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Correlation Coefficient between Possibility Neutrosophic Soft Sets

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Abstract: In this study, we propose a new method to compute the correlation coefficient between possibility neutrosophic soft sets and give some properties related to this method. Finally, we give an example related to the proposed method.

Keywords: Soft set, neutrosophic soft set, possibility neutrosophic soft set, correlation coefficient

1 Introduction

Soft set theory was proposed by Molodtsov [20] to be an alternative to the fuzzy set theory [30] and intuitionistic fuzzy set theory [2] as a mathematical tool for dealing with uncertainty in 1999. Maji et al. [22,23] introduced some new operations between two soft sets and applied soft set theory to decision making problem in 2003. Fuzzy sets were combined with soft sets by Maji et al. [21]. Then the works on soft set theory and their applications have progressed rapidly [1,8,9,13,27].

Neutrosophic set was defined by Smarandache [26], as a new mathematical tool for dealing with problems involving incomplete, indeterminant, inconsistent knowledge. Maji[24] combined the concept of soft set and neutrosophic set together by introducing a new concept called neutrosophic soft set and gave an application of neutrosophic soft set in decision making problems. Karaaslan [18] redefined concept and operations of neutrosophic soft sets different from Maji's neutrosophic soft set definition and operations. Recently, the properties and applications on the neutrosophic soft sets have been studied increasingly [4,5,6,11,12,28].

Possibility fuzzy soft sets and operations defined on these sets were first introduced by Alkhazaleh et al. [3]. In 2012, the concept of possibility intuitionistic fuzzy soft set and its operations were defined by Bashir et al. [7]. Also, Bashir et al. [7] discussed the similarity measure of two possibility intuitionistic fuzzy soft sets and they gave an application of the proposed similarity measure. In 2014, the concept of possibility neutrosophic soft set and its operations were defined by Karaaslan [19]. In 2012 Hanafy and Salama [14,15,17] introduced and studied some operations on neutrosophic sets and investigated the correlation of neutrosophic data[14]. Murthy and Pal[25] have discussed and investigated the concept of correlation in fuzzy set[10,29]. In 2013, Hanafy et al.[16] proposed another method to calculate the correlation coefficient of neutrosophic sets.

In this study, after giving some definitions related to the possibility neutrosophic soft sets (PNS-set), we propose a new method to compute the correlation coefficient between possibility neutrosophic soft sets. The value obtained as a result of the application of proposed method tells us the strength of relationship between the neutrosophic soft sets and whether the neutrosophic sets are positively or negatively related. Finally, we give an example related to the proposed method.

2 Preliminary

In this section, we recall some required definitions related to the PNS-sets [18,19].

Throughout paper U is an initial universe, E is a set of parameters and Λ is an index set.

Definition 2.1[18] A neutrosophic soft set (or namely *ns*-set) f over U is a neutrosophic set valued function from E to $\mathcal{N}(\mathcal{U})$. It can be written as

$$f = \left\{ \left(e, \{\langle u, t_{f(e)}(u), i_{f(e)}(u), f_{f(e)}(u)\rangle : u \in U\}\right) : e \in E \right\}$$

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where, $\mathscr{N}(\mathscr{U})$ denotes set of all neutrosophic sets over U. Note that if $f(e) = \{\langle u, 0, 1, 1\rangle : u \in U\}$, the element $(e, f(e))$ is not appeared in the neutrosophic soft set f. Set of all *ns*-sets over U is denoted by $\mathbb{NS}$.

Definition 2.2[19] Let U be an initial universe, E be a parameter set, $\mathscr{N}(\mathscr{U})$ be the collection of all neutrosophic sets of U and I^U is collection of all fuzzy subset of U. A possibility neutrosophic soft set (*PNS*-set) f_μ over U is defined by the set of ordered pairs

$$f_\mu = \left\{\left(e_i, \{(\frac{u_j}{f(e_i)(u_j)}, \mu(e_i)(u_j)) : u_j \in U\}\right) : e_i \in E\right\}$$

where, $i, j \in \Lambda$, f is a mapping given by $f : E \to \mathscr{N}(\mathscr{U})$ and $\mu(e_i)$ is a fuzzy set such that $\mu : E \to I^U$. Here, $\tilde{f}_\mu$ is a mapping defined by $f_\mu : E \to \mathscr{N}(\mathscr{U}) \times I^U$.
For each parameter $e_i \in E$, $f(e_i) = \{\langle u_j, t_{f(e_i)}(u_j), i_{f(e_i)}(u_j), f_{f(e_i)}(u_j)\rangle : u_j \in U\}$ indicates neutrosophic value set of parameter e_i, where $t, i, f : U \to [0,1]$ are the membership functions of truth, indeterminacy and falsity of the element $u_j \in U$, respectively. For each $u_j \in U$ and $e_i \in E$, $0 \le t_{f(e_i)}(u_j) + i_{f(e_i)}(u_j) + f_{f(e_i)}(u_j) \le 3$. Also $\mu(e_i)$, degrees of possibility of belongingness of elements of U in $f(e_i)$. So we can write

$$f_\mu(e_i) = \left\{\left(\frac{u_1}{f(e_i)(u_1)}, \mu(e_i)(u_1)\right), \left(\frac{u_2}{f(e_i)(u_2)}, \mu(e_i)(u_2)\right), ..., \left(\frac{u_n}{f(e_i)(u_n)}, \mu(e_i)(u_n)\right)\right\}.$$

From now on, we will show the set of all possibility neutrosophic soft sets over U with $\mathscr{PN}(U,E)$ such that E is the parameter set.

Example 1.[19] Let $U = \{u_1, u_2, u_3\}$ be a set of three cars. Let $E = \{e_1, e_2, e_3\}$ be a set of qualities where e_1 =cheap, e_2 =equipment, e_3 =fuel consumption and let $\mu : E \to I^U$. We define a function $f_\mu : E \to \mathscr{N}(\mathscr{U}) \times I^U$ as follows:

$$f_\mu = \left\{\begin{array}{l} f_\mu(e_1) = \left\{\left(\frac{u_1}{(0.5,0.2,0.6)}, 0.8\right), \left(\frac{u_2}{(0.7,0.3,0.5)}, 0.4\right), \left(\frac{u_3}{(0.4,0.5,0.8)}, 0.7\right)\right\} \\ f_\mu(e_2) = \left\{\left(\frac{u_1}{(0.8,0.4,0.5)}, 0.6\right), \left(\frac{u_2}{(0.5,0.7,0.2)}, 0.8\right), \left(\frac{u_3}{(0.7,0.3,0.9)}, 0.4\right)\right\} \\ f_\mu(e_3) = \left\{\left(\frac{u_1}{(0.6,0.7,0.5)}, 0.2\right), \left(\frac{u_2}{(0.5,0.3,0.7)}, 0.6\right), \left(\frac{u_3}{(0.6,0.5,0.4)}, 0.5\right)\right\} \end{array}\right\}.$$

Also we can define a function $g_\nu : E \to \mathscr{N}(\mathscr{U}) \times I^U$ as follows:

$$g_\nu = \left\{\begin{array}{l} g_\nu(e_1) = \left\{\left(\frac{u_1}{(0.6,0.3,0.8)}, 0.4\right), \left(\frac{u_2}{(0.6,0.5,0.5)}, 0.7\right), \left(\frac{u_3}{(0.2,0.6,0.4)}, 0.8\right)\right\} \\ g_\nu(e_2) = \left\{\left(\frac{u_1}{(0.5,0.4,0.3)}, 0.3\right), \left(\frac{u_2}{(0.4,0.6,0.5)}, 0.6\right), \left(\frac{u_3}{(0.7,0.2,0.5)}, 0.8\right)\right\} \\ g_\nu(e_3) = \left\{\left(\frac{u_1}{(0.7,0.5,0.3)}, 0.8\right), \left(\frac{u_2}{(0.4,0.4,0.6)}, 0.5\right), \left(\frac{u_3}{(0.8,0.5,0.3)}, 0.6\right)\right\} \end{array}\right\}.$$

For the purpose of storing a possibility neutrosophic soft set in a computer, we can use matrix notation of possibility neutrosophic soft set f_μ. For example, matrix notation of possibility neutrosophic soft set f_μ can be written as follows: for $m, n \in \Lambda$,

$$f_\mu = \begin{pmatrix} (\langle 0.5,0.2,0.6\rangle,0.8) & (\langle 0.7,0.3,0.5\rangle,0.4) & (\langle 0.4,0.5,0.8\rangle,0.7) \\ (\langle 0.8,0.4,0.5\rangle,0.6) & (\langle 0.5,0.7,0.2\rangle,0.8) & (\langle 0.7,0.3,0.9\rangle,0.4) \\ (\langle 0.6,0.7,0.5\rangle,0.2) & (\langle 0.5,0.3,0.7\rangle,0.6) & (\langle 0.6,0.5,0.4\rangle,0.5) \end{pmatrix}$$

where the $m-$th row vector shows $f(e_m)$ and $n-$th column vector shows u_n.

Definition 2.3[19] Let $f_\mu \in \mathscr{PN}(U,E)$, where $f_\mu(e_i) = \{(f(e_i)(u_j), \mu(e_i)(u_j)) : e_i \in E, u_j \in U\}$ and $f(e_i) = \{\langle u, t_{f(e_i)}(u_j), i_{f(e_i)}(u_j), f_{f(e_i)}(u_j)\rangle\}$ for all $e_i \in E$, $u \in U$. Then for $e_i \in E$ and $u_j \in U$,

1. f^t_μ is said to be the truth-membership part of f_μ,
$$f^t_\mu = \{(f^t_{ij}(e_i), \mu_{ij}(e_i))\},$$
$$f^t_{ij}(e_i) = \{(u_j, t_{f(e_i)}(u_j))\},$$
$$\mu_{ij}(e_i) = \{(u_j, \mu(e_i)(u_j))\}$$
2. f^i_μ is said to be the indeterminacy-membership part of f_μ,
$$f^i_\mu = \{(f^i_{ij}(e_i), \mu_{ij}(e_i))\},$$
$$f^i_{ij}(e_i) = \{(u_j, i_{f(e_i)}(u_j))\},$$
$$\mu_{ij}(e_i) = \{(u_j, \mu(e_i)(u_j))\}$$
3. f^f_μ is said to be the falsity-membership part of f_μ,
$$f^f_\mu = \{(f^f_{ij}(e_i), \mu_{ij}(e_i))\},$$
$$f^f_{ij}(e_i) = \{(u_j, f_{f(e_i)}(u_j))\},$$
$$\mu_{ij}(e_i) = \{(u_j, \mu(e_i)(u_j))\}.$$

We can write a possibility neutrosophic soft set in the form $f_\mu = (f^t_\mu, f^i_\mu, f^f_\mu)$.

3 Correlation coefficient between possibility neutrosophic soft sets

In this section, we define the correlation coefficient between two *PNS*-sets.

Definition 3.1 Let f_μ and g_ν be two *PNS*-sets. The correlation coefficient $c_{f_\mu - PNS}(f_\mu, g_\nu)$ between two *PNS*-sets, f_μ and g_ν, is

$$c_{f_\mu - PNS}(f_\mu, g_\nu) = \frac{1}{3}(c_t(f_\mu, g_\nu) + c_i(f_\mu, g_\nu) + c_f(f_\mu, g_\nu))c_p(\mu, \nu)$$

where

$c_p(\mu,\nu)=$
$\frac{1}{|E|}\sum_{e\in E}\left(\frac{\sum_{i=1}^{n}(\mu(e)(u_i)-\overline{\mu(e)})(\nu(e)(u_i)-\overline{\nu(e)})}{\sqrt{\sum_{i=1}^{n}(\mu(e)(u_i)-\overline{\mu(e)})^2\sum_{i=1}^{n}(\nu(e)(u_i)-\overline{\nu(e)})^2}}\right),$

$c_t(f_\mu,g_\nu)=$
$\frac{1}{|E|}\sum_{e_k\in E}\left(\frac{\sum_{j=1}^{n}(f^t_{kj}(e_k)-\overline{f^t_{kj}(e_k)})(g^t_{kj}(e_k)-\overline{g^t_{kj}(e_k)})}{\sqrt{\sum_{j=1}^{n}(f^t_{kj}(e_k)-\overline{f^t_{kj}(e_k)})^2\sum_{j=1}^{n}(g^t_{kj}(e_k)-\overline{g^t_{kj}(e_k)})^2}}\right),$

$c_i(f_\mu,g_\nu)=$
$\frac{1}{|E|}\sum_{e_k\in E}\left(\frac{\sum_{j=1}^{n}(f^i_{kj}(e_k)-\overline{f^i_{kj}(e_k)})(g^i_{kj}(e_k)-\overline{g^i_{kj}(e_k)})}{\sqrt{\sum_{j=1}^{n}(f^i_{kj}(e_k)-\overline{f^i_{kj}(e_k)})^2\sum_{j=1}^{n}(g^i_{kj}(e_k)-\overline{g^i_{kj}(e_k)})^2}}\right),$

$c_f(f_\mu,g_\nu)=$
$\frac{1}{|E|}\sum_{e_k\in E}\left(\frac{\sum_{j=1}^{n}(f^f_{kj}(e_k)-\overline{f^f_{kj}(e_k)})(g^f_{ij}(e_k)-\overline{g^f_{kj}(e_k)})}{\sqrt{\sum_{j=1}^{n}(f^f_{kj}(e_k)-\overline{f^f_{kj}(e_k)})^2\sum_{j=1}^{n}(g^f_{kj}(e_k)-\overline{g^f_{kj}(e_k)})^2}}\right)$

and

$$\overline{\mu(e)}=\frac{1}{|U|}\sum_{j=1}^{n}\mu(e)(u_j),$$

$$\overline{\nu(e)}=\frac{1}{|U|}\sum_{j=1}^{n}\nu(e)(u_j),$$

$$\overline{f^t_{kj}(e_k)}=\frac{1}{|U|}\sum_{j=1}^{n}t_f(e_k)(u_j),$$

$$\overline{g^t_{kj}(e_k)}=\frac{1}{|U|}\sum_{j=1}^{n}t_g(e_k)(u_j),$$

$$\overline{f^i_{kj}(e_k)}=\frac{1}{|U|}\sum_{j=1}^{n}i_f(e_k)(u_j),$$

$$\overline{g^i_{kj}(e_k)}=\frac{1}{|U|}\sum_{j=1}^{n}i_g(e_k)(u_j),$$

$$\overline{f^f_{kj}(e_k)}=\frac{1}{|U|}\sum_{j=1}^{n}f_f(e_k)(u_j),$$

$$\overline{g^f_{kj}(e_k)}=\frac{1}{|U|}\sum_{j=1}^{n}f_g(e_k)(u_j).$$

The correlation coefficient between *PNS*-sets ranges in $[-1,1]$, which can correlate possibility neutrosophic soft sets. So, we define the following correlation measure.

Example 2. Let us consider *PNS*-sets f_μ and g_ν in Example 1. Then, we can compute the correlation between *PNS*-sets f_μ and g_ν as follow:

$$\overline{\mu(e_1)}=\frac{1}{3}(0.8+0.7+0.4)=0.63,$$

$$\overline{\mu(e_2)}=\frac{1}{3}(0.6+0.8+0.4)=0.60,$$

$$\overline{\mu(e_3)}=\frac{1}{3}(0.2+0.6+0.5)=0.43,$$

$$\overline{\nu(e_1)}=\frac{1}{3}(0.4+0.7+0.8)=0.63,$$

$$\overline{\nu(e_2)}=\frac{1}{3}(0.3+0.6+0.8)=0.57,$$

$$\overline{\nu(e_3)}=\frac{1}{3}(0.8+0.5+0.6)=0.63$$

$$c_p(\mu,\nu)=\frac{1}{|3|}\sum_{e\in E}\left(\frac{\sum_{i=1}^{3}(\mu(e)(u_i)-\overline{\mu(e)})(\nu(e)(u_i)-\overline{\nu(e)})}{\sqrt{\sum_{i=1}^{n}(\mu(e)(u_i)-\overline{\mu(e)})^2\sum_{i=1}^{n}(\nu(e)(u_i)-\overline{\nu(e)})^2}}\right)$$

and

$$\begin{aligned}c_p(\mu,\nu) &= \frac{1}{3}\Big(\frac{(0.17)(-0.23)+(-0.23)(0.07)+(0.07)(0.17)}{\sqrt{((0.17)^2+(0.07)^2+(-0.23)^2)((-0.23)^2+(0.07)^2+(0.17)^2)}}\\ &+\frac{(0)(-0.27)+(0.20)(0.03)+(-0.20)(0.23)}{\sqrt{((0)^2+(0.20)^2+(-0.20)^2)((-0.27)^2+(0.03)^2+(0.23)^2)}}\\ &+\frac{(-0.23)(0.17)+(0.17)(-0.13)+(0.07)(-0.03)}{\sqrt{((-0.23)^2+(0.17)^2+(0.07)^2)((0.17)^2+(-0.13)^2+(-0.03)^2)}}\Big)\\ &= -0.650,\end{aligned}$$

$$\begin{aligned}c_t(f_\mu,g_\nu) &= \frac{1}{3}\Big(\frac{(-0.033)(0.133)+(0.167)(0.133)+(-0.133)(-0.267)}{\sqrt{((-0.033)^2+(0.167)^2+(-0.133)^2)((0.133)^2+(0.133)^2+(-0.267)^2)}}\\ &+\frac{(0.133)(-0.033)+(-0.167)(-0.133)+(0.033)(0.167)}{\sqrt{((0.133)^2+(-0.167)^2+(0.033)^2)((-0.033)^2+(-0.133)^2+(0.167)^2)}}\\ &+\frac{(0.033)(0.067)+(-0.067)(-0.233)+(0.033)(0.167)}{\sqrt{((0.033)^2+(-0.067)^2+(0.033)^2)((0.067)^2+(-0.233)^2+(0.167)^2)}}\Big)\\ &= 0.743.\end{aligned}$$

Similarly, we have $c_i(f_\mu,g_\nu)=0.657$ and $c_f(f_\mu,g_\nu)=0.203$.
Consequently,

$$\begin{aligned}c_{f_\mu-PNS}(f_\mu,g_\nu) &= \frac{1}{3}(0.743+0.657+0.203)(-0.650)\\ &= -0,348.\end{aligned}$$

It shows that neutrosophic soft sets f_μ and g_ν have a bad negatively correlated coefficient.

Proposition 3.1 Let f_μ and g_ν be two *PNS*-sets. The correlation coefficient $c_{f_\mu-PNS}(f_\mu,g_\nu)$ fulfills the following properties:

1.$c_{f_\mu-PNS}(f_\mu,g_\nu)=c_{f_\mu-PNS}(g_\nu,f_\mu)$,
2.If $f_\mu=g_\nu$ then $c_{f_\mu-PNS}(f_\mu,g_\nu)=1$,
3.$|c_{f_\mu-PNS}(f_\mu,g_\nu)|\leq 1$.

Proof. The proof is clear from Definition 3.

4 Conclusion

In this paper we have introduced correlation coefficient between the *PNS*-sets. An example of the proposed method has been given to compare relation between *PNS*-sets. In future, these seem to have natural applications.

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