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Sliding Mode Control for A Class of Uncertain Time-Delay Systems

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Abstract: This paper investigates the stability and stabilization problems for a class of uncertain time-delay systems. For exploring the stability problem, the Lyapunov-Krasovskii function (LKF) method and Leibniz-Newton formula are adopted to analyze the stability problems of a class of uncertain time-delay systems. In addition, the proposed delay-dependent stability conditions for a class of time-delay unforced systems can be formulated by linear matrix inequalities (LMIs). By examining the stabilization problem, based on the sliding mode control scheme, some assumptions, and some transformations, the delay-dependent stabilization condition for uncertain time-delay system is propounded to guarantee the asymptotic stabilization of uncertain time-delay system in this paper. Moreover, based on the Schur complement formula and some variable transformations, the delay-dependent stabilization conditions of the uncertain time-delay system can be presented in terms of linear matrix inequality (LMI) form. Finally, a numerical example is illustrated to demonstrate the effectiveness and validity of the proposed control scheme.

Keywords: Sliding mode control, time-delay, linear matrix inequalities

1 Introduction

It is known that time-delay phenomenon always exists in many physical and engineering systems, for example, manual controls, pollution dynamic models, rolling hills, neural networks systems, and inferred grinding models. Therefore, the problems of stability analysis and controller design for nonlinear systems with time delay have gained considerable research attention [1,2,3] in the past few years. In general, the stabilization for a delay systems are divided into two categories; delay-independent and delay-dependent cases. The delay-independent approach usually is derived from the standard Lyapunov-Krasovskii function to obtain the stability condition, which provides feasible solutions irrespective of the size of delay. Since the time delay is not taken into consideration in the process of designing controllers, the delay times are allowed to be arbitrarily large. But the disadvantage of delay-independent approach is that its stability/stabilization condition generally more conservative than delay-dependent ones especially when the size of the delay is small. For this reason, delay-dependent stabilization for time-delay systems based on Lyapunov-Krasovksii functional approach are discussed in many studies [4,5,6]. But for some systems, one cannot addresses the stabilization condition directly, and one has to transform the original system into a more fitting form for further analysis by using the Lyapunov-Krasovskii technique. Therefore, there are some studies that adopt some transformations to obtain the equivalent equation for original system [7,8].

Among the various robust control methods for uncertain system, the well-known sliding mode control (SMC) [9,10,11] has been recognized as an effective robust control approach for uncertain systems. SMC can be regarded a special type of variable structure control. The property of SMC is that it provides discontinuous control laws to drive the system states to a specified sliding surface and to keep them on the sliding surface. Besides this, the closed-loop response becomes totally insensitive to a particular class of uncertainties. Moreover, it provides a systematic control design method for nonlinear system. Therefore, SMC technique has been widely applied to many uncertain nonlinear systems [12,13,14,15,16,17]. Over the past few years, a considerable number of studies have been made on SMC for time-delay system [18,19,20,21,22,23].

Motivated by the above discussion, this paper will explore the stabilization problems of a class of uncertain

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time-delay systems. The main contributions of this paper are highlighted as following: i) designing a sliding mode controller for the uncertain time-delay system; ii) Describing the delay-dependent stabilization conditions for the uncertain time-delay system via LMIs; iii) Utilizing the proposed method for a numerical example with time-delay.

This paper is organized to the subsequent sections: In Section II, a stability condition for a delay system which is proposed. In Section III, a sliding control method is propounded for a class of uncertain time-delay system. In Section IV, a simulation is shown to illustrate the proposed method. Finally, in Section V conclusion is drawn.

Notation: The notations in this paper are quite standard. R^n and $R^{n\times m}$ denote, respectively, the n -dimensional Euclidean space and the set of all $n\times m$ real matrices. A^T denotes the transpose of matrix A. $X\leq Y$ or $X<Y$, respectively, where X and Y are symmetric matrices, means that $X-Y$ is negative semi-definite or negative definite, respectively. I is the identity matrix with a compatible dimension (without confusion).

2 Stability Analysis for Time-Delay System

Considering the following time-delay system:

$$\begin{aligned}\dot{x}(t)=&Ax(t)+A_dx(t-\tau(t))\\ x(t)=&\upsilon(t),\ \ t\in[-t_0\ \ 0]\end{aligned}\tag{1}$$

where $x(t)\in R^n$ is the state, $A\in R^{n\times n}$ and $A_d\in R^{n\times n}$ are the already known real constant matrices, $\tau(t)$ is the time delay, satisfied $\tau(t)\leq\tau_M$ and $\dot{\tau}(t)\leq\tau_D$, and $\upsilon(t)$ is the initial condition.

Assumption 1. [7] The parameter uncertainties considered here are norm-bounded and presented by the form:

$$[\Delta A(t)\ \ \Delta A_d(t)]=MF(t)[N\ \ N_d]$$

where M, N, and N_d are the already known real constant matrices of appropriate dimensions; $F(t)\in R^{p\times q}$ is an unknown matrix function with Lebesgue-measurable elements and satisfies:

$$F^T(t)F(t)\leq I,\ \forall t.$$

Assumption 2. [9] The pair (A,B) is controllable for the following time-delay system, where $rank(B)=m$

$$\begin{aligned}\dot{x}(t)=&[A+\Delta A(t)]x(t)\\ &+[A_d+\Delta A_d(t)]x(t-\tau(t))+Bu(t)\end{aligned}\tag{2}$$

Lemma 1. [9] For a given symmetric matrix D, and there exist M, N, and $F(t)$ are the matrices with suitable dimension, where the $F(t)$ satisfies the $F^T(t)F(t)\leq I$, then

$$D+MF(t)N+N^TF^T(t)M^T<0$$

if and only if there exists a scalar $\lambda>0$ which is able to satisfy:

$$D+\lambda MM^T+\lambda N^TIN<0$$

Lemma 2. [12] Given any matrix $H(t)\in R^{p\times q}$ such that $H^T(t)H(t)\leq I$, then

$$2x^TH(t)y\leq x^Tx+y^Ty$$

for all $x\in R^p$ and $y\in R^q$.

Theorem 1. For a given real constant $\varepsilon>0$, the system (1) is asymptotically stable if there exist some symmetric positive definite matrices: P_0, P_1, R_1 such that the following linear matrix inequality (LMI) condition holds:

$$\sum=\begin{bmatrix}\Sigma_{11}&\Sigma_{12}&\Sigma_{13}\\ *&\Sigma_{22}&\Sigma_{23}\\ *&*&\Sigma_{33}\end{bmatrix}<0\tag{3}$$

where $\Sigma_{11}=P_0A+A^TP_0+P_1-R_1$, $\Sigma_{12}=P_0A_d+R_1$, $\Sigma_{13}=\varepsilon A^TP_0$, $\Sigma_{22}=-(1-\tau_D)P_0-R_1$, $\Sigma_{23}=\varepsilon A_d^TP_0$, $\Sigma_{33}=-2\varepsilon P_0+\tau_M^2R_1$, and $*$ stands for the symmetric form in the matrix.

Proof. At first, one chooses a Lyapunov-Krasovskii functional:

$$\begin{aligned}V(x_t)=&x^T(t)P_0x(t)+\int_{t-\tau(t)}^{t}x^T(s)P_1x(s)ds\\ &+\tau_M\int_{t-\tau_M}^{t}(s-(t-\tau_M))\dot{x}^T(s)R_1\dot{x}(s)ds\end{aligned}\tag{4}$$

The time derivatives of $V(x_t)$ becomes

$$\begin{aligned}\dot{V}(x_t)=&x^T(t)(P_0A+A^TP_0)x(t)+2x^T(t)P_0A_dx(t-\tau(t))\\ &+x^T(t)P_1x(t)-(1-\dot{\tau}(t))x^T(t-\tau(t))P_1x(t-\tau(t))\\ &+\tau_M^2\dot{x}^T(t)R_1\dot{x}(t)-\tau_M\int_{t-\tau_M}^{t}\dot{x}^T(s)R_1\dot{x}(s)ds.\end{aligned}\tag{5}$$

By the inequality in [24], we have

$$\begin{aligned}&-\tau_M\int_{t-\tau_M}^{t}\dot{x}^T(s)R_1\dot{x}(s)ds\\ &\leq-\tau(t)\int_{t-\tau(t)}^{t}\dot{x}^T(s)R_1\dot{x}(s)ds\\ &\leq-(\int_{t-\tau(t)}^{t}\dot{x}(s)ds)^TR_1(\int_{t-\tau(t)}^{t}\dot{x}(s)ds).\end{aligned}\tag{6}$$

According to the Leibniz-Newton formula, we can obtain

$$\int_{t-\tau(t)}^{t}\dot{x}(s)ds=x(t)-x(t-\tau(t)).\tag{7}$$

From the (6) and (7), we have the following inequality:

$$\begin{aligned}&\tau_M\int_{t-\tau_M}^{t}\dot{x}^TR_1\dot{x}(s)ds\\ &\leq-(x(t)-x(t-\tau(t)))^TR_1(x(t)-x(t-\tau(t))).\end{aligned}\tag{8}$$

Besides, we can choose a scalar $\varepsilon > 0$ such that

$$-2\varepsilon\dot{x}^T P_0\dot{x}(t) + \varepsilon\dot{x}^T P_0\dot{x}(t)[Ax(t) + A_d x(t-\tau(t))] \\ \varepsilon[Ax(t) + A_d x(t-\tau(t))]^T P_0\dot{x}(t) = 0 \quad (9)$$

Concludomg the equations (5), (8), and (9), we can obtain the following result

$$\begin{aligned}\dot{V}(x_t) \leq & x^T(t)(P_0A + A^TP_0)x(t) + 2x^T(t)P_0A_dx(t-\tau(t)) \\ & + x^TP_1x(t) - (1-\tau_D(t))x^T(t-\tau(t))P_1x(t-\tau(t)) \\ & + \tau_M^2\dot{x}^T(t)R_1\dot{x}(t) - (x(t)-x(t-\tau(t)))^TR_1(x(t) \\ & - x(t-\tau(t))) - 2\varepsilon\dot{x}^T(t)P_0\dot{x}(t) + \varepsilon\dot{x}^T(t)P_0[Ax(t) \\ & + A_dx(t-\tau(t))] + \varepsilon\dot{x}^T(t)P_0[Ax(t) + A_dx(t-\tau(t))] \\ & + \varepsilon[Ax(t) + A_dx(t-\tau(t))]^TP_0\dot{x}(t) \\ & = [\psi^T \cdot \sum \cdot \psi] \end{aligned} \quad (10)$$

where $\psi^T = [x^T(t) \quad x^T(t-\tau(t)) \quad \dot{x}^T(t)]$. Clearly, if the LMI (3) hold, the system (1) is asymptotically stable. This completes the proof of the theorem. □

3 Sliding Mode Control for Uncertain Time-Delay System

In this section, we will discuss the stabilization problem of time-delay system with uncertainties. Before discussing the stabilization problem of time-delay system with uncertainties, we have to select an appropriate sliding surface, while the system state remains on the sliding surface, the desired performance can be achieved.

Firstly, consider the time-delay system with uncertainties

$$\begin{aligned}\dot{x}(t) = & [A + \Delta A(t)]x(t) \\ & + [A_d + \Delta A_d(t)]x(t-\tau(t)) + Bu(t) \\ x(t) = & \upsilon(t), \ \ t \in [-t_0 \ \ 0]\end{aligned} \quad (11)$$

where $A \in R^{n\times n}$, $A_d \in R^{n\times n}$, $\tau(t)$, and τ_M are the same as those in (1); $u(t)$ is the input control; B is the already known real constant matrix with appropriate dimension; $\Delta A(t) \in R^{n\times n}$ and $\Delta A_d(t) \in R^{n\times n}$ satisfied the Assumption 1.

From the Assumption 2, we can know there exists a which is a real nonsingular such that

$$TB = \begin{bmatrix} 0_{(n-m)} \\ \bar{B} \end{bmatrix}$$

where $\bar{B} \in R^{m\times m}$ is nonsingular and satisfies the following singular value decomposition (SVD)

$$\bar{B} = U\begin{bmatrix} \omega_{m\times m} \\ 0_{(n-m)\times m}\end{bmatrix}V^T, \ \ U = [U_1 \ \ U_2] \\ T = col\{U_2^T \ \ U_1^T\}$$

where $U_1 \in R^{n\times m}$ and $U_2 \in R^{n\times(n-m)}$ are both unitary matrices and is a diagonal positive-definite matrix.

By utilizing the transformation $\eta(t) = Tx(t)$ [9], we can transform the system (11) into:

$$\begin{aligned}\dot{\eta}_1(t) = & (\bar{A}_{11} + \Delta\bar{A}_{11})\eta_1(t) + (\bar{A}_{12} + \Delta\bar{A}_{12})\eta_2(t) \\ & + (\bar{A}_{d11} + \Delta\bar{A}_{d11})\eta_1(t-\tau(t)) \\ & + (\bar{A}_{d12} + \Delta\bar{A}_{d12})\eta_2(t-\tau(t))\end{aligned} \quad (12)$$

$$\begin{aligned}\dot{\eta}_2(t) = & (\bar{A}_{21} + \Delta\bar{A}_{21})\eta_1(t) + (\bar{A}_{22} + \Delta\bar{A}_{22})\eta_2(t) \\ & + \bar{B}u(t) + (\bar{A}_{d21} + \Delta\bar{A}_{d21})\eta_1(t-\tau(t)) \\ & + (\bar{A}_{d22} + \Delta\bar{A}_{d22})\eta_2(t-\tau(t))\end{aligned} \quad (13)$$

where $\bar{A}_{11} = U_2^TAU_2, \bar{A}_{12} = U_2^TAU_1, \bar{A}_{d11} = U_2^TA_dU_2, \bar{A}_{d12} = U_2^TA_dU_1, \Delta\bar{A}_{11} = U_2^TMF(t)NU_2, \Delta\bar{A}_{12} = U_2^TMF(t)NU_1, \Delta\bar{A}_{d11} = U_2^TMF(t)N_dU_2, \Delta\bar{A}_{d12} = U_2^TMF(t)N_dU_1, \bar{A}_{21} = U_1^TAU_1, \bar{A}_{22} = U_1^TAU_2, \bar{A}_{d21} = U_1^TA_dU_1, \bar{A}_{d22} = U_1^TA_dU_2, \Delta\bar{A}_{21} = U_1^TMF(t)NU_1, \Delta\bar{A}_{22} = U_1^TMF(t)NU_2, \Delta\bar{A}_{d21} = U_1^TMF(t)N_dU_1, \Delta\bar{A}_{d22} = U_1^TMF(t)N_dU_2, \eta_1 \in R^{n-m}, \eta_1 \in R^m, \bar{B} = \omega V^T$.

Here, we can design the sliding surface for the (12) and (13) as

$$S(t) = Q\eta_1(t) + \eta 2(t) = 0 \quad (14)$$

where $Q \in R^{m\times(n-m)}$ is a real matrix.

From (14), we can substitute the result $\eta_2(t) = -Q\eta_1(t)$ into the (13), and then we can obtain the following sliding mode equation:

$$\begin{aligned}\dot{\eta}_1(t) = & [\bar{A}_{11} + \Delta\bar{A}_{11} - (\bar{A}_{12} + \Delta\bar{A}_{12})Q]\eta_1(t) \\ & + [\bar{A}_{d11} + \Delta\bar{A}_{d11} - (\bar{A}_{d12} + \Delta\bar{A}_{d12})Q] \\ & \times \eta_1(t-\tau(t))\end{aligned} \quad (15)$$

By the result of [25], $S(t) = 0$ and $\dot{S}(t) = 0$, we can get the following equivalent control law

$$\begin{aligned}u_{eq}(t) = & -\bar{B}^{-1}\{Q[\bar{A}_{11}\eta_1(t) + \bar{A}_{12}\eta_2(t) \\ & + \bar{A}_{d11}\eta_1(t-\tau(t)) + \bar{A}_{d12}\eta_2(t-\tau(t))] \\ & + \bar{A}_{21}\eta_1(t) + \bar{A}_{22}\eta_2(t) + \bar{A}_{d21}\eta_1(t-\tau(t)) \\ & + \bar{A}_{d22}\eta_2(t-\tau(t))\}\end{aligned} \quad (16)$$

Now, we will explore the stability condition of (15) and conclude the result as following theorem.

Theorem 2. Given a real constant $\varepsilon > 0$, if there exist some symmetric positive definite matrices: P_0, P_1, and R_1 such that the following LMIs hold:

$$\hat{\Xi} = \begin{bmatrix} \hat{\Xi}_{11} & \Xi_{12} & \hat{\Xi}_{13} & \Xi_{14} \\ * & \Xi_{22} & \Xi_{23} & \Xi_{24} \\ * & * & \hat{\Xi}_{33} & \Xi_{34} \\ * & * & * & \Xi_{44}\end{bmatrix} < 0 \quad (17)$$

then the sliding mode dynamics (15) is asymptotically stable with sliding surface

$$S(t) = Q\eta_1(t) + \eta_2(t)$$

where $\hat{\Xi}_{11} = \Xi_{11} + \lambda U_2^T MM^T U_2, \Xi_{12} = (\bar{A}_{d11}\bar{P}_0 - \bar{A}_{d12}L) + \bar{R}_1, \hat{\Xi}_{13} = \Xi_{13} + \varepsilon\lambda U_2^T MM^T U_2, \Xi_{14} = (U_2\bar{P}_0 - U_1L)^T N^T, \Xi_{22} = -(1-\tau_D)\bar{P}_1 - \bar{R}_1, \Xi_{23} = \varepsilon(\bar{A}_{d11}\bar{P}_0 - \bar{A}_{d12}L)^T, \Xi_{24} = (U_2\bar{P}_0 - U_1L)^T N_d^T, \hat{\Xi}_{33} = \Xi_{33} + \varepsilon\lambda U_2^T MM^T U_2, \Xi_{11} = (\bar{A}_{11}\bar{P}_0 - \bar{A}_{12}L) + (\bar{A}_{11}\bar{P}_0 - \bar{A}_{12}L)^T + \bar{P}_1 - \bar{R}_1, \Xi_{13} = \varepsilon(\bar{A}_{11}\bar{P}_0 - \bar{A}_{12}L)^T, \Xi_{33} = -2\varepsilon\bar{P}_0 + \tau_M^2\bar{R}_1, \bar{P}_0 = P_0^{-1}, \Xi_{34} = 0, \Xi_{44} = -\lambda I, \bar{P}_1 = P_0^{-1}P_1P_0^{-1}, \bar{R}_1 = P_0^{-1}R_1P_0^{-1}, Q = LP_0^{-1}$.

Proof. Firstly, pre- and post-multiply the matrix $\sum$ by $\Gamma = diag[P_0^{-1}\ P_0^{-1}\ P_0^{-1}] > 0$ and define the A and A_d as

$$A = [\bar{A}_{11} + \Delta\bar{A}_{11} - (\bar{A}_{12} + \Delta\bar{A}_{12})Q]$$
$$A_d = [\bar{A}_{d11} + \Delta\bar{A}_{d11} - (\bar{A}_{d12} + \Delta\bar{A}_{d12})Q],$$

then we can obtain the (18)

$$\hat{\sum} = \begin{bmatrix} \hat{\Sigma}_{11} & \hat{\Sigma}_{12} & \hat{\Sigma}_{13} \\ * & \hat{\Sigma}_{22} & \hat{\Sigma}_{23} \\ * & * & \hat{\Sigma}_{33} \end{bmatrix} < 0 \tag{18}$$

where $\hat{\Sigma}_{11} = [\bar{A}_{11} + \Delta\bar{A}_{11} - (\bar{A}_{12} + \Delta\bar{A}_{12})Q]\bar{P}_0 + \bar{P}_0[\bar{A}_{11} + \Delta\bar{A}_{11} - (\bar{A}_{12} + \Delta\bar{A}_{12})Q]^T + \bar{P}_1 - \bar{R}_1, \hat{\Sigma}_{12} = [\bar{A}_{d11} + \Delta\bar{A}_{d11} - (\bar{A}_{d12} + \Delta\bar{A}_{d12})Q]\bar{P}_0 + \bar{R}_1, \hat{\Sigma}_{13} = \varepsilon\bar{P}_0[\bar{A}_{11} + \Delta\bar{A}_{11} - (\bar{A}_{12} + \Delta\bar{A}_{12})Q]^T, \hat{\Sigma}_{22} = -(1-\tau_D)\bar{P}_1 - \bar{R}_1, \hat{\Sigma}_{23} = \varepsilon\bar{P}_0A_d^T, \hat{\Sigma}_{33} = -2\varepsilon\bar{P}_0 + \tau_M^2\bar{R}_1$.

By Assumption 1, we can obtain (19) from (18)

$$\Xi + \begin{bmatrix} U_2^T M \\ 0 \\ \varepsilon U_2^T M \end{bmatrix} F(t) \begin{bmatrix} (U_2^T\bar{P}_0 - U_1L)^T N^T \\ (U_2^T\bar{P}_0 - U_1L)^T N_d^T \\ 0 \end{bmatrix} + [\Theta]^T < 0 \tag{19}$$

where $\Xi = \begin{bmatrix} \Xi_{11} & \Xi_{12} & \Xi_{13} \\ * & \Xi_{22} & \Xi_{23} \\ * & * & \Xi_{33} \end{bmatrix}$, and the $[\Theta]^T$ means the transposition of second term in the inequality (19).

By utilizing the Lemma 1 and Schur complement to (19), we can obtain the (17).This completes the proof of the theorem. □

After designing the sliding surface, the next step of the SMC design procedure is to design a feedback control law such that the reachability of the specified sliding surface (14) is ensured.

Theorem 3. Consider the system (11) subject to the Assumption 1 and Assumption 2. If LMIs (17) with sliding surface (14), where Q is given by(17) .The control input is given as:

$$u(t) = u_1(t) + u_2(t) + u_3(t) \tag{20}$$

where $u_1(t) = u_{eq}(t)$,

$$\begin{aligned} u_2(t) = -\bar{B}^{-1}\Big\{\frac{1}{2}\Big[&4\left\|(QU_2^T M)\right\|^2 + \|NU_2\eta_1(t)\|^2 \\ &+ \|NU_1\eta_2(t)\|^2 + \|N_dU_2\eta_1(t-\tau(t))\|^2 \\ &+ \|N_dU_1\eta_2(t-\tau(t))\|^2 + 4\left\|(U_1^T M)\right\|^2 \\ &+ \|NU_1\eta_1(t)\|^2 + \|NU_2\eta_2(t)\|^2 + \|N_dU_1\eta_1(t-\tau(t))\|^2 \\ &+ \|N_dU_1\eta_2(t-\tau(t))\|^2\Big]\Big\} sign(S(t)) \end{aligned}$$

,

$u_3(t) = -\bar{B}^{-1}[kS(t) + \sigma sign(S)], k > 0, \sigma > 0$. Then, the all signals involves in closed-loop system (11) with the control input (20) are uniformly ultimately bounded.

Proof. According to the concept of sliding mode control, we know that the reaching condition of sliding mode control is $S^T(t)\dot{S}(t) < 0$. From the sliding surface (14) and substituting the control input (20) into $\dot{S}(t)$ we can obtain

$$\dot{S}(t) = \pi_1 - \pi_2 - \pi_3$$

where

$$\begin{aligned} \pi_1 = &Q\Big[U_2^T MFNU_2\eta_1(t) + U_2^T MFNU_1\eta_2(t) \\ &+ U_2^T MFN_dU_2\eta_1(t-\tau(t)) + U_2^T MFN_dU_1\eta_2(t-\tau(t))\Big] \\ &+ U_1^T MFNU_1\eta_1(t) + U_1^T MFNU_1\eta_2(t) \\ &+ U_1^T MFN_dU_2\eta_2(t-\tau(t)) \end{aligned}$$

$$\begin{aligned} \pi_2 = &\frac{1}{2}\Big[4\left\|(QU_2^T M)\right\|^2 + \|NU_2\eta_1(t)\|^2 \\ &+ \|NU_1\eta_2(t)\|^2 + \|N_dU_2\eta_1(t-\tau(t))\|^2 \\ &+ \|N_dU_1\eta_2(t-\tau(t))\|^2 + 4\left\|(U_1^T M)\right\|^2 \\ &+ \|NU_1\eta_1(t)\|^2 + \|NU_2\eta_2(t)\|^2 + \|N_dU_1\eta_1(t-\tau(t))\|^2 \\ &+ \|N_dU_1\eta_2(t-\tau(t))\|^2\Big] sign(S(t)) \end{aligned}$$

$\pi_3 = kS(t) + \sigma sign(S), k > 0, \sigma > 0$

By the Lemma 2, we have $\pi_1 \leq \pi_2$. Clearly, if $S(t) > 0$, then $\dot{S}(t) \leq -\pi_3 < 0$. On the contrary, if $S(t) < 0$, then $\dot{S}(t) \geq -\pi_3 > 0$. Concluding the above discussions, we can know that $S^T(t)\dot{S}(t) < 0$. This completes the proof of the theorem. □

4 Simulation

In this section, we will apply the proposed method to design a sliding mode controller for an uncertain time-delay system. Firstly, consider the following uncertain time-delay system:

$$\begin{aligned} \dot{x}(t) = &[A + \Delta A(t)]x(t) \\ &+ [A_d + \Delta A_d(t)]x(t-\tau(t)) + Bu(t) \end{aligned} \tag{21}$$

where $A = \begin{bmatrix} 0 & 1 \\ -2 & -3 \end{bmatrix}$, $A_d = \begin{bmatrix} 0 & 0.1 \\ -0.2 & -0.3 \end{bmatrix}$, $B = \begin{bmatrix} 0 \\ 4.7 \end{bmatrix}$.
Based on Assumption 1, we can define the matrices M, N, and N_d as follows:

$$M = \begin{bmatrix} 0.2 & 0 \\ 0 & 0.2 \end{bmatrix}, \; N = N_d = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}, \; \tau(t) = 0.5, \; \tau_M = 1$$

In this example, we choose $\tau(t) = 0.5$, $\tau_M = 1$, $\underline{k} = 0.5$, $\sigma = 0.5$ and utilizing the Theorem 2 with and $\bar{B} = B$ and $T = I$, then we can figure out $P_0 = 0.9697$, $L = 1.0670$. Substituting these values into (20), then we can get the control input. The simulation results of applying the sliding mode controller to the time-delay uncertain system (21) under three different initial conditions $[-1.5 \quad -0.7]$, $[1.1 \quad 0.5]$, and $[-2 \quad 1.3]$ are shown in Fig. 1 and Fig. 2. From these simulation results, we can find that the designed sliding mode controller ensures the robust asymptotic stability of the closed-loop system, and the states are regulated to zero after few seconds. Fig. 3 shows the total control input u with $k = 0.3$ and $\sigma = 0.3$, and the Fig. 4, Fig. 5, and Fig. 6 show the u_1, u_2, and u_3 respectively.

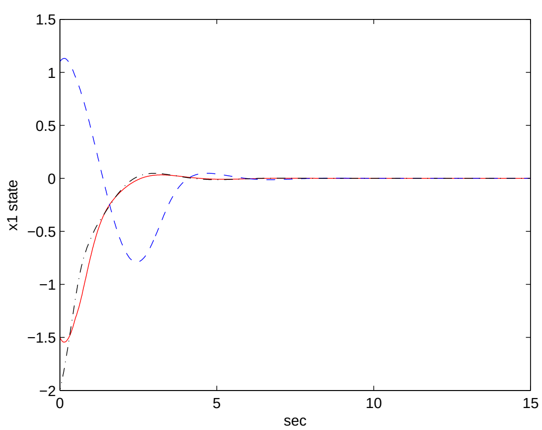

Fig. 1: State responses of x_1 under three initial conditions $[-1.5 \quad -0.7]$, $[1.1 \;\; 0.5]$ and $[-2 \;\; 1.3]$.

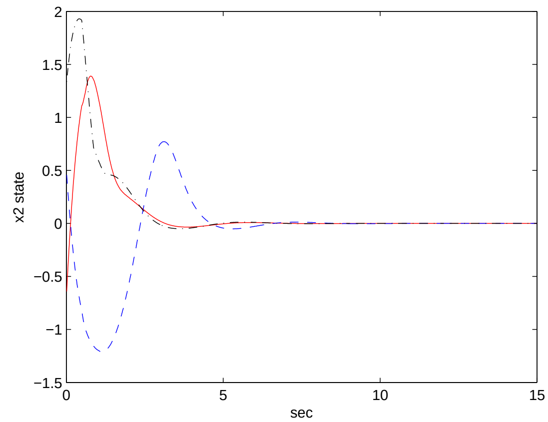

Fig. 2: State responses of x_2 under three initial conditions $[-1.5 \quad -0.7]$, $[1.1 \;\; 0.5]$ and $[-2 \;\; 1.3]$.

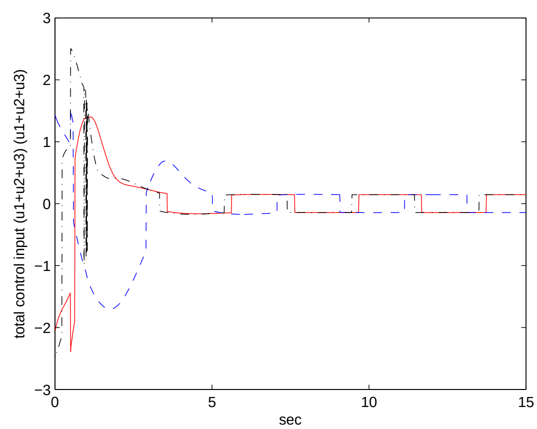

Fig. 3: Total control input u under three initial conditions $[-1.5 \quad -0.7]$, $[1.1 \;\; 0.5]$ and $[-2 \;\; 1.3]$.

5 Conclusion

In this paper, the stability and stabilization problems for a class of uncertain time-delay systems are explored. By utilizing the Lyapunov-Krasovskii function (LKF) method and Leibniz-Newton formula, the proposed delay-dependent stability condition for a class of time-delay unforced system can be formulated by linear matrix inequalities (LMIs). For the stabilization problem, based on the sliding mode control scheme, the delay-dependent stabilization condition for uncertain time-delay system is presented to guarantee the asymptotic stabilization of uncertain time-delay system in this paper. Finally, an uncertain nonlinear system with time-delay is illustrated to demonstrate the effectiveness and feasibility of the proposed control scheme.

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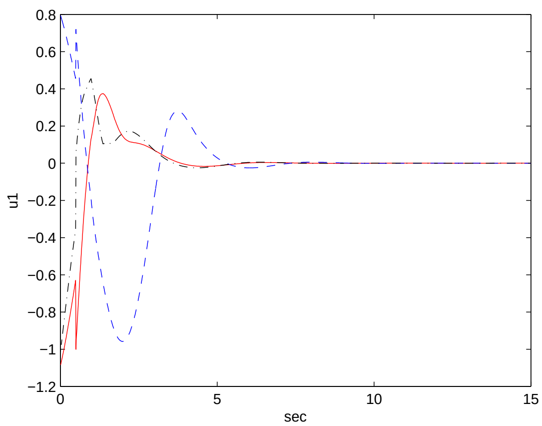

Fig. 4: Control input u_1 for three initial conditions $[-1.5 \ \ -0.7]$, $[1.1 \ \ 0.5]$ and $[-2 \ \ 1.3]$.

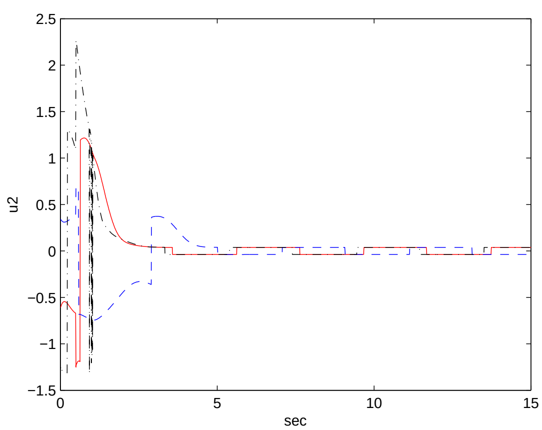

Fig. 5: Control input u_2 for three initial conditions $[-1.5 \ \ -0.7]$, $[1.1 \ \ 0.5]$ and $[-2 \ \ 1.3]$.

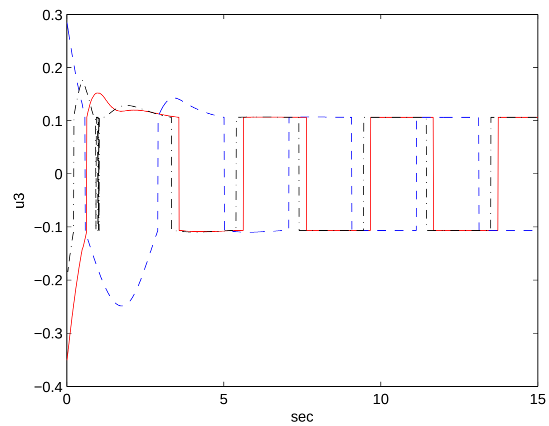

Fig. 6: Control input u_3 for three initial conditions $[-1.5 \ \ -0.7]$, $[1.1 \ \ 0.5]$ and $[-2 \ \ 1.3]$.

References

[1] V.L. Kharitonov, and D. Melchor-Aguilar, On delay-dependent stability conditions for time-delay systems. System & Control Letters. **46**, 173-179 (2002).

[2] E. Fridman, and U. Shaked, Delay-dependent stability and control: constant and time-varying delays. International Journal of Control. **76**, 48-60 (2003).

[3] W.H. Chen, Z.H. Guan, and X. Lu, Delay-dependent output feedback stabilisation of Markovian jump system with time-delay. IEE Proceedings Control Theory and Applications. **151**, 561-566 (2004).

[4] X.P. Guan, and C.L. Chen, Delay-dependent guaranteed cost control for T-S fuzzy systems with time delays. IEEE Trans Fuzzy System. **12**, 236-249 (2004).

[5] S. Xu, and J. Lam, Improved delay-dependent stability criteria for time-delay systems. IEEE Transcation on Automatic Control. **50**, 384-387(2005).

[6] K. Shao, H. Gao, and M. Gao, Delay-dependent controller design for nonlinear system with time-varying delay based on neural networks. IEEE International Conference on Industrial Technology. 1-5 (2008).

[7] S. Xu, J. Lam, and X. Mao, Delay-dependent control and filtering for uncertain Markovian jump systems with time-varying delays. IEEE Transactions on Circuits and Systems I: Regular Papers. **54**, 2070-2077 (2007).

[8] M. Wu, Y. He, J.H. She, and G.P. Liu, Delay-dependent criteria for robust stability of time-varying delay systems. Automatica. **40**, 1435-1439 (2004).

[9] G.H. Wang, L.G. Wu, and H.J. Gao, Delay-dependent adaptive sliding mode control of uncertain systems with state and input delays. Machine Learning and Cybernetic. **1**, 454-460 (2005).

[10] K.C. Veluvolu, Y.C. Soh, and W. Cao, Discrete-time sliding mode observer design for a class of uncertain nonlinear systems with application to bioprocess. Control, Automation, Robotics and Vision Conference. **2**, 855-860 (2004).

[11] K.C. Veluvolu, Y.C. Soh, and W. Cao, Robust observer with sliding mode estimation for nonlinear uncertain systems. IET Control Theory & Application. **1**, 1533-1540(2007).

[12] M. Yan, J. Yi, Y. Liu, J. Wei, and X. Huang, Sliding mode control algorithm based on T-S fuzzy model for a class of nonlinear systems. IEEE International Conference on Automation and Logistics. 664-669 (2009).

[13] Z. Chen, C. Shan, and H. Zhu, Adaptive fuzzy sliding mode control algorithm for a non-affine nonlinear system. IEEE Transactions on Industrial Informatics. **3**, 302-311 (2007).

[14] H. Hu, and P.Y. Woo, Fuzzy supervisory sliding-mode and neural-network control for robotic manipulators. IEEE Transactions on Industrial Electronics. **53**, 929-940 (2006).

[15] R.J. Wai, Fuzzy sliding-mode control using adaptive tuning technique. IEEE Transactions on Industrial Electronics. **54**, 586-594 (2007).

[16] H.H. Choi, LMI-based sliding surface design for integral sliding mode control of mismatched uncertain systems. IEEE Transactions on Automatic Control. **52**, 736-742 (2007).

[17] J. Zhang, P. Shi, and Y. Xia, Robust adaptive sliding-mode control for fuzzy systems with mismatched uncertainties. IEEE Transactions on Fuzzy Systems. **81**, 700-711 (2010).

[18] Y. He, S. Wang, M. Yan, and Y. Jing, Delay-dependent sliding mode control for Mismatched uncertain Markovian jump systems with time-delay. Chinese Control and Decision Conference. 1117-1122 (2010).

[19] Y. Niu, J. Lam, and X. Wang, Sliding-mode control for uncertain neutral delay systems. IEE Proceedings Control Theory and Applications. **151**, 38-44 (2004).

[20] J. Xiang, H. Su, J. Chu, and K. Zhang, LMI approach to robust delay dependent/independent sliding mode control of uncertain time-delay systems. IEEE International Conference on Systems, Man and Cybernetics. **3**, 2266-2271 (2003).

[21] H.P. Pang, and L.P. Wang, Global robust sliding mode control for a class of uncertain time-delay systems. International Conference on Machine Learning and Cybernetics. **2**, 910-915 (2010).

[22] Y. Xia, and Y. Jia, Robust sliding-mode control for uncertain time-delay systems: an LMI approach. IEEE Transactions on Automatic Control. **48**, 1086-1091 (2003).

[23] Y. Xia, M. Fu, H. Yang, and G.P. Liu, Robust sliding-mode control for uncertain time-delay systems based on delta operator. IEEE Transactions on Industrial Electronics. **56**, 3646-3655 (2009).

[24] K. Gu, V.L. Kharitonov, and J. Chen Stability of time-delay systems. Boston, Birkhauser, (2003).

[25] W.B. Gao, and J.C. Hung, Variable structure control of non-linear system: A new approach. IEEE Transaction on Industrial Electronics. **40**, 45-55 (1993).

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