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Energy decay to an abstract coupled system of extensible beams models

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Abstract: In this work we study existence of solutions for an abstract coupled system of nonlinear equations of extensible beams models and present the exponential decay for full energy of the system.

Keywords: Nonlinear beam equation, abstract coupled system, existence and uniqueness of solution, asymptotic behavior.

1. Introduction

A mathematical model for the transverse deflection of an extensible beam of length L whose ends are held at fixed distance apart is equation

$$u_{tt} + \alpha u_{xxxx} + \left(\beta \int_0^L u_\xi^2(\xi, t)\, d\xi\right)(-u_{xx}) = 0$$

which has been proposed by Woinowsky-Krieger [14], where α is a positive constant, β is a constant, not necessarily positive, and the nonlinear term represents the change in the tension of the beam due to its extensibility. The abstract formulation of this model, that was studied by Medeiros [8] is the equation

$$u'' + \alpha A^2 u + M(|A^{1/2}u|^2 Au = 0$$

where A is a linear operator in a Hilbert space H and M a real function. We use the standard Lebesgue space and Sobolev space with their usual notation and properties as in [6] and in this sense $(\cdot\,, \cdot)$ and $|\cdot|$ denote respectively the inner product and norm in H. The extensible beam or plate was studied by several authors, like Eisley (1964) [5], Dickey (1970) [4], Ball (1973)[1], Menzala(1980)[9], Brito (1984)[3], Biler (1986) [2], Pereira (1989) [11] and more recently, by Rivera (2008) [12] and Ma (2010) [7].

In this paper we prove the existence, uniqueness and exponential decay of solutions for an abstract coupled system of Woinowsky-Krieger model of nonlinear equations of the beam. The importance of this work is to present the abstract formulation of the coupled system and to apply the Theorem of Nakao to obtain the asymptotic behavior. In this direction, for $\delta_1 > 0$ and $\delta_2 > 0$ follows the problem that we consider here,

$$\begin{aligned}
&K_1 u'' + A_1^2 u \\
&+M\left(\left|A_1^{1/2}u\right|^2 + \left|A_2^{1/2}v\right|^2\right) A_1 u + \delta_1 u' = 0 \qquad (1)\\
&K_2 v'' + A_2^2 v \\
&+M\left(\left|A_1^{1/2}u\right|^2 + \left|A_2^{1/2}v\right|^2\right) A_2 v + \delta_2 v' = 0 \qquad (2)\\
&u(0) = u_0, \qquad v(0) = v_0 \qquad (3)\\
&(K_1 u')(0) = K_1^{1/2} u_1, \quad (K_2 v')(0) = K_2^{1/2} v_1 \qquad (4)
\end{aligned}$$

where $M \in C^0[0, +\infty)$, with $M(\lambda) \geq 0 \;\; \forall\, \lambda > 0$ and K_i, $i = 1, 2$ are symmetrical linear operators in H with $(K_i w, w) > 0, \forall \;\; w \in H$, $A_i, i = 1, 2$ are self-adjoint and positive linear operator, with domain $\mathcal{D}(A_i)$ dense in H, that is, there exist positive constants m_i, $i = 1, 2$, such that $(A_1 v, v) \geq m_1 |v|^2$, $\forall\; v \in \mathcal{D}(A_1)$, $(A_2 w, w) \geq m_2 |w|^2$, $\forall\; w \in \mathcal{D}(A_2)$.

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In section 2 we study the existence of solution by Faedo-Galerkin's method and in section 3 we prove the stability of system by Nakao's theorem.

2. Existence of solution

In this section, we will prove that for

$$(u_0, v_0) \in \mathcal{D}(A_1) \times \mathcal{D}(A_2) \text{ and } (u_1, v_1) \in H \times H,$$

there exists a unique solution (u, v) of (1) - (4) in

$$L^\infty([0,T), \mathcal{D}(A_1)) \times L^\infty([0,T], \mathcal{D}(A_2))$$

where the time T depend only of initial data. We make use of Faedo-Galerkin approximation to prove the existence of weak solution and in this direction we consider the following result.

Theorem 1.

For $(u_0, v_0) \in \mathcal{D}(A_1) \times \mathcal{D}(A_2),\ (u_1, v_1) \in H \times H,$

there exist functions $u, v : R^+ \longrightarrow H$*, such that*

$$(u, v) \in L^\infty([0,T), \mathcal{D}(A_1)) \times L^\infty([0,T], \mathcal{D}(A_2))$$
$$(K_1 u', K_2 v') \in [L^\infty([0,T), H)]^2$$
$$(u', v') \in [L^2([0,T), H)]^2$$

satisfying in the sense of $\mathcal{D}'(R^+)$
$\forall\ w \in \mathcal{D}(A_1),$

$$\frac{d}{dt}(K_1 u', w) + (A_1 u, A_1 w) + M\left(\left|A_1^{\frac{1}{2}} u\right|^2 + \left|A_2^{\frac{1}{2}} v\right|^2\right)(A_1 u, w) + \delta_1(u', w) = 0, \quad (5)$$

and $\forall\ z \in \mathcal{D}(A_2),$

$$\frac{d}{dt}(K_2 v', z) + (A_2 v, A_2 z) + M\left(\left|A_1^{\frac{1}{2}} u\right|^2 + \left|A_2^{\frac{1}{2}} v\right|^2\right)(A_2 v, z) + \delta_2(v', z) = 0, \quad (6)$$

with initial conditions

$$(u(0), v(0)) = (u_0, v_0),$$
$$(K_1 u(0), K_2 v(0)) = \left(K_1^{\frac{1}{2}} u_1, K_2^{\frac{1}{2}} v_1\right).$$

Proof. **Approximate problem**
Let $(w_\nu)_{\nu \in N}$ and $(z_\nu)_{\nu \in N}$, Hilbert's basis of $\mathcal{D}(A_1)$ and $\mathcal{D}(A_2)$ respectively. We take

$$V_m = [w_1, ..., w_m] \text{ and } W_m = [z_1, ..., z_m]$$

so we have that

$$u_m(t) = \sum_{j=1}^m g_{jm}(t) w_j \in V_m$$

and

$$v_m(t) = \sum_{j=1}^m h_{jm}(t) z_j \in W_m$$

satisfies $\forall\ w \in V_m$ and $\forall\ z \in W_m$

$$(K_1 u_m''(t), w) + (A_1 u_m(t), A_1 w) + M\left(\left|A_1^{\frac{1}{2}} u_m(t)\right|^2 + \left|A_2^{\frac{1}{2}} v_m(t)\right|^2\right)(A_1 u_m(t), w) + \delta_1(u_m'(t), w) = 0, \quad (7)$$

$$(K_2 v_m''(t), z) + (A_2 v_m(t), A_2 z) + M\left(\left|A_1^{\frac{1}{2}} u_m(t)\right|^2 + \left|A_2^{\frac{1}{2}} v_m(t)\right|^2\right)(A_2 v_m(t), z) + \delta_2(v_m'(t), z) = 0, \quad (8)$$

and

$$(u_m(0), v_m(0)) \longrightarrow (u_0, v_0), \quad (9)$$

$$(K_1 u_m'(0), K_2 v_m'(0)) \longrightarrow \left(K_1^{\frac{1}{2}} u_1, K_2^{\frac{1}{2}} v_1\right), \quad (10)$$

in $\mathcal{D}(A_1) \times \mathcal{D}(A_2)$ and $H \times H$ respectively.

Now by the Carathéodory's theorem, $u_m(t)$ and $v_m(t)$ are defined just in the interval $[0, T_m]$ with $0 < T_m < T$ and we need to prolong for the interval $[0, T],\ 0 < T < \infty$.

Priori estimates
Taking $w = u_m'(t)$ and $z = v_m'(t)$ in (7) and (8) we respectively obtain

$$\frac{1}{2}\frac{d}{dt}\left[|K_1^{\frac{1}{2}} u_m'|^2 + |A_1 u_m|^2\right] + M\left(\left|A_1^{\frac{1}{2}} u_m\right|^2 + \left|A_2^{\frac{1}{2}} v_m\right|^2\right)(A_1 u_m, u_m') + \delta_1 |u_m'|^2 = 0, \quad (11)$$

$$\frac{1}{2}\frac{d}{dt}\left[|K_2^{\frac{1}{2}} v_m'|^2 + |A_2 v_m|^2\right] + M\left(\left|A_1^{\frac{1}{2}} u_m\right|^2 + \left|A_2^{\frac{1}{2}} v_m\right|^2\right)(A_2 v_m, v_m') + \delta_2 |v_m'|^2 = 0. \quad (12)$$

Now we observe that

$$M\left(\left|A_1^{\frac{1}{2}} u_m\right|^2 + \left|A_2^{\frac{1}{2}} v_m\right|^2\right)(A_1 u_m, u_m') = \frac{1}{2} M\left(\left|A_1^{\frac{1}{2}} u_m\right|^2 + \left|A_2^{\frac{1}{2}} v_m\right|^2\right)\frac{d}{dt}|A_1^{\frac{1}{2}} u_m|^2,$$

and

$$M\left(\left|A_1^{\frac{1}{2}} u_m\right|^2 + \left|A_2^{\frac{1}{2}} v_m\right|^2\right)(A_2 v_m, v_m') = \frac{1}{2} M\left(\left|A_1^{\frac{1}{2}} u_m\right|^2 + \left|A_2^{\frac{1}{2}} v_m\right|^2\right)\frac{d}{dt}|A_2^{\frac{1}{2}} v_m|^2.$$

Now adding and using $\widehat{M}(\lambda) = \int_0^\lambda M(s)ds$, follows from (12), (12) that

$$\frac{1}{2}\frac{d}{dt}\left[|K_1^{\frac{1}{2}}u_m'|^2+|A_1u_m|^2+|K_2^{\frac{1}{2}}v_m'|^2\right.$$
$$\left.+|A_2v_m|^2+\widehat{M}\left(\left|A_1^{\frac{1}{2}}u_m\right|^2+\left|A_2^{\frac{1}{2}}v_m\right|^2\right)\right]$$
$$+\delta_1|u_m'|^2+\delta_2|v_m'|^2=0.$$

From the equation above, after integration from 0 to t, $0\leq t\leq T_m$, we obtain

$$|K_1^{\frac{1}{2}}u_m'|^2+|A_1u_m|^2+|K_2^{\frac{1}{2}}v_m'|^2+|A_2v_m|^2$$
$$+\widehat{M}\left(\left|A_1^{\frac{1}{2}}u_m\right|^2+\left|A_2^{\frac{1}{2}}v_m\right|^2\right)$$
$$+2\int_0^t(\delta_1|u_m'|^2+\delta_2|v_m'|^2)ds$$
$$=|K_1^{\frac{1}{2}}u_{1m}|^2+|A_1u_{0m}|^2+|K_2^{\frac{1}{2}}v_{1m}|^2+|A_2v_{0m}|^2$$
$$+\widehat{M}\left(\left|A_1^{\frac{1}{2}}u_{0m}\right|^2+\left|A_2^{\frac{1}{2}}v_m\right|^2\right)\leq C_1,$$

with $C_1>0$, constant independent of m and t. Then we have

$$\left(K_1^{\frac{1}{2}}u_m'\right),\left(K_2^{\frac{1}{2}}v_m'\right)\ \text{ limited in } L^\infty([0,T);H),\qquad(13)$$
$$(u_m)\ \text{ limited in } L^\infty([0,T);\mathcal{D}(A_1)),\qquad(14)$$
$$(v_m)\ \text{ limited in } L^\infty([0,T);\mathcal{D}(A_2)).\qquad(15)$$

Since that $K_i^{\frac{1}{2}}$, $i=1,2$, are limited from H in H using (13) we get

$$(K_1u_m'),(K_2v_m')\ \text{ limited in } L^\infty([0,T);H).\qquad(16)$$

Passage to the limit

Using the estimates (14) - (15), we can extract subsequences of $(u_m)_{m\in N}$, $(v_m)_{m\in N}$, (which we denote with the same symbol) so that

$$u_m\overset{\star}{\rightharpoonup}u \text{ weak star in } L^\infty([0,T);\mathcal{D}(A_1))\qquad(17)$$
$$v_m\overset{\star}{\rightharpoonup}v \text{ weak star in } L^\infty([0,T);\mathcal{D}(A_2))\qquad(18)$$
$$u_m'\rightharpoonup u\ \text{ weak in } L^\infty([0,T);H)\qquad(19)$$
$$v_m'\rightharpoonup v'\ \text{ weak in } L^\infty([0,T);H)\qquad(20)$$
$$K_1u_m'\rightharpoonup K_1u'\ \text{ weak in } L^\infty([0,T);H)\qquad(21)$$
$$K_2v_m'\rightharpoonup K_2v'\ \text{ weak in } L^\infty([0,T);H)\qquad(22)$$

From (17)-(20) and Aubin-Lions's compacteness theorem, see [6] follows that

$$u_m\longrightarrow u\ \text{ strongly in } L^2([0,T);\mathcal{D}(A_1^{\frac{1}{2}})),\qquad(23)$$
$$v_m\longrightarrow v\ \text{ strongly in } L^2([0,T);\mathcal{D}(A_2^{\frac{1}{2}})).\qquad(24)$$

By continuity of M we have the following weak convergence in $L^2([0,T);H)$,

$$M\left(\left|A_1^{\frac{1}{2}}u_m\right|^2+\left|A_2^{\frac{1}{2}}v_m\right|^2\right)A_1u_m$$
$$\rightharpoonup M\left(\left|A_1^{\frac{1}{2}}u\right|^2+\left|A_2^{\frac{1}{2}}v\right|^2\right)A_1u,\qquad(25)$$

$$M\left(\left|A_1^{\frac{1}{2}}u_m\right|^2+\left|A_2^{\frac{1}{2}}v_m\right|^2\right)A_2v_m$$
$$\rightharpoonup M\left(\left|A_1^{\frac{1}{2}}u\right|^2+\left|A_2^{\frac{1}{2}}v\right|^2\right)A_2v.\qquad(26)$$

These convergence implies that we can take limits in the approximate problem (7)-(10) and then we concludes the existence of weak solution. Finally since that M is locally Lipschitz, the uniqueness of solution can be proved as usual, by Ladyzhenskaya's method [13].

3. Stability of solutions

First we introduce the Nakao's theorem

Theorem 2. *Let $E(t)$ be a nonnegative function on $[0,\infty)$ satisfying*

$$s\in[t,t+1]\sup E(s)\leq C_0(\,E(t)-E(t+1)\,)$$

where C_0 is a positive constant. Then there exist C positive constant such that

$$E(t)\leq Ce^{-w\,t}\quad \textit{with}\quad w=\frac{1}{C_0+1}.$$

Proof. See page 748 of [10].

Now we will to prove our principal result

Theorem 3. *For $t\geq 1$ and $(u_0,v_0)\in\mathcal{D}(A_1)\times\mathcal{D}(A_2)$, $(u_1,v_1)\in H\times H$, the full energy of (1)-(4) defined by*

$$E(t)=\frac{1}{2}\left[\left|K_1^{\frac{1}{2}}u'\right|^2+|A_1u|^2+\left|K_2^{\frac{1}{2}}v'\right|^2+|A_2v|^2\right.$$
$$\left.+\widehat{M}\left(\left|A_1^{\frac{1}{2}}u\right|^2+\left|A_2^{\frac{1}{2}}v\right|^2\right)\right]$$

satisfies

$$E(t)\leq C\,e^{-wt}.$$

Proof. Using (12) and (12) we obtain after passage to limit

$$\frac{1}{2}\frac{d}{dt}\left[|K_1^{\frac{1}{2}}u'|^2+|A_1u|^2+|K_2^{\frac{1}{2}}v'|^2+|A_2v|^2\right.$$
$$\left.+\ \widehat{M}\left(\left|A_1^{\frac{1}{2}}u\right|^2+\left|A_2^{\frac{1}{2}}v\right|^2\right)\right]+\delta_1|u'|^2+\delta_2|v'|^2=0.$$

Performing integration from t_1 to t_2, with $0<t_1<t_2$ we get

$$E(t_2)+\int_{t_1}^{t_2}\left[\delta_1|u'(s)|^2+\delta_2|v'(s)|^2\right]ds=E(t_1)\qquad(27)$$

and for all $t>0$

$$E(t+1)+\int_t^{t+1}\left[\delta_1|u'(s)|^2+\delta_2|v'(s)|^2\right]ds=E(t).$$

Now, defining $C_2[E(t)-E(t+1)]\equiv F^2(t)$ we obtain

$$\int_t^{t+1}\left[|u'(s)|^2+|v'(s)|^2\right]ds\leq F^2(t),\qquad(28)$$

with
$$C_2=\frac{1}{\min\{\delta_1,\delta_2\}}.$$

Then, we can choose
$t_1\in\left[t,t+\frac{1}{4}\right]$ and $t_2\in\left[t+\frac{3}{4},t+1\right]$, such that

$$|u'(t_1)|^2+|v'(t_1)|^2\le 4F^2(t),$$

and so

$$|u'(t_1)|+|v'(t_1)|\le 4F(t). \tag{29}$$

Similarly we get

$$|u'(t_2)|+|v'(t_2)|\le 4F(t). \tag{30}$$

Making $w=u(t)$ and $z=v(t)$ in (7) and (8) we obtain respectively

$$\begin{aligned}&(K_1u''(t),u(t))+|A_1u(t)|^2\\&+M\left(\left|A_1^{\frac12}u(t)\right|^2+\left|A_2^{\frac12}v(t)\right|^2\right)\left|A_1^{\frac12}u(t)\right|^2\\&+\delta_1(u'(t),u(t))=0,\end{aligned}$$

$$\begin{aligned}&(K_2v''(t),v(t))+|A_2v(t)|^2+\\&M\left(\left|A_1^{\frac12}u(t)\right|^2+\left|A_2^{\frac12}v(t)\right|^2\right)\left|A_2^{\frac12}v(t)\right|^2\\&+\delta_2(v'(t),v(t))=0,\end{aligned}$$

and adding

$$\begin{aligned}&\frac{d}{dt}(K_1u'(t),u(t))-\left|K_1^{\frac12}u'(t)\right|^2+|A_1u(t)|^2\\&+M\left(\left|A_1^{\frac12}u(t)\right|^2+\left|A_2^{\frac12}v(t)\right|^2\right)\left|A_1^{\frac12}u(t)\right|^2\\&+\frac{d}{dt}(K_2v'(t),v(t))-\left|K_2^{\frac12}v'(t)\right|^2++|A_2v(t)|^2\\&+M\left(\left|A_1^{\frac12}u(t)\right|^2+\left|A_2^{\frac12}v(t)\right|^2\right)\left|A_2^{\frac12}v(t)\right|^2\\&+\delta_1(u'(t),u(t))+\delta_2(v'(t),v(t))=0.\end{aligned}$$

Integrating from t_1 to t_2 we have

$$\begin{aligned}&\int_{t_1}^{t_2}\Big[|A_1u(t)|^2+|A_2v(t)|^2\\&+M\left(\left|A_1^{\frac12}u(t)\right|^2+\left|A_2^{\frac12}v(t)\right|^2\right)\left|A_1^{1/2}u(t)\right|^2\\&+M\left(\left|A_1^{\frac12}u(t)\right|^2+\left|A_2^{1/2}v(t)\right|^2\right)\left|A_2^{\frac12}v(t)\right|^2\Big]dt\\&=(K_1u'(t_1),u(t_1))-(K_1u'(t_2),u(t_2)\\&+(K_2v'(t_1),v(t_1))-(K_2v'(t_2),v(t_2))\\&+\int_{t_1}^{t_2}\Big[\left|K_1u'(t)\right|^2+\left|K_2v'(t)\right|^2\\&-\delta_1(u'(t),u(t))-\delta_2(v'(t),v(t))\Big]dt.\end{aligned}$$

Using $M\in C^0[0,+\infty)$, with $M(\lambda)\ge 0$ we get

$$\begin{aligned}&\int_{t_1}^{t_2}|A_1u(t)|^2+|A_2v(t)|^2dtq\\&\le M_1\int_{t_1}^{t_2}|u'(t)|^2dt+M_2\int_{t_1}^{t_2}|v'(t)|^2dt\\&+C_3\delta_1\int_{t_1}^{t_2}|u'(t)||A_1u(t)|dt+C_4\delta_2\int_{t_1}^{t_2}|v'(t)||A_2v(t)|dt\\&+M_1|u'(t_1)||u(t_1)|+M_1|u'(t_2)||u(t_2)|\\&+M_2|v'(t_1)||v(t_1)|+M_2|v'(t_2)||v(t_2)|,\end{aligned}$$

where $M_i=\|K_i\|_{\mathcal{L}(H)}$, $i=1,2$ and C_3,C_4 are constants such that $|u(t)|\le C_3|A_1u(t)|$ and $|v(t)|\le C_4|A_2v(t)|$.

Using (29) and (30) we obtain for $s\in[t,t+1]$

$$\begin{aligned}&\int_{t_1}^{t_2}\left[|A_1u(t)|^2+|A_2v(t)|^2\right]\,dt\le M_1F^2(t)+M_2F^2(t)\\&+\frac{C_3^2\delta_1^2}{2}\int_{t_1}^{t_2}|u'(t)|^2dt+\frac12\int_{t_1}^{t_2}|A_1u(t)|^2dt\\&+\frac{C_4^2\delta_2^2}{2}\int_{t_1}^{t_2}|v'(t)|^2dt+\frac12\int_{t_1}^{t_2}|A_2v(t)|^2dt\\&+M_1C_3\,\sup|A_1u(s)|(|u'(t_1)|+|u'(t_2)|)\\&+M_2C_4\,\sup|A_2v(s)|(|v'(t_1)|+|v'(t_2)|).\end{aligned}$$

Defining

$$\begin{aligned}&C_4[F^2(t)+\sup|A_1u(s)|(|u'(t_1)|+|u'(t_2)|)\\&+\sup|A_2v(s)|(|v'(t_1)|+|v'(t_2)|)]\equiv G^2(t)\end{aligned}$$

we get

$$\int_{t_1}^{t_2}\left[|A_1u(t)|^2+|A_2v(t)|^2\right]\le G^2(t), \tag{31}$$

where

$$C_4=\max\{2M_0,2M_1C_3,2M_2C_4\},$$

with

$$M_0=\left(M_1+M_2+\frac{C_3^2\delta_1^2}{2}+\frac{C_4^2\delta_2^2}{2}\right).$$

From (28) and (31) we obtain

$$\begin{aligned}&\int_{t_1}^{t_2}\left[(|A_1u(t)|^2+|A_2v(t)|^2)\right.\\&\left.+\delta_1|u'(t)|^2+\delta_2|v'(t)|^2\right]dt\le C_5F^2(t)+G^2(t),\end{aligned}$$

where $C_5=\delta_1+\delta_2$.

Then, there exist $t^*\in[t_1,t_2]$ such that

$$\begin{aligned}&|A_1u(t^*)|^2+|A_2v(t^*)|^2+\delta_1|u'(t^*)|^2+\delta_2|v'(t^*)|^2\\&\le 2[C_5F^2(t)+G^2(t)].\end{aligned} \tag{32}$$

Observe that

$$\begin{aligned}
&\widehat{M}\left(|A_1^{1/2}u(t^*)|^2 + |A_2^{1/2}v(t^*)|^2\right) \\
&= \int_0^{|A_1^{1/2}u(t^*)|^2+|A_2^{1/2}v(t^*)|^2} M(s)ds \\
&\le m_0\left(|A_1^{1/2}u(t^*)|^2 + |A_2^{1/2}v(t^*)|^2\right) \\
&\le m_0\, C_6\left(|A_1u(t^*)|^2 + |A_2v(t^*)|^2\right), \qquad (33)
\end{aligned}$$

where $m_0 = \max M(s)$ that is finite and C_6 is a constant such that

$$|A_1^{\frac{1}{2}}u(t)|^2 \le C_6|A_1u(t)|^2 \text{ and } |A_2^{\frac{1}{2}}v(t)|^2 \le C_6|A_2v(t)|^2.$$

Now, using the energy functional and (32), (33), we obtain

$$E(t^*) \le C_7\left[F^2(t) + G^2(t)\right]. \qquad (34)$$

From (27), (28) and (34) we have

$$E(t^{t_1}) = E(t^*) + \int_{t_1}^{t^*}\left[\delta_1|u'(s)|^2 + \delta_2|v'(s)|^2\right]ds,$$

so

$$E(t) \le \int_1^{t+1}\left[\delta_1|u'(s)|^2 + \delta_2|v'(s)|^2\right]ds + E(t^*),$$

and then for $s \in [t, t+1]$

$$\begin{aligned}
\sup E(s) &\le E(t^*) + \int_t^{t+1}\left[\delta_1|u'(s)|^2 + \delta_2|v'(s)|^2\right]ds \\
&\le C_7\left[F^2(t) + G^2(t)\right] + C_5F^2(t),
\end{aligned}$$

so we obtain

$$\sup E(s) \le C_8F^2(t) + \frac{1}{2}\sup E(s).$$

From last inequality we have

$$\sup E(s) \le 2C_8F^2(t) = C_0\left[E(t) - E(t+1)\right],$$

and then

$$E(t) \le C_0\left[E(t) - E(t+1)\right].$$

Finally, by theorem 3.1 we concludes that

$$E(t) \le Ce^{-w\,t} \quad \text{with} \quad w = \frac{1}{C_0+1}.$$

4. Conclusion

In this paper, we employed the Nakao's method to analyze the asymptotic behaviour for an abstract coupled system of nonlinear equations of extensible beams models. We prove that when the time t goes to infinity the system has a exponential decay.

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