

An Abductive Framework for Knowledge Base Dynamics

Radhakrishnan Delhibabu*

Informatik 5, Knowledge-Based Systems Group, RWTH Aachen, Germany

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Abstract: The dynamics of belief and knowledge is one of the major components of any autonomous system that should be able to incorporate new pieces of information. We introduced the knowledge base dynamics to deal with two important points: first, to handle belief states that need not be deductively closed; and the second point is the ability to declare certain parts of the belief as immutable. In this paper, we address another, radically new approach to this problem. This approach is very close to the Hansson's dyadic representation of belief. Here, we consider the immutable part as defining a new logical system. By a logical system, we mean that it defines its own consequence relation and closure operator. Based on this, we provide an abductive framework for knowledge base dynamics.

Keywords: AGM, Immutable, Integrity Constraint, Knowledge Base Dynamics, Abduction.

1 Introduction

Over the last three decades [17], abduction has been embraced in AI as a non-monotonic reasoning paradigm to address some of the limitations of deductive reasoning in classical logic. The role of abduction has been demonstrated in a variety of applications. It has been proposed as a reasoning paradigm in AI for diagnosis, natural language understanding, default reasoning, planning, knowledge assimilation and belief revision, multi-agent systems and other problems (see [49]).

In the concept of knowledge assimilation and belief revision (see [40]), when a new item of information is added to a knowledge base, inconsistency can result. Revision means modifying the knowledge base in order to maintain consistency, while keeping the new information and removing (contraction) or not removing the least possible previous information. In our case, update means revision and contraction, that is insertion and deletion in database perspective. The previous work [10, 11] makes connections with contraction from knowledge base dynamics.

Our knowledge base dynamics is defined in two parts: an immutable part (formulae or sentences) and updatable part (literals) (for definition and properties see works of Nebel [37] and Segerberg [50]). Knowledge bases have a set of integrity constraints (see the definitions in later section). In the case of finite knowledge bases, it is

sometimes hard to see how the update relations should be modified to accomplish certain knowledge base updates.

Example 1. Consider a database with an (immutable) rule that a staff member is a person who is currently working in a research group under a chair. Additional (updatable) facts are that matthias and gerhard are group chairs, and delhibabu and aravindan are staff members in group info1. Our first integrity constraint (IC) is that each research group has only one chair ie. $\forall x, y, z (y=z) \leftarrow \text{group_chair}(x, y) \wedge \text{group_chair}(x, z)$. Second integrity constraint is that a person can be a chair for only one research group ie. $\forall x, y, z (y=z) \leftarrow \text{group_chair}(y, x) \wedge \text{group_chair}(z, x)$.

Immutable part: $\text{staff_chair}(X, Y) \leftarrow$
 $\text{staff_group}(X, Z), \text{group_chair}(Z, Y)$.

Updatable part: $\text{group_chair}(\text{infor1}, \text{matthias}) \leftarrow$
 $\text{group_chair}(\text{infor2}, \text{gerhard}) \leftarrow$
 $\text{staff_group}(\text{delhibabu}, \text{infor1}) \leftarrow$
 $\text{staff_group}(\text{aravindan}, \text{infor1}) \leftarrow$

Suppose we want to update this database with the information, $\text{staff_chair}(\text{aravindan}, \text{gerhard})$; From the immutable part, we can deduce that this can be achieved by asserting $\text{staff_group}(\text{aravindan}, Z) \wedge \text{group_chair}(Z, \text{gerhard})$

If we are restricted to definite clauses, there are three plausible ways to do this. When dealing with the revision

* Corresponding author e-mail: delhibabu@kbsg.rwth-aachen.de

of a knowledge base (both insertions and deletions), there are other ways to change a knowledge base and it has to be performed automatically also. Considering the information, change is precious and must be preserved as much as possible. The *principle of minimal change* [23, 48] can provide a reasonable strategy. On the other hand, practical implementations have to handle contradictory, uncertain, or imprecise information, so several problems can arise: how to define efficient change in the style of AGM [1]; what result has to be chosen [27,31,35]; and finally, according to a practical point of view, what computational model to support for knowledge base revision has to be provided?

Since knowledge base change is one of the main problems arising in knowledge representation, it has been tackled according to several points of view. In this article, we consider the immutable part as defining a new logical system. By a logical system, we mean that it defines its own consequence relation and closure operator. Based on this, we provide an abductive framework for belief dynamics (see [3, 8, 52]).

The rest of paper is organized as follows: First we start with preliminaries along with the concept of logical system and properties of consequences operator. In Section 3, we introduce knowledge base dynamics with our logical system. In Section 4, we explore the relationship of knowledge base dynamics with coherence approach. In Section 5, we present how knowledge base dynamics can be realized using abductive explanations. In Section 6, we give brief overview of related works. In Section 7, we make conclusions with a summary of our contribution as well as a discussion of future directions of investigation.

2 Preliminaries

A first order language consists of an alphabet \mathcal{A} of a language \mathcal{L} . We assume a countable universe of variables Var , ranged over x, y, z , and a countable universe of relation (i.e predicate) symbols, ranged over by \mathcal{A} . The following grammar defines FOL, the language of first order logic with equality and binary relations:

$$\phi ::= x = x \mid a(x, x) \mid \neg\phi \mid \bigvee\phi \mid \bigwedge\phi \mid \exists X : \phi.$$

Here $\phi \subseteq FOL$ and $X \subseteq Var$ are finite sets of formulae and variables, respectively.

Definition 1(Normal Logic Program [22]). *By an alphabet \mathcal{A} of a language \mathcal{L} we mean disjoint sets of constants, predicate symbols, and function symbols, with at least one constant. In addition, any alphabet is assumed to contain a countably infinite set of distinguished variable symbols. A term over \mathcal{A} is defined recursively as either a variable, a constant or an expression of the form $f(t_1, \dots, t_n)$ where f is a function symbol of \mathcal{A} , n its arity, and the t_i are terms. An atom over \mathcal{A} is an expression of the form $P(t_1, \dots, t_n)$ where P*

is a predicate symbol of \mathcal{A} and the t_i are terms. A literal is either an atom A or its default negation $\text{not } A$. We dub default literals those of the form $\text{not } A$. A term (resp. atom, literal) is said ground if it does not contain variables. The set of all ground terms (resp. atoms) of \mathcal{A} is called the Herbrand universe (resp. base) of \mathcal{A} . A Normal Logic Program is a possibly infinite set of rules (with no infinite descending chains of syntactical dependency) of the form:

$$H \leftarrow B_1, \dots, B_n, \text{not } C_1, \dots, \text{not } C_m, \text{ (with } m, n \geq 0 \text{ and finite)}$$

Where H , the B_i and the C_j are atoms, and each rule stands for all its ground instances. In conformity with the standard convention, we write rules of the form $H \leftarrow$ also simply as H (known as fact). An NLP P is called definite if none of its rules contain default literals. H is the head of the rule r , denoted by $\text{head}(r)$, and $\text{body}(r)$ denotes the set $\{B_1, \dots, B_n, \text{not } C_1, \dots, \text{not } C_m\}$ of all the literals in the body of r .

When doing problem modeling with logic programs, rules of the form

$$\perp \leftarrow B_1, \dots, B_n, \text{not } C_1, \dots, \text{not } C_m, \text{ (with } m, n \geq 0 \text{ and finite)}$$

with a non-empty body are known as a type of integrity constraints (ICs), specifically denials, and they are normally used to prune out unwanted candidate solutions. We abuse the *not* default negation notation applying it to non-empty sets of literals too: we write $\text{not } S$ to denote $\{\text{not } s : s \in S\}$, and confound $\text{not not } a \equiv a$. When S is an arbitrary, non-empty set of literals $S = \{B_1, \dots, B_n, \text{not } C_1, \dots, \text{not } C_m\}$ we use:

- S^+ denotes the set $\{B_1, \dots, B_n\}$ of positive literals in S .
- S^- denotes the set $\{\text{not } C_1, \dots, \text{not } C_m\}$ of negative literals in S .
- $|S| = S^+ \cup (\text{not } S^-)$ denotes the set $\{B_1, \dots, B_n, C_1, \dots, C_m\}$ of atoms of S .

As expected, we say a set of literals S is consistent iff $S^+ \cap |S^-| = \emptyset$. We also write $\text{heads}(P)$ to denote the set of heads of non-IC rules of a (possibly constrained) program P , i.e., $\text{heads}(P) = \{\text{head}(r) : r \in P\} \setminus \{\perp\}$, and $\text{facts}(P)$ to denote the set of facts of P - $\text{facts}(P) = \{\text{head}(r) : r \in P \wedge \text{body}(r) = \emptyset\}$.

Definition 2(Level mapping[4]). *Let P be a normal logic program and B_P its Herbrand base. A level mapping for P is a function $\|\cdot\|: B_P \rightarrow \mathbb{N}$ of ground atoms to natural numbers. The mapping $\|\cdot\|$ is also extended to ground literals by assigning $\|\neg A\| = \|A\|$ for all ground atoms $A \in B_P$. For every literal ground L , $\|L\|$ is called as the level of L in P .*

Definition 3(Acyclic program [4]). Let P be a normal logic program and \parallel a level mapping for P . P is called as *acyclic with respect to \parallel* if for every ground clause $H \leftarrow L_1, \dots, L_n$ (with $n \geq 0$ and *finit*) in P the level of A is higher than the level of every L_i ($1 \leq i \leq n$). Moreover P is called *acyclic* if P is acyclic with respect to some level mapping for P .

Unlike knowledge base dynamics, where knowledge is defined as a set of sentences, here we wish to define a knowledge base KB wrt a language \mathcal{L} , as an abductive framework $\langle P, Ab, IC, K \rangle$, where,

- * P is an acyclic normal logic program with all abducibles in P at level 0 and no non-abducible at level 0. P is referred to as a *logical system*. This in conjunction with the integrity constraints corresponds to immutable part of the knowledge base, here P is defined by immutable part. This is discussed further in the next subsection;
- * Ab is a set of atoms from \mathcal{L} , called the *abducibles*. This notion is required in an abductive framework, and this corresponds to the atoms that may appear in the updatable part of the knowledge;
- * IC is the set of *integrity constraints*, a set of sentences from language \mathcal{L} . This specifies the integrity of a knowledge base and forms a part of the knowledge that can not be modified over time;
- * K is a set of sentences from \mathcal{L} . It is the *current knowledge*, and the only part of KB that changes over time. This corresponds to the updatable part of the knowledge base. The main requirement here is that no sentence in K can have an atom that does not appear in Ab .

2.1 Logical system

The main idea of our approach is to consider the immutable part of the knowledge to define a new logical system. By a logical system, we mean that P defines its own consequence relation \models_P and its closure Cn_P . Given P , we have the Herbrand Base HB_P and G_P , the ground instantiation of P .

An *abductive interpretation* I is a set of abducibles, i.e. $I \subseteq Ab$. How I interprets all the ground atoms of L ¹ is defined, inductively on the level of atoms wrt P , as follows:

- *An atom A at level 0 (note that only abducibles are at level 0) is interpreted as: A is true in I iff $A \in I$, else it is false in I .
- *An atom A at level n is interpreted as: A is true in I iff \exists clause $A \leftarrow L_1, \dots, L_k$ in G_P s.t. $\forall L_j$ ($1 \leq j \leq k$) if L_j is an atom then L_j is true in I , else if L_j is a negative literal $\neg B_j$, then B_j is false in I .

¹ the set of all the ground atoms of L , in fact depends of L , and is given as HB_P , the Herbrand Base of P

This interpretation of ground atoms can be extended, in the usual way, to interpret sentences in L , as follows (where α and β are sentences):

- * $\neg\alpha$ is true in I iff α is false in I .
- * $\alpha \wedge \beta$ is true in I iff both α and β are true in I .
- * $\alpha \vee \beta$ is true in I iff either α is true in I or β is true in I .
- * $\forall\alpha$ is true in I iff all ground instantiations of α are true in I .
- * $\exists\alpha$ is true in I iff some ground instantiation of α is true in I .

Given a sentence α in L , an abductive interpretation I is said to be an *abductive model* of α iff α is true in I . Extending this to a set of sentences K , I is a abductive model of K iff I is an abductive model of every sentence α in K .

Given a set of sentences K and a sentence α , α is said to be a *P-consequence* of K , written as $K \models_P \alpha$, iff every abductive model of K is an abductive model of α also. Putting it in other words, let $Mod(K)$ be the set of all abductive models of K . Then α is a *P-consequence* of K iff α is true in all abductive interpretations in $Mod(K)$. The *consequence operator* Cn_P is then defined as $Cn_P(K) = \{\alpha \mid K \models_P \alpha\} = \{\alpha \mid \alpha \text{ is true in all abductive interpretations in } Mod(K)\}$. K is said to be *P-consistent* iff there is no expression α s.t. $\alpha \in Cn_P(K)$ and $\neg\alpha \in Cn_P(K)$. Two sentences α and β are said to be *P-equivalent* to each other, written as $\alpha \equiv \beta$, iff they have the same set of abductive models, i.e. $Mod(\alpha) = Mod(\beta)$.

2.2 Properties of consequences operator

Since a new consequence operator is defined, it is reasonable, to ask whether it satisfies certain properties that are required in the knowledge base dynamics context. Here, we observe that all the required properties, listed by various researchers in knowledge base dynamics, are satisfied by the defined consequence operator. The following propositions follow from the above definitions, and can be verified easily.

Cn_P satisfies inclusion, i.e. $K \subseteq Cn_P(K)$.

Cn_P satisfies iteration, i.e. $Cn_P(K) = Cn_P(Cn_P(K))$.

Another interesting property is *monotony*, i.e. if $K \subseteq K'$, then $Cn_P(K) \subseteq Cn_P(K')$. Cn_P satisfies monotony. To see this, first observe that $Mod(K') \subseteq Mod(K)$.

Cn_P satisfies *superclassicality*, i.e. if α can be derived from K by first order classical logic, then $\alpha \in Cn_P(K)$.

Cn_P satisfies *deduction*, i.e. if $\beta \in Cn_P(K \cup \{\alpha\})$, then $(\beta \leftarrow \alpha) \in Cn_P(K)$.

Cn_P satisfies *compactness*, i.e. if $\alpha \in Cn_P(K)$, then $\alpha \in Cn_P(K')$ for some finite subset K' of K .

2.3 Statics of a knowledge base

The statics of a knowledge base KB , is given by the current knowledge K and the integrity constraints IC . An abductive interpretation M is an abductive model of KB iff it is an abductive model of $K \cup IC$. Let $Mod(KB)$ be the set of all abductive models of KB . The *belief set* represented by KB , written as KB^\bullet is given as,

$$KB^\bullet = Cn_P(K \cup IC) = \{\alpha \mid \alpha \text{ is true in every abductive}$$

model of $KB\}$. A belief (represented by a sentence in \mathcal{L}) α is *accepted* in KB iff $\alpha \in KB^{bullet}$ (i.e. α is true in every model of KB). α is *rejected* in KB iff $\neg\alpha \in KB^\bullet$ (i.e. α is false in every model of KB). Note that there may exist a sentence α s.t. α is neither accepted nor rejected in KB (i.e. α is true in some but not all models of KB), and so KB represents a partial description of the world.

Two knowledge bases KB_1 and KB_2 are said to be *equivalent* to each other, written as $KB_1 \equiv KB_2$, iff they are based on the same logical system and their current knowledge are P -equivalent, i.e. $P_1 = P_2$, $Ab_1 = Ab_2$, $IC_1 = IC_2$ and $K_1 \equiv K_2$. Obviously, two equivalent knowledge bases KB_1 and KB_2 represent the same belief set, i.e. $KB_1^\bullet = KB_2^\bullet$.

3 Knowledge base dynamics

In AGM [1] three kinds of belief dynamics are defined: expansion, contraction and revision. We consider all of them, one by one, in the sequel.

3.1 Expansion

Let α be new information that has to be added to a knowledge base KB . Suppose $\neg\alpha$ is not accepted in KB . Then, obviously α is P -consistent with IC , and KB can be *expanded* by α , by modifying K as follows:

$$KB + \alpha \equiv \langle P, Ab, IC, K \cup \{\alpha\} \rangle$$

Note that we do not force the presence of α in the new K , but only say that α must be in the belief set represented by the expanded knowledge base. If in case $\neg\alpha$ is accepted in KB (in other words, α is inconsistent with IC), then expansion of KB by α results in a inconsistent knowledge base with no abductive models, i.e. $(KB + \alpha)^\bullet$ is the set of all sentences in \mathcal{L} .

Putting it in model-theoretic terms, KB can be expanded by a sentence α , when α is not false in all models of KB . The expansion is defined as:

$$Mod(KB + \alpha) = Mod(KB) \cap Mod(\alpha).$$

If α is false in all models of KB , then clearly $Mod(KB + \alpha)$ is empty, implying that expanded knowledge base is inconsistent.

3.2 Revision

As usual, for revising and contracting a knowledge base, the rationality of the change is discussed first. Later a construction is provided that complies with the proposed rationality postulates.

Rationality postulates

Let $KB = \langle P, Ab, IC, K \rangle$ be revised by a sentence α to result in a new knowledge base $KB \dot{+} \alpha = \langle P', Ab', IC', K' \rangle$.

When a knowledge base is revised, we do not (generally) wish to modify the underlying logical system P or the set of abducibles Ab . This is referred to as *inferential constancy* by Hansson [21, 22].

- (+1)(*Inferential constancy*) $P' = P$ and $Ab' = Ab, IC' = IC$.
- (+2)(*Success*) α is accepted in $KB \dot{+} \alpha$, i.e. α is true in all models of $KB \dot{+} \alpha$.
- (+3)(*Consistency*) α is satisfiable and P -consistent with IC iff $KB \dot{+} \alpha$ is P -consistent, i.e. $Mod(\{\alpha\} \cup IC)$ is not empty iff $Mod(KB \dot{+} \alpha)$ is not empty.
- (+4)(*Vacuity*) If $\neg\alpha$ is not accepted in KB , then $KB \dot{+} \alpha \equiv KB + \alpha$, i.e. if α is not false in all models of KB , then $Mod(KB \dot{+} \alpha) = Mod(KB) \cap Mod(\alpha)$.
- (+5)(*Preservation*) If $KB \equiv KB'$ and $\alpha \equiv \beta$, then $KB \dot{+} \alpha \equiv KB' \dot{+} \beta$, i.e. if $Mod(KB) = Mod(KB')$ and $Mod(\alpha) = Mod(\beta)$, then $Mod(KB \dot{+} \alpha) = Mod(KB \dot{+} \beta)$.
- (+6)(*Extended Vacuity 1*) $(KB \dot{+} \alpha) + \beta$ implies $KB \dot{+} (\alpha \wedge \beta)$, i.e. $(Mod(KB \dot{+} \alpha) \cap Mod(\beta)) \subseteq Mod(KB \dot{+} (\alpha \wedge \beta))$.
- (+7)(*Extended Vacuity 2*) If $\neg\beta$ is not accepted in $(KB \dot{+} \alpha)$, then $KB \dot{+} (\alpha \wedge \beta)$ implies $(KB \dot{+} \alpha) + \beta$, i.e. if β is not false in all models of $KB \dot{+} \alpha$, then $Mod(KB \dot{+} (\alpha \wedge \beta)) \subseteq (Mod(KB \dot{+} \alpha) \cap Mod(\beta))$.

Construction

Let \mathcal{S} stand for the set of all abductive interpretations that are consistent with IC , i.e. $\mathcal{S} = Mod(IC)$. We do not consider abductive interpretations that are not models of IC , simply because IC does not change during revision. Observe that when IC is empty, \mathcal{S} is the set of all abductive interpretations. Given a knowledge base KB , and two abductive interpretations I_1 and I_2 from \mathcal{S} , we can compare how close these interpretations are to KB by using an order \leq_{KB} among abductive interpretations in \mathcal{S} . $I_1 <_{KB} I_2$ iff $I_1 \leq_{KB} I_2$ and $I_2 \not\leq_{KB} I_1$.

Let $\mathcal{F} \subseteq \mathcal{S}$. An abductive interpretation $I \in \mathcal{F}$ is minimal in \mathcal{F} wrt \leq_{KB} if there is no $I' \in \mathcal{F}$ s.t. $I' <_{KB} I$. Let, $Min(\mathcal{F}, \leq_{KB}) = \{I \mid I \text{ is minimal in } \mathcal{F} \text{ wrt } \leq_{KB}\}$.

For any knowledge base KB , the following are desired properties of \leq_{KB} :

- (≤ 1)(*Pre-order*) \leq_{KB} is a *pre-order*, i.e. it is transitive and reflexive.
- (≤ 2)(*Connectivity*) \leq_{KB} is *total* in \mathcal{S} , i.e. $\forall I_1, I_2 \in \mathcal{S}$: either $I_1 \leq_{KB} I_2$ or $I_2 \leq_{KB} I_1$.

- (≤ 3)(Faithfulness) \leq_{KB} is faithful to KB, i.e. $I \in \text{Min}(\mathcal{S}, \leq_{KB})$ iff $I \in \text{Mod}(KB)$.
- (≤ 4)(Minimality) For any non-empty subset \mathcal{F} of \mathcal{S} , $\text{Min}(\mathcal{F}, \leq_{KB})$ is not empty.
- (≤ 5)(Preservance)] For any knowledge base KB' , if $KB \equiv KB'$ then $\leq_{KB} = \leq_{KB'}$.

Let KB (and consequently K) be revised by a sentence α , and \leq_{KB} be a rational order that satisfies (≤ 1) to (≤ 5). Then the abductive models of the revised knowledge base are given precisely by: $\text{Min}(\text{Mod}(\{\alpha\} \cup IC), \leq_{KB})$. Note that, this construction does not say what the resulting K is, but merely says what should be the abductive models of the new knowledge base.

Representation theorem

Now, we proceed to show that revision of KB by α , as constructed above, satisfies all the rationality postulates stipulated in the beginning of this section. This is formalized by the following lemma.

Lemma 1. Let KB be a knowledge base, \leq_{KB} an order among \mathcal{S} that satisfies (≤ 1) to (≤ 5). Let a revision operator $\dot{+}$ be defined as: for any sentence α , $\text{Mod}(KB \dot{+} \alpha) = \text{Min}(\text{Mod}(\{\alpha\} \cup IC), \leq_{KB})$. Then $\dot{+}$ satisfies all the rationality postulates for revision ($\dot{+}1$) to ($\dot{+}7$).

Proof.

- ($\dot{+}1$) $P' = P$ and $Ab' = Ab$ and $IC' = IC$
This is satisfied obviously, since our construction does not touch P and Ab , and IC follows from every abductive interpretation in $\text{Mod}(KB \dot{+} \alpha)$.
- ($\dot{+}2$) α is accepted in $KB \dot{+} \alpha$
Note that every abductive interpretation $M \in \text{Mod}(KB \dot{+} \alpha)$ is a model of α . Hence α is accepted in $KB \dot{+} \alpha$.
- ($\dot{+}3$) α is satisfiable and P -consistent with IC iff $KB \dot{+} \alpha$ is P -consistent.
If part: If $KB \dot{+} \alpha$ is P -consistent, then $\text{Mod}(KB \dot{+} \alpha)$ is not empty. This implies that $\text{Mod}(\{\alpha\} \cup IC)$ is not empty, and hence α is satisfiable and P -consistent with IC .
Only if part: If α is satisfiable and P -consistent with IC , then $\text{Mod}(\{\alpha\} \cup IC)$ is not empty, and (≤ 4) ensures that $\text{Mod}(KB \dot{+} \alpha)$ is not empty. Thus, $KB \dot{+} \alpha$ is P -consistent.
- ($\dot{+}4$) If $\neg\alpha$ is not accepted in KB , then $KB \dot{+} \alpha \equiv KB + \alpha$.
We have to establish that $\text{Min}(\text{Mod}(\{\alpha\} \cup IC), \leq_{KB}) = \text{Mod}(KB) \cap \text{Mod}(\alpha)$. Since $\neg\alpha$ is not accepted in KB , $\text{Mod}(KB) \cap \text{Mod}(\alpha)$ is not empty. The required result follows immediately from the fact that \leq_{KB} is faithful to KB (i.e. satisfies ≤ 3), which selects only and all those models of α which are also models of KB .
- ($\dot{+}5$) If $KB \equiv KB'$ and $\alpha \equiv \beta$ then $KB \dot{+} \alpha = KB' \dot{+} \beta$
(≤ 5) ensures that $\leq_{KB} = \leq_{KB'}$. The required result follows immediately from this and the fact that $\text{Mod}(\alpha) = \text{Mod}(\beta)$.

- ($\dot{+}6$) $(KB \dot{+} \alpha) + \beta$ implies $KB \dot{+} (\alpha \wedge \beta)$.
We consider this in two cases. When $\neg\beta$ is accepted in $KB \dot{+} \alpha$, $(KB \dot{+} \alpha) + \beta$ is the set of all sentences from \mathcal{L} , and the postulate follows immediately. Instead when $\neg\beta$ is not accepted in $KB \dot{+} \alpha$, this postulates coincides with the next one.
- ($\dot{+}7$) If $\neg\beta$ is not accepted in $KB \dot{+} \alpha$, then $KB \dot{+} (\alpha \wedge \beta)$ implies $(KB \dot{+} \alpha) + \beta$.

Together with the second case of previous postulate, we need to show that $KB \dot{+} (\alpha \wedge \beta) = (KB \dot{+} \alpha) + \beta$. In other words, we have to establish that $\text{Min}(\text{Mod}(\{\alpha \wedge \beta\} \cup IC), \leq_{KB}) = \text{Mod}(KB \dot{+} \alpha) \cap \text{Mod}(\beta)$. For the sake of simplicity, let us represent $\text{Min}(\text{Mod}(\{\alpha \wedge \beta\} \cup IC), \leq_{KB})$ by P , and $\text{Mod}(KB \dot{+} \alpha) \cap \text{Mod}(\beta)$, which is the same as $\text{Min}(\text{Mod}(\{\alpha\} \cup IC), \leq_{KB}) \cap \text{Mod}(\beta)$, by Q . The required result is obtained in two parts:

- 1) \forall (abductive interpretation) M : if $M \in P$, then $M \in Q$
Obviously $M \in \text{Mod}(\beta)$. Assume that $M \notin \text{Min}(\text{Mod}(\{\alpha\} \cup IC), \leq_{KB})$. This can happen in two cases, and we show that both the cases lead to contradiction.
Case A: No model of β is selected by \leq_{KB} from $\text{Mod}(\{\alpha\} \cup IC)$. But this contradicts our initial condition that $\neg\beta$ is not accepted in $KB \dot{+} \alpha$.
Case B: Some model, say M' , of β is selected by \leq_{KB} from $\text{Mod}(\{\alpha\} \cup IC)$. Since M is not selected, it follows that $M' <_{KB} M$. But then this contradicts our initial assumption that $M \in P$. So, $P \subseteq Q$.
- 2) \forall (abductive interpretation) M : if $M \in Q$, then $M \in P$
 $M \in Q$ implies that M is a model of both α and β , and M is selected by \leq_{KB} from $\text{Mod}(\{\alpha\} \cup IC)$. Note that $\text{Mod}(\{\alpha \wedge \beta\} \cup IC) \subseteq \text{Mod}(\{\alpha\} \cup IC)$. Since M is selected by \leq_{KB} in a bigger set (i.e. $\text{Mod}(\{\alpha\} \cup IC)$), \leq_{KB} must select M from its subset $\text{Mod}(\{\alpha \wedge \beta\} \cup IC)$ also. Hence $Q \subseteq P$. ■

But, that is not all. Any rational revision of KB by α , that satisfies all the rationality postulates, can be constructed by our construction method, and this is formalized below.

Lemma 2. Let KB be a knowledge base and $\dot{+}$ a revision operator that satisfies all the rationality postulates for revision ($\dot{+}1$) to ($\dot{+}7$). Then, there exists an order \leq_{KB} among \mathcal{S} , that satisfies (≤ 1) to (≤ 5), and for any sentence α , $\text{Mod}(KB \dot{+} \alpha)$ is given in $\text{Min}(\text{Mod}(\{\alpha\} \cup IC), \leq_{KB})$.

Proof. Let us construct an order \leq_{KB} among interpretations in \mathcal{S} as follows: For any two abductive interpretations I and I' in \mathcal{S} , define $I \leq_{KB} I'$ iff either $I \in \text{Mod}(KB)$ or $I \in \text{Mod}(KB \dot{+} \text{form}(I, I'))$, where $\text{form}(I, I')$ stands for sentence whose only models are I and I' . We will show that \leq_{KB} thus constructed satisfies

(≤ 1) to (≤ 5) and
 $Min(Mod(\{\alpha\} \cup IC), \leq_{KB}) = Mod(KB \dot{+} \alpha)$.

First, we show that $Min(Mod(\{\alpha\} \cup IC), \leq_{KB}) = Mod(KB \dot{+} \alpha)$. Suppose α is not satisfiable, i.e. $Mod(\alpha)$ is empty, or α does not satisfy IC , then there are no abductive models of $\{\alpha\} \cup IC$, and hence $Min(Mod(\{\alpha\} \cup IC), \leq_{KB})$ is empty. From ($\dot{+}3$), we infer that $Mod(KB \dot{+} \alpha)$ is also empty. When α is satisfiable and α satisfies IC , the required result is obtained in two parts:

1) If $I \in Min(Mod(\{\alpha\} \cup IC), \leq_{KB})$, then $I \in Mod(KB \dot{+} \alpha)$

Since α is satisfiable and consistent with IC , ($\dot{+}3$) implies that there exists at least one model, say I' , for $KB \dot{+} \alpha$. From ($\dot{+}1$), it is clear that I' is a model of IC , from ($\dot{+}2$) we also get that I' is a model of α , and consequently $I \leq_{KB} I'$ (because $I \in Min(Mod(\{\alpha\} \cup IC), \leq_{KB})$). Suppose $I \in Mod(KB)$, then ($\dot{+}4$) immediately gives $I \in Mod(KB \dot{+} \alpha)$. If not, from our definition of \leq_{KB} , it is clear that $I \in Mod(KB \dot{+} form(I, I'))$. Note that $\alpha \wedge form(I, I') \equiv form(I, I')$, since both I and I' are models of α . From ($\dot{+}6$) and ($\dot{+}7$), we get $Mod(KB \dot{+} \alpha) \cap \{I, I'\} = Mod(KB \dot{+} form(I, I'))$. Since $I \in Mod(KB \dot{+} form(I, I'))$, it immediately follows that $I \in Mod(KB \dot{+} \alpha)$.

2) If $I \in Mod(KB \dot{+} \alpha)$, then $I \in Min(Mod(\{\alpha\} \cup IC), \leq_{KB})$.

From ($\dot{+}1$) we get I is a model of IC , and from ($\dot{+}2$), we obtain $I \in Mod(\alpha)$. Suppose $I \in Mod(KB)$, then from our definition of \leq_{KB} , we get $I \leq_{KB} I'$, for any other model I' of α and IC , and hence $I \in Min(Mod(\{\alpha\} \cup IC), \leq_{KB})$. Instead, if I is not a model of KB , then, to get the required result, we should show that $I \in Mod(KB \dot{+} form(I, I'))$, for every model I' of α and IC . As we have observed previously, from ($\dot{+}6$) and ($\dot{+}7$), we get $Mod(KB \dot{+} \alpha) \cap \{I, I'\} = Mod(KB \dot{+} form(I, I'))$. Since $I \in Mod(KB \dot{+} \alpha)$, it immediately follows that $I \in Mod(KB \dot{+} form(I, I'))$. Hence $I \leq_{KB} I'$ for any model I' of α and IC , and consequently, $I \in Min(Mod(\{\alpha\} \cup IC), \leq_{KB})$.

Now we proceed to show that the order \leq_{KB} among \mathcal{S} , constructed as per our definition, satisfies all the rationality axioms (≤ 1) to (≤ 5).

(≤ 1) \leq_{KB} is a pre-order.

Note that we need to consider only abductive interpretations from \mathcal{S} . From ($\dot{+}2$) and ($\dot{+}3$), we have $Mod(KB \dot{+} form(I, I')) = \{I\}$, and so $I \leq_{KB} I$. Thus \leq_{KB} satisfies reflexivity. Let $I_1 \in Mod(IC)$ and $I_2 \notin Mod(IC)$. Clearly, it is possible that two interpretations I_1 and I_2 are not models of KB , and $Mod(KB \dot{+} form(I_1, I_2)) = \{I_1\}$. So, $I_1 \leq_{KB} I_2$ does not necessarily imply $I_2 \leq_{KB} I_1$, and thus \leq_{KB} satisfies anti-symmetry.

To show the transitivity, we have to prove that $I_1 \leq_{KB} I_3$, when $I_1 \leq_{KB} I_2$ and $I_2 \leq_{KB} I_3$ hold. Suppose $I_1 \in Mod(KB)$, then $I_1 \leq_{KB} I_3$ follows immediately from our definition of \leq_{KB} . On the other case, when $I_1 \notin Mod(KB)$, we first observe that $I_1 \in Mod(KB \dot{+} form(I_1, I_2))$, which follows from definition of \leq_{KB} and $I_1 \leq_{KB} I_2$. Also observe that $I_2 \notin Mod(KB)$. If I_2 were a model of KB , then it follows from ($\dot{+}4$) that $Mod(KB \dot{+} form(I_1, I_2)) = Mod(KB) \cap \{I_1, I_2\} = \{I_2\}$, which is a contradiction, and so $I_2 \notin Mod(KB)$. This, together with $I_2 \leq_{KB} I_3$, implies that $I_2 \in Mod(KB \dot{+} form(I_2, I_3))$. Now consider $Mod(KB \dot{+} form(I_1, I_2, I_3))$. Since $\dot{+}$ satisfies ($\dot{+}2$) and ($\dot{+}3$), it follows that this is a non-empty subset of $\{I_1, I_2, I_3\}$. We claim that $Mod(KB \dot{+} form(I_1, I_2, I_3)) \cap \{I_1, I_2\}$ can not be empty. If it is empty, then it means that $Mod(KB \dot{+} form(I_1, I_2, I_3)) = \{I_3\}$. Since $\dot{+}$ satisfies ($\dot{+}6$) and ($\dot{+}7$), this further implies that $Mod(KB \dot{+} form(I_2, I_3)) = Mod(KB \dot{+} form(I_1, I_2, I_3)) \cap \{I_2, I_3\} = \{I_3\}$. This contradicts our observation that $I_2 \in Mod(KB \dot{+} form(I_2, I_3))$, and so $Mod(KB \dot{+} form(I_1, I_2, I_3)) \cap \{I_1, I_2\}$ can not be empty. Using ($\dot{+}6$) and ($\dot{+}7$) again, we get $Mod(KB \dot{+} form(I_1, I_2)) = Mod(KB \dot{+} form(I_1, I_2, I_3)) \cap \{I_1, I_2\}$. Since we know that $I_1 \in Mod(KB \dot{+} form(I_1, I_2))$, it follows that $I_1 \in Mod(KB \dot{+} form(I_1, I_2, I_3))$. From ($\dot{+}6$) and ($\dot{+}7$) we also get $Mod(KB \dot{+} form(I_1, I_3)) = Mod(KB \dot{+} form(I_1, I_2, I_3)) \cap \{I_1, I_3\}$, which clearly implies that $I_1 \in Mod(KB \dot{+} form(I_1, I_3))$. From our definition of \leq_{KB} , we now obtain $I_1 \leq_{KB} I_3$. Thus, \leq_{KB} is a pre-order.

(≤ 2) \leq_{KB} is total.

Since $\dot{+}$ satisfies ($\dot{+}2$) and ($\dot{+}3$), for any two abductive interpretations I and I' in \mathcal{S} , it follows that $Mod(KB \dot{+} form(I, I'))$ is a non-empty subset of $\{I, I'\}$. Hence, \leq_{KB} is total.

(≤ 3) \leq_{KB} is faithful to KB .

From our definition of \leq_{KB} , it follows that $\forall I_1, I_2 \in Mod(KB) : I_1 <_{KB} I_2$ does not hold. Suppose $I_1 \in Mod(KB)$ and $I_2 \notin Mod(KB)$. Then, we have $I_1 \leq_{KB} I_2$. Since $\dot{+}$ satisfies ($\dot{+}4$), we also have $Mod(KB \dot{+} form(I_1, I_2)) = \{I_1\}$. Thus, from our definition of \leq_{KB} , we can not have $I_2 \leq_{KB} I_1$. So, if $I_1 \in Mod(KB)$ and $I_2 \notin Mod(KB)$, then $I_1 <_{KB} I_2$ holds. Thus, \leq_{KB} is faithful to KB .

(≤ 4) For any non-empty subset \mathcal{F} of \mathcal{S} , $Min(\mathcal{F}, \leq_{KB})$ is not empty.

Let α be a sentence such that $Mod(\{\alpha\} \cup IC) = \mathcal{F}$. We have already shown that $Mod(KB \dot{+} \alpha) = Min(\mathcal{F}, \leq_{KB})$. Since, $\dot{+}$ satisfies ($\dot{+}3$), it follows that $Mod(KB \dot{+} \alpha)$ is not empty, and thus $Min(\mathcal{F}, \leq_{KB})$ is not empty.

(≤ 5) If $KB \equiv KB'$, then $\leq_{KB} = \leq_{KB'}$.

This follows immediately from the fact that $\dot{+}$ satisfies ($\dot{+}5$).

Thus, the order among interpretations \leq_{KB} , constructed as per our definition, satisfies (≤ 1) to (≤ 5) , and $Mod(KB \dot{+} \alpha) = Min(Mod(\{\alpha\} \cup IC), \leq_{KB})$. ■

So, we have a one to one correspondence between the axiomatization and the construction, which is highly desirable, and this is summarized by the following *representation theorem*.

Theorem 1. Let KB be revised by α , and $KB \dot{+} \alpha$ be obtained by the construction discussed above. Then, $\dot{+}$ is a revision operator iff it satisfies all the rationality postulates $(\dot{+}1)$ to $(\dot{+}7)$.

Proof. Follows from Lemma 1. and Lemma 2. ■

3.3 Contraction

Contraction of a sentence from a knowledge base KB is studied in the same way as that of revision. We first discuss the rationality of change during contraction and proceed to provide a construction for contraction using duality between revision and contraction.

Rationality Postulates

Let $KB = \langle P, Ab, IC, K \rangle$ be contracted by a sentence α to result in a new knowledge base $KB \dot{-} \alpha = \langle P', Ab', IC', K' \rangle$.

- $(\dot{-}1)$ (Inferential Constancy) $P' = P$ and $Ab' = Ab$ and $IC' = IC$.
- $(\dot{-}2)$ (Success) If $\alpha \notin Cn_P(KB)$, then α is not accepted in $KB \dot{-} \alpha$, i.e. if α is not true in all the abductive interpretations, then α is not true in all abductive interpretations in $Mod(KB \dot{-} \alpha)$.
- $(\dot{-}3)$ (Inclusion) \forall (belief) β : if β is accepted in $KB \dot{-} \alpha$, then β is accepted in KB , i.e. $Mod(KB) \subseteq Mod(KB \dot{-} \alpha)$.
- $(\dot{-}4)$ (Vacuity) If α is not accepted in KB , then $KB \dot{-} \alpha = KB$, i.e. if α is not true in all the abductive models of KB , then $Mod(KB \dot{-} \alpha) = Mod(KB)$.
- $(\dot{-}5)$ (Recovery) $(KB \dot{-} \alpha) \dot{+} \alpha$ implies KB , i.e. $Mod(KB \dot{-} \alpha) \cap Mod(\alpha) \subseteq Mod(KB)$.
- $(\dot{-}6)$ (Preservation) If $KB \equiv KB'$ and $\alpha \equiv \beta$, then $KB \dot{-} \alpha = KB' \dot{-} \beta$, i.e. if $Mod(KB) = Mod(KB')$ and $Mod(\alpha) = Mod(\beta)$, then $Mod(KB \dot{-} \alpha) = Mod(KB' \dot{-} \beta)$.
- $(\dot{-}7)$ (Conjunction 1) $KB \dot{-} (\alpha \wedge \beta)$ implies $KB \dot{-} \alpha \cap KB \dot{-} \beta$, i.e. $Mod(KB \dot{-} (\alpha \wedge \beta)) \subseteq Mod(KB \dot{-} \alpha) \cap Mod(KB \dot{-} \beta)$.
- $(\dot{-}8)$ (Conjunction 2) If α is not accepted in $KB \dot{-} (\alpha \wedge \beta)$, then $KB \dot{-} \alpha$ implies $KB \dot{-} (\alpha \wedge \beta)$, i.e. if α is not true in all the models of $KB \dot{-} (\alpha \wedge \beta)$, then $Mod(KB \dot{-} \alpha) \subseteq Mod(KB \dot{-} (\alpha \wedge \beta))$.

Before providing a construction for contraction, we wish to study the duality between revision and contraction. The Levi and Harper identities still holds in our case, and is discussed in the sequel.

Relationship between contraction and revision

Contraction and revision are related to each other. Given a contraction function $\dot{-}$, a revision function $\dot{+}$ can be obtained as follows:

$$(Levi Identity) \quad Mod(KB \dot{+} \alpha) = Mod(KB \dot{-} \neg \alpha) \cap Mod(\alpha)$$

The following theorem formally states that Levi identity holds in our approach.

Theorem 2. Let $\dot{-}$ be a contraction operator that satisfies all the rationality postulates $(\dot{-}1)$ to $(\dot{-}8)$. Then, the revision function $\dot{+}$, obtained from $\dot{-}$ using the Levi Identity, satisfies all the rationality postulates $(\dot{+}1)$ to $(\dot{+}7)$. ■

Similarly, a contraction function $\dot{-}$ can be constructed using the given revision function $\dot{+}$ as follows:

$$(Harper Identity) \quad Mod(KB \dot{-} \alpha) = Mod(KB) \cup Mod(KB \dot{+} \neg \alpha)$$

Theorem 3. Let $\dot{+}$ be a revision operator that satisfies all the rationality postulates $(\dot{+}1)$ to $(\dot{+}7)$. Then, the contraction function $\dot{-}$, obtained from $\dot{+}$ using the Harper Identity, satisfies all the rationality postulates $(\dot{-}1)$ to $(\dot{-}8)$. ■

Construction

Given the construction for revision, based on order among interpretation in \mathcal{S} , a construction for contraction can be provided as:

$$Mod(KB \dot{-} \alpha) = Mod(KB) \cup Min(Mod(\{\neg \alpha\} \cup IC), \leq_{KB}),$$

where \leq_{KB} is the relation among interpretations in \mathcal{S} that satisfies the rationality axioms (≤ 1) to (≤ 5) . As in the case of revision, this construction says what should be the models of the resulting knowledge base, and does not explicitly say what the resulting knowledge base is.

Representation theorem

Since the construction for contraction is based on a rational contraction for revision, the following lemmata and theorem follow obviously.

Lemma 3. Let KB be a knowledge base, \leq_{KB} an order among \mathcal{S} that satisfies (≤ 1) to (≤ 5) . Let a contraction operator $\dot{-}$ be defined as: for any sentence α , $Mod(KB \dot{-} \alpha) = Mod(KB) \cup Min(Mod(\{\neg \alpha\} \cup IC), \leq_{KB})$. Then $\dot{-}$ satisfies all the rationality postulates for contraction $(\dot{-}1)$ to $(\dot{-}8)$.

Proof. Follows from Theorem 1 and Theorem 3. ■

Lemma 4. Let KB be a knowledge base and $\dot{-}$ a contraction operator that satisfies all the rationality postulates for contraction ($\dot{-}1$) to ($\dot{-}8$). Then, there exists an order \leq_{KB} among \mathcal{S} , that satisfies (≤ 1) to (≤ 5), and for any sentence α , $Mod(KB \dot{-} \alpha)$ is given as $Mod(KB) \cup Min(Mod(\{\neg\alpha\} \cup IC), \leq_{KB})$.

Proof. Follows from Theorem 1 and Theorem 3. ■

Theorem 4. Let KB be contracted by α , and $KB \dot{-} \alpha$ be obtained by the construction discussed above. Then $\dot{-}$ is a contraction operator iff it satisfies all the rationality postulates ($\dot{-}1$) to ($\dot{-}8$).

Proof. Follows from Lemma 3 and Lemma 4. ■

4 Relationship with the coherence approach of AGM

Given knowledge base $KB = \langle P, Ab, IC, K \rangle$ represents a belief set KB^\bullet that is closed under Cn_P . We have defined how KB can be expanded, revised, or contracted. The question now is: *does our foundational approach (wrt classical first-order logic) on KB coincide with coherence approach (wrt our consequence operator Cn_P) of AGM on KB^\bullet ?* There is a problem in answering this question (similar practical problem [5]), since our approach, we require IC to be immutable, and only the current knowledge K is allowed to change. On the contrary, AGM approach treat every sentence in KB^\bullet equally, and can throw out sentences from $Cn_P(IC)$. One way to solve this problem is to assume that sentences in $Cn_P(IC)$ are more entrenched than others. However, one-to-one correspondence can be established, when IC is empty. The key is our consequence operator Cn_P , and in the following, we show that coherence approach of AGM with this consequence operator, is exactly same as our foundational approach, when IC is empty.

4.1 Expansion

Expansion in AGM (see [1])- framework is defined as $KB\#\alpha = Cn_P(KB^\bullet \cup \{\alpha\})$, is easy to see that this is equivalent to our definition of expansion (when IC is empty), and is formalized below.

Theorem 5. Let $KB + \alpha$ be an expansion of KB by α (as defined in section 3.2). Then $(KB + \alpha)^\bullet = KB\#\alpha$.

Proof. By our definition of expansion, $(KB + \alpha)^\bullet = Cn_P(IC \cup K \cup \{\alpha\})$, which is clearly the same set as $Cn_P(KB^\bullet \cup \{\alpha\})$. ■

4.2 Revision

AGM puts forward rationality postulates ($*1$) to ($*8$) to be satisfied by a revision operator on KB^\bullet . reproduced below:

- ($*1$)(Closure) $KB^\bullet * \alpha$ is a belief set.
- ($*2$)(Success) $\alpha \in KB^\bullet * \alpha$.
- ($*3$)(Expansion 1) $KB^\bullet * \alpha \subseteq KB^\bullet \#\alpha$.
- ($*4$)(Expansion 2) If $\neg\alpha \notin KB^\bullet$, then $KB^\bullet \#\alpha \subseteq KB^\bullet * \alpha$.
- ($*5$)(Consistency) $KB^\bullet * \alpha$ is inconsistent iff $\vdash \neg\alpha$.
- ($*6$)(Preservation) If $\vdash \alpha \leftrightarrow \beta$, then $KB^\bullet * \alpha = KB^\bullet * \beta$.
- ($*7$)(Conjunction 1) $KB^\bullet * (\alpha \wedge \beta) \subseteq (KB^\bullet * \alpha)\#\beta$.
- ($*8$)(Conjunction 2) If $\neg\beta \notin KB^\bullet * \alpha$, then, $(KB^\bullet * \alpha)\#\beta \subseteq KB^\bullet * (\alpha \wedge \beta)$.

The equivalence between our approach and AGM approach is brought out by the following two theorems.

Theorem 6. Let KB a knowledge base with an empty IC and $\dot{+}$ be a revision function that satisfies all the rationality postulates ($\dot{+}1$) to ($\dot{+}7$). Let a revision operator $*$ on KB^\bullet be defined as: for any sentence α , $KB^\bullet * \alpha = (KB \dot{+} \alpha)^\bullet$. The revision operator $*$, thus defined satisfies all the AGM-postulates for revision ($*1$) to ($*8$).

Proof.

- ($*1$) $KB^\bullet * \alpha$ is a belief set.
This follows immediately, because $(KB \dot{+} \alpha)^\bullet$ is closed wrt Cn_P .
- ($*2$) $\alpha \in KB^\bullet * \alpha$.
This follows from the fact that $\dot{+}$ satisfies ($\dot{+}2$).
- ($*3$) $KB^\bullet * \alpha \subseteq KB^\bullet \#\alpha$.
- ($*4$) If $\neg\alpha \notin KB^\bullet$, then $KB^\bullet \#\alpha \subseteq KB^\bullet * \alpha$.
These two postulates follow from ($\dot{+}4$) and theorem 5.
- ($*5$) $KB^\bullet * \alpha$ is inconsistent iff $\vdash \neg\alpha$.
This follows from from ($\dot{+}3$) and our assumption that IC is empty.
- ($*6$) If $\vdash \alpha \leftrightarrow \beta$, then $KB^\bullet * \alpha = KB^\bullet * \beta$.
This corresponds to ($\dot{+}5$).
- ($*7$) $KB^\bullet * (\alpha \wedge \beta) \subseteq (KB^\bullet * \alpha)\#\beta$. This follows from ($\dot{+}6$) and theorem 5.
- ($*8$) If $\neg\beta \notin KB^\bullet * \alpha$, then, $(KB^\bullet * \alpha)\#\beta \subseteq KB^\bullet * (\alpha \wedge \beta)$.
This follows from ($\dot{+}7$) and theorem 5. ■

Theorem 7. Let KB a knowledge base with an empty IC and $*$ a revision operator that satisfies all the AGM-postulates ($*1$) to ($*8$). Let a revision function $\dot{+}$ on KB be defined as: for any sentence α , $(KB \dot{+} \alpha)^\bullet = KB^\bullet * \alpha$. The revision function $\dot{+}$, thus defined, satisfies all the rationality postulates ($\dot{+}1$) to ($\dot{+}7$).

Proof.

- ($\dot{+}1$) P, Ab and IC do not change.
Obvious.
- ($\dot{+}2$) α is accepted in $KB \dot{+} \alpha$.
Follows from ($*2$).

- (+3) If α is satisfiable and consistent with IC , then $KB \dot{+} \alpha$ is consistent.
Since we have assumed IC to be empty, this directly corresponds to (*5).
- (+4) If $\neg\alpha$ is not accepted in KB , then $KB \dot{+} \alpha \equiv KB + \alpha$.
Follows from (*3) and (*4).
- (+5) If $KB \equiv KB'$ and $\alpha \equiv \beta$, then $KB \dot{+} \alpha \equiv KB' \dot{+} \beta$.
Since $KB \equiv KB'$ they represent same belief set, i.e. $KB^\bullet = KB'^\bullet$. Now, this postulate follows immediately from (*6).
- (+6) $(KB \dot{+} \alpha) + \beta$ implies $KB \dot{+} (\alpha \wedge \beta)$.
Corresponds to (*7).
- (+7) If $\neg\beta$ is not accepted in $KB \dot{+} \alpha$, then $KB \dot{+} (\alpha \wedge \beta)$ implies $(KB \dot{+} \alpha) + \beta$.
Corresponds to (*8). ■

4.3 Contraction

AGM puts forward rationality postulates (−1) to (−8) to be satisfied by a contraction operator on closed set KB^\bullet , reproduced below:

- (−1) (Closure) $KB^\bullet - \alpha$ is a belief set.
- (−2) (Inclusion) $KB^\bullet - \alpha \subseteq KB^\bullet$.
- (−3) (Vacuity) If $\alpha \notin KB^\bullet$, then $KB^\bullet - \alpha = KB^\bullet$.
- (−4) (Success) If $\not\vdash \alpha$, then $\alpha \notin KB^\bullet - \alpha$.
- (−5) (Preservation) If $\vdash \alpha \leftrightarrow \beta$, then $KB^\bullet - \alpha = KB^\bullet - \beta$.
- (−6) (Recovery) $KB^\bullet \subseteq (KB^\bullet - \alpha) + \alpha$.
- (−7) (Conjunction 1) $KB^\bullet - \alpha \cap KB^\bullet - \beta \subseteq KB^\bullet - (\alpha \wedge \beta)$.
- (−8) (Conjunction 2) If $\alpha \notin KB^\bullet - (\alpha \wedge \beta)$, then $KB^\bullet - (\alpha \wedge \beta) \subseteq KB^\bullet - \alpha$.

As in the case of revision, the equivalence is brought out by the following theorems. Since contraction is constructed in terms of revision, these theorems are trivial.

Theorem 8. Let KB be a knowledge base with an empty IC and $\dot{-}$ be a contraction function that satisfies all the rationality postulates (−1) to (−8). Let a contraction operator $-$ on KB^\bullet be defined as: for any sentence α , $KB^\bullet - \alpha = (KB \dot{-} \alpha)^\bullet$. The contraction operator $-$, thus defined, satisfies all the AGM - postulates for contraction (−1) to (−8).

Proof. Follows from Theorem 2 and Theorem 6. ■

Theorem 9. Let KB be a knowledge base with an empty IC and $-$ be a contraction operator that satisfies all the AGM - postulates (−1) to (−8). Let a contraction function $\dot{-}$ on KB be defined as: for any sentence α , $(KB \dot{-} \alpha)^\bullet = KB^\bullet - \alpha$. The contraction function $\dot{-}$, thus defined, satisfies all the rationality postulates (−1) to (−8).

Proof. Follows from Theorem 3 and Theorem 7. ■

5 Realizing knowledge base dynamics using abductive explanations

In this section, we explore how belief dynamics can be realized in practice (see [7, 10, 11]). Here, we will see how revision can be implemented based on the construction using models of revising sentence and an order among them. The notion of abduction proves to be useful and is explained in the sequel.

Let α be a sentence in \mathcal{L} . An *abductive explanation* for α wrt KB is a set of abductive literals ² Δ s.t. Δ consistent with IC and $\Delta \models_P \alpha$ (that is $\alpha \in Cn_P(\Delta)$). Further Δ is said to be *minimal* iff no proper subset of Δ is an abductive explanation for α .

The basic idea to implement revision of a knowledge base KB by a sentence α , is to realize $Mod(\{\alpha\} \cup IC)$ in terms of abductive explanations for α wrt KB . We first provide a useful lemma.

Definition 4. Let KB be a knowledge base, α a sentence, and Δ_1 and Δ_2 be two minimal abductive explanations for α wrt KB . Then, the disjunction of Δ_1 and Δ_2 , written as $\Delta_1 \vee \Delta_2$, is given as:

$$\Delta_1 \vee \Delta_2 = (\Delta_1 \cap \Delta_2) \cup \{\alpha \vee \beta \mid \alpha \in \Delta_1 \setminus \Delta_2 \text{ and } \beta \in \Delta_2 \setminus \Delta_1\}.$$

Extending this to Δ^\bullet , a set of minimal abductive explanations for α wrt KB , $\vee \Delta^\bullet$ is given by the disjunction of all elements of Δ^\bullet .

Lemma 5. Let KB be a knowledge base, α a sentence, and Δ_1 and Δ_2 be two minimal abductive explanations for α wrt KB . Then, $Mod(\Delta_1 \vee \Delta_2) = Mod(\Delta_1) \cup Mod(\Delta_2)$.

Proof. First we show that every model of Δ_1 is a model of $\Delta_1 \vee \Delta_2$. Clearly, a model M of Δ_1 satisfies all the sentences in $(\Delta_1 \cap \Delta_2)$. The other sentences in $(\Delta_1 \vee \Delta_2)$ are of the form $\alpha \vee \beta$, where α is from Δ_1 and β is from Δ_2 . Since M is a model of Δ_1 , α is true in M , and hence all such sentences are satisfied by M . Hence M is a model of $\Delta_1 \vee \Delta_2$ too. Similarly, it can be shown that every model of Δ_2 is a model of $\Delta_1 \vee \Delta_2$ too.

Now, it remains to be shown that every model M of $\Delta_1 \vee \Delta_2$ is either a model of Δ_1 or a model of Δ_2 . We will now show that if M is not a model of Δ_2 , then it must be a model of Δ_1 . Since M satisfies all the sentences in $(\Delta_1 \cap \Delta_2)$, we need only to show that M also satisfies all the sentences in $\Delta_1 \setminus \Delta_2$. For every element $\alpha \in \Delta_1 \setminus \Delta_2$: there exists a subset of $(\Delta_1 \vee \Delta_2)$, $\{\alpha \vee \beta \mid \beta \in \Delta_2 \setminus \Delta_2\}$. M satisfies all the sentences in this subset. Suppose M does not satisfy α , then it must satisfy all $\beta \in \Delta_1 \setminus \Delta_2$. This implies that M is a model of Δ_2 , which is a contradictory to our assumption. Hence M must satisfy α , and thus a model Δ_1 . Similarly, it can be shown that M must be a model of Δ_2 if it is not a model of Δ_1 . ■

² An abductive literal is either an abducible A from Ab , or its negation $\neg A$.

As one would expect, all the models of revising sentence α can be realized in terms abductive explanations for α , and the relationship is precisely stated below.

Lemma 6. *Let KB be a knowledge base, α a sentence, and Δ^\bullet the set of all minimal abductive explanations for α wrt KB . Then $Mod(\{\alpha\} \cup IC) = Mod(\vee \Delta^\bullet)$.*

Proof. It can be easily verified that every model M of a minimal abductive explanation is also a model of α . Since every minimal abductive explanation satisfies IC , M is a model of $\alpha \cup IC$. It remains to be shown that every model M of $\{\alpha\} \cup IC$ is a model of one of the minimal abductive explanations for α wrt KB . This can be verified by observing that a minimal abductive explanation for α wrt KB can be obtained from M . ■

Thus, we have a way to generate all the models of $\{\alpha\} \cup IC$, and we just need to select a subset of this based on an order that satisfies (≤ 1) to (≤ 5) . Suppose we have such an order that satisfies all the required postulates, then this order can be mapped to a particular set of abductive explanations for α wrt KB . This is stated precisely in the following theorem. An important implication of this theorem is that there is no need to compute all the abductive explanations for α wrt KB . However, it does not say which abductive explanations need to be computed.

Theorem 10. *Let KB be a knowledge base, and \leq_{KB} be an order among abductive interpretations in \mathcal{S} that satisfies all the rationality axioms (≤ 1) to (≤ 5) . Then, for every sentence α , there exists Δ^\bullet a set of minimal abductive explanations for α wrt KB , s.t. $Min(Mod(\{\alpha\} \cup IC), \leq_{KB})$ is a subset of $Mod(\vee \Delta^\bullet)$, and this does not hold for any proper subset of Δ^\bullet .*

Proof. From Lemma 6. and Lemma 5., it is clear that $Mod(\{\alpha\} \cup IC)$ is the union of all the models of all minimal abductive explanations of α wrt KB . Min selects a subset of this, and the theorem follows immediately. ■

The above theorem 10. is still not very useful in realizing revision. We need to have an order among all the interpretations that satisfies all the required axioms, and need to compute all the abductive explanations for α wrt KB . The need to compute all abductive explanations arises from the fact that the converse of the above theorem does not hold in general. This scheme requires an universal order \leq , in the sense that same order can be used for any knowledge base. Otherwise, it would be necessary to specify the new order to be used for further modifying $(KB \dot{+} \alpha)$. However, even if the order can be worked out, it is not desirable to demand all abductive explanations of α wrt KB be computed. So, it is desirable to work out, when the converse of the above theorem is true. The following theorem says that, suppose α is rejected in KB , then revision of KB by α can be worked out in terms of some abductive explanations for α wrt KB .

Theorem 11. *Let KB be a knowledge base, and a revision function $\dot{+}$ be defined as: for any sentence α that is rejected in KB , $Mod(KB \dot{+} \alpha)$ is a non-empty subset of $Mod(\vee \Delta^\bullet)$, where Δ^\bullet is a set of all minimal abductive explanations for α wrt KB . Then, there exists an order \leq_{KB} among abductive interpretations in \mathcal{S} , s.t. \leq_{KB} satisfies all the rationality axioms (≤ 1) to (≤ 5) and $Mod(KB \dot{+} \alpha) = Min(Mod(\{\alpha\} \cup IC), \leq_{KB})$.*

Proof. It is easy to define a pre-order s.t. every model of $Mod(KB \dot{+} \alpha)$ is strictly minimal than all other interpretations. It is easy to verify that such a pre-order satisfies (≤ 1) to (≤ 5) . In particular, since α is rejected in KB , (≤ 3) faithfulness is satisfied, and since non-empty subset of $Mod(\vee \Delta^\bullet)$ is selected, (≤ 4) is also satisfied. ■

An important corollary of this theorem is that, revision of KB by α can be realized just by computing one abductive explanation of α wrt KB , and is stated below.

Corollary 1. *Let KB be a knowledge base, and a revision function $\dot{+}$ be defined as: for any sentence α that is rejected in KB , $Mod(KB \dot{+} \alpha)$ is a non-empty subset of $Mod(\Delta)$, where Δ is an abductive explanations for α wrt KB . Then, there exists an order \leq_{KB} among abductive interpretations in \mathcal{S} , s.t. \leq_{KB} satisfies all the rationality axioms (≤ 1) to (≤ 5) and $Mod(KB \dot{+} \alpha) = Min(Mod(\{\alpha\} \cup IC), \leq_{KB})$. ■*

The precondition that α is rejected in KB is not a serious limitation in various applications such as database updates and diagnosis, where close world assumption is employed to infer negative information. For example, in diagnosis it is generally assumed that all components are functioning normally, unless otherwise there is specific information against it. Hence, a knowledge base in diagnosis either accepts or rejects normality of a component, and there is no "don't know" third state. In other words, in these applications the knowledge base is assumed to be complete. Hence, when such a complete knowledge base is revised by α , either α is already accepted in KB or rejected in KB , and so the above scheme works fine.

6 Related Works

We begin by recalling previous work on view deletion. Chandrabose [10, 11], defines a contraction operator in view deletion with respect to a set of formulae or sentences using Hansson's [22] belief change. Similar to our [?, 15, 16] approach, he focused on set of formulae or sentences in knowledge base revision for view update wrt. insertion and deletion and formulae are considered at the same level. Chandrabose proposed different ways to change knowledge base via only database deletion, devising particular postulate which is shown to be necessary and sufficient for such an update process.

Our knowledge base consists of two parts, immutable part and updatable part, but focus is on principle of minimal change. There are more related works on that topic. Eiter [19] is focusing on revision from different perspective - prime implication. Segerberg [50] defined new modeling for belief revision in terms of irrevocability on prioritized revision. Hansson [22] constructed five types of non-prioritized belief revision. Makinson [33] developed dialogue form of revision AGM. Papini [41] defined a new version of knowledge base revision.

We are bridging gap between philosophical work, paying little attention to computational aspects of database work [34,51]. In such a case, Hansson's [22] kernel change is related with abductive method. Aliseda's [2] book on abductive reasoning is one of the motivation keys. Christiansen's [12,13] work on dynamics of abductive logic grammars exactly fits our minimal change (insertion and deletion).

In general, our abduction theory is related to knowledge base dynamics (see how abduction theory is related with other applications, respectively, reasoning [6, 43,44], update [46,47], equivalence [24,44,45] and problem solving [25,32]). More similar to our work is paper presented by Bessant et al. [7], local search-based heuristic technique that empirically proves to be often viable, even in the context of very large propositional applications. Laurent et al. [28] parented updating deductive databases in which every insertion or deletion of a fact can be performed in a deterministic way.

Furthermore, and at a first sight more related to our work, some work has been done on "core-retainment" (same as our immutable part) in the model of language splitting introduced by Parikh [39]. More recently, Doukari [18], Özçep [38] and Wu, et al. [53] applied similar ideas for dealing with knowledge base dynamics. These works represent motivation keys for our future work. Second, we are dealing with how to change minimally in the theory of "principle of minimal change", but current focus is on finding second best abductive explanation [29] and 2-valued minimal hypothesis for each normal program [42]. Finally, when we presented knowledge base change in abduction framework, we did not talk about compilability and complexity (see the works of Liberatore [30] and Zanuttini [54]).

7 Conclusion

The main contribution of this work lies in showing how abductive framework deals with knowledge base dynamics via belief change operation. We consider the immutable part as defining a new logical system. By a logical system, we mean that it defines its own consequence relation and closure operator. We presented that relationship of the coherence approach of AGM with this consequence operator is exactly same as our foundational approach, when IC is empty.

We believe that knowledge base dynamics can also be applied to other applications such as view maintenance, diagnosis, and we plan to explore it in further works [9]. Still, a lot of developments are possible, for improving existing operators or for defining new classes of change operators. As immediate extension, question raises: is there any *real life application for AGM in 25 year theory?* [20]. The revision and update are more challenging in knowledge base dynamic, so we can extend the theory to combine results similar to Konieczny's [26] and Nayak's [36].

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R. Delhibabu is a lecture at Kazan Federal University (KFU) and visiting faculty at IT University Innopolis, Kazan, Russia. He received his joint Ph.D. degree from RWTH Aachen, Germany and Anna University Chennai, India, for the support of India4EU by the European

Commission and SSN college of engineering research funds. His main research interests in the area of cognitive modeling such as addiction, artificial consciousness, affective computing, decision making and time perception. He is also interested in applied logic, in particularly belief revision, cognitive robotics, database theory, distributed computing and machine learning.