

Improvement in Estimating the Finite Population Mean Under Maximum and Minimum Values in Double Sampling Scheme

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Abstract: This research article, presents a ratio estimator for the estimation of finite population mean of the study variable under double sampling scheme when there is unusually low and unusually high values and analyzes their properties. The expressions for bias and MSE of the proposed estimator are derived up to first order of approximation. Also efficiency conditions are carried out with the other existing estimators. Numerical results show that the proposed formula is more efficient than the usual unbiased estimator and the usual ratio estimator in double sampling.

Keywords: Bias, Mean Square Error (MSE), Double Sampling, Ratio estimator, Efficiency, Low and High values.

1 Introduction

In the field of survey sampling we have seen that a number of sampling procedures depend on the advanced knowledge about an auxiliary character x_i . One should prefer stratified random sampling if stratum weight w_h is given or have a prior knowledge of the frequency distribution of a closely related auxiliary variate x_i for effecting optimal stratification of a closely related auxiliary character x for effecting optimum stratification. Similarly ratio, product and regression method of estimation requires prior knowledge of the population parameters of the ancillary variable say x_i . While using probability proportional to size sampling (pps) information on population of the auxiliary variable x_i is required for knowing the probability p_i values. But when such information is not easily available or when the auxiliary information is lacking the above designs and methods of estimation does not provide efficient results with a decreasing efficiency. In such a situation it is more meaningful, convenient and economical to take a larger initial sample to collect certain items of information from a sample constituting only a part of the original sample by using simple random sampling scheme in which the auxiliary variable x_i alone is measured the purpose of taking an initial large sample is to furnish a good estimate of the auxiliary variable x_i or some function of the auxiliary variable like population total, population mean, population variance, population standard deviation, population coefficient of variation, population quantiles and frequency distribution etc, of the x_i values such sampling technique is called double sampling or two phase sampling. The purpose and the advantages of taking a large preliminary sample is furnish a good estimate of the ancillary variable or the frequency distribution of x_i , and which is used for estimating the population parameters. A real life example in this context is if we want to estimate the number of leaves in some area, it could be quite time consuming to count the number of leaves in a number of 18x18 inch quadrats, whereas a visual estimate of the number of leaves in such an area is relatively simple to make. If a specific relationship between the visual and actual number of leaves in a quadrat can be established, we can make proficient use of the visual estimates (even if they are highly biased) to improve the precision of the estimate of the total number of leaves through two phase sampling scheme.

In literature for estimating the population mean of the study variable using double sampling, various authors have worked. Bowley (1926) and Neyman (1934, 38) are the early statisticians who worked in the field of survey sampling (stratified random sampling). Hansen and Hurwitz (1943) firstly incorporate auxiliary information in probability proportional to size sampling. Sukhatme (1962) have developed a general ratio-type estimator. Chand (1975) have

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suggested two chain ratio-type estimators to estimate the population mean using two auxiliary variables. Das and Tripathi (1978) worked on the estimation of population variance and Kiregyera (1980,84) have proposed a chain ratio-type and a regression-type estimator using the knowledge of two auxiliary variables. Srivastava et al. (1990) proposed a generalized chain estimator for the estimation of population mean of the study variable. Bahl and Tuteja (1991) have proposed exponential ratio and product-type estimators to estimate the population mean using single auxiliary variable. Armstrong and St-Jean (1993) provides application in double sampling. Singh and Espejo (2003) proposed a class of ratio-product estimators for estimating the unknown population parameter in double sampling. Singh and Vishwakarma (2007) considered a modified exponential ratio and product estimators of the estimation of population mean. Further Srivastava (1970), Cochran (1977), Searls (1964), Mohanty (1967), Bedi (1985), Sahoo et al. (1994a), Singh et al. (2006), Samiuddin and hanif (2007), Singh et al. (2007), Singh et al. (2011a), Singh et al. (2011), Singh and Choudhury (2012), Khare et al. (2013) proposed a generalized chain ratio in regression estimator for population mean using two auxiliary characters in sample survey. Khan and Shabbir (2013) proposed a ratio-type estimator for the estimation of population variance using the knowledge of quartiles and its functions as auxiliary information. Recently Hossain and Khan (2014) worked on the estimation of population mean using maximum and minimum values under simple random sampling by incorporating the knowledge of two auxiliary variables.

Consider a finite population of size N of different units $U = \{U_1, U_2, U_3, \dots, U_N\}$. Let y and x be the study and the auxiliary variable with corresponding values y_i and x_i respectively for i -th unit $i = \{1, 2, 3, \dots, N\}$ is defined on a finite population U . Let $\bar{Y} = (1/N) \sum_{i=1}^N y_i$ and $\bar{X} = (1/N) \sum_{i=1}^N x_i$ be the corresponding population means of the study as well as auxiliary variable respectively. Also let $S_y^2 = (1/N-1) \sum_{i=1}^N (y_i - \bar{Y})^2$ and $S_x^2 = (1/N-1) \sum_{i=1}^N (x_i - \bar{X})^2$ be the corresponding population variances of the study as well as auxiliary variable respectively and let C_y and C_x be the coefficient of variation of the study as well as auxiliary variable respectively, and ρ_{yx} be the correlation coefficient between x and y . In order to estimate the unknown population variance by using simple random sampling scheme we take a sample of size n units from the population U by using simple random sample without replacement.

Let y and x be the study and the auxiliary variable with corresponding values y_i and x_i respectively for i -th unit $i = \{1, 2, 3, \dots, n\}$ in the sample.

Let $\bar{y} = (1/n) \sum_{i=1}^n y_i$ and $\bar{x} = (1/n) \sum_{i=1}^n x_i$ be the corresponding sample means of the study as well as auxiliary variable respectively. Also let $\hat{S}_y^2 = (1/n-1) \sum_{i=1}^n (y_i - \bar{y})^2$ and $\hat{S}_x^2 = (1/n-1) \sum_{i=1}^n (x_i - \bar{x})^2$ be the corresponding sample variances of the study as well as auxiliary variable respectively.

where $C_y = \frac{S_y}{\bar{Y}}$ and $C_x = \frac{S_x}{\bar{X}}$ are the coefficients of variation of the study and auxiliary variables respectively. Let $S_{yx} = \frac{\sum_{i=1}^N (y_i - \bar{Y})(x_i - \bar{X})}{N-1}$ be the co-variances between their respective subscripts respectively.

The usual unbiased estimator to estimate the population mean of the study variable is

$$\bar{y} = \frac{1}{n} \sum_{i=1}^n x_i \quad (1)$$

The variance of the estimator \bar{y} up to first order of approximation is, given by

$$V(\bar{y}) = \theta \bar{Y}^2 C_y^2 \quad (2)$$

In many real data sets there would be unusually high y_{\max} or would be unusually low values y_{\min} and to estimate the population parameters without considering these information is very sensitive in either the case the result will be either over estimated or under estimated. In order to handle this situation Sarndal (1972), suggested the following unbiased estimator for the estimation of finite population mean and is, given by

$$\bar{y}_S = \begin{cases} (\bar{y} + c) & \text{if sample contains } y_{\min} \text{ and } y_{\max} \\ (\bar{y} - c) & \text{if sample contains } y_{\max} \text{ and } y_{\min} \\ \bar{y} & \text{for all other samples,} \end{cases} \quad (3)$$

where c is an unknown constant, whose values is to be find for minimum variance.

The minimum variance of the estimator \bar{y}_S up to first order of approximation is, given as

$$\text{var}(\bar{y}_S)_{\min} = \text{var}(\bar{y}) - \frac{\theta (y_{\max} - y_{\min})^2}{2(N-1)} \quad (4)$$

where the optimum value of c_{opt} is

$$c_{opt} = \frac{(y_{\max} - y_{\min})}{2n} \quad (5)$$

The usual classical ratio estimator in double sampling is, given by

$$\bar{y}_r = \bar{y} \frac{\bar{x}'}{\bar{x}} \tag{6}$$

The mean squared error of the ratio estimator is given as follows

$$MSE(\bar{y}_r) = \bar{Y}^2 \left[\theta C_y^2 + \theta_2 C_x (C_x - 2\rho_{yx} C_y) \right] \tag{7}$$

where $\theta = \frac{1}{n} - \frac{1}{N}$, $\theta' = \frac{1}{n'} - \frac{1}{N}$ and $\theta_2 = \frac{1}{n} - \frac{1}{n'}$.

2 The Proposed Strategy and its Properties

Consider a finite population $U = \{U_1, U_2, U_3, \dots, U_N\}$ of size N different units. In the first phase, we draw a large sample of size $n' (n' < N)$ from a population U by using simple random sample without replacement sampling (SRSWOR) scheme and estimate the population mean of the auxiliary variable x . In the second phase, we draw a sample (subsample) of size n from first phase sample of size n' , i.e. $(n < n')$ by using SRSWOR or directly from the population U and observed both the study and the auxiliary variables respectively.

On the lines of Sarndal (1972), we propose the following estimator to estimate the finite population mean of the study variable, given by

$$\bar{y}_m = \bar{y}_{C_{11}} \frac{\bar{x}'_{C_{21}}}{\bar{x}_{C_{21}}} \tag{8}$$

Ancillary information can increase the precision of the estimates of the unknown population parameters. When these information is positively related to the variable of interest, in such circumstances ratio method of estimation provides efficient results for unknown population parameters. Using ratio estimator in double sampling with an additional information on low and high values for both the study and the auxiliary variables. In such a strategy, the selection of the higher value of the auxiliary variable the higher the value of study variable is to be expected, and the lower the value of auxiliary variable the lower the value of study variable is to be expected, using such type of information our proposed estimator becomes.

Further Simplifying, we get.

$$\bar{y}_m = \begin{cases} (\bar{y} + c_1) \frac{(\bar{x}' + c_2)}{(\bar{x} + c_2)} & \text{if sample contains } y_{\min} \text{ and } x_{\min}. \\ (\bar{y} - c_1) \frac{(\bar{x}' - c_2)}{(\bar{x} - c_2)} & \text{if sample contains } y_{\max} \text{ and } x_{\max}. \\ \bar{y} \frac{\bar{x}'}{\bar{x}} & \text{for all other samples,} \end{cases} \tag{9}$$

where $(\bar{y}_{C_{11}} = \bar{y} + c_1, \bar{x}'_{C_{21}} = \bar{x}' + c_2, \bar{x}_{C_{21}} = \bar{x} + c_2)$. If sample contains low y_{\min}

and low x_{\min} values, $(\bar{y}_{C_{11}} = \bar{y} - c_1, \bar{x}'_{C_{21}} = \bar{x}' - c_2, \bar{x}_{C_{21}} = \bar{x} - c_2)$. If sample contains high y_{\max} and high x_{\max} values and

$(\bar{y}_{C_{11}} = \bar{y}, \bar{x}'_{C_{21}} = \bar{x}', \bar{x}_{C_{21}} = \bar{x})$ for all other samples. where c_1 and c_2 are unknown constants, whose value is to be determined for optimality conditions.

We define the following relative error terms and their expectations for obtain the properties of estimators.

Let $e_0 = \frac{\bar{y} - Y}{\bar{y}}$, $e_1 = \frac{\bar{x} - X}{\bar{x}}$, and $e'_1 = \frac{\bar{x}' - X'}{\bar{x}'}$, such that $E(e_0) = E(e_1) = E(e'_1) = 0$.

$$E(e_0^2) = \frac{\theta}{\bar{Y}^2} \left[S_y^2 - \frac{2nc_1}{N-1} (\Delta y - nc_1) \right],$$

$$E(e_1^2) = \frac{\theta}{\bar{X}^2} \left[S_x^2 - \frac{2nc_2}{N-1} (\Delta x - nc_2) \right],$$

$$E(e_1'^2) = \frac{\theta'}{\bar{X}'^2} \left[S_x'^2 - \frac{2n'c_2}{N-1} (\Delta x - n'c_2) \right],$$

$$E(e_1 e'_1) = \frac{\theta'}{\bar{X}^2} \left[S_x^2 - \frac{2n'c_2}{N-1} (\Delta x - n'c_2) \right],$$

$$E(e_0e_1) = \frac{\theta}{\bar{X}\bar{Y}} \left[S_{yx} - \frac{n}{N-1} \{c_2\Delta y + c_1\Delta x - 2nc_1c_2\} \right],$$

and

$$E(e_0e'_1) = \frac{\theta'}{\bar{X}\bar{Y}} \left[S_{yx} - \frac{n'}{N-1} \{c_2\Delta y + c_1\Delta x - 2n'c_1c_2\} \right],$$

where $\Delta y = y_{\max} - y_{\min}$ and $\Delta x = x_{\max} - x_{\min}$.

The bias of the estimator up to first order of approximation by neglecting terms of e' 's having power greater than two, we have

$$\begin{aligned} \text{Bias}(\bar{y}_m) &= \bar{Y} (\theta_2 C_x^2 - \theta_2 C_{xy}) \\ &\quad - \frac{R}{(N-1)} \left\{ \begin{aligned} &\frac{2c_2}{\bar{X}} \{ (n\theta - n'\theta') \Delta x - c_2 (n^2\theta - n'^2\theta') \} \\ &- (n\theta - n'\theta') (c_2\Delta y + c_1\Delta x) \\ &+ 2c_1c_2 (n^2\theta - n'^2\theta') \end{aligned} \right\}, \end{aligned} \quad (10)$$

where $R = \frac{\bar{Y}}{\bar{X}}$.

where n' is the size of the first phase sample and \bar{x}' is the corresponding first phase sample mean.

The minimum mean squared error of \bar{y}_m , estimator up to first order of approximation after neglecting terms of e' 's having power greater than two, we have

$$\begin{aligned} \text{MSE}(\bar{y}_m)_{\min} &= \text{MSE}(\bar{y}_r) \\ &\quad - \frac{1}{2n'N(N-1)} \left\{ \begin{aligned} &\Delta y [(N-n)\Delta y + 2R(n-n')\Delta x] \\ &+ \frac{(n-n')[(N-n)\Delta y - n'R\Delta x]^2}{n'(N-n'-n)} \end{aligned} \right\} \end{aligned} \quad (11)$$

where $\text{MSE}(\bar{y}_r) = \bar{Y}^2 (\theta C_y^2 + \theta_2 C_x (C_x - 2\rho_{yx} C_y))$ and the optimum values of the unknown constants are, given by

$$c_{1opt} = \frac{\Delta y}{2n'},$$

and

$$c_{2opt} = \left[\frac{(N-n)\Delta y - n'R\Delta x}{2n'R(N-n'-n)} \right].$$

3 Efficiency Comparison

To compare the efficiencies of various competing estimators with propose estimator, we require mean square errors up to first order of approximation.

The variance of the usual unbiased estimator \bar{y} using simple random sampling (SRSWOR) is, given by

The proposed estimator \bar{y}_m is more efficient than the other existing estimators if

(i) By (2) and (11),

$$\text{MSE}(\bar{y}_m)_{\min} < \text{MSE}(\bar{y}) \text{ if}$$

$$\rho_{yx} > \frac{C_x}{2C_y} - \frac{1}{4n'\theta_2 N(N-1)\bar{Y}^2 C_x C_y} \left\{ \begin{aligned} &\Delta y [(N-n)\Delta y + 2R(n-n')\Delta x] \\ &+ \frac{(n-n')[(N-n)\Delta y - n'R\Delta x]^2}{n'(N-n'-n)} \end{aligned} \right\}.$$

The proposed estimator \bar{y}_m is more efficient than the other existing estimators if

(ii) By (7) and (11),

$$\text{MSE}(\bar{y}_m)_{\min} < \text{MSE}(\bar{y}_r) \text{ if}$$

$$\left[\frac{(y_{\max} - y_{\min}) ((N-n)(y_{\max} - y_{\min}) - 2R(n'-n)(x_{\max} - x_{\min}))}{(n'-n)((N-n)(y_{\max} - y_{\min}) - n'R(x_{\max} - x_{\min}))^2} \right] > 0.$$

4 Numerical Illustration

To examine the performance of the proposed estimator and to verify its theoretical results with the other existing estimators, we have considered six natural populations from Agricultural Statistics Washington (US). The description and the necessary data statistics of the populations are given below.

Population 1. (Source: Agricultural Statistics (1999) [10] Washington, DC.).

Y: Estimated number of fish caught during 1995 and X: Estimated number of fish caught during 1994.

$$N = 69, n' = 36, n = 20, \bar{Y} = 4514.899, \bar{X} = 4954.435,$$

$$S_y^2 = 37199578, S_x^2 = 49829270, S_{yx} = 41335932.850, C_y^2 = 1.8249,$$

$$C_x^2 = 2.0300, y_{\max} = 30027, y_{\min} = 23, x_{\max} = 38007,$$

$$x_{\min} = 32, \rho_{yx} = 0.9601.$$

Population 2. (Source: Agricultural Statistics (1999) [10] Washington, DC.).

Y: Estimated number of fish caught during 1995 and X: Estimated number of fish caught during 1993.

$$N = 69, n' = 36, n = 20, \bar{Y} = 4514.899, \bar{X} = 4591.072,$$

$$S_y^2 = 37199578, S_x^2 = 39881874, S_{yx} = 36838026.141, C_y^2 = 1.8249,$$

$$C_x^2 = 1.892, y_{\max} = 30027, y_{\min} = 23, x_{\max} = 34060,$$

$$x_{\min} = 35, \rho_{yx} = 0.9564.$$

Population 3. (Source: Agricultural Statistics (1998) [11] Washington, US.).

Y: Season average price per pound during 1996 and X: Season average price per pound during 1995.

$$N = 36, n' = 22, n = 12, \bar{Y} = 0.2033, \bar{X} = 0.1856,$$

$$S_y^2 = 0.0065, S_x^2 = 0.0057, S_{yx} = 0.0053, C_y^2 = 0.156,$$

$$C_x^2 = 0.164, y_{\max} = 0.452, y_{\min} = 0.101, x_{\max} = 0.403,$$

$$x_{\min} = 0.071, \rho_{yx} = 0.8775.$$

Population 4. (Source: Agricultural Statistics (1998) [11] Washington, US.). Y: Season average price per pound during 1996 and X: Season average price per pound during 1994.

$$N = 36, n' = 22, n = 12, \bar{Y} = 0.2033, \bar{X} = 0.1708,$$

$$S_y^2 = 0.0065, S_x^2 = 0.0042, S_{yx} = 0.0044, C_y^2 = 0.156,$$

$$C_x^2 = 0.138, y_{\max} = 0.452, y_{\min} = 0.101, x_{\max} = 0.334,$$

$$x_{\min} = 0.078, \rho_{yx} = 0.8577.$$

Population 5. (Source: Agricultural Statistics (1999) [10] Washington, DC.).

Y: Estimated number of fish caught during 1995 and X: Estimated number of fish caught during 1994.

$$N = 69, n' = 35, n = 20, \bar{Y} = 4514.899, \bar{X} = 4954.435,$$

$$S_y^2 = 37199578, S_{yx} = 41335932.85, S_x^2 = 49829270, C_y^2 = 1.8249,$$

$$C_x^2 = 2.030, y_{\max} = 30027, y_{\min} = 23, x_{\max} = 38007,$$

$$x_{\min} = 32, \rho_{yx} = 0.9601.$$

Population 6. (Source: Agricultural Statistics (1999) [10] Washington, DC.).

Y: Estimated number of fish caught during 1995 and X: Estimated number of fish caught during 1992.

$$N = 69, n' = 35, n = 20, \bar{Y} = 4514.899, \bar{X} = 4230.174,$$

$$S_y^2 = 37199578, S_{yx} = 32395255.07, S_x^2 = 31010599, C_y^2 = 1.8249,$$

$$C_x^2 = 1.7329, y_{\max} = 30027, y_{\min} = 23, x_{\max} = 38933,$$

$$x_{\min} = 5, \rho_{yx} = 0.9538.$$

The mean squared error (MSE) of the proposed and the existing estimators are shown in **Table 1**.

Table 1. MSE of the existing and the proposed estimators.

Estimator	Population #:					
	1	2	3	4	5	6
\bar{y}	1320903.481	1320903.481	0.000360	0.000360	1320903.481	1320903.481
\bar{y}_r	566683.9008	568722.3321	0.0001777	0.000182	589106.6451	597260.3703
\bar{y}_m	540166.8784	534151.5045	0.0001716	0.000179	573031.6473	577818.1979

5 Conclusion

In this article, we have proposed a ratio estimator for the estimation of the finite population mean under maximum and minimum values of the study and the auxiliary variables respectively, using Two Phase Sampling Scheme. We have found some theoretical conditions under which the proposed strategy is shown to be more efficient than the usual unbiased estimator and the ratio estimators. These theoretical conditions are shown numerically in **Table1** where we have observed that the performance of the proposed estimator is better than the usual unbiased estimator and the ratio estimators. Having the largest gain in competence the proposed estimator appeared to be the best one among all the estimators and would work very well in practical surveys.

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