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Dual to Separate Product Type Exponential Estimator in Sample Surveys

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Abstract: This paper addresses the problem of estimation of finite population mean in case of post-stratification. In this paper dual to separate product type exponential estimator is proposed using the same approach adopted by Srivenkataramana [15] and Bandhyopadhyaya [2]. Conditions under which the proposed estimator is more efficient than usual unbiased estimator, usual separate product type estimator, dual to separate product type estimator and separate product type exponential estimator are obtained. The bias and mean squared error expressions are obtained upto the first degree of approximation. An empirical study has been carried out to demonstrate the performance of the proposed estimator.

Keywords: Separate estimator; Post-stratification; Bias, Mean squared error

1 Introduction

Cochran [4] and Robson [11] envisaged classical ratio and product estimators which were studied in case of post stratification by Ige and Tripathi [6]. Tailor et al. [23] proposed dual to Ige and Trapthi [6] ratio and product estimators. Chouhan [3] studied the Bhal and Tuteja [1] product type exponential estimator in case of post-stratification. Srivenkataramana [15] and Bandhyopadhyaya [2] envisaged dual to classical ratio and product estimators using transformation on auxiliary variate. Tailor and Tailor [22] proposed dual to Bhal and Tuteja [1] product type exponential estimator. The problem of estimating the population parameters has been discussed by various researchers including Tailor and Lone [17,20,21], Lone and Tailor [9],Lone et al.[7] , Lone et al.[10],Singh et al. [12] and Singh et al.[13].At the estimation stage separate type estimators have been discussed by few researchers including Vishwakarma and Singh [24], Yadav et al. [25],Tailor and Lone [19], Chouhan [3],Tailor and Lone [18], Lone and Tailor [8]. Consider a finite population $U=(U_1,U_2,...,U_N)$ of size N . A sample of size n is drawn from population U using simple random sampling without replacement. After selecting the sample, it is observed that which units belong to h^{th} stratum. Let n_h be the size of the sample falling in h^{th} stratum such that $\sum_{h=1}^{L} n_h = n$. Here it is assumed that *n* is so large that possibility of n_h being zero is very small.

Let x_{hi} be the observation on i^{th} unit that fall in h^{th} stratum for auxiliary variate *x* and y_{hi} be the observation on i^{th} unit that fall in h^{th} stratum for study variate *y*, then

$\bar{X}_h = \frac{1}{N_h}\sum_{i=1}^{N_h} x_{hi}$: h^{th} stratum mean for auxiliary variate *x*,

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$\bar{Y}_h = \frac{1}{N_h}\sum_{i=1}^{N_h} y_{hi}$: h^{th} stratum mean for study variate y,

$\bar{X} = \frac{1}{N}\sum_{h=1}^{L}\sum_{i=1}^{N_h} x_{hi} = \sum_{h=1}^{L} W_h \bar{X}_h$: Population mean of auxiliary variate x,

$\bar{Y} = \frac{1}{N}\sum_{h=1}^{L}\sum_{i=1}^{N_h} y_{hi} = \frac{1}{N}\sum_{h=1}^{L} N_h \bar{Y}_h = \sum_{h=1}^{L} W_h \bar{Y}_h$: Population mean of the study variate y

In case of post-stratification, usual unbiased estimator of population mean $\bar{Y}$ is defined as

$$\bar{y}_{PS} = \sum_{h=1}^{L} W_h \bar{y}_h , \tag{1.1}$$

where

$W_h = \frac{N_h}{N}$ is the weight of the h^{th} stratum and $\bar{y}_h = \frac{1}{n_h}\sum_{i=1}^{n_h} y_{hi}$ is sample mean of n_h sample units that fall in the h^{th} stratum.

Using the results from Stephen [16] the variance of $\bar{y}_{PS}$ to the first degree of approximation is obtained as

$$Var(\bar{y}_{PS}) = \left(\frac{1}{n} - \frac{1}{N}\right)\sum_{h=1}^{L} W_h S_{yh}^2 + \frac{1}{n^2}\sum_{h=1}^{L}(1 - W_h) S_{yh}^2 , \tag{1.2}$$

where $S_{yh}^2 = \frac{1}{N_h - 1}\sum_{i=1}^{N_h}(y_{hi} - \bar{Y}_h)^2$.

When the correlation between the study variate y and the auxiliary variate x is negative, Ige and Tripathi [6] proposed a product type estimator in case of post-stratification as

$$\hat{\bar{Y}}_{PPS} = \bar{y}_{PS}\left(\frac{\bar{x}_{PS}}{\bar{X}}\right). \tag{1.3}$$

The separate version of Ige and Tripathi [6] product type estimator can be written as

$$\hat{\bar{Y}}_{PPS}^{S} = \sum_{h=1}^{L} W_h \bar{y}_h \left(\frac{\bar{x}_h}{\bar{X}_h}\right). \tag{1.4}$$

Upto the first degree of approximation, mean squared error of $\hat{\bar{Y}}_{PPS}^{S}$ is obtained as

$$MSE\left(\hat{\bar{Y}}_{PPS}^{S}\right) = \left(\frac{1}{n} - \frac{1}{N}\right)\left[\sum_{h=1}^{L} W_h S_{yh}^2 + \sum_{h=1}^{L} W_h R_h^2 S_{xh}^2 + 2\sum_{h=1}^{L} W_h R_h S_{yxh}\right] \tag{1.5}$$

Following Srivenkataramana [15] and Bandhyopadhyayh [2] transformation, we define dual to separate product type estimator in case of post-stratification as

$$\hat{\bar{Y}}_{PPS}^{*} = \sum_{h=1}^{L} W_h \bar{y}_h \left(\frac{\bar{X}_h}{\bar{x}_h^{*}}\right), \tag{1.6}$$

where $\bar{x}_h^* = \dfrac{N_h \bar{X}_h - n_h \bar{x}_h}{N_h - n_h}$.

To the first degree of approximation, bias and mean squared error of dual to separate product type estimator are obtained as

$$B(\hat{\bar{Y}}_{PPS}^{*}) = \left(\frac{1}{n} - \frac{1}{N}\right)\sum_{h=1}^{L} \bar{Y}_h \left(C_{xh}^2\, a_h^2 + a_h \rho_{xyh}\, C_{xh}\, C_{yh}\right), \tag{1.7}$$

and

$$MSE\left(\hat{\bar{Y}}_{PPS}^{*}\right) = \left(\frac{1}{n} - \frac{1}{N}\right)\left[\sum_{h=1}^{L} W_h S_{yh}^2 + \sum_{h=1}^{L} W_h R_h^2 a_h^2 S_{xh}^2 + 2\sum_{h=1}^{L} W_h a_h R_h S_{yxh}\right], \tag{1.8}$$

where $R_h = \dfrac{\bar{Y}_h}{\bar{X}_h}$ and $a_h = \dfrac{n_h}{N_h - n_h}$.

In case of negative correlation coefficient between the study variate y and the auxiliary variate x, Bahl and Tuteja [1] proposed product type exponential estimator for population mean as

$$\hat{\bar{Y}}_{1P} = \bar{y} \exp\left(\frac{\bar{x} - \bar{X}}{\bar{x} + \bar{X}}\right). \tag{1.9}$$

Motivated by Bahl and Tuteja [1], Singh et al. [14] proposed product type exponential estimator in stratified random sampling as

$$\hat{\bar{Y}}_{St}^{Pe} = \bar{y}_{st} \exp\left(\frac{\bar{x}_{st} - \bar{X}}{\bar{x}_{st} + \bar{X}}\right). \tag{1.10}$$

Chouhan [3] proposed product type exponential estimator in case of post-stratification as

$$\hat{\bar{Y}}_{PS}^{Pe} = \bar{y}_{PS} \exp\left(\frac{\bar{x}_{PS} - \bar{X}}{\bar{x}_{PS} + \bar{X}}\right). \tag{1.11}$$

Up to the first degree of approximation, the bias and mean squared error of estimator $\hat{\bar{Y}}_{PS}^{Pe}$ are obtained as

$$B\left(\hat{\bar{Y}}_{PS}^{Pe}\right) = \left(\frac{1}{n} - \frac{1}{N}\right)\sum_{h=1}^{L} \frac{1}{\bar{X}}\left(\frac{3}{8} R S_{xh}^2 - \frac{1}{2} S_{yxh}\right), \tag{1.12}$$

and

$$MSE\left(\hat{\bar{Y}}_{PS}^{Pe}\right)=\left(\frac{1}{n}-\frac{1}{N}\right)\sum_{h=1}^{L}W_h\left(S_{yh}^2+\frac{1}{4}R^2S_{xh}^2+RS_{yxh}\right) \quad . \tag{1.13}$$

where $R=\frac{\bar{Y}}{\bar{X}}$.

Here we define the separate version of Chouhan [3] product type exponential estimator as

$$\hat{\bar{Y}}_{PS}^{SPe}=\sum_{h=1}^{L}W_h\,\bar{y}_h\left(\frac{\bar{x}_h-\bar{X}_h}{\bar{x}_h+\bar{X}_h}\right). \tag{1.14}$$

Upto the first degree of approximation mean squared error of $\hat{\bar{Y}}_{PS}^{SPe}$ is obtained as

$$MSE\left(\hat{\bar{Y}}_{PS}^{SPe}\right)=\left(\frac{1}{n}-\frac{1}{N}\right)\sum_{h=1}^{L}W_h\left(S_{yh}^2+\frac{1}{4}R_h^2S_{xh}^2+R_hS_{yxh}\right) \quad . \tag{1.15}$$

where $S_{xh}^2=\frac{1}{N_h-1}\sum_{i=1}^{N_h}\left(x_{hi}-\bar{X}_h\right)^2$ and $S_{yxh}=\frac{1}{N_h-1}\sum_{i=1}^{N_h}\left(y_{hi}-\bar{Y}_h\right)\left(x_{hi}-\bar{X}_h\right).$

2. Proposed Estimator

Using transformation $x_i^*=\frac{N\bar{X}-nx_i}{N-n}$ on auxiliary variate x, Srivenkataramana [15] and Bandhyopadhyayh [2] defined dual to classical product estimator as

$$\hat{\bar{Y}}_{2P}^{*}=\bar{y}\left(\frac{\bar{X}}{\bar{x}^*}\right) \quad . \tag{2.1}$$

where $\bar{x}^*=\frac{N\bar{X}-n\bar{x}}{N-n}$ is unbiased estimator of population mean $\bar{X}$.

Motivated by Srivenkataramana [15] and Bondyopadhyayh [2], Tailor and Tailor [22] proposed dual to Bahl and Tuteja [1] product type exponential estimator as

$$\hat{\bar{Y}}_{3P}=\bar{y}\left(\frac{\bar{X}-\bar{x}^*}{\bar{X}+\bar{x}^*}\right). \tag{2.2}$$

By using the same transformation adopted by Srivenkataramana [15] and Bondyopadhyayh [2], we propose dual to separate product type exponential estimator for population mean $\bar{Y}$ in case of post-stratification as

$$\hat{\bar{Y}}_{PS}^{*Pe}=\sum_{h=1}^{L}W_h\bar{y}_h\exp\left(\frac{\bar{X}_h-\bar{x}_h^*}{\bar{X}_h+\bar{x}_h^*}\right). \tag{2.3}$$

To obtain the bias and mean squared error of suggested estimator $\hat{\bar{Y}}_{PS}^{*Pe}$ we write

$\bar{y}_h = \bar{Y}_h(1+e_{0h})$, $\bar{x}_h = \bar{X}_h(1+e_{1h})$ such that

$E(e_{0h}) = E(e_{1h}) = 0$,

$$E(e_{0h}^2) = \left(\frac{1}{nW_h} - \frac{1}{N_h}\right)C_{yh}^2,$$

$$E(e_{1h}^2) = \left(\frac{1}{nW_h} - \frac{1}{N_h}\right)C_{xh}^2,$$

$$E(e_{0h}e_{1h}) = \left(\frac{1}{nW_h} - \frac{1}{N_h}\right)\rho_{yxh}C_{yh}C_{xh},$$

Expressing (2.3) in terms of e_{ih} 's, we have

$$\hat{\bar{Y}}_{PS}^{*Pe} = \sum_{h=1}^{L} W_h\bar{Y}_h\,(1+e_{0h})\,\exp\left(\frac{a_h\bar{X}_h e_{1h}}{2\bar{X}_h - a_h\bar{X}_h e_{1h}}\right)$$

$$\hat{\bar{Y}}_{PS}^{*Pe} = \sum_{h=1}^{L} W_h\bar{Y}_h\,(1+e_{0h})\,\exp\left\{\frac{a_h e_{1h}}{2}\left(1-\frac{a_h e_{1h}}{2}\right)^{-1}\right\}$$

$$\hat{\bar{Y}}_{PS}^{*Pe} = \sum_{h=1}^{L} W_h\bar{Y}_h\,(1+e_{0h})\left[1+\frac{1}{2}a_h e_{1h}\left(1-\frac{a_h e_{1h}}{2}\right)^{-1} + \frac{1}{2}\left\{\frac{a_h^2 e_{1h}^2}{4}\left(1-\frac{a_h e_{1h}}{2}\right)^{-2}\right\} + \ldots.\right]$$

$$\hat{\bar{Y}}_{PS}^{*Pe} = \sum_{h=1}^{L} W_h\bar{Y}_h(1+e_{0h})\left[1+\frac{a_h e_{1h}}{2} + \frac{a_h^2 e_{1h}^2}{8} + \ldots\right]$$

$$\left(\hat{\bar{Y}}_{PS}^{*Pe} - \bar{Y}\right) = \sum_{h=1}^{L} W_h\bar{Y}_h\left[e_{0h} + \frac{a_h e_{1h}}{2} + \frac{3a_h^2 e_{1h}^2}{8} + \frac{a_h e_{0h} e_{1h}}{2}\right] \tag{2.4}$$

Taking expectation both sides to (2.4), we get the bias of the proposed estimator $\hat{\bar{Y}}_{PS}^{*Pe}$ upto the first degree of approximation is obtained as

$$B(\hat{\bar{Y}}_{PS}^{*Pe}) = \left(\frac{1}{n} - \frac{1}{N}\right)\sum_{h=1}^{L}\frac{a_h}{4\bar{X}_h}\left[\frac{3}{2}R_h\, a_h S_{xh}^2 + 2S_{yxh}\right]. \tag{2.5}$$

Squaring both sides of (2.4) and then taking expectation, we get the mean squared error of the proposed estimator $\hat{\bar{Y}}_{PS}^{*Pe}$ upto the first degree of approximation as

$$MSE\left(\hat{\bar{Y}}_{PS}^{*Pe}\right) = \left(\frac{1}{n} - \frac{1}{N}\right)\left[\sum_{h=1}^{L} W_h S_{yh}^2 + \frac{1}{4}\sum_{h=1}^{L} W_h a_h^2 R_h^2 S_{xh}^2 + \sum_{h=1}^{L} W_h a_h R_h S_{yxh}\right]. \tag{2.6}$$

3. Efficiency Comparisons of the Proposed Estimator $\hat{\bar{Y}}_{PS}^{*Pe}$ with $\bar{y}_{PS}$, $\hat{\bar{Y}}_{PPS}^{S}$, $\hat{\bar{Y}}_{PPS}^{*}$ and $\hat{\bar{Y}}_{PS}^{SPe}$

Comparing (1.2) and (2.6), it is observed that the proposed estimator $\hat{\bar{Y}}_{PS}^{*Pe}$ would be more efficient than the usual unbiased estimator $\bar{y}_{PS}$ i.e.

$MSE\left(\hat{\bar{Y}}_{PS}^{*Pe}\right) < V\left(\bar{y}_{PS}\right)$ if

$$\sum_{h=1}^{L} W_h a_h^2 R_h^2 S_{xh}^2 + 4\sum_{h=1}^{L} W_h a_h R_h S_{yxh} < 0 \tag{3.1}$$

From (1.5) and (2.6), it is concluded that the proposed estimator $\hat{\bar{Y}}_{PS}^{*Pe}$ would be more efficient than $\hat{\bar{Y}}_{PPS}^{S}$ i.e.

$MSE\left(\hat{\bar{Y}}_{PS}^{*Pe}\right) < MSE\left(\hat{\bar{Y}}_{PPS}^{S}\right)$ if

$$\sum_{h=1}^{L} W_h R_h^2 S_{xh}^2 \left(a_h^2 - 4\right) + 4\sum_{h=1}^{L} W_h R_h S_{yxh} \left(a_h - 2\right) < 0 \tag{3.2}$$

From (1.8) and (2.6), it is observed that the proposed estimator $\hat{\bar{Y}}_{PS}^{*Pe}$ would be more efficient than $\hat{\bar{Y}}_{PPS}^{*}$ i.e.

$MSE\left(\hat{\bar{Y}}_{PS}^{*Pe}\right) < MSE\left(\hat{\bar{Y}}_{PPS}^{*}\right)$ if

$$-3\sum_{h=1}^{L} W_h a_h^2 R_h^2 S_{xh}^2 - 4\sum_{h=1}^{L} W_h a_h R_h S_{yxh} < 0 . \tag{3.3}$$

From expression (1.15) and (2.6), it is concluded that the proposed estimator $\hat{\bar{Y}}_{PS}^{*Pe}$ would be more efficient than $\hat{\bar{Y}}_{PS}^{SPe}$ i.e.

$MSE\left(\hat{\bar{Y}}_{PS}^{*Pe}\right) < MSE\left(\hat{\bar{Y}}_{PS}^{SPe}\right)$ if

$$\sum_{h=1}^{L} W_h R_h^2 S_{xh}^2 \left(a_h^2 - 1\right) + 4\sum_{h=1}^{L} W_h R_h S_{yxh} \left(a_h - 1\right) < 0 \tag{3.4}$$

4. Empirical Study

To exhibit the performance of the proposed estimator in comparison to other considered estimators, a population data set is being considered. In this data set x :Total annual sunshine hours and y :Snowy days. The description of population is given below:

Table 4.1 - Population- I **[**Source: [5]**]**

Constants	n_h	N_h	$\bar{X}_h$	$\bar{Y}_h$	$S_{x_1}^2$	$S_{y_1}^2$	ρ_{yxh}	S_{yxh}
Stratum I	4	10	1629.99	142.8	10438.71	37.31	-0.38	-239.25
Stratum II	4	10	2035.96	91.0	10662.63	43.16	-0.35	-240.45

Table 4.2 Percent relative Efficiency of $\bar{y}_{PS}$, $\hat{\bar{Y}}_{PPS}^{S}$, $\hat{\bar{Y}}_{PPS}^{*}$, $\hat{\bar{Y}}_{PS}^{SPe}$ and $\hat{\bar{Y}}_{PS}^{*Pe}$ with respect to $\bar{y}_{PS}$

Estimators	$\bar{y}_{PS}$	$\hat{\bar{Y}}_{PPS}^{S}$	$\hat{\bar{Y}}_{PPS}^{*}$	$\hat{\bar{Y}}_{PS}^{SPe}$	$\hat{\bar{Y}}_{PS}^{*Pe}$
PRE	**100.00**	**67.89**	**95.17**	**108.58**	**113.99**

5. Conclusion

Table 4.2 exhibits that there is a significant gain in efficiency by using the proposed estimator $\hat{\bar{Y}}_{PS}^{*Pe}$ over usual unbiased estimator $\bar{y}_{PS}$,usual separate product type estimator $\hat{\bar{Y}}_{PPS}^{S}$, dual to separate product type estimator $\hat{\bar{Y}}_{PPS}^{*}$ and separate product type exponential estimator $\hat{\bar{Y}}_{PS}^{SPe}$. Section 3 provides the conditions under which the proposed estimator has less mean squared error in comparison to other considered estimators. Thus the proposed estimator $\hat{\bar{Y}}_{PS}^{*Pe}$ is recommended for use in practice if the conditions obtained in section 3 are satisfied.

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