac{dZ(N)}{dN}\begin{cases} <0 & (0<N<\Omega) \quad (23a)\\ =0 & N=\Omega \quad (23b)\\ >0 & (N>\Omega). \quad (23c)\end{cases}$$

Equations (23a) to (23c) show that $Z(N)$ is decreasing on $(0,\Omega]$ and increasing on $[\Omega,\infty)$. Let

$$N_1^* = \lfloor\Omega\rfloor = \text{the greatest integer} \leqq \Omega. \tag{24}$$

Consequently, we have

$$Z(N^*) = \min\{Z(N_1^*), Z(N_1^*+1)\}. \tag{25}$$

Then

$$Q^* = Q^*(N^*),$$

which is determined by the equation (18).
If $\overline{N}^*$ in the equation (14) is not an integer, Kim and Ha [18] do not illustrate why they take

$$N^* = N^-(N_1^*) \qquad \text{or} \qquad N^* = N^+(N_1^*+1).$$

Equations (23a) to (23c), (24) and (25) give us the concrete reason.

Case 2. $H_B + H_S\left(\frac{2D}{P}-1\right) \leqq 0$.
Under this circumstance, the equation (20) implies that

$$\frac{dZ(N)}{dN} > 0. \tag{26}$$

Equation (26) shows that $Z(N)$ is increasing on $[1,\infty)$. The optimal solution (Q^*,N^*) of $TC(Q,N)$ will be $N^*=1$ and $Q^*=Q^*(1)$.

Combining Cases 1 and 2, we have the following results.

Theorem 1.(A) *If* $H_B + H_S\left(\frac{2D}{P}-1\right) > 0$, *the optimal solution* (Q^*,N^*) *for* $TC(Q,N)$ *can then be expressed as follows*:
$N^* = N_1^*$ *or* N_1^*+1 *according to the equations* (24) *and* (25),
and
$Q^* = Q^*(N^*)$ *which is determined by the equations* (18).
(B) *If* $H_B + H_S\left(\frac{2D}{P}-1\right) \leqq 0$, *then the optimal solution of* $TC(Q,N)$ *can be expressed as follows*:
$N^* = 1$
and
$Q^* = Q^*(1)$ *which is determined by the equation* (18).

Kim and Ha [18, p. 3] always assumed that the buyer's holding cost H_B would be greater than the supplier's holding cost H_S. However, if technology advances in the future, from the point of actual practice, three cases may occur as follows:

Case 3. $H_B > H_S$;

Case 4. $H_B = H_S$;

Case 5. $H_B < H_S$.

Kim and Ha [18] only discussed Case (3). Cases 4 and 5 were not discussed by Kim and Ha [18]. This paper explores all of the Cases 3 to 5 in order to generalize and enlarge the applications of Kim and Ha [18].

5 The Single-Setup-Multiple-Delivery (SSMD) Model

Under the single-setup-multiple-delivery (SSMD) model, the buyer's order quantity is manufactured at one setup and shipped in equal amounts over multiple deliveries. Kim and Ha [18, p. 3] indicated that splitting the order quantity into multiple small lots is consistent with JIT implementation. This section will explore when the SSMD policy is more beneficial than the single-delivery policy.

Theorem 2.(A) *If*

$$(A+CS)\left[H_B+H_S\left(\frac{2D}{P}-1\right)\right] < FH_S\left(1-\frac{D}{P}\right),$$

then $N^*=1$ *and the single-delivery policy is better.*

(B) *If*

$$FH_S\left(1-\frac{D}{P}\right) \leqq (A+CS)\left[H_B+H_S\left(\frac{2D}{P}-1\right)\right] < 2FH_S\left(1-\frac{D}{P}\right),$$

then $N^*=1$ *and the single-delivery policy is better.*

(C) *If*

$$(A+CS)\left[H_B+H_S\left(\frac{2D}{P}-1\right)\right] = 2FH_S\left(1-\frac{D}{P}\right),$$

then $N^*=1$ *or* 2 *and the single-delivery policy and SSMD policy are undifferentiated.*

(D) *If*

$$2FH_S\left(1-\frac{D}{P}\right) < (A+CS)\left[H_B+H_S\left(\frac{2D}{P}-1\right)\right] < 4FH_S\left(1-\frac{D}{P}\right),$$

then $N^*=2$ *and the SSMD policy is better.*

(E) *If*

$$4FH_S\left(1-\frac{D}{P}\right) \leqq (A+CS)\left[H_B+H_S\left(\frac{2D}{P}-1\right)\right],$$

then $N^*\geqq 2$ *and the SSMD policy is better.*

Proof.(A) If

$$(A+CS)\left[H_B+H_S\left(\frac{2D}{P}-1\right)\right] < FH_S\left(1-\frac{D}{P}\right),$$

there are two cases to occur as follows:

(1) If

$$\left[H_B+H_S\left(\frac{2D}{P}-1\right)\right] \leqq 0,$$

then Theorem 1(B) implies that $N^*=1$.

(2) If

$$\left[H_B+H_S\left(\frac{2D}{P}-1\right)\right] > 0,$$

then the equations (20), (22), (24) and (25) imply that

$$\Omega < 1, \quad N_1^*=0, \quad Z(1) < Z(0)=\infty$$

and

$$Z(N^*)=\min\{Z(0),Z(1)\}=Z(1).$$

So, obviously, we have $N^*=1$.

By combining (A1) and (A2), we have $N^*=1$. So, the single-delivery policy is better.

(B) If

$$FH_S\left(1-\frac{D}{P}\right) \leqq (A+CS)\left[H_B+H_S\left(\frac{2D}{P}-1\right)\right] < 2FH_S\left(1-\frac{D}{P}\right),$$

the equations (20), (21), (24) and (25) imply that

$$1 \leqq \Omega < 2, \quad N_1^*=1, \quad Z(1) < Z(2)$$

and

$$Z(N^*)=\min\{Z(1),Z(2)\}=Z(1).$$

So, $N^*=1$ and the single-delivery policy is better.

(C) If

$$(A+CS)\left[H_B+H_S\left(\frac{2D}{P}-1\right)\right] = 2FH_S\left(1-\frac{D}{P}\right),$$

the equations (20), (22), (24) and (25) imply that

$$1 \leqq \Omega < 2, \quad N_1^*=1, \quad Z(1) < Z(2)$$

and

$$Z(N^*)=\min\{Z(1),Z(2)\}=Z(1)=Z(2).$$

So, we have $N^*=1$ or 2. Both the single-delivery policy and the SSMD policy are undifferentiated.

(D) If

$$2FH_S\left(1-\frac{D}{P}\right) < (A+CS)\left[H_B+H_S\left(\frac{2D}{P}-1\right)\right] < 4FH_S\left(1-\frac{D}{P}\right),$$

the equations (20), (22), (24) and (25) imply that

$$1 \leqq \Omega < 2, \quad N_1^*=1, \quad Z(1) > Z(2)$$

and

$$Z(N^*)=\min\{Z(1),Z(2)\}=Z(2).$$

So, clearly, $N^*=2$ and the SSMD policy is better.

(E) If

$$4FH_S\left(1-\frac{D}{P}\right) \leqq (A+CS)\left[H_B+H_S\left(\frac{2D}{P}-1\right)\right],$$

then the equations (22), (24), and (25) imply that

$$2 \leqq \Omega, \quad N_1^* \geqq 2$$

and

$$Z(N^*) = \min\{Z(N_1^*), Z(N_1^*+1)\}.$$

So, we get $N^* \geqq 2$ and the SSMD policy is better.

Again, by combining the arguments in (A) to (E), we complete the proof of Theorem 2.

If

$$H_B + H_S\left(\frac{2D}{P}-1\right) > 0,$$

then the equation (18) yields

$$\frac{dQ^*(N)}{dN} = \frac{\{FH_S(1-D/P)N^2 + [H_B+H_S(2D/P-1)](A+CS+2FN)\}}{\{[N(1-D/P)+(2D/P-1)]H_S+H_B\}^2} > 0. \quad (27)$$

Equation (27) illustrates that $Q^*(N)$ is increasing on $[1,\infty)$. Hence, clearly, Theorem 2 and the equation (27) reveal the following result.

Theorem 3. *Suppose that* $H_B + H_S\left(\frac{2D}{P}-1\right) > 0$. *Then*

(A) $Q^*(N)$ *is increasing with respect to* $N \geqq 1$.

(B) $Q_{\min} = Q^*(2)$ *is the minimum order quantity that makes the SSMD policy favorable over the single-delivery policy.*

Comparing Fact 1 in Kim and Ha [18, p. 6] and Theorem 3 in this paper, we have the following observations:

(O1) Example 4 in Section 7 demonstrates that Fact 1(b) in Kim and Ha [18, p. 6] is not necessary true, in general.

(O2) Let

$$\overline{Q}_{\min} = \sqrt{\frac{2DFN}{H_B + H_S(2D/P-1)}}. \quad (28)$$

Kim and Ha [18, p. 6] take $\overline{Q}_{\min}$ as the minimum order quantity such that the SSMD policy is favorable over the single-delivery policy if $Q \geqq \overline{Q}_{\min}$. However, the equation (28) reveals that $\overline{Q}_{\min}$ is a function of N. In fact, it is a variable, but not a constant. However, this paper presents a deterministic minimum order quantity from a different point of view:

$$\overline{Q}_{\min} = \sqrt{\frac{4D(A+CS+2F)}{H_B+H_S}}, \quad (29)$$

such that the SSMD policy is favorable over the single-delivery policy.

(O3) If more frequent deliveries occur, the corresponding optimal minimal order quantity warrants more.

6 The Convergence of the Delivery Size

In Kim and Ha [18, p. 6], the optimal delivery size q^* is obtained by dividing $\overline{Q}^*$ by $\overline{N}^*$ from the equations (14) and (15) as follows:

$$q^* = \frac{\overline{Q}^*}{\overline{N}^*} = \sqrt{\frac{2DFP}{(H_B-H_S)P+2DH_S}}. \quad (30)$$

However, if $\overline{N}^*$ is not an integer, the definition of the optimal delivery size q^* is not appropriate. The correct definition of the optimal delivery size should be expressed as follows:

$$\begin{aligned}\overline{q}(N) &= \text{The optimal delivery size when the number of deliveries is } N\\ &= \frac{\text{The optimal order quantity when the number of deliveries is } N}{N}\\ &= \frac{Q^*(N)}{N}\\ &= \sqrt{\frac{2DN(A+CS+NF)}{N^2\{[N(1-D/P)+(2D/P-1)]H_S+H_B\}}}.\end{aligned} \quad (31)$$

Equation (31) shows that $\overline{q}(N)$ is a function of N and

$$\lim_{N\to\infty}\{\overline{q}(N)\} = 0. \quad (32)$$

Equation (32) demonstrates that Theorem 1 and Corollary 1 in Kim and Ha [18, p. 7] are wrong.

7 A Set of Numerical Examples

*Example 2.*Demand rate, $D = 4800$ units/year; Production rate, $P = \$19200$ units/year; Ordering cost, $A = \$25$/cycle; Setup cost, $CS = \$600$/cycle; Transportation, $F = \$50$/trip; Handling and receiving cost is $V = \$1$/unit; Holding cost for buyer is $H_B = \$7$/unit/year; Holding cost for supplier is $H_S = \$6$/unit/year.

Therefore, we have

$$H_B - H_S(1-2D/P) = 4 > 0$$

and Theorem 1(A) is applied. We have $\Omega = 3.33$, $N_1^* = \lfloor\Omega\rfloor = 3$, $Z(3) = 1508.33 < 1525 = Z(4)$ and $q^* = 346.410$. Equations (17) and (24) reveal $N^* = 3$, $Q^* = Q^*(N^*) = 1129$ and $TC(1129,3) = 11387.86$. However, by applying the procedure developed in Section 3.1 of Kim and Ha [18], we obtain $N^* = 3$, $Q^* = 1155$ and $TC(1155,3) = 11389.53 > 11387.86$. Therefore, the optimal solution obtained by this paper is better than that of Kim and Ha [18]. Furthermore, the equations (2) and (3) yield

$$\frac{\partial TC(1155,3)}{\partial N} = -48.88 \neq 0 \quad (33)$$

and

$$\frac{\partial TC(1155,3)}{\partial Q} = 0.128 \neq 0. \quad (34)$$

Equations (33) and (34) indicate that the equations (12) and (13) are not satisfied by Kim and Ha's optimum solution $(1155,3)$. It is not appropriate to locate the optimal solution (Q^*,N^*) by solving the equations (12) and (13) simultaneously. Furthermore, since

$$4FH_S\left(1-\frac{D}{P}\right) \leqq (A+CS)\left[H_B+H_S\left(\frac{2D}{P}-1\right)\right],$$

Theorem 2(E) implies that the SSMD policy is better. It matches the above result ($N^*=3$). Table 1 illustrates the theoretical results of Theorem 3 and Section 6 in this paper.

Table 1: Theoretical Results of Theorem 3 and Section 6 for Example 2

N	1	2	3	4	…	100	…	200000
$Q^*(N)$	873	1034	1129	1200	…	3449	…	146064
$\overline{q}(N)$	873	517	376	300	…	34.49	…	0.73
$TC(Q^*(N),N)$	12222	11526	11387	11400	…	20458	…	662089

Example 3. If the values of H_S and D are changed from 6 and 4800 into 8.5 and 1200, respectively, the values for other parameters in Example 1 remain unchanged. So, $H_B+H_S(2D/P-1)=-0.4375<0$ and q^* does not exist. Theorem 1(B) is applied. We have $N^*=1$, $Q^*=Q^*(N^*)=464$, $TC(464,1)=4692.94$. Since $H_B<H_S$, the solution procedure described in Section 3.1 of Kim and Ha [18] cannot be applied to search for the optimal solution (Q^*,N^*). Since

$$(A+CS)\left[H_B+H_S\left(\frac{2D}{P}-1\right)\right] < FH_S\left(1-\frac{D}{P}\right),$$

Theorem 2(A) implies that the single-delivery policy is better. It matches the above result ($N^*=1$). Table 2 illustrates the theoretical results of Theorem 3 and Section 6 in this paper.

Table 2: Theoretical Results of Theorem 3 and Section 6 for Example 3

N	1	2	3	4	…	100	…	200000
$Q^*(N)$	464	474	488	502	…	1301	…	54881
$\overline{q}(N)$	464	237	163	125.5	…	13.01	…	0.27
$TC(Q^*(N),N)$	4693	4872	5015	5144.8	…	11569	…	438534

Example 4. If the values of H_B and F are changed from 7 and 50 into 6.1 and 220, respectively, those of the other parameters in Example 1 remain unchanged. So, $H_B-H_S(2D/P-1)=3.1>0$ and Theorem 1(A) is applied. We have $\Omega=1.4$, $N_1^*=\lfloor\Omega\rfloor=1$, $Z(1)=2927.5<2948.75=Z(2)$ and $q^*=825.403$. Equations (17) and (24) reveal $N^*=1$, $Q^*=Q^*(1)=1033$, and $TC(1033,1)=12651.83$. However, applying the procedure developed in Section 3.1 of Kim and Ha [18], we obtain $Q^*=1155$, $N^*=1$ and $TC(1155,1)=12701.69>12651.83$. Therefore, the optimal solution obtained by this paper is better. Since

$$FH_S\left(1-\frac{D}{P}\right) \leqq (A+CS)\left[H_B+H_S\left(\frac{2D}{P}-1\right)\right] < 2FH_S\left(1-\frac{D}{P}\right),$$

Theorem 2(B) implies that the single-delivery policy is better. It matches the above result ($N^*=1$). Since $H_B>H_S$ and $N^*=1$, this example demonstrates that Fact 1(b) in Kim and Ha [18, p. 6] is not necessarily true. Table 3 illustrates the theoretical results of Theorem 3 and Section 6 in this paper.

Table 3: Theoretical Results of Theorem 3 and Section 6 for Example 4

N	1	2	3	4	…	100	…	200000
$Q^*(N)$	1033	1300	1494	1655	…	6924	…	306378
$\overline{q}(N)$	1033	650	498	414	…	69.24	…	1.5
$TC(Q^*(N),N)$	12651	12665	13062	13530	…	36170	…	1383507

8 Concluding Remarks and Observations

An inventory problem consists of two parts: (1) the modeling and (2) the solution procedure.

(A) In modeling, Kim and Ha [18, p. 3] assumed that $H_B>H_S$. However, this paper does not need this assumption. This paper has enlarged the applications of Kim and Ha's inventory model in [18].

(B) In the solution procedure Kim and Ha [18] based their approach on the convexity of $TC(Q,N)$. However, in fact, this paper shows that $TC(Q,N)$ is not necessarily convex, such that the validity of Kim and Ha's solution procedure in [18] is questionable from the mathematical viewpoint. Hence, this paper developed an analytical solution procedure free of using convexity to correct and improve Kim and Ha's approach in [18].

In addition, Kim and Ha [18] showed that the optimal delivery size is unique and the delivery size q converges to the unique delivery size $q^*>0$ as N approaches infinity. However, Section 6 in this paper demonstrates that the delivery size $\overline{q}(N)$ is a function of N and converges to zero. This observation will contradict Kim and Ha's conclusion that the convergence in delivery size can offer insights on the standardization of transportation vehicle size issue. Furthermore, this paper gives sufficient conditions and the minimum order quantity $Q_{\min}$ that make the SSMD policy favorable over the single-delivery policy. Incorporating the above arguments, we conclude that this paper improves on Kim and Ha's approach in [18].

References

[1] M. Avriel, *Nonlinear Programming*: *Analysis and Methods*, Prentice-Hall Incorporated, Englewood Cliffs, New Jersey, 1976.

[2] M. Ben-Daya, R. As'ad and M. Seliaman, An integrated production inventory model with raw material replenishment considerations in a three layer supply chain, *Internat. J. Prod. Econ.* **143** (2013), 53–61.

[3] L.-H. Chen and F.-S. Kang, Coordination between vendor and buyer trade credit and items of imperfect quality, *Internat. J. Prod. Econ.* **123** (2010), 52–61.

[4] K.-J. Chung and L. E. Cárdenas-Barrón, The simplified solution procedure for deteriorating items under stock-dependent demand and two-level trade credit in the supply chain management, *Appl. Math. Modelling* **37** (2013), 4653–4660.

[5] K.-J. Chung, K.-L. Hou and S.-P. Lan, The optimal production cycle time in an integrated production-inventory model for decaying raw materials, *Appl. Math. Modelling* **33** (2009), 1–10.

[6] K.-J. Chung, S.-D. Lin and H. M. Srivastava, The complete solution procedures for the mathematical analysis of some families of optimal inventory models with order-size dependent trade credit and deterministic and constant demand, *Appl. Math. Comput.* **219** (2012), 141–156.

[7] K.-J. Chung, S.-D. Lin and H. M. Srivastava, The inventory models under conditional trade credit in a supply chain system, *Appl. Math. Modelling* **37** (2013), 10036–10052.

[8] S Minner, Multiple-supplier inventory models in supply chain management: A review, International Journal of Production Economics, Elsevier, **81-82**, 265-279 (2003).

[9] M. Deloof, Does working capital management affect profitability in Belgian firms?, *J. Bus. Finance Account.* **30** (2003), 573–588.

[10] C. H. Glock, The joint economic lot size model: a review, *Internat. J. Prod. Econ.* **135** (2012), 671–686.

[11] S. K. Goyal, An integrated inventory model for a single supplier-single customer problem, *Internat. J. Prod. Res.* **15** (1976), 107–111.

[12] M. A. Hoque, Generalized single-vendor multi-buyer integrated inventory supply chain models with a better synchronization, *Internat. J. Prod. Econ.* **131** (2011), 463–472.

[13] M. A. Hoque, An alternative optimal solution technique for a single-vendor single-buyer integrated production inventory mode, *Internat. J. Prod. Res.* **47** (2009), 4063–4076.

[14] C.-K. Huang, An optimal policy for a single-vendor single buyer integrated production-inventory problem with process unreliability consideration, *Internat. J. Prod. Econ.* **91** (2004), 91–98.

[15]; C.-K. Huang, D.-M. Tsai, J.-C. Wu and K.-J. Chung, An integrated vendor-buyer inventory model with order-processing cost reduction and permissible delay in payments, *European J. Oper. Res.* **202** (2010), 473–478.

[16] M. Y. Jaber, M. Bonney and A. L. Guiffrida, Coordinating a three-level supply chain with learning-based continuous improvement, *Internat. J. Prod. Econ.* **127** (2010), 27–38.

[17] P. N. Joglekar, Comment on a quantity discount pricing model to increase supplier profits, *Management Sci.* **34** (1988), 1391–1398.

[18] S.-L. Kim and D. Ha, A JIT lot-splitting model for supply chain management: Enhancing buyer-supplier linkage, *Internat. J. Prod. Econ.* **86** (2003), 1–10.

[19] S. Lee and D. Kim, An optimal policy for a single-vendor single-buyer integrated production-distribution model with both deteriorating and defective items, *Internat. J. Prod. Econ.* **147** (2014), 161–170.

[20] T.-Y. Lin, Coordination policy with overlapped delivery between supplier and buyer considering quantity discounts and disposal cost, *Comput. Industr. Engrg.* **66** (2013), 53–62.

[21] T.-Y. Lin and D.-H. Yeh, Optimal coordination policy for supply chain system under imperfect quality consideration, *J. Marine Sci. Tech.* **18** (2010), 449–457.

[22] M. Omar, R. Sarker and W. A. M. Othman, A just-in-time three-level integrated manufacturing system for linearly time-varying demand process, *Appl. Math. Modelling* **37** (2013), 1275–1281.

[23] A. Ramachandran and M. Jankiraman, The relationship between working capital management efficiency and EBIT, *Managing Global Tradition.* **7** (2009), 61–74.

[24] H. Rau and B. C. OuYang, An optimal batch size for integrated production-inventory policy in a supply chain, *European J. Oper. Res.* **185** (2008), 619–634.

[25] M. K. Salameh and M. Y. Jaber, Economic production quantity model for items with imperfect quality, *Internat. J. Prod. Econ.* **64** (2000), 59–64.

[26] H. M. Srivastava and B. R. K. Kashyap, *Special Functions in Queuing Theory and Related Stochastic Processes*, Academic Press, New York and London, 1982.

[27] C. Yan, A. Banerjee and L. Yang, An integrated production-distribution model for a deteriorating inventory item, *Internat. J. Prod. Econ.* **133** (2011), 228–232.

[28] P.-C. Yang and H.-M. Wee, Economic ordering policy of deteriorated item for vendor and buyer: an integrated approach, *Prod. Plan. Control* **11** (2000), 474–480.

[29] B. Yıldız and M. Ustaoğlu, Optimal production model for EVs manufacturing process in Turkey: A comparable case of EMQ/JIT production models, *Procedia - Soc. Behavior. Sci.* **58** (2012), 1482–1490.

[30] L. Zavanella and S. Zanoni, A one-vendor multi-buyer integrated production-inventory model:

The 'Consignment Stock' case, *Internat. J. Prod. Econ.* **118** (2009), 225–232.

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