Journal of Statistics Applications & Probability Letters
An International Journal

http://dx.doi.org/10.12785/jsapl/020105

Improved Estimators in Simple Random Sampling When Study Variable is an Attribute

*Prayas Sharma and Rajesh Singh**

Department of Statistics, Banaras Hindu University, Varanasi (U.P.), India

Received: 2 Oct. 2014, Revised: 11 Nov. 2014, Accepted: 15 Nov. 2014
Published online: 1 Jan. 2015

Abstract: This paper addresses the problem of estimating the population mean in the presence of auxiliary information when study variable itself is qualitative in nature. Bias and MSE expressions of the class of estimators are derived up to the first order approximation. The proposed estimators have been compared with the traditional estimator and several other estimators considered by [15]Singh. In addition, we substantiated this theoretical claim by an empirical study to show the superiority of these estimators.

Keywords: Attribute, point bi-serial, mean square error, simple random sampling.

1 Introduction

In the theory of sample surveys, it is usual to make use of the auxiliary information at the estimation stage in order to improve the precision or accuracy of an estimator of unknown population parameter of interest. Ratio, product and regression methods of estimation are good examples in this context. Several authors including [19], [20], [5], [6], [14], [12] and [17] suggested estimators using known population parameters of an auxiliary variable. But there may be many practical situations when auxiliary information is not available directly but is qualitative in nature, that is, auxiliary information is available in the form of an attribute. For example:
(a) The height of a person may depend on the fact that whether the person is male or female.
(b) The efficiency of a Dog may depend on the particular breed of that Dog.
(c) The yield of wheat crop produced may depend on a particular variety of wheat, etc.
(see [3])

In these situations by taking the advantage of point bi-serial correlation between the study variable y and the auxiliary attribute ϕ along with the prior knowledge of the population parameter of auxiliary attribute, the estimators of population parameter of interest can be constructed.

In many situations, study variable is generally ignored not only by ratio scale variables that are essentially qualitative, or nominal scale, in nature, such as sex, race, colour, religion, nationality, geographical region, political upheavals (see [2]). Taking into consideration the point bi-serial correlation between auxiliary attribute and study variable, several authors including [3], [4], [14], [7], [12], [15], [1], [13],[21], [22],[10,11] and [9,8] proposed improved estimators of population parameters under different situations . All the others have implicitly assumed that the study variable Y is quantitative whereas the auxiliary variable is qualitative.

There may be situations when study variable itself is qualitative in nature. For example, consider U.S. presidential elections. Assume that there are two political parties, Democratic and Republican. The dependent variable here is the vote choice between two political parties. Suppose we let $Y = 1$, if the vote is for a Democratic candidate and $Y = 0$, if the vote is for republican candidate. Some of the variables used in the vote choice are growth rate of GDP, unemployment and inflation rates, whether the candidate is running for re-election, etc. For the present purposes, the important thing is to note that the study variable is a qualitative variable. One can think several other examples where the

* Corresponding author e-mail: rsinghstat@gmail.com

study variable is qualitative in nature. Thus, a family either owns a house or it does not, it has disability insurance or it does not, both husband and wife are in the labour force or only one suppose is, etc. In this paper we propose estimators in which study variable itself is qualitative in nature. (see [2]).

Consider a sample of size n drawn by simple random sampling without replacement (SRSWOR) from a population of size N. Let ϕ_i and x_i denote the observations on variable ϕ and x respectively for i^{th} unit (i=1,2,3N).$\phi_i = 1$, if i^{th} unit of population possesses attribute ϕ and $\phi_i = 0$, otherwise. Let $A = \sum_{i=1}^{N} \phi_i$ and $a = \sum_{i=1}^{n} \phi_i$, denotes the total number of units in the population and sample possessing attribute ϕ respectively, $P = \frac{A}{N}$ and $p = \frac{a}{n}$, denotes the proportion of units in the population and sample, respectively, possessing attribute ϕ.
Let us define,

$$e_p = \frac{(p-P)}{P}, e_1 = \frac{(\bar{x}-\bar{X})}{X}, e_3 = \frac{\left(s_x^2 - S_x^2\right)}{S_x^2}$$

Such that,

$$E(e_i) = 0, (i = p, 1, 3)$$

and

$$E(e_p^2) = fC_p^2, E(e_1^2) = fC_x^2, E(e_3^2) = f(\lambda_{04} - 1)$$
$$E(e_1 e_p) = f\rho_{pb} C_p C_x, E(e_3 e_p) = fC_p \lambda_{12}, E(e_1 e_3) = fC_x \lambda_{03}$$

where,

$$f = \left(\frac{1}{n} - \frac{1}{N}\right), C_p^2 = \frac{S_p^2}{P^2}, C_x^2 = \frac{S_x^2}{X^2}$$

and ρ_{pb} is the point bi-serial correlation coefficient.

2 Estimators in Literature

[15] proposed the following ratio-type estimator for estimating unknown population mean when study variable is an attribute, as

$$t_a = \left(\frac{P}{\bar{x}}\right)\bar{X} \tag{1}$$

The bias and MSE expressions of the estimator t_a, to the first order of approximation is respectively, given by

$$B(t_a) = f\left(\frac{C_x^2}{2} - \rho_{pb} C_p C_x\right) \tag{2}$$

$$MSE(t_a) = fP^2\left(C_p^2 + C_x^2 - 2\rho_{pb} C_p C_x\right) \tag{3}$$

[15] proposed a general class of estimator as,

$$t_b = H(p, u) \tag{4}$$

where $u = \frac{\bar{x}}{X}$ and $H(u, p)$ is a parametric equation of p and u such that

$$H(p, 1) = P, \forall P \tag{5}$$

and satisfying the following regulations:
(i) whatever be the sample chosen, the point (p,u) assumes that the values in a bounded closed convex subset R_2 of the two-dimensional real space that contain the point (p,1).

(ii) the function H(p,u) is a continuous and bounded in R_2 .
(iii) the first and second order partial derivatives of H(p,u) exist and are continuous as well as bounded in R_2.
where,

$$H_1 = \left.\frac{\partial H}{\partial u}\right|_{p=P,u=1}, \qquad H_2 = \frac{1}{2}\left.\frac{\partial^2 H}{\partial u^2}\right|_{p=P,u=1},$$

$$H_3 = \frac{1}{2}\left.\frac{\partial^2 H}{\partial p \partial u}\right|_{p=P,u=1} \quad and \quad H_4 = \frac{1}{2}\left.\frac{\partial^2 H}{\partial p^2}\right|_{p=P,u=1}$$

the bias and minimum MSE of the estimator t_b are respectively, given by

$$B(t_b) = f\left(P\rho_{pb}C_pC_xH_3 + C_x^2H_2 + P^2C_y^2H_4\right) \tag{6}$$

$$MSE(t_b)_{min} = fP^2C_p^2\left(1-\rho_{pb}^2\right) \tag{7}$$

[15] proposed another family of estimator for estimating P, as

$$t_c = [q_1P + q_2(\bar{X}-\bar{x})]\left[\frac{a\bar{X}+b}{a\bar{x}+b}\right]^{\alpha} exp\left[\frac{(a\bar{X}+b)-(a\bar{x}+b)}{(a\bar{X}+b)+(a\bar{x}+b)}\right]^{\beta} \tag{8}$$

The bias and minimum MSE of the estimator t_c to the first order of approximation, are respectively, given as

$$Bias(t_c) = P(q_1-1) + f\left[(q_2\bar{X}B + q_1PA)C_x^2 - q_1PB\rho C_pC_x\right] \tag{9}$$

$$MSE(t_c)_{min} = \left[P^2 - \frac{\Delta_1\Delta_5^2 + \Delta_3\Delta_4^2 - 2\Delta_2\Delta_4\Delta_5}{\Delta_1\Delta_3 - \Delta_2^2}\right] \tag{10}$$

where,

$$M_1 = P^2f\left(C_p^2 + B^2C_x^2 - 2B\rho C_pC_x\right), \;\; M_2 = \bar{X}fC_x^2, \;\; M_3 = P^2f\left(AC_x^2 - 2B\rho C_pC_x\right),$$

$$M_4 = P\bar{X}f\left(-BC_x^2 + \rho C_pC_x\right), \;\; M_5 = \bar{X}Pf\left(-Bc_x^2\right)$$

here the optimum values of q_1 and q_2 are given as

$$q_1^* = \frac{\Delta_1\Delta_4 - \Delta_2\Delta_5}{\Delta_1\Delta_3 - \Delta_2^2} \quad and \quad q_2^* = \frac{\Delta_1\Delta_5 - \Delta_2\Delta_4}{\Delta_1\Delta_3 - \Delta_2^2} \tag{11}$$

where,

$$\Delta_1 = \left(P^2 + M_1 + 2M_3\right), \Delta_2 = (-M_4 - M_5), \Delta_3 = (M_2), \Delta_4 = \left(P^2 + M_3\right), \Delta_5 = (-M_5)$$

3 Proposed Estimators

The following estimator is proposed

$$t_1 = p\left(\frac{\bar{x}}{\bar{X}}\right)^{\alpha}\left(\frac{s_x^2}{S_x^2}\right)^{\beta} \tag{12}$$

where α and β are suitably chosen constants to be determined such that MSE of the estimator t_1 is minimum.
The bias and MSE of the estimator t_1 to the first order of approximation are respectively, given by

$$Bias(t_1) = \alpha\left(\frac{\alpha+1}{2}\right)C_p^2 + \beta\left(\frac{\beta+1}{2}\right)C_x^2 + \alpha\beta C_x\lambda_{03} + \alpha\rho_{px}C_pC_x + \beta C_p\lambda_{12} \tag{13}$$

$$MSE(t_1) = P^2f\left[C_p^2 + \alpha^2C_x^2 + \beta^2(\lambda_{04}-1) + 2\alpha\rho_{px}C_pC_x + 2\beta C_p\lambda_{12} + 2\alpha\beta C_x\lambda_{03}\right] \tag{14}$$

Differentiating equation (14) partially with respect to α and β , equating them to zero, we get the optimum values of α and β respectively, as

$$\alpha^* = \frac{C_p\left\{\lambda_{03}\lambda_{12} - \rho_{px}(\lambda_{04}-1)\right\}}{C_x\left\{(\lambda_{04}-1) - \lambda_{03}^2\right\}} \qquad \beta^* = \frac{C_p\left\{\rho_{px}\lambda_{03} - \lambda_{12}\right\}}{\left\{(\lambda_{04}-1) - \lambda_{03}^2\right\}} \tag{15}$$

Putting the optimum values of α and β from equation (15) into equation (14), we get the minimum MSE of the estimator t_1 as

$$MSE(t_1)_{min} = P^2 f C_p^2\left[1 - \rho_{px}^2 - \frac{(\lambda_{03}\rho_{px} - \lambda_{12})^2}{(\lambda_{04} - 1 - \lambda_{03}^2)}\right] \tag{16}$$

Following [18], we propose a general family of estimators for estimating P, as

$$t_2 = H(p,u,v) \tag{17}$$

where $u = \frac{\bar{x}}{\bar{X}}, v = \frac{s_x^2}{S_x^2}$ and $H(p,u,v)$ is a parametric function of p, u and v such that

$$H(p,1,1) = P, \forall P \tag{18}$$

and satisfying following regulations:
(iv) whatever be the sample chosen, the point (p,u,v) assumes the values in a closed convex subset R_3 of the three-dimensional real space containing the point (p,1,1).
(v) the function H(p,u,v) is a continuous and bounded in R_3 .
(vi) the first and second order partial derivatives of H(p,u,v) exist and are continuous as well as bounded in R_3.
Expanding H(p,u,v) about the point (P,1,1) in a second order Taylor series we have

$$t_2 = H(p,u,v) = [P + (p-P), 1 + (u-1), 1 + (v-1)] \tag{19}$$

$$(t_2 - P) = \left[Pe_0 + e_1H_1 + e_3H_2 + Pe_0^2H_3 + e_1^2H_4 + e_3^2H_5 + Pe_0e_1H_6 + e_1e_3H_7 + Pe_0e_3H_8\right] \tag{20}$$

where,

$$\left.\frac{\partial H}{\partial p}\right|_{p=P,u=1} = 1$$

$$H_1 = \left.\frac{\partial H}{\partial u}\right|_{p=P,u=1}, \qquad H_2 = \left.\frac{\partial H}{\partial v}\right|_{p=P,u=1},$$

$$H_3 = \frac{1}{2}\left.\frac{\partial^2 H}{\partial p^2}\right|_{p=P,u=1,v=1}, \qquad H_4 = \frac{1}{2}\left.\frac{\partial^2 H}{\partial u^2}\right|_{p=P,u=1,v=1},$$

$$H_5 = \frac{1}{2}\left.\frac{\partial^2 H}{\partial v^2}\right|_{p=P,u=1,v=1}, \qquad H_6 = \frac{1}{2}\left.\frac{\partial^2 H}{\partial p\partial u}\right|_{p=P,u=1,v=1},$$

$$H_7 = \frac{1}{2}\left.\frac{\partial^2 H}{\partial u\partial v}\right|_{p=P,u=1,v=1}, \qquad H_8 = \frac{1}{2}\left.\frac{\partial^2 H}{\partial p\partial v}\right|_{p=P,u=1,v=1},$$

Taking expectations both sides of (20), we get the bias of the estimator t_2 to the first order of approximation, as

$$B(t_2) = f\left[PC_p^2H_3 + C_x^2H_4 + (\lambda_{04}-1)H_5 + P\rho_{px}C_pC_xH_6 + C_x\lambda_{03}H_7 + PC_p\lambda_{12}H_8\right] \tag{21}$$

Squaring both sides of (20) and neglecting terms of e's having power greater than two, we have

$$(t_2 - P)^2 = \left[Pe_0^2 + e_1^2H_1^2 + e_3^2H_2^2 + 2Pe_0e_1H_1 + 2Pe_0e_3H_2 + 2e_1e_3H_1H_2\right] \tag{22}$$

Taking expectations of both sides of (22), we get the MSE of the wider class of estimator t_2 as

$$MSE(t_2) = f\left[PC_p^2 + C_x^2H_1^2 + (\lambda_{04}-1)H_2^2 + 2P\rho_{px}C_pC_xH_1 + 2PC_p\lambda_{12}H_2 + 2C_x\lambda_{03}H_1H_2\right] \quad (23)$$

On differentiating (23) with respect to H_1 and H_2 equating to zero, respectively we obtain the optimum values of H_1 and H_2, as

$$H_1^* = \frac{C_p\left\{\lambda_{03}\lambda_{12} - \rho_{px}(\lambda_{04}-1)\right\}}{C_x\left\{(\lambda_{04}-1) - \lambda_{03}^2\right\}} \qquad H_2^* = \frac{C_p\left\{\rho_{px}\lambda_{03} - \lambda_{12}\right\}}{\left\{(\lambda_{04}-1) - \lambda_{03}^2\right\}} \quad (24)$$

On substituting the values of H_1^* and H_2^* from (24) in (23), we obtain the minimum MSE of the estimator t_2, as

$$MSE(t_2)_{min} = P^2fC_p^2\left[1 - \rho_{px}^2 - \frac{(\lambda_{03}\rho_{px} - \lambda_{12})^2}{(\lambda_{04} - 1 - \lambda_{03}^2)}\right] \quad (25)$$

We propose another improved family of estimators for estimating P, as

$$t_3 = m_1p\left[\frac{\bar{X}}{\gamma\bar{x} + (1-\gamma)\bar{X}}\right]^g + m_2pexp\left[\frac{\delta(S_x^2 - s_x^2)}{(S_x^2 + s_x^2)}\right] \quad (26)$$

where γ is suitable constant. g and δ are constants that can takes values (0,1,-1) for designing different estimators; and m_1 and m_2 are suitable chosen constants to be determined such that mean square error (MSE) of the class of estimator t_3 is minimum.
Expressing the class of estimators t_3 at equation (26) in terms of e's, we have

$$t_3 = m_1P(1+e_0)(1+\gamma e_1)^{-g} + m_2P(1+e_0)exp\left[\frac{-\delta e_3}{2}\left(1+\frac{e_3}{2}\right)^{-1}\right] \quad (27)$$

Simplifying equation (27) and retaining terms up to the first order of approximation, we have

$$t_3 - P = -P\left[1 - m_1\left(1 - \gamma g e_0e_1 + \frac{g(g+1)}{2}\gamma^2e_1^2\right) - m_2\left(1 - \frac{\delta}{2}e_0e_3 + \frac{\delta(\delta+1)}{8}e_3^2\right)\right] \quad (28)$$

Taking expectations both sides of equation (28), we get the bias of the estimator t_3 to the first order of approximation, as

$$Bias(t_3) = -P\left[1 - m_1B - m_2E\right] \quad (29)$$

where,

$$B = \left\{1 - \gamma g f\rho_{px}C_pC_x + \frac{g(g+1)}{2}\gamma^2fC_x^2\right\} \quad (30)$$

$$E = \left\{1 - \frac{\delta}{2}f\rho_{px}C_pC_x + \frac{\delta(\delta+1)}{8}fC_x^2\right\}$$

Squaring both sides of equation (28) and neglecting the terms having power greater than two, and then taking expectations both sides, we get the MSE of the estimator t_3 to the first order of approximation, as

$$MSE(t_3) = P^2\left[1 + m_1^2A + m_2^2C + 2m_1m_2D - 2m_1B - 2m_2E\right] \quad (31)$$

where,

$$A = \left\{1 + f\left(C_p^2 - 4\gamma g\rho_{px}C_pC_x\right) + \gamma^2g(2g+1)C_x^2)\right\}$$

$$C = \left\{1 + f\left(C_p^2 - 2\delta C_p\lambda_{12} + \frac{\left\{\delta^2 + \delta(\delta+2)\right\}(\lambda_{04}-1)}{4}\right)\right\}$$

$$D = \left\{1 + f\left(C_p^2 - \delta C_p\lambda_{12} + \frac{\delta(\delta+2)}{8}(\lambda_{04}-1) - 2\gamma g\rho_{px}C_pC_x + \frac{\gamma\delta g}{2}C_x\lambda_{03} + \frac{g(g+1)}{2}\gamma^2C_x^2\right)\right\}$$

And B and E are the same as defined earlier in (30).
The MSE of the class of estimator t_3 at equation (31) is minimised for the optimum values of m_1 and m_2 given as

$$m_1^* = \frac{(BC - DE)}{(AC - D^2)} \qquad m_2^* = \frac{(AE - BD)}{(AC - D^2)} \tag{32}$$

Putting equations (32) in (29) and(31), we get the resulting minimum bias and MSE of the proposed class of estimators t_3, respectively, as

$$Bias(t_3)_{min} = -P\left[1 - \frac{B^2C - 2BDE + AE^2}{AC - D^2}\right] \tag{33}$$

$$MSE(t_3)_{min} = P^2\left[1 - \frac{B^2C - 2BDE + AE^2}{AC - D^2}\right] \tag{34}$$

4 Efficiency Comparisons

First we compare the MSE of proposed estimators t_1 and t_2 with usual estimator,

$$MSE(t_1)_{min} = MSE(t_2)_{min} \leq V(p)$$

If,

$$fC_p^2\left[1 - \rho_{px}^2 - \frac{(\lambda_{03}\rho_{px} - \lambda_{12})^2}{(\lambda_{04} - 1 - \lambda_{03}^2)}\right] \leq fC_p^2 \tag{35}$$

On solving we observed that above conditions holds always true.
Now, we compare the efficiency of proposed estimator t_3 with usual estimator,

$$MSE(t_3)_{min} \leq V(p)$$

If,

$$\left[1 - \frac{B^2C - 2BDE + AE^2}{(AC - D^2)}\right] \leq fC_p^2 \tag{36}$$

On solving we observed that above conditions holds always true.
Next we compare the efficiency of proposed estimator t_3 with wider class of estimator t_2.

$$MSE(t_3)_{min} \leq MSE(t_2)_{min} = MSE(t_1)_{min}$$

If,

$$\left[1 - \frac{B^2C - 2BDE + AE^2}{(AC - D^2)}\right] \leq fC_p^2\left[1 - \rho_{px}^2 - \frac{(\lambda_{03}\rho_{px} - \lambda_{12})^2}{(\lambda_{04} - 1 - \lambda_{03}^2)}\right] \tag{37}$$

Finally, we compare the efficiency of proposed estimator t_3 with class of estimator t_c proposed by [15] as

$$MSE(t_3)_{min} \leq MSE(t_c)_{min}$$

If,

$$P^2\left[1 - \frac{B^2C - 2BDE + AE^2}{(AC - D^2)}\right] \leq \left[P^2 - \frac{\Delta_1\Delta_5^2 + \Delta_3\Delta_4^2 - 2\Delta_2\Delta_4\Delta_5}{\Delta_1\Delta_3 - \Delta_2^2}\right] \tag{38}$$

5 Empirical study

Data Statistics: The data used for empirical study has been taken from [2]-pg, 601. Using raw data, we have calculated the following values.
where,
Y Home ownership.
X Income (thousands of dollars).

n	N	P	$\bar{X}$	ρ_{pb}	C_p	C_x	λ_{12}	λ_{04}	λ_{03}
11	40	0.525	14.4	0.897	0.963	0.308	-0.118	1.75	-0.153

The following Table shows comparison between some existing estimators and proposed estimators with respect to usual estimator.

Table 1: Percent relative efficiency of proposed estimators with respect to usual estimator.

Estimators	p	t_3	t_b	t_c	t_1	t_2	t_3		
							$g=1,\delta=1$	$g=1,\delta=-1$	$g=0,\delta=1$
PRE	100	189.38	511.79	518.05	513.92	513.92	685.51	199.20	141.23

Table 1 exhibits that the percent relative efficiencies of the proposed estimators t_1, t_2 and t_3 along with percent relative efficiencies of some existing estimators in case of qualitative characteristics. Here we observed that the MSE's of the proposed estimators t_1 and t_2 are similar at their optimums. The proposed estimators t_1 and t_2 are better than the usual estimator p and general class of estimator t_b but less efficient than the minimum MSE of the class of estimator t_c (due to [15]) and proposed class of estimator t_3 for the given data set. Under empirical comparison the performance of the proposed class of estimator t_3 is analyzed for the different values of g and δ. It was found that the proposed class of estimator t_3 is most efficient for the choice g=1, $\delta = 1$ among all the estimators considered in this paper for the estimation of P.

6 Conclusion

This paper deals with an interesting and new problem which involves the estimation of the qualitative characteristics. In my cognizance this problem is new and only [15] has suggested some estimators for such situation. In this article we have proposed three estimators when study variable is itself an attribute using the quantitative auxiliary information. We have determined the bias and mean square error of the proposed estimators to the first order of approximation. In theoretical and empirical efficiency comparisons, it has been shown that the proposed estimator t_3 is more efficient than the estimators considered here. Since only [15] has considered such situation and proposed some estimator and we have compared with the available estimators due to [15] therefore, we can say that our proposed estimator t_3 is most efficient estimator among all the estimators available for the estimation of the qualitative characteristics.

Acknowledgement

The authors are grateful to the anonymous six referees for a careful checking of the details and for helpful comments that improved this paper.

References

[1] A.M. Abd-Elfattah, E.A. Sherpeny, S.M. Mohamed and O.F. Abdou, Improvement estimating the population mean in simple random sampling using information on auxiliary attribute, Applied mathematics and computation (2010),doi:10.1016/j.amc.2009.12.041.

[2] D. N. Gujarati and Sangeetha, Basic economtrics, Tata McGraw Hill, (2007).

[3] V.D. Naik and P.C. Gupta, A note on estimation of mean with known population proportion of an auxiliary character, Jour. Ind. Soc. Agr. Stat., **48(2)**,151-158,(1996).

[4] H. S. Jhajj, M. K. Sharma and L. K. Grover, A family of estimators of population mean using information on auxiliary attribute, Pak. J. Statist., **22 (1)**, 43-50,(2006).

[5] H. Kadilar and Cingi, A new estimator using two auxiliary variables, Applied Mathematics and Computation **162**, 901-908,(2005).

[6] M. Khoshnevisan, R. Singh, P. Chauhan, N. Sawan and F. Smarandache, A general family of estimators for estimating population mean using known value of some population parameter(s), Far East Journal of Theoretical Statistics,**22**, 181-191,(2007).

[7] J. Shabbir and S. Gupta, On estimating the finite population mean with known population proportion of an auxiliary variable, Pakistan Journal of Statistics, **23(1)**, 1-9, (2007).

[8] P. Sharma and R. Singh, Improved Ratio Type Estimators Using Two Auxiliary Variables under Second Order Approximation, Mathematical Journal of Interdisciplinary Sciences, **2(2)**,193-204, (2014).

[9] P. Sharma and R. Singh, Improved Estimators for Simple random sampling and Stratified random sampling Under Second order of Approximation, Statistics in Transition-new series, **14(3)**, 379-390, (2013).

[10] P. Sharma, A. Sanaullah, H. Verma and R. Singh, Some Exponential Ratio-Product Estimators using information on Auxiliary Attributes under Second Order Approximation, International Journal of Statistics and Economics, **12(3)**,58-66, (2013a).

[11] P. Sharma, R. Singh and Min-kim Jong, Study of Some Improved Ratio Type Estimators Using information on Auxiliary Attributes under Second Order Approximation, Journal of Scientific Research, **57**, 138-146, (2013b).

[12] R. Singh, P. Chauhan, N. Sawan and F. Smarandache, Ratio estimators in simple random sampling using information on auxiliary attribute, Pak. J. Stat. Oper. Res., **4(1)**,47-53, (2008).

[13] H.P. Singh and R.S.Solanki, Improved estimation of population mean in simple random sampling using information on auxiliary attribute, Appl. Math. Comput., **218**, 7798-7812, (2012).

[14] R. Singh, P. Cauhan, N. Sawan, and F. Smarandache, Auxiliary information and a priori values in construction of improved estimators, Renaissance High Press, (2007).

[15] R. Singh, M. Kumar and F. Smarandache, Ratio estimators in simple random sampling when study variable is an attribute. World Applied Sciences Journal, **11(15)**, 586-589, (2010).

[16] R. Singh, M. Kumar and F. Smarandache, Almost unbiased estimator for Estimating population mean using known value of some population parameter(s), Pak. J. Stat. Oper. Res., **4(2)** 63-76, (2008).

[17] R. Singh and M. Kumar, A note on transformations on auxiliary variable in survey sampling, MASA, **6(1)**, 17-19, (2011).

[18] S.K. Srivastava, and H.S. Jhajj, A class of estimators of population mean in survey sampling using auxiliary information, Biometrika, **68**, 341-343, (1989).

[19] L. N. Upadhyaya and H. P. Singh, , Use of transformed auxiliary variable in estimating the finite population mean, Biom. Jour., **41**, 627-636, (1999).

[20] W.A. Abu-Dayyeh, M.S. Ahmed, R.A. Ahmed, H.A. Muttlak, Some estimators of a finite population mean using auxiliary information, Applied Mathematics and Computation, **139**, 287-298, (2003).

[21] H. Verma, R. Singh and F. Smarandache, Some Improved Estimators of Population Mean Using Information on Two Auxiliary Attributes, Educational Publishing and Journal of Matter Regularity (Beijing),17-24,(2013).

[22] H. K. Verma, P. Sharma and R. Singh, Improved Estimator of Finite Population Mean Using Auxiliary Attribute in Stratified Random Sampling, Journal of Scientific Research, **58**, 99-105, (2014).