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Estimation of Stress-Strength Reliability Model Using Finite Mixture of Two Parameter Lindley Distributions

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Abstact: The term "stress- strength reliability" when it appears in statistical literature typically refers to the quantity $P(X > Y)$, where a system with random strength X is subjected to a random strength Y such that a system fails, if the stress exceeds the strength. Stress –Strength reliability is considered in this paper where the strength follows finite mixture of two parameter Lindley distribution and stress follows exponential, Lindley distribution and mixture of two parameter Lindley distribution. The general expressions for the reliabilities of a system are obtained. Estimates of parameters are obtained by maximum likelihood estimation method. For different values of stress and strength parameters, reliability has been obtained. Special cases are also discussed.

Key Words: Lindley distribution, Exponential distribution, Reliability function and Maximum likelihood estimation.

1 Introduction

The problem of increasing reliability of any system become more significant in many fields of industry, transport, communications technology, etc, with the complex mechanization and automation of industrial processes. Underestimation and overestimation of factors associated with reliability may engender great losses. The term "stress-strength reliability" when it appears in statistical literature, typically refers to the quantity R=P(Y < X), where a system with random strength X is subject to a random stress Y such that the system fails if the stress exceeds the strength. The term stress-strength was first introduced by Church and Harris. Some authors have considered different choices for stress and strength distributions. The stress-strength reliability and its estimation problems for several distributions are discussed in the works of Church and Harris [8], Woodward and Kelley [18], Beg and Singh [14], Awad and Gharraf [1], Surles and Padgett [6,7], Raqab and Kundu [13], Mokhlis[15], and Saraçoğlu et al. [3]. Recently, Kotz et al. [17] have presented a review of all methods and results on the stress-strength reliability in the last four decades. Adil H. khan and T.R Jan[2]obtained Bayes estimators of the parameters of the Consul, Geeta and Size-biased Geeta distributions and associated reliability function.

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Lindley [4] proposed "Lindley distribution" (LD) in the context of Bayesian statistics, as a counter example of fudicial statistics. Lindley distribution belongs to an exponential family and it can be written as a mixture of an exponential and a gamma distribution with shape parameter 2. Therefore, many properties of the mixture distribution can be translated for the Lindley distribution. The Lindley distribution, in spite of little attention in the statistical literature, is important for studying stress-strength reliability modeling. Besides, some researchers have proposed new classes of distributions based on modifications of the Lindley distribution, including also their properties. Sankaran [12] introduced the discrete Poisson-Lindley distribution by combining the Poisson and Lindley distributions. Adamidis and Loukas [9] introduced a two-parameter lifetime distribution with decreasing failure rate by compounding exponential and geometric distributions, which was named exponential geometric (EG) distribution. Ghitany et al. [11] investigated most of the statistical properties of the Lindley distribution, showing this distribution may provide a better fitting than the exponential distribution. Bakouch et al. [5] introduced a new extension of the Lindley distribution, called extended Lindley (EL) distribution, which offers a more flexible model for lifetime data.

A two-parameter Lindley distribution with parameters $\lambda$and α is defined by its probability density function (p.d.f.)

$$f(x;\alpha,\lambda)=\frac{\lambda^2}{\lambda+\alpha}(1+\alpha x)e^{-\lambda x} \quad ; \quad x>0,\lambda>0,\alpha>-\lambda \tag{1}$$

It can easily be seen that at $\alpha=1$, the distribution (1) reduces to the one parameter LD and at $\alpha=0$, it reduces to the exponential distribution with parameters λ.

In this paper we consider three cases

1) Stress follows exponential distribution and strength follows finite mixture of Lindley distributions.
2) Stress follows Lindley distribution and strength follows finite mixture of Lindley distributions.
3) Stress and strength both follows finite mixture of Lindley distributions.

We discuss the estimation procedure for finite mixture of two parameter Lindley distributions by the method of maximum likelihood estimation and also estimation of stress-strength reliability.

2 Statistical model

In this model propose that strength (X) and stress (Y) are independent random variables and the values of strength and stress are non-negative.The reliability of a component with strength X and stress Y imposed on it is given by

$$R=P(X>Y)=\int_o^\infty\left[\int_0^x g(y)dy\right]f(x)dx \tag{2}$$

where $f(x)$ and $g(y)$ are pdf of strength and stress respectively.

A finite mixture of 2-parameter Lindley distributions with k components can be represented as

$$f(x)=p_1f_1(x)+p_2f_2(x)+\cdots+p_kf_k(x)\,;p_i>0, i=1,2,\dots,k\sum_{i=1}^{k}p_i=1$$

The r^{th} moment of mixture of two 2- parameter Lindley distributions is

$$E(x^r)=\int_0^\infty x^r\left(p_1\frac{\lambda_1^2}{\lambda_1+\alpha_1}(1+\alpha_1x)e^{-\lambda_1x}+p_2\frac{\lambda_2^2}{\lambda_2+\alpha_2}(1+\alpha_2x)e^{-\lambda_2x}\right)dx$$

$$\text{where}, p_1+p_2=1$$

$$= \frac{p_1\lambda_1^2}{\lambda_1+\alpha_1}\left[\frac{\Gamma(r+1)}{\lambda_1^{r+1}}+\alpha_1\frac{\Gamma(r+2)}{\lambda_1^{r+2}}\right]+\frac{p_2\lambda_2^2}{\lambda_2+\alpha_2}\left[\frac{\Gamma(r+1)}{\lambda_2^{r+1}}+\alpha_2\frac{\Gamma(r+2)}{\lambda_2^{r+2}}\right]$$

$$\text{When } r=1,\ E(x)=\frac{p_1}{\lambda_1+\alpha_1}\left[1+\frac{2\alpha_1}{\lambda_1}\right]+\frac{p_2}{\lambda_2+\alpha_2}\left[1+\frac{2\alpha_2}{\lambda_2}\right];\ p_1+p_2=1$$

$$\text{When } r=1,\quad E(x^2)=\frac{2p_1}{\lambda_1(\lambda_1+\alpha_1)}\left[1+\frac{3\alpha_1}{\lambda_1}\right]+\frac{2p_2}{\lambda_2(\lambda_2+\alpha_2)}\left[1+\frac{3\alpha_2}{\lambda_2}\right]$$

Thus variance is given by

$$V(x)=\frac{2p_1}{\lambda_1(\lambda_1+\alpha_1)}\left[1+\frac{3\alpha_1}{\lambda_1}\right]+\frac{2p_2}{\lambda_2(\lambda_2+\alpha_2)}\left[1+\frac{3\alpha_2}{\lambda_2}\right]\qquad-\left[\frac{p_1}{\lambda_1+\alpha_1}\left[1+\frac{2\alpha_1}{\lambda_1}\right]+\frac{p_2}{\lambda_2+\alpha_2}\left[1+\frac{2\alpha_2}{\lambda_2}\right]\right]^2$$

3 Reliability Computations

Let X be the strength of the k-components with probability density functions $f_i(x); i=1,2,\dots,k$. The pdf of X which follows finite mixture of two parameter Lindley distributions is

$$f_i(x)=p_i\frac{\lambda_i^2}{\lambda_i+\alpha_i}(1+\alpha_i x)e^{-\lambda_i x};\ \ x>0,\lambda_i>0,\alpha_i>-\lambda_i,p_i>0,i=1,2,\dots,k,\sum_{i=1}^{k}p_i=1$$

Case I: The stress Y follows exponential distribution

As Y follows exponential distribution, pdf of Y is given by

$$g(y)=\lambda e^{-\lambda y},\lambda>0. y>0$$

For two components k = 2 and

$$f(x)=p_1\frac{\lambda_1^2}{\lambda_1+\alpha_1}(1+\alpha_1 x)e^{-\lambda_1 x}+p_2\frac{\lambda_2^2}{\lambda_2+\alpha_2}(1+\alpha_2 x)e^{-\lambda_2 x}\ ;\ \ p_1+p_2=1,\lambda_1,\lambda_2,x>0$$

As X and Y are independent then from (2), Reliability function R_2 is

$$R_2=\int_0^{\infty}\int_0^{x}\lambda e^{-\lambda y}\left[p_1\frac{\lambda_1^2}{\lambda_1+\alpha_1}(1+\alpha_1 x)e^{-\lambda_1 x}+p_2\frac{\lambda_2^2}{\lambda_2+\alpha_2}(1+\alpha_2 x)e^{-\lambda_2 x}\right]dxdy$$

$$=-\frac{p_1\lambda_1^2}{\lambda_1+\alpha_1}\left[\left(\frac{1}{\lambda+\lambda_1}+\frac{1}{\lambda_1}\right)+\alpha_1\left(\frac{1}{(\lambda+\lambda_1)^2}+\frac{1}{\lambda_1^2}\right)\right]-\frac{p_2\lambda_2^2}{\lambda_2+\alpha_2}\left[\left(\frac{1}{\lambda+\lambda_2}+\frac{1}{\lambda_2}\right)+\alpha_2\left(\frac{1}{(\lambda+\lambda_2)^2}+\frac{1}{\lambda_2^2}\right)\right]$$

$$=-\sum_{i=1}^{2}\frac{p_i\lambda_i^2}{\lambda_i+\alpha_i}\left[\left(\frac{1}{\lambda+\lambda_i}+\frac{1}{\lambda_i}\right)+\alpha_i\left(\frac{1}{(\lambda+\lambda_i)^2}+\frac{1}{\lambda_i^2}\right)\right];\qquad\sum_{i=1}^{2}p_i=1$$

For three components k = 3,we have

$$f(x)=p_1\frac{\lambda_1^2}{\lambda_1+\alpha_1}(1+\alpha_1 x)e^{-\lambda_1 x}+p_2\frac{\lambda_2^2}{\lambda_2+\alpha_2}(1+\alpha_2 x)e^{-\lambda_2 x}+p_3\frac{\lambda_3^2}{\lambda_3+\alpha_3}(1+\alpha_3 x)e^{-\lambda_3 x}$$

$$; \quad p_1 + p_2 + p_3 = 1, \lambda_1, \lambda_2, \lambda_3, x > 0$$

$$R_3 = \int_0^\infty \int_0^x \lambda e^{-\lambda y} \left[\frac{p_1 \lambda_1^2}{\lambda_1 + \alpha_1} (1 + \alpha_1 x) e^{-\lambda_1 x} + \frac{p_2 \lambda_2^2}{\lambda_2 + \alpha_2} (1 + \alpha_2 x) e^{-\lambda_2 x} + \frac{p_3 \lambda_3^2}{\lambda_3 + \alpha_3} (1 + \alpha_3 x) e^{-\lambda_3 x} \right] dxdy$$

$$= -\frac{p_1 \lambda_1^2}{\lambda_1 + \alpha_1} \left[\left(\frac{1}{\lambda + \lambda_1} - \frac{1}{\lambda_1} \right) + \alpha_1 \left(\frac{1}{(\lambda + \lambda_1)^2} - \frac{1}{\lambda_1^2} \right) \right] - \frac{p_2 \lambda_2^2}{\lambda_2 + \alpha_2} \left[\left(\frac{1}{\lambda + \lambda_2} - \frac{1}{\lambda_2} \right) + \alpha_2 \left(\frac{1}{(\lambda + \lambda_2)^2} - \frac{1}{\lambda_2^2} \right) \right]$$

$$- \frac{p_3 \lambda_3^2}{\lambda_3 + \alpha_3} \left[\left(\frac{1}{\lambda + \lambda_3} - \frac{1}{\lambda_3} \right) + \alpha_3 \left(\frac{1}{(\lambda + \lambda_3)^2} - \frac{1}{\lambda_3^2} \right) \right]$$

$$R_3 = -\sum_{i=1}^{3} \frac{p_i \lambda_i^2}{\lambda_i + \alpha_i} \left[\left(\frac{1}{\lambda + \lambda_i} - \frac{1}{\lambda_i} \right) + \alpha_i \left(\frac{1}{(\lambda + \lambda_i)^2} - \frac{1}{\lambda_i^2} \right) \right] ; \qquad \sum_{i=1}^{3} p_i = 1$$

In general for k-components, $f(x) = p_1 f_1(x) + p_2 f_2(x) + \cdots + p_k f_k(x)$; $\sum_{i=1}^{k} p_i = 1$, we have

$$R_k = -\sum_{i=1}^{k} \frac{p_i \lambda_i^2}{\lambda_i + \alpha_i} \left[\left(\frac{1}{\lambda + \lambda_i} - \frac{1}{\lambda_i} \right) + \alpha_i \left(\frac{1}{(\lambda + \lambda_i)^2} - \frac{1}{\lambda_i^2} \right) \right]$$

$$R_k = \sum_{i=1}^{k} \frac{p_i \lambda \lambda_i}{(\lambda_i + \alpha_i)(\lambda + \lambda_i)} \left[1 + \alpha_i \left(\frac{1}{\lambda_i} + \frac{1}{\lambda + \lambda_i} \right) \right]$$

Special case

1. When $\alpha_i = 0$, two parameter Lindley distribution reduces to exponential distribution and then R_k is the reliability function when X follows exponential distribution and Y follows mixture of exponential distributions and is given as

$$R_k = \sum_{i=1}^{k} p_i \frac{\lambda}{\lambda + \lambda_i} = 1 - \sum_{i=1}^{k} p_i \frac{\lambda_i}{\lambda + \lambda_i} ; \qquad \sum_{i=1}^{k} p_i = 1 \text{ (See Sandhya and. Umamaheswari, [10])}$$

2. When $\alpha_i = 1$, two parameter Lindley distribution reduces to one parameter Lindleyl distribution and then R_k is the reliability function when X follows exponential distribution and Y follows mixture of one parameter Lindley distributions and is given as

$$R_k = \sum_{i=1}^{k} \frac{p_i \lambda \lambda_i}{(\lambda_i + 1)(\lambda + \lambda_i)} \left[1 + \frac{1}{\lambda_i} + \frac{1}{\lambda + \lambda_i} \right]$$

Case II: The stress Y followstwo parameter Lindley distribution

As Y follows Lindley distribution, pdf of Y is given by

$$g(y) = \frac{\lambda^2}{\lambda + \alpha} (1 + \alpha y) e^{-\lambda y} \quad ; \quad y > 0, \lambda > 0, \alpha > -\lambda$$

For two components k = 2

$$f(x) = p_1 \frac{\lambda_1^2}{\lambda_1 + \alpha_1} (1 + \alpha_1 x) e^{-\lambda_1 x} + p_2 \frac{\lambda_2^2}{\lambda_2 + \alpha_2} (1 + \alpha_2 x) e^{-\lambda_2 x} \; ; \; p_1 + p_2 = 1, \lambda_1, \lambda_2, x > 0$$

As X and Y are independent then from (2), Reliability function R_2 is

$$R_2 = \int_0^\infty \int_0^x \frac{\lambda^2}{\lambda+\alpha}(1+\alpha y)e^{-\lambda y}\left[p_1\frac{\lambda_1^2}{\lambda_1+\alpha_1}(1+\alpha_1 x)e^{-\lambda_1 x} + p_2\frac{\lambda_2^2}{\lambda_2+\alpha_2}(1+\alpha_2 x)e^{-\lambda_2 x}\right]dxdy$$

$$R_2 = \sum_{i=1}^{2}\int_0^\infty \int_0^x \frac{p_i\lambda^2\lambda_i^2}{(\lambda+\alpha)(\lambda_i+\alpha_i)}(1+\alpha y)e^{-\lambda y}(1+\alpha_i x)e^{-\lambda_i x}\,dxdy$$

$$R_2 = \sum_{i=1}^{2}\frac{p_i\lambda_i^2}{(\lambda+\alpha)(\lambda_i+\alpha_i)}\left\{\lambda\left[\left(\frac{1}{\lambda_i}-\frac{1}{\lambda+\lambda_i}\right)+\alpha_i\left(\frac{1}{\lambda_i^2}-\frac{1}{(\lambda+\lambda_i)^2}\right)\right] + \alpha\left[\left(\frac{1}{\lambda_i}-\frac{1}{\lambda+\lambda_i}-\frac{\lambda}{(\lambda+\lambda_i)^2}\right)+\alpha_i\left(\frac{1}{\lambda_i^2}-\frac{1}{(\lambda+\lambda_i)^2}-\frac{2\lambda}{(\lambda+\lambda_i)^3}\right)\right]\right\}$$

For three components k = 3, we have

$$f(x) = p_1\frac{\lambda_1^2}{\lambda_1+\alpha_1}(1+\alpha_1 x)e^{-\lambda_1 x} + p_2\frac{\lambda_2^2}{\lambda_2+\alpha_2}(1+\alpha_2 x)e^{-\lambda_2 x} + p_3\frac{\lambda_3^2}{\lambda_3+\alpha_3}(1+\alpha_3 x)e^{-\lambda_3 x}$$

$$;\ p_1+p_2+p_3 = 1, \lambda_1, \lambda_2, \lambda_3, x > 0$$

$$R_3 = \int_0^\infty \int_0^x \frac{\lambda^2}{\lambda+\alpha}(1+\alpha y)e^{-\lambda y}\left[\frac{p_1\lambda_1^2}{\lambda_1+\alpha_1}(1+\alpha_1 x)e^{-\lambda_1 x} + \frac{p_2\lambda_2^2}{\lambda_2+\alpha_2}(1+\alpha_2 x)e^{-\lambda_2 x} + \frac{p_3\lambda_3^2}{\lambda_3+\alpha_3}(1+\alpha_3 x)e^{-\lambda_3 x}\right]dxdy$$

$$R_3 = \sum_{i=1}^{3}\frac{p_i\lambda_i^2}{(\lambda+\alpha)(\lambda_i+\alpha_i)}\left\{\lambda\left[\left(\frac{1}{\lambda_i}-\frac{1}{\lambda+\lambda_i}\right)+\alpha_i\left(\frac{1}{\lambda_i^2}-\frac{1}{(\lambda+\lambda_i)^2}\right)\right] + \alpha\left[\left(\frac{1}{\lambda_i}-\frac{1}{\lambda+\lambda_i}-\frac{\lambda}{(\lambda+\lambda_i)^2}\right)+\alpha_i\left(\frac{1}{\lambda_i^2}-\frac{1}{(\lambda+\lambda_i)^2}-\frac{2\lambda}{(\lambda+\lambda_i)^3}\right)\right]\right\}$$

In general for k-components,

$$f(x) = p_1 f_1(x) + p_2 f_2(x) + \cdots + p_k f_k(x)\ ;\quad \textstyle\sum_{i=1}^{k} p_i = 1, \text{ we have}$$

$$R_k = \sum_{i=1}^{k}\frac{p_i\lambda_i^2}{(\lambda+\alpha)(\lambda_i+\alpha_i)}\left\{\lambda\left[\left(\frac{1}{\lambda_i}-\frac{1}{\lambda+\lambda_i}\right)+\alpha_i\left(\frac{1}{\lambda_i^2}-\frac{1}{(\lambda+\lambda_i)^2}\right)\right] + \alpha\left[\left(\frac{1}{\lambda_i}-\frac{1}{\lambda+\lambda_i}-\frac{\lambda}{(\lambda+\lambda_i)^2}\right)+\alpha_i\left(\frac{1}{\lambda_i^2}-\frac{1}{(\lambda+\lambda_i)^2}-\frac{2\lambda}{(\lambda+\lambda_i)^3}\right)\right]\right\}$$

$$R_k = \sum_{i=1}^{k}\frac{\lambda p_i\lambda_i^2}{(\lambda+\alpha)(\lambda_i+\alpha_i)}\left\{\frac{\lambda+\alpha}{\lambda_i(\lambda+\lambda_i)}\left[1+\alpha_i\left(\frac{1}{\lambda_i}+\frac{1}{\lambda+\lambda_i}\right)\right]-\frac{\alpha}{(\lambda+\lambda_i)^2}\left(1+\frac{2\alpha_i}{\lambda+\lambda_i}\right)\right\}$$

Special case

1) When $\alpha = \alpha_i = 0$, generalized Lindley distribution reduces to exponential distribution and then R_k is the reliability function when X follows exponential distribution and Y follows mixture of exponential distributions and is given as

$$R_k = 1 - \sum_{i=1}^{k} p_i\frac{\lambda_i}{\lambda+\lambda_i}\ ;\qquad \sum_{i=1}^{k} p_i = 1$$

(See K Sandhya and T.S. Umamaheswari, [10])

2) When $\alpha = \alpha_i = 1$, two parameter Lindley distribution reduces to one parameter Lindley distribution and then R_k is the reliability function when X follows one parameter Lindley distribution and Y follows mixture of one parameter Lindley distribution and is given as

$$R_k = \sum_{i=1}^{k} \frac{\lambda p_i \lambda_i^2}{(\lambda+1)(\lambda_i+1)} \left\{ \frac{\lambda+1}{\lambda_i(\lambda+\lambda_i)} \left[1 + \frac{1}{\lambda_i} + \frac{1}{\lambda+\lambda_i}\right] - \frac{1}{(\lambda+\lambda_i)^2}\left(1 + \frac{2}{\lambda+\lambda_i}\right) \right\}$$

Case III: The stress Y follows mixture of two parameter Lindley distributions

As X and Y both follows mixture of Lindley distributions, pdf of X andY is given by

For two components, k = 2

$$f(x) = p_1 \frac{\lambda_1^2}{\lambda_1+\alpha_1}(1+\alpha_1 x)e^{-\lambda_1 x} + p_2 \frac{\lambda_2^2}{\lambda_2+\alpha_2}(1+\alpha_2 x)e^{-\lambda_2 x} \;;\; p_1 + p_2 = 1, \lambda_1, \lambda_2, x > 0$$

$$g(y) = p_3 \frac{\lambda_3^2}{\lambda_3+\alpha_3}(1+\alpha_3 y)e^{-\lambda_3 y} + p_4 \frac{\lambda_4^2}{\lambda_4+\alpha_4}(1+\alpha_4 y)e^{-\lambda_4 y} \;;\; p_3 + p_4 = 1, \lambda_3, \lambda_4, y > 0$$

As X and Y are independent then from (2), Reliability function R_2 is

$$R_2 = \int_0^\infty \int_o^x \left(p_1 \frac{\lambda_1^2}{\lambda_1+\alpha_1}(1+\alpha_1 x)e^{-\lambda_1 x} + p_2 \frac{\lambda_2^2}{\lambda_2+\alpha_2}(1+\alpha_2 x)e^{-\lambda_2 x}\right)\left(p_3 \frac{\lambda_3^2}{\lambda_3+\alpha_3}(1+\alpha_3 y)e^{-\lambda_3 y} + p_4 \frac{\lambda_4^2}{\lambda_4+\alpha_4}(1+\alpha_4 y)e^{-\lambda_4 y}\right) dxdy$$

$$R_2 = \sum_{j=i+2}^{4} \sum_{i=1}^{2} \int_o^\infty \int_0^x \frac{p_i p_j \lambda_i^2 \lambda_j^2}{(\lambda_i+\alpha_i)(\lambda_j+\alpha_j)} (1+\alpha_i x)\,(1+\alpha_j y) e^{-\lambda_i x} e^{-jy} dxdy$$

$$R_2 = \sum_{j=i+2}^{4} \sum_{i=1}^{2} \frac{p_i p_j \lambda_i^2}{(\lambda_i+\alpha_i)(\lambda_j+\alpha_j)} \left\{ \lambda_j \left[\left(\frac{1}{\lambda_i} - \frac{1}{\lambda_i+\lambda_j}\right) + \alpha_i\left(\frac{1}{\lambda_i^2} - \frac{1}{(\lambda_i+\lambda_j)^2}\right)\right] + \alpha_j \left[\left(\frac{1}{\lambda_i} - \frac{1}{\lambda_i+\lambda_j} - \frac{\lambda_j}{(\lambda_i+\lambda_j)^2}\right) + \alpha_i\left(\frac{1}{\lambda_i^2} - \frac{1}{(\lambda_i+\lambda_j)^2} - \frac{2\lambda_j}{(\lambda_i+\lambda_j)^3}\right)\right] \right\}$$

For three components k = 3, we have

$$f(x) = p_1 \frac{\lambda_1^2}{\lambda_1+\alpha_1}(1+\alpha_1 x)e^{-\lambda_1 x} + p_2 \frac{\lambda_2^2}{\lambda_2+\alpha_2}(1+\alpha_2 x)e^{-\lambda_2 x} + p_3 \frac{\lambda_3^2}{\lambda_3+\alpha_3}(1+\alpha_3 x)e^{-\lambda_3 x}$$

$$;\; p_1 + p_2 + p_3 = 1, \lambda_1, \lambda_2, \lambda_3, x > 0$$

$$g(y) = p_4 \frac{\lambda_4^2}{\lambda_4+\alpha_4}(1+\alpha_4 y)e^{-\lambda_4 y} + p_5 \frac{\lambda_5^2}{\lambda_5+\alpha_5}(1+\alpha_5 y)e^{-\lambda_5 y} + p_3 \frac{\lambda_6^2}{\lambda_6+\alpha_6}(1+\alpha_6 y)e^{-\lambda_6 y}$$

$$;\; p_4 + p_5 + p_6 = 1, \lambda_4, \lambda_5, \lambda_6, y > 0$$

X and Y are independent then from (I), Reliability function R is

$$R_3 = \int_0^\infty \int_o^x \left(\frac{p_1\lambda_1^2}{\lambda_1+\alpha_1}(1+\alpha_1 x)e^{-\lambda_1 x} + \frac{p_2\lambda_2^2}{\lambda_2+\alpha_2}(1+\alpha_2 x)e^{-\lambda_2 x} + \frac{p_3\lambda_3^2}{\lambda_3+\alpha_3}(1+\alpha_3 x)e^{-\lambda_3 x} \right)$$

$$\times \left(\frac{p_4\lambda_4^2}{\lambda_4+\alpha_4}(1+\alpha_4 y)e^{-\lambda_4 y} + \frac{p_5\lambda_5^2}{\lambda_5+\alpha_5}(1+\alpha_5 y)e^{-\lambda_5 y} + \frac{p_3\lambda_6^2}{\lambda_6+\alpha_6}(1+\alpha_6 y)e^{-\lambda_6 y} \right) dxdy$$

$$R_3 = \sum_{j=i+3}^{6} \sum_{i=1}^{3} \int_o^\infty \int_0^x \frac{p_i p_j \lambda_i^2 \lambda_j^2}{(\lambda_i+\alpha_i)(\lambda_j+\alpha_j)} (1+\alpha_i x)\,(1+\alpha_j y) e^{-\lambda_i x} e^{-jy} dxdy$$

$$R_3 = \sum_{j=i+3}^{6} \sum_{i=1}^{3} \frac{p_i p_j \lambda_i^2}{(\lambda_i+\alpha_i)(\lambda_j+\alpha_j)} \Bigg\{ \lambda_j \left[\left(\frac{1}{\lambda_i} - \frac{1}{\lambda_i+\lambda_j} \right) + \alpha_i \left(\frac{1}{\lambda_i^2} - \frac{1}{(\lambda_i+\lambda_j)^2} \right) \right]$$
$$+ \alpha_j \left[\left(\frac{1}{\lambda_i} - \frac{1}{\lambda_i+\lambda_j} - \frac{\lambda_j}{(\lambda_i+\lambda_j)^2} \right) + \alpha_i \left(\frac{1}{\lambda_i^2} - \frac{1}{(\lambda_i+\lambda_j)^2} - \frac{2\lambda_j}{(\lambda_i+\lambda_j)^3} \right) \right] \Bigg\}$$

In general for k-components,

$$f(x) = p_1 f_1(x) + p_2 f_2(x) + \cdots + p_k f_k(x)\,; \quad \sum_{i=1}^{k} p_i = 1$$

$$g(y) = p_{k+1} f_{k+1}(y) + p_{k+2} f_{k+2}(y) + \cdots + p_{2k} f_{2k}(y)\,; \quad \sum_{i=k+1}^{2k} p_i = 1$$

$$R_k = \int_0^\infty \int_o^x \big(p_1 f_1(x) + p_2 f_2(x) + \cdots + p_k f_k(x)\big)\,\big(p_{k+1} f_{k+1}(y) + p_{k+2} f_{k+2}(y) + \cdots + p_{2k} f_{2k}(y)\big) dxdy$$

$$R_k = \sum_{j=i+2}^{2k} \sum_{i=1}^{k} \int_o^\infty \int_0^x \frac{p_i p_j \lambda_i^2 \lambda_j^2}{(\lambda_i+\alpha_i)(\lambda_j+\alpha_j)} (1+\alpha_i x)\,(1+\alpha_j y) e^{-\lambda_i x} e^{-jy} dxdy$$

$$R_k = \sum_{j=i+k}^{2k} \sum_{i=1}^{k} \frac{p_i p_j \lambda_i^2}{(\lambda_i+\alpha_i)(\lambda_j+\alpha_j)} \Bigg\{ \lambda_j \left[\left(\frac{1}{\lambda_i} - \frac{1}{\lambda_i+\lambda_j} \right) + \alpha_i \left(\frac{1}{\lambda_i^2} - \frac{1}{(\lambda_i+\lambda_j)^2} \right) \right]$$
$$+ \alpha_j \left[\left(\frac{1}{\lambda_i} - \frac{1}{\lambda_i+\lambda_j} - \frac{\lambda_j}{(\lambda_i+\lambda_j)^2} \right) + \alpha_i \left(\frac{1}{\lambda_i^2} - \frac{1}{(\lambda_i+\lambda_j)^2} - \frac{2\lambda_j}{(\lambda_i+\lambda_j)^3} \right) \right] \Bigg\}$$

$$R_k = \sum_{j=i+k}^{2k} \sum_{i=1}^{k} \frac{p_i p_j \lambda_i^2 \lambda_j}{(\lambda_i+\alpha_i)(\lambda_j+\alpha_j)} \left\{ \frac{\lambda_j+\alpha_j}{\lambda_i(\lambda_i+\lambda_j)} \left[1 + \alpha_i \left(\frac{1}{\lambda_i} + \frac{1}{\lambda_i+\lambda_j} \right) \right] - \frac{\alpha_j}{(\lambda_i+\lambda_j)^2} \left(1 + \frac{2\alpha_i}{\lambda_i+\lambda_j} \right) \right\}$$

Special case

1) When $\alpha_i = \alpha_j = 0$, generalized Lindley distribution reduces to exponential distribution and then R_k is the reliability function when X and Y both follows mixture of exponential distributions and is given as

$$R_k = 1 - \sum_{j=i+k}^{2k} \sum_{i=1}^{k} p_i p_j \frac{\lambda_i}{(\lambda_i+\lambda_j)} \quad ; \quad \sum_{j=k+1}^{2k} p_j = \sum_{i=1}^{k} p_i = 1$$

(See K Sandhya and T.S. Umamaheswari, [10])

2) When$\alpha_i = \alpha_j = 1$, two parameter Lindley distribution reduces to one parameter Lindley distribution and then R_k is the reliability function when X follows mixture of one parameter Lindley distribution and Y follows mixture of one parameter Lindley distribution and is given as

$$R_k = \sum_{j=i+k}^{2k}\sum_{i=1}^{k}\frac{p_i p_j \lambda_i^2 \lambda_j}{(\lambda_i+1)(\lambda_j+1)}\left\{\frac{\lambda_j+1}{\lambda_i(\lambda_j+\lambda_i)}\left[1+\frac{1}{\lambda_i}+\frac{1}{\lambda_j+\lambda_i}\right]-\frac{1}{(\lambda_j+\lambda_i)^2}\left(1+\frac{2}{\lambda_j+\lambda_i}\right)\right\}$$

4 Hazard Rate

Let$s(t)$ be the survival function of a component then the survival function of the model

For two components is given by

$$S(t) = p_1\left(\frac{\lambda_1+\alpha_1+\lambda_1\alpha_1}{\lambda_1+\alpha_1}\right)e^{-\lambda_1 t} + p_2\left(\frac{\lambda_2+\alpha_2+\lambda_2\alpha_2}{\lambda_2+\alpha_2}\right)e^{-\lambda_2 t}$$

then

$$h(t) = \frac{f(t)}{S(t)} = \frac{p_1\frac{\lambda_1^2}{\lambda_1+\alpha_1}(1+\alpha_1 t)e^{-\lambda_1 t} + p_2\frac{\lambda_2^2}{\lambda_2+\alpha_2}(1+\alpha_2 t)e^{-\lambda_2 t}}{p_1\left(\frac{\lambda_1+\alpha_1+\lambda_1\alpha_1}{\lambda_1+\alpha_1}\right)e^{-\lambda_1 t} + p_2\left(\frac{\lambda_2+\alpha_2+\lambda_2\alpha_2}{\lambda_2+\alpha_2}\right)e^{-\lambda_2 t}}$$

In general for k-components

$$h(t) = \frac{\sum_{i=1}^{k} p_i \frac{\lambda_i^2}{\lambda_i+\alpha_i}(1+\alpha_i x)e^{-\lambda_i x}}{\sum_{i=1}^{k} p_i\left(\frac{\lambda_i+\alpha_i+\lambda_i\alpha_i}{\lambda_i+\alpha_i}\right)e^{-\lambda_i x}}$$

5 Estimation of Parameters

$$L(\lambda_1,\lambda_2,p_1/\hat{y}) = \prod_{j=1}^{n}\left[p_1\frac{\lambda_1^2}{\lambda_1+\alpha_1}(1+\alpha_1 y_j)e^{-\lambda_1 y_j} + p_2\frac{\lambda_2^2}{\lambda_2+\alpha_2}(1+\alpha_2 y_j)e^{-\lambda_2 y_j}\right]$$

$$= \frac{n!}{n_1!\,n_2!}\left(\frac{p_1\lambda_1^2}{\lambda_1+\alpha_1}\right)^{n_1}\left(\frac{p_2\lambda_2^2}{\lambda_2+\alpha_2}\right)^{n_2}\prod_{j=1}^{n_1}(1+\alpha_1 y_{1j})e^{-\lambda_1 y_{1j}}\prod_{j=1}^{n_2}(1+\alpha_2 y_{2j})e^{-\lambda_2 y_{2j}}$$

$$\log L(\lambda_1,\lambda_2,p_1/\hat{y}) = \log\frac{n!}{n_1!\,n_2!} + n_1\log p_1\lambda_1^2 - n_1\log(\lambda_1+\alpha_1) + n_2\log p_2\lambda_2^2 - n_2\log(\lambda_2+\alpha_2)$$

$$-\lambda_1\sum_{j=1}^{n_1} y_{1j} - \lambda_2\sum_{j=1}^{n_2} y_{2j} + \sum_{j=1}^{n_1}\log(1+\alpha_1 y_{1j}) + \sum_{j=1}^{n_2}\log(1+\alpha_2 y_{2j})$$

Now,

$$\frac{\partial \log L}{\partial \lambda_1} = 0$$

$$\Rightarrow \qquad \lambda_1^2 - \left(\frac{n_1}{\sum_{j=1}^{n_1} y_{1j}} - \alpha_1\right)\lambda_1 - \frac{2n_1\alpha_1}{\sum_{j=1}^{n_1} y_{1j}} = 0$$

$$\Rightarrow \quad \hat{\lambda}_1 = \frac{1}{2}\left[\left(\frac{n_1}{\sum_{j=1}^{n_1} y_{1j}} - \alpha_1\right) + \sqrt{\left(\frac{n_1}{\sum_{j=1}^{n_1} y_{1j}} - \alpha_1\right)^2 + \frac{8n_1\alpha_1}{\sum_{j=1}^{n_1} y_{1j}}}\right]$$

Similarly,

$$\hat{\lambda}_2 = \frac{1}{2}\left[\left(\frac{n_2}{\sum_{j=1}^{n_1} y_{2j}} - \alpha_2\right) + \sqrt{\left(\frac{n_2}{\sum_{j=1}^{n_2} y_{2j}} - \alpha_1\right)^2 + \frac{8n_2\alpha_2}{\sum_{j=1}^{n_2} y_{2j}}}\right]$$

and, $$\frac{\partial \log L}{\partial p_1} = 0$$

$$\Rightarrow \quad \hat{p}_1 = \frac{n_1}{n}$$

and $n = n_1 + n_2$

Generalizing the above results for k-components we get

$$\hat{\lambda}_i = \frac{1}{2}\left[\left(\frac{n_i}{\sum_{j=1}^{n_i} y_{ij}} - \alpha_i\right) + \sqrt{\left(\frac{n_i}{\sum_{j=1}^{n_i} y_{ij}} - \alpha_i\right)^2 + \frac{8n_i\alpha_i}{\sum_{j=1}^{n_i} y_{ij}}}\right] \text{ and } \hat{p}_i = \frac{n_i}{n} \quad ; \quad n = \sum_{i=1}^{k} n_i$$

Estimation of Stress-Strength Reliability

1) The M.L.E of R when the strength X follows finite mixture of Lindley distributions with parameter λ_i, α_i(known), and p_i and stress Y follows exponential distribution with parameter λ is given as

$$\hat{R}_k = \sum_{i=1}^{k} \frac{\hat{p}_i\hat{\lambda}_i^2\lambda}{(\hat{\lambda}_i + \alpha_i)(\lambda + \hat{\lambda}_i)}\left[1 + \alpha_i\left(\frac{1}{\hat{\lambda}_i} + \frac{1}{\lambda + \hat{\lambda}_i}\right)\right]$$

2) The M.L.E of R when the strength X follows finite mixture of Lindley distributions with parameter λ_i, α_i(known), and p_i and stress Y follows Lindley distribution with parameter λ and α (known) is given as

$$\hat{R}_k = \sum_{i=1}^{k} \frac{\hat{p}_i\hat{\lambda}_i^2\lambda}{(\lambda + \alpha)(\hat{\lambda}_i + \alpha_i)}\left\{\frac{\lambda + \alpha}{\hat{\lambda}_i(\lambda + \hat{\lambda}_i)}\left[1 + \alpha_i\left(\frac{1}{\hat{\lambda}_i} + \frac{1}{\lambda + \hat{\lambda}_i}\right)\right] - \frac{\alpha}{(\lambda + \hat{\lambda}_i)^2}\left(1 + \frac{2\alpha_i}{\lambda + \hat{\lambda}_i}\right)\right\}$$

3) The M.L.E of R when the strength X and Y follows finite mixture of Lindley distributions with parameter λ_i, α_i(known), and p_i is given as

$$\hat{R}_k = \sum_{j=i+1}^{2k}\sum_{i=1}^{k} \frac{\hat{p}_i\hat{p}_j\hat{\lambda}_i^2\hat{\lambda}_j}{(\lambda_i + \alpha_i)(\hat{\lambda}_j + \alpha_j)}\left\{\frac{\hat{\lambda}_j + \alpha_j}{\hat{\lambda}_i(\hat{\lambda}_i + \hat{\lambda}_j)}\left[1 + \alpha_i\left(\frac{1}{\hat{\lambda}_i} + \frac{1}{\hat{\lambda}_i + \hat{\lambda}_j}\right)\right] - \frac{\alpha_j}{(\hat{\lambda}_i + \hat{\lambda}_j)^2}\left(1 + \frac{2\alpha_i}{\hat{\lambda}_i + \hat{\lambda}_j}\right)\right\}$$

6 Numerical Evaluation

For some specific values of the parameters involved in the expression of R in two component systems, we have evaluated system reliability R for different casesof two parameter Lindley distribution. We consider a data on life to death of two groups of leukaemia patients which is given in table 7. Table 8 provides the values of estimates for different values of α_1 and α_2 by M.L.E method. Also, table 9 and 10 provides estimates of survival function and hazard rate function, respectively, for different values of $\alpha_1$and α_2 at various time points.

Case I: Strength has mixture of two parameter Lindley distribution and Stress has Exponential distribution:

Table 1: Variation in R for constant Stress

p_1	$\lambda_1 = \lambda_2$	$\alpha_1 = \alpha_2$	λ	R
0.1	0.1	0.5	0.6	0.9591
0.1	0.2	0.5	0.6	0.8839
0.1	0.3	0.5	0.6	0.8055
0.1	0.4	0.5	0.6	0.7333
0.1	0.5	0.5	0.6	0.6694
0.1	0.6	0.5	0.6	0.6136
0.1	0.7	0.5	0.6	0.5650
0.1	0.8	0.5	0.6	0.5227
0.1	0.9	0.5	0.6	0.4857
0.1	1	0.5	0.6	0.4531

Table 2: Variation in R for constant Strength

p_1	$\lambda_1 = \lambda_2$	$\alpha_1 = \alpha_2$	λ	R
0.1	0.4	0.5	0.1	0.2888
0.1	0.4	0.5	0.2	0.4567
0.1	0.4	0.5	0.3	0.5646
0.1	0.4	0.5	0.4	0.6388
0.1	0.4	0.5	0.5	0.6927
0.1	0.4	0.5	0.6	0.7333
0.1	0.4	0.5	0.7	0.7649
0.1	0.4	0.5	0.8	0.7901
0.1	0.4	0.5	0.9	0.8106
0.1	0.4	0.5	1	0.8276

Case II: Strength has mixture of two parameter Lindley distribution and Stress has Lindley distributions:

Table 3: Variation in R for constant Strength

p_1	$\lambda_1 = \lambda_2$	$\alpha_1 = \alpha_2$	λ	R
0.1	0.1	0.5	0.1	0.5000
0.1	0.1	0.5	0.2	0.7372
0.1	0.1	0.5	0.3	0.8378
0.1	0.1	0.5	0.4	0.8888
0.1	0.1	0.5	0.5	0.9182
0.1	0.1	0.5	0.6	0.9366
0.1	0.1	0.5	0.7	0.9490
0.1	0.1	0.5	0.8	0.9578
0.1	0.1	0.5	0.9	0.9642
0.1	0.1	0.5	1	0.9691

Table 4: Variation in R for constant Stress

p_1	$\lambda_1 = \lambda_2$	$\alpha_1 = \alpha_2$	λ	R
0.1	0.1	0.5	0.6	0.9366
0.1	0.2	0.5	0.6	0.8291
0.1	0.3	0.5	0.6	0.7255
0.1	0.4	0.5	0.6	0.6363
0.1	0.5	0.5	0.6	0.5618
0.1	0.6	0.5	0.6	0.5000
0.1	0.7	0.5	0.6	0.4485
0.1	0.8	0.5	0.6	0.4053
0.1	0.9	0.5	0.6	0.3688
0.1	1	0.5	0.6	0.3377

Case III: Stress -Strength has mixture of two parameter Lindley distributions:

Table 5: Variation in R for constant Stress

$p_1 = p_3$	$\lambda_1 = \lambda_2$	$\alpha_1 = \alpha_2 = \alpha_3 = \alpha_4$	$\lambda_3 = \lambda_4$	R
0.1	0.1	0.5	0.2	0.7372
0.1	0.2	0.5	0.2	0.5000
0.1	0.3	0.5	0.2	0.3771
0.1	0.4	0.5	0.2	0.2686
0.1	0.5	0.5	0.2	0.2107
0.1	0.6	0.5	0.2	0.1708

Table 6: Variation in R for constant Strength

$p_1 = p_3$	$\lambda_1 = \lambda_2$	$\alpha_1 = \alpha_2 = \alpha_3 = \alpha_4$	$\lambda_3 = \lambda_4$	R
0.1	0.6	0.5	0.1	0.0633
0.1	0.6	0.5	0.2	0.1708
0.1	0.6	0.5	0.3	0.2744
0.1	0.6	0.5	0.4	0.3636
0.1	0.6	0.5	0.5	0.4381
0.1	0.6	0.5	0.6	0.5000

0.1	0.7	0.5	0.2	0.1422	0.1	0.6	0.5	0.7	0.5514
0.1	0.8	0.5	0.2	0.1208	0.1	0.6	0.5	0.8	0.5946
0.1	0.9	0.5	0.2	0.1045	0.1	0.6	0.5	0.9	0.6311
0.1	1	0.5	0.2	0.0917	0.1	0.6	0.5	1	0.6622

The following data are taken from Feigl and Zelen [16]. They refer to 33 leukaemia patients, claasified as either “AG positive” or “AG negative” (positive values being defined by the 3presence of Auer rods and/or significant granulature of the leukaemic cells in the bone marrow at diagnosis, and negative values if both Auer rods and granulature are absent). The initial white blood cell counts and the survival times in weeks are given. We are interested in what we can say about how long a patient is likely to survive, given the white blood cell count (WBC) and the AG group.

Table 7: Survival times of leukaemia patients

AG positive		AG Negative	
WBC	TIME(t)	WBC	TIME(t)
2300	65	4400	56
750	156	3000	65
4300	100	4000	17
2600	134	1500	7
6000	16	9000	16
10500	108	5300	22
10000	121	10000	3
17000	4	19000	4
5400	39	27000	2
7000	143	28000	3
9400	56	31000	8
32000	26	26000	4
35000	22	21000	3
100000	1	79000	30
100000	1	100000	4
52000	5	100000	43
100000	65		

Table 8**:** Estimation of parameters of survival times of leukaemia patients

$\alpha_1 = \alpha_2$	$\hat{\lambda}_1$	$\hat{\lambda}_2$	$\hat{p}_1$
0	0.01600	0.05574	0.515
0.1	0.02846	0.08576	0.515
0.2	0.02993	0.09371	0.515
0.3	0.03053	0.09779	0.515
0.4	0.03086	0.10031	0.515
0.5	0.03107	0.10204	0.515
0.6	0.03122	0.10330	0.515
0.7	0.03132	0.10427	0.515
0.8	0.03141	0.10502	0.515
0.9	0.03147	0.10564	0.515
1	0.03152	0.10614	0.515

Table 9: Maximum Likelihood estimate of survival probability at various time points for different values of $\alpha_1 = \alpha_2$

$R(t)$

T \ $\alpha_1 = \alpha_2$	0	0.2	0.4	0.6	0.8	1
1	0.96553	0.99379	0.99300	0.99263	0.99242	0.99228
15	0.61531	0.46854	0.45260	0.44602	0.44241	0.44017
30	0.40977	0.24798	0.23665	0.23227	0.22995	0.22855
45	0.29015	0.14575	0.13824	0.13544	0.13398	0.13312
60	0.21430	0.08996	0.08463	0.08267	0.08166	0.08107
75	0.16253	0.05666	0.05274	0.05131	0.05057	0.05014
90	0.12523	0.03598	0.03308	0.03202	0.03148	0.03117
105	0.09737	0.02292	0.02079	0.02003	0.01963	0.01941
120	0.07610	0.01462	0.01308	0.01253	0.01225	0.01209

Table 10: Maximum Likelihood estimate of Hazard rate at various time points

$H(t)$

T \ $\alpha_1 = \alpha_2$	0	0.2	0.4	0.6	0.8	1
1	0.0349	0.0183	0.0140	0.0119	0.0107	0.0100
15	0.0295	0.0413	0.0446	0.0462	0.0471	0.0477
30	0.0248	0.0476	0.0512	0.0526	0.0534	0.0538
45	0.0214	0.0504	0.0537	0.0548	0.0554	0.0558
60	0.0191	0.0557	0.0600	0.0613	0.0621	0.0626
75	0.0179	0.0635	0.0692	0.0713	0.0726	0.0732
90	0.0170	0.0731	0.0804	0.0834	0.0848	0.0856
105	0.0165	0.0837	0.0923	0.0958	0.0978	0.0990
120	0.0163	0.0944	0.1047	0.1093	0.1110	0.1125

Table 10: Maximum Likelihood estimate of Hazard rate at various time points

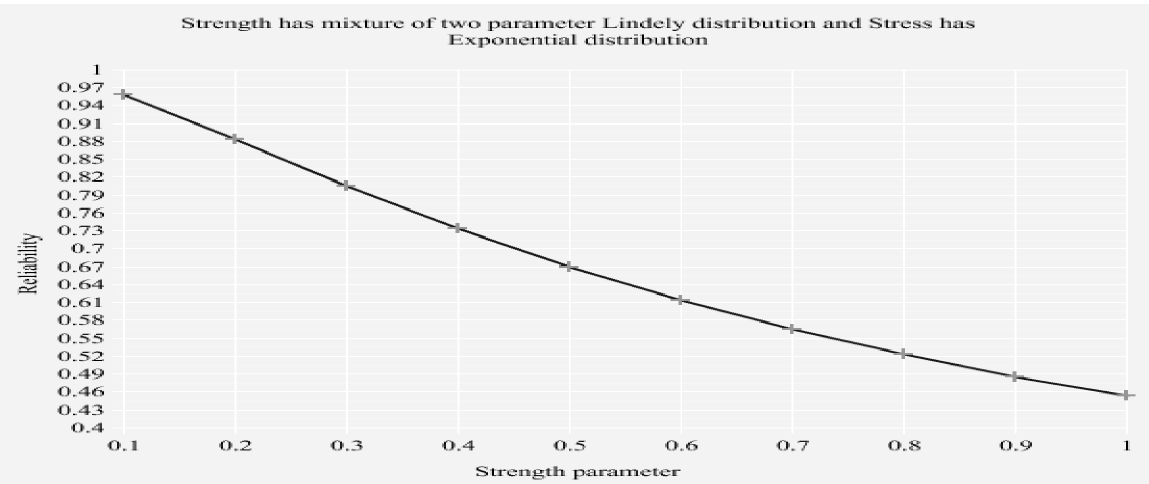

Fig. 1: Variation in R for constant Stress

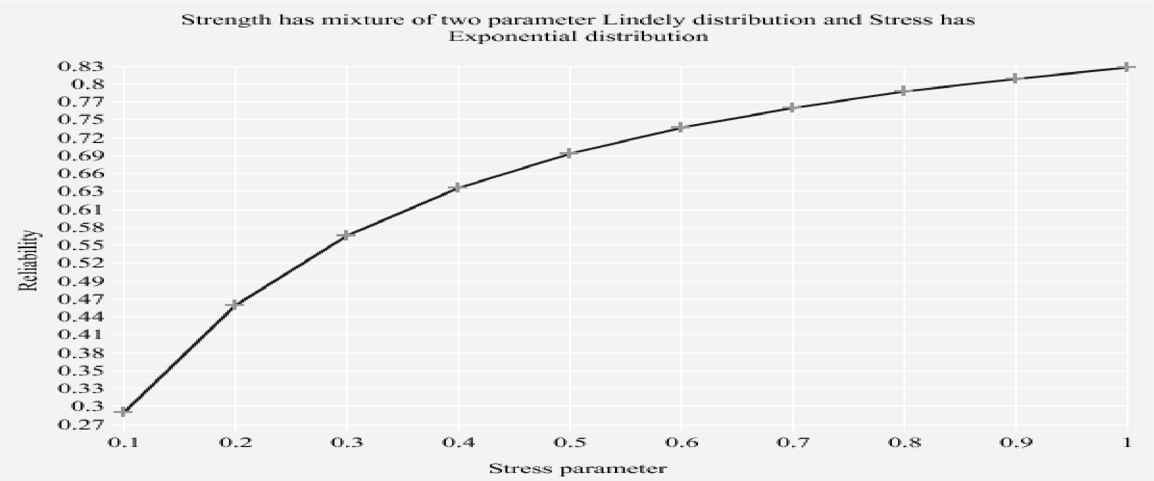

Fig. 2: Variation in R for constant Strength

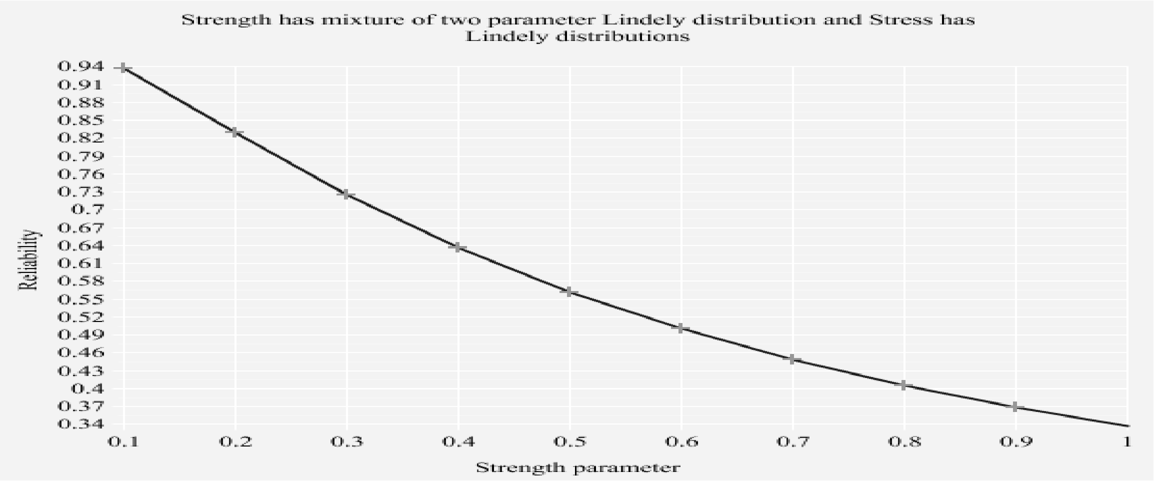

Fig. 3: Variation in R for constant Stress

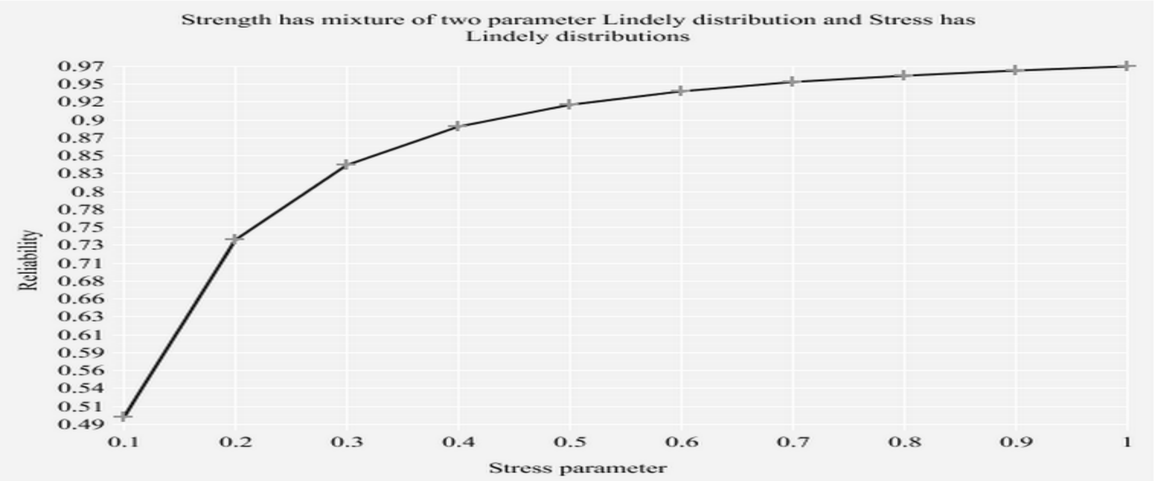

Fig. 4: Variation in R for constant Strength

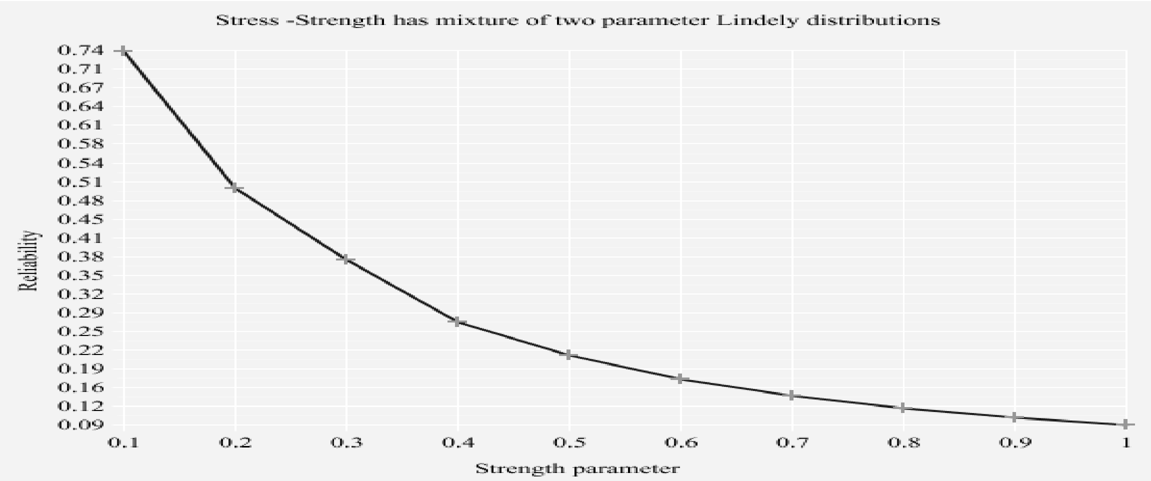

Fig. 5: Variation in R for constant Stress

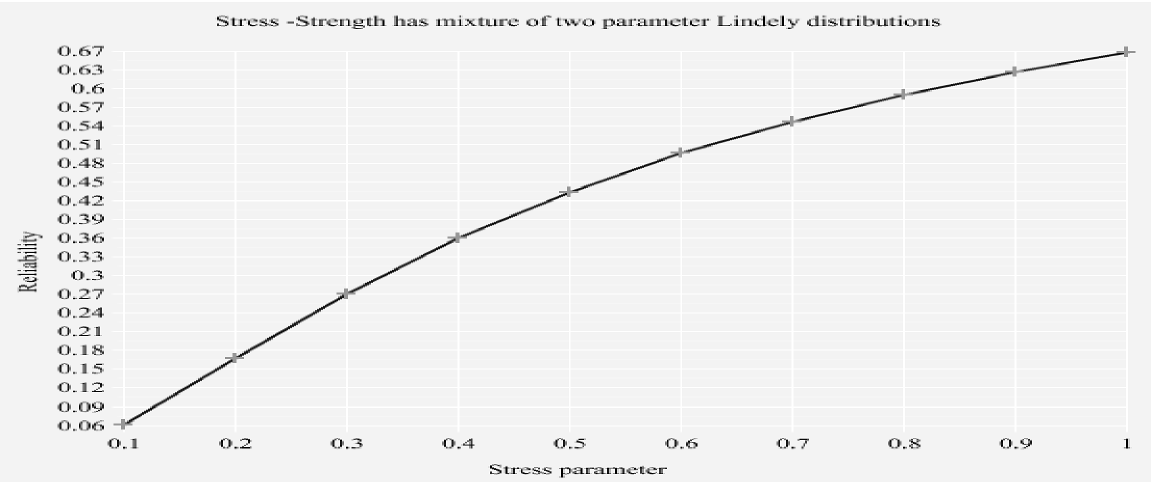

Fig. 6: Variation in R for constant Strength

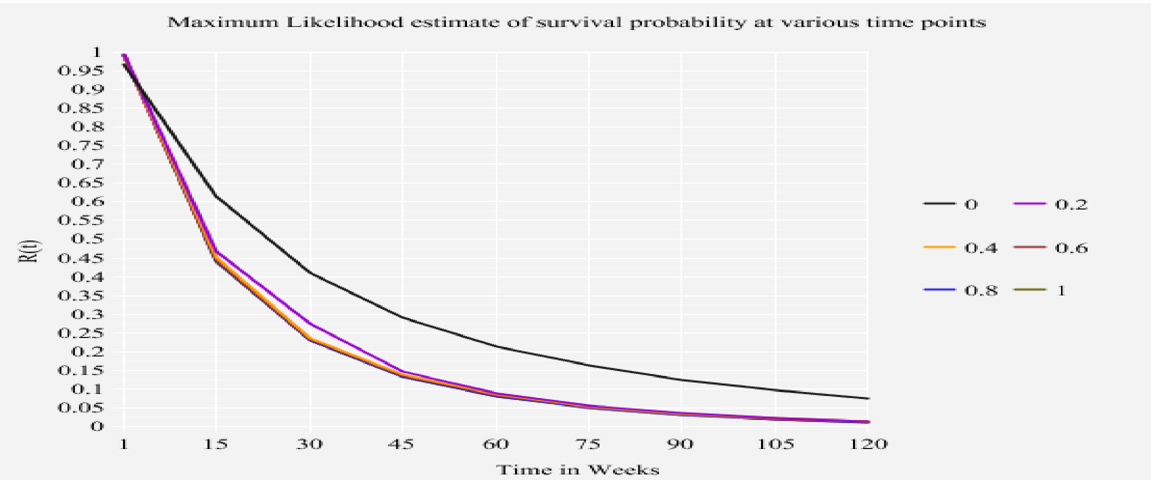

Fig. 7: Survival probability at various time points for different values of $\alpha_1 = \alpha_2$

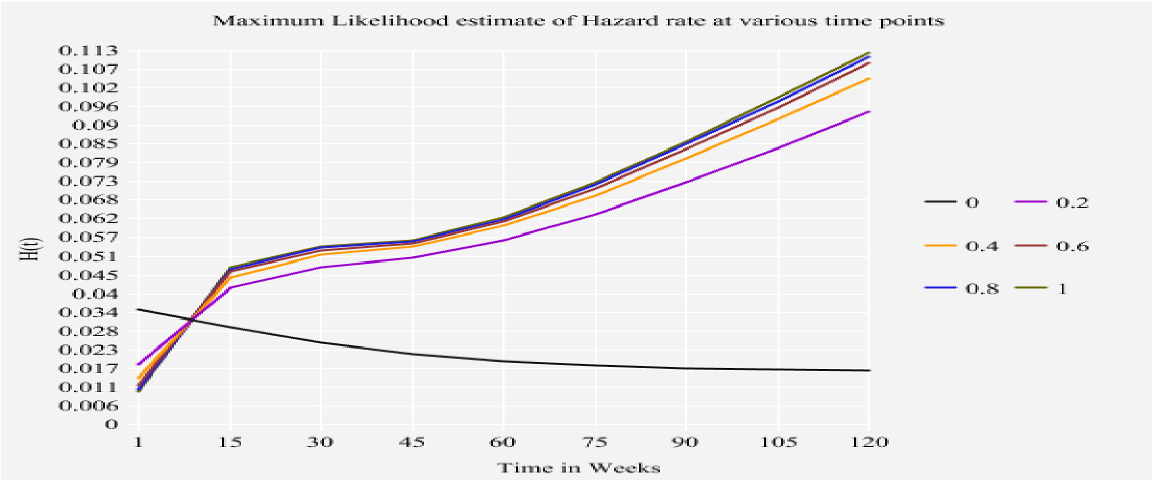

Fig. 8: Hazard rate at various time points for different values of $\alpha_1 = \alpha_2$

7. Conclusions

In the proposed model, we have studied the stress-strength reliability for two parameter Lindley distribution when strength follows finite mixture of two parameter Lindley distribution and stress follows exponential, Lindley distribution and finite mixture of two parameter Lindley distribution. Estimates of parameters are obtained by maximum likelihood estimation method. The numerical evaluation indicates that when the stress has exponential and Lindley distributions then reliability increases when stress increases and vice versa. And when stress follows finite mixture of two parameter Lindley distribution reliability increases with increases in strength and decreases with decreases in stress. At the end we have discussed real life example and plot the graphs of reliability function and hazard rate for various values of α.

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