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H_∞ Fuzzy Control for a Class of Networked Control System

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Abstract: This paper presents the H_∞ fuzzy controller design for a class of networked control system with “network transmission delay” and “data packet dropout.” First, we apply a Takagi-Sugeno fuzzy model (T-S fuzzy model) to approach the input-output relation of the networked control system (NCS). Based on the T-S fuzzy model, we propose a H_∞ fuzzy controller design to inhibit the effect of “network transmission delay” and “data packet dropout,” and to guarantee the H_∞ control performance of the overall system. Based on Lyapunov stability theorem, with the chosen Lyapunov-Krasovskii function, we analyze the stability and the robustness of the overall system, and obtain the Lyapunov stability criterion. After that, by using the Schur complements, the Lyapunov stability criterion would be presented in the linear matrix inequality format to solve the control problem more efficiently. Finally, the computer simulations are given to demonstrate the validity and the performance of the proposed control strategy.

Keywords: H_∞ control theory, fuzzy control, networked control system

1 Introduction

Nowadays, increasing attention has been paid to design of networked control system (NCS) [1,2,3,4,5], which is a spatially distributed system for which the connection between sensors, actuators, and controllers is supported by a share communication network. In NCS, data is transmitted in atomic unit called data packets. It means that the system states and the output signals are sampled by the sensors and transmitted to the controllers by the data packets, the controller signals are transmitted to the actuators by the data packets, and the actuators would control the controlled plants according to the receiving data packets.

Due to the introduction of the share communication network, two major challenges in NCSs still to be fully addressed are the effects of both “network transmission delay” and “data packet dropout” on the system performance [1,2,3,4,5]. To treat the mentioned effects, several methods have been proposed. Seiler, Sengupta, Shi and Yub adopted H_∞ control theory to stabilize NSC and analyzed the system robustness [2,5]. Huang and Nguang applied state feedback control method to treat the control problem of NSC [3]. Peng and Yang used T-S fuzzy control technique to guarantee the stability of NSC [4].

Over the past decades, H_∞ fuzzy control [6,7,8,9] has been applied as an effective approach for nonlinear system. For a given nonlinear system, the objective of H_∞ fuzzy control is to design a controller so that the gain (the induced L_2-norm) from the unmodeled dynamics, the external disturbances, etc., to the system states must be equal or less than a prescribed attenuation level. Generally, the H_∞ fuzzy control design problem can be characterized in terms of a linear matrix inequality (LMI) problem or an eigenvalue problem (EVP) [10,11,12,13, 14].

In this paper, we proposed the H_∞ fuzzy controller design for a class of the nonlinear networked control systems. First, “network transmission delay” and “data packet dropout” of the NCS would be analyzed. Then, Takagi-Sugeno fuzzy model (T-S fuzzy model, [4,6,7,8, 9]) would be applied to describe the input-output relation of NSC. Based on the T-S fuzzy model, the H_∞ fuzzy controller is proposed to inhibit the effect of “network transmission delay” and “data packet dropout” to guarantee the H_∞ control performance of the overall system. Choosing the suitable Lyapunov-Krasovskii

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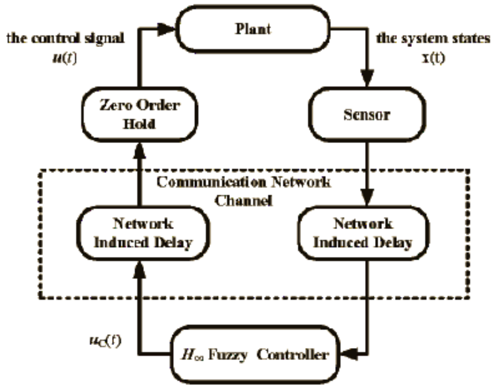

Fig. 1: The framework of the H_∞ fuzzy controller for a class of the networked control system

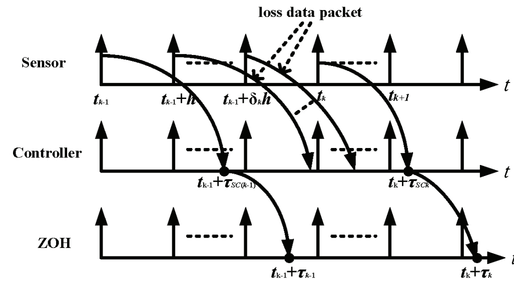

Fig. 2: NSC timing diagram with "network transmission delay" and "data packet dropout"

function, based on using Lyapunov stability theorem, we analyze the stability and the robustness of the overall system. Therefore, we can obtain the Lyapunov stability criterion. Next, the Lyapunov stability criterion would be transformed into a certain form of LMI characterized in terms of LMI problem or an eigenvalue problem. After that, by using the convex optimization technique, the control problem would be solved more efficiently.

This paper is organized as follows: In section 2, we will describe the problem formulation. Section 3 will present the proposed control strategy, and analyze the stability and robustness of the overall system. In section 4, computer simulation will be given to illustrate the control performance of the proposed control strategy. Section 5 will conclude this paper.

2 Problem Formulation

This paper considers a class of the networked control system, where the framework shown in **Fig.** 1 [1,2,3,4,5]. This system is composed of the following five parts: a sensor, a controller, a zero-order-holder (ZOH), applied as an actuator, a controller, a communication network channel, and a controlled plant.

The controlled plant can be described as the T-S fuzzy model [4,6,7,8,9], in which the lth fuzzy rule is formulated in the following form:

Plant Rule:

IF q_1 is $F^l_{q_1}$ and ... and $F^l_{q_k}$,

$$\text{THEN } \dot{\mathbf{x}}(t) = \mathbf{A}_l\mathbf{x}(t) + \mathbf{B}_l\mathbf{u}(t) + \boldsymbol{\omega}(t) \tag{1}$$

where l=1,2,...,L, L is the number of the fuzzy rules, $\mathbf{x}(t) = [x(t), \dot{x}(t), \ldots, x^{(n-1)}(t)] \in \Re^n$ is the state vector, $\mathbf{u}(t) \in \Re^m$, $\boldsymbol{\omega}(t) \in \Re^n$ is the external disturbance, q_h $(h = 1,2,\ldots,k)$ are the premise variables, $F^l_{q_h}(h = 1,2,\ldots,k;$ $l = 1,2,\ldots,L)$ are the fuzzy sets associated with q_h $(h = 1,2,\ldots,k)$, and $\mathbf{A}_l \in \Re^{(n\times n)}$ and $\mathbf{B}_l \in \Re^{(n\times m)}$ are known constant matrices. The initial condition of controlled plant is given as $\mathbf{x}(t_0) = \mathbf{x}_0$. Without loss of generality, we make the following assumption:

Assumption 1 [6,12]: The external disturbance $\boldsymbol{\omega}(t)$ is bounded. That is, $\boldsymbol{\omega}(t) \in L_2[0,t_f]$, $\forall t_f \in [0,\infty)$.

Since the controller is connected with the sensor or ZOH via a share communication network channel, it will be caused the network transmission delay and the data packet dropout (shown in **Fig.** 2). Therefore, there are essentially the following time delays:

(1) τ_{sck}: at every sampling instant t_k, the time delays between the sensor and the controller.
(2) τ_{czk}: at every sampling instant t_k, time delays between the controller and ZOH.

Define the network transmission delay as $\tau_k = \tau_{sck} + \tau_{czk}$ at every sampling instant t_k and make the following assumption:

Assumption 2 [18]: The network transmission delay τ_k is bounded as $\tau_{min} \le \tau_k \le \tau_{max}$, where τ_{min} is lower bound of τ_k and τ_{max} is upper bound of τ_k.

Also, to treat the data packet dropout, we define the following data packet dropout number:

(1) η_{sck}: at every sampling instant t_k, the data packet dropout number between the sensor and the controller.
(2) η_{czk}: at every sampling instant t_k, the data packet dropout number between the controller and ZOH.

Define the network transmission delay as $\eta_k = \eta_{sck} + \eta_{czk}$ at every sampling instant t_k and make the following assumption:

Assumption 3 [18]: The network transmission delay η_k is bounded as $\eta_{min} \le \eta_k \le \eta_{max}$, where η_{min} is lower bound of η_k and η_{max} is upper bound of η_k.

Let h is the sampling period. We have

$$t_{k+1} - t_k = (\eta_k + 1)h + \tau_{k+1} - \tau_k \tag{2}$$

The objective of this paper is to design a controller such that the effect of $\boldsymbol{\omega}(t)$ on the system state vector $\mathbf{x}(t)$ would be attenuated below a prescribed attenuation level ρ $(0 < \rho < 1)$. Thus, the following H_∞ control performance related to the system state vector $\mathbf{x}(t)$ is requested:

$$\int_{t_0}^{t_f} \mathbf{x}^T(t)\mathbf{R}\mathbf{x}(t)dt \le V(t_0) + \rho \int_{t_0}^{t_f} \boldsymbol{\omega}^T(t)\boldsymbol{\omega}(t)dt \quad (3)$$

for all $\boldsymbol{\omega}(t) \in L_2[0,t_f]\,\forall t_f \in [0,\infty)$, where $\mathbf{R}$ is a symmetric positive definite weighting matrices and $V(t_0)$ is positive.

It was seen that if the system starts with initial conditions, $V(t_0) = 0$. The H_∞ control performance [6,7,8,9] can be expressed as

$$\int_{t_0}^{t_f} \mathbf{x}^T(t)\mathbf{R}\mathbf{x}(t)dt \le \rho \int_{t_0}^{t_f} \boldsymbol{\omega}^T(t)\boldsymbol{\omega}(t)dt$$

or equivalently

$$\frac{\|\mathbf{x}(t)\|_{\mathbf{R}}}{\|\boldsymbol{\omega}(t)\|} \le \rho$$

where $\|\mathbf{x}(t)\|_{\mathbf{R}} = \left(\int_{t_0}^{t_f} \mathbf{x}^T(t)\mathbf{R}\mathbf{x}(t)dt\right)^{1/2}$. It means that the L_2-gain from $\boldsymbol{\omega}(t)$ to $\mathbf{x}$(t) must be equal or less than a prescribed attenuation level ρ.

3 H_∞ Fuzzy Controller Design

To achieve the objective, we design the H_∞ fuzzy controller based on T-S fuzzy model. Here, the lth fuzzy rule of the H_∞ fuzzy controller is formulated in the following form:

Control Rule:

IF q_1 is $F^l_{q_1}$and ... and $F^l_{q_k}$,

THEN $\mathbf{u}_C(t) = \mathbf{K}_l\mathbf{x}(t - \tau_{sck} - (\eta_{sck}+1)h) \quad (4)$

where l=1,2,...,L, L is the number of the fuzzy rules, $\mathbf{u}_C(t)$ is the output of the H_∞ fuzzy controller, $\mathbf{K}_l \in \Re^{(n\times n)}$ is the controller gain matrix.

By using the singleton fuzzifier, the product inference and the center average defuzzifier [14], the output of the T-S fuzzy model (1) and the H_∞ fuzzy controller (4) can be respectively obtained by

$$\dot{\mathbf{x}}(t) = \sum_{l=1}^{L} \zeta^l(\mathbf{A}_l\mathbf{x}(t) + \mathbf{B}_l)\mathbf{u}(t) + \boldsymbol{\omega}(t)) \quad (5)$$

$$\mathbf{u}_C(t) = \sum_{l=1}^{L} \zeta^l(\mathbf{K}_l\mathbf{x}(t - \tau_{sck} - (\eta_{sck}+1)h)) \quad (6)$$

where

$$\zeta^l = \prod_{i=1}^{L}\mu_{F^l_{q_i}}(q_i)/\sum_{l=1}^{L}\prod_{i=1}^{L}\mu_{F^l_{q_i}}(q_i) \quad l = 1,2,...,L.$$

Thus, the output of ZOH can be expressed as

$$\mathbf{u}(t_k) = \sum_{l=1}^{L} \zeta^l(\mathbf{K}_l\mathbf{x}(t_k - \tau_k - (\eta_k+1)h)) \quad (7)$$

for $t_k \le t \le t_{k+1}$. Substituting (7) into (5) yields

$$\dot{\mathbf{x}}(t) = \sum_{l=1}^{L}\sum_{i=1}^{L} \zeta^l(\mathbf{A}_l\mathbf{x}(t) + \mathbf{B}_l\mathbf{K}_i\mathbf{x}(t_k - \tau_k - (\eta_k+1)h)) + \boldsymbol{\omega}(t) \quad (8)$$

for $t_k \le t \le t_{k+1}$. Since

$$t_{k+1} - t_k - (\eta_k+1)h = t - \tau_{min} - \tau(t) \quad (9)$$

where

$$\tau(t) = t - t_k - \tau_{min} + \tau_k + (\eta_k+1)h$$

in which $t_k \le t \le t_{k+1}$. By (2), we can obtain

$$0 \le \tau(t) \le \kappa \quad (10)$$

where

$$\kappa = \tau_{max} - \tau_{min} + 2(\eta_{max}+1)h \quad (11)$$

Substituting (11) into (8) yields, for $t_k \le t \le t_{k+1}$,

$$\dot{\mathbf{x}}(t) = \sum_{l=1}^{L}\sum_{i=1}^{L} \zeta^l(\mathbf{A}_l\mathbf{x}(t) + \mathbf{B}_l\mathbf{K}_i\mathbf{x}(t - \tau_{min} - \tau(t))) + \boldsymbol{\omega}(t) \quad (12)$$

Consequently, we have the following main result: **Theorem.** Consider a class of networked control system with the controlled plant which is described by the T-S fuzzy model as (1). Suppose that **Assumptions 1-3** are satisfied, $\mathbf{R}$ is a given symmetric positive definite weighting matrix, and $0 < \rho < 1$ is the design constant serving as an attenuation level. Let the symmetric positive definite matrices $\mathbf{P}$, $\mathbf{Q}$, $\mathbf{M}_1$ and $\mathbf{M}_2$ and a positive constant ρ be the solution of the following quadratic matrix inequalities:

$$\Pi_{1l} - \mathbf{R} < 0 \quad (13a)$$
$$\Pi_{2l} < 0 \quad (13b)$$
$$\lambda_{max}(\Pi_{3)} < \rho \quad (13c)$$

where

$$\Pi_{1l} = \mathbf{A}_l^T\mathbf{P} + \mathbf{P}\mathbf{A}_l + (\sigma^{-1}\mathbf{I} + \mathbf{I})\mathbf{P}\mathbf{P} + \tau_{min}(\mathbf{I} + \mathbf{M}_1 + \mathbf{M}_1\mathbf{M}_1) + \kappa(\mathbf{I} + \mathbf{M}_2 + \mathbf{M}_2\mathbf{M}_2) + \mathbf{Q}$$

$$\Pi_{2l} = \mathbf{K}_i^T\mathbf{B}_l^T(\mathbf{I} + 2\tau_{min}\mathbf{I} + 2\kappa\mathbf{I} + \tau_{min}\mathrm{M}_1 + \kappa\mathbf{M}_2)\mathbf{B}_l\mathbf{K}_i - \mathbf{Q}$$

$$\Pi_3 = \rho + \tau_{min}(\mathbf{M}_1 + 2\mathbf{M}_1\mathbf{M}_1) + \kappa(\mathbf{M}_2 + 2\mathbf{M}_2\mathbf{M}_2)$$

where $l = 1,2,...,L, i = 1,2,...,L$, and $\lambda_{max}(\Pi_3)$ denotes the maximum eigenvalue of Π_3. Therefore, if the H_∞ fuzzy controller is given as (4), the H_∞ control performance given as (3) for the overall system can be guaranteed.

Proof. Consider the Lyapunov-Krasovskii function candidate [16,17] as

$$V(t) = V_1(t) + V_2(t) + V_3(t) \quad (14)$$

for $t_k \le t \le t_{k+1}$, where

$$V_1(t) = \mathbf{x}^T(t)\mathbf{P}\mathbf{x}(t)$$

$$V_2(t) = \int_{t-\tau_{min}-\tau(t)}^{t} \mathbf{x}^T(\phi)\mathbf{P}\mathbf{x}(\phi)d\phi$$

$$V_3(t) = \int_{-\tau_{min}}^{0}\int_{t-\tau_{min}-\tau(t)}^{t} \dot{\mathbf{x}}^T(\phi)\mathbf{M}_1\dot{\mathbf{x}}(\phi)d\phi d\varphi + \int_{-\tau_{min}-\kappa}^{-\tau_{min}}\int_{t-\varphi}^{t} \dot{\mathbf{x}}^T(\phi)\mathbf{M}_2\dot{\mathbf{x}}(\phi)d\phi d\varphi$$

Then, for $t_k \le t \le t_{k+1}$, the time derivative of $V_1(t)$ can be obtained as

$$\begin{aligned}\dot{V}_1(t) &= \dot{\mathbf{x}}^T(t)\mathbf{P}\mathbf{x}(t) + \mathbf{x}^T(t)\mathbf{P}\dot{\mathbf{x}}(t)\\ &= \sum_{l=1}^{L}\sum_{i=1}^{L}\zeta^l\zeta^i(\mathbf{x}^T(t)\mathbf{A}_l^T\mathbf{P}\mathbf{x}(t)\\ &\quad +\mathbf{x}^T(t)\mathbf{P}\mathbf{A}_l\mathbf{x}(t)\\ &\quad +\mathbf{x}^T(t-\tau_{min}-\tau(t))\mathbf{K}_i^T\mathbf{B}_l^T\mathbf{P}\mathbf{x}(t)\\ &\quad +\mathbf{x}^T(t)\mathbf{P}\mathbf{B}_l\mathbf{K}_i\mathbf{x}(t-\tau_{min}-\tau(t))\\ &\quad +\boldsymbol{\omega}^T(t)\mathbf{P}\mathbf{x}(t)+\mathbf{x}^T(t)\mathbf{P}\boldsymbol{\omega}(t))\\ &\le \sum_{l=1}^{L}\sum_{i=1}^{L}\zeta^l\zeta^i(\mathbf{x}^T(t)(\mathbf{A}_l^T\mathbf{P}+\mathbf{P}\mathbf{A}_l+(\rho^{-1}\mathbf{I}+\mathbf{I})\mathbf{P}\mathbf{P})\mathbf{x}(t)\\ &\quad +\mathbf{x}^T(t-\tau_{min}-\tau(t))\mathbf{K}_i^T\mathbf{B}_l^T\mathbf{B}_l\mathbf{K}_i\mathbf{x}(t-\tau_{min}-\tau(t))\\ &\quad +\sigma\boldsymbol{\omega}^T(t)\boldsymbol{\omega}(t))\end{aligned} \tag{15}$$

By using Newton-Leibniz formula [18], we obtain

$$\dot{V}_2(t) = \mathbf{x}^T(t)\mathbf{Q}\mathbf{x}(t) - \mathbf{x}^T(t-\tau_{min}-\tau(t))\mathbf{Q}\mathbf{x}(t-\tau_{min}-\tau(t)) \tag{16}$$

for $t_k \le t \le t_{k+1}$. Moreover, the time derivative of $V_3(t)$can be obtained as

$$\begin{aligned}\dot{V}_3(t) &= \tau_{min}\dot{\mathbf{x}}^T(t)\mathbf{M}_1\dot{\mathbf{x}}(t) - \int_{t-\tau_{min}}^{t}\dot{\mathbf{x}}^T(\phi)\mathbf{M}_1\dot{\mathbf{x}}(\phi)d\phi\\ &\quad +\kappa\dot{\mathbf{x}}^T(t)\mathbf{M}_2\dot{\mathbf{x}}(t) - \int_{t-\tau_{min}-\kappa}^{t}\dot{\mathbf{x}}^T(\phi)\mathbf{M}_2\dot{\mathbf{x}}(\phi)d\phi\\ &\le \tau_{min}\dot{\mathbf{x}}^T(t)\mathbf{M}_1\dot{\mathbf{x}}(t)+\kappa\dot{\mathbf{x}}^T(t)\mathbf{M}_2\dot{\mathbf{x}}(t)\end{aligned} \tag{17}$$

for $t_k \le t \le t_{k+1}$. From (12), we have

$$\begin{aligned}\dot{\mathbf{x}}^T(t)\mathbf{M}_1\dot{\mathbf{x}}(t) &= \sum_{l=1}^{L}\sum_{i=1}^{L}\zeta^l\zeta^i(\mathbf{x}^T(t)\mathbf{A}_l^T+\mathbf{x}^T(t-\tau_{min}\\ &\quad -\tau(t))\mathbf{K}_i^T\mathbf{B}_l^T+\boldsymbol{\omega}^T(t))\mathbf{M}_1(\mathbf{A}_l\mathbf{x}(t)\\ &\quad +\mathbf{B}_l\mathbf{K}_i\mathbf{x}(t-\tau_{min}-\tau(t))+\boldsymbol{\omega}(t))\\ &\le \sum_{l=1}^{L}\sum_{i=1}^{L}\zeta^l\zeta^i(\mathbf{x}^T(t)\mathbf{A}_l^T(\mathbf{I}+\mathbf{M}_1+\mathbf{M}_1\mathbf{M}_1)\mathbf{A}_l\mathbf{x}(t)\\ &\quad +\mathbf{x}^T(t-\tau_{min}-\tau(t))\mathbf{K}_i^T\mathbf{B}_l^T(\mathbf{I}\\ &\quad +2\mathbf{M}_1)\mathbf{B}_l\mathbf{K}_i\mathbf{x}(t-\tau_{min}-\tau(t))\\ &\quad +\boldsymbol{\omega}^T(t)(\mathbf{M}_1+2\mathbf{M}_1\mathbf{M}_1)\boldsymbol{\omega}(t))\end{aligned}$$

and

$$\begin{aligned}\dot{\mathbf{x}}^T(t)\mathbf{M}_2\dot{\mathbf{x}}(t) &\le \sum_{l=1}^{L}\sum_{i=1}^{L}\zeta^l\zeta^i(\mathbf{x}^T(t)\mathbf{A}_l^T(\mathbf{I}\\ &\quad +\mathbf{M}_2+\mathbf{M}_2\mathbf{M}_2)\mathbf{A}_l\mathbf{x}(t)\\ &\quad +\mathbf{x}^T(t-\tau_{min}-\tau(t))\mathbf{K}_i^T\mathbf{B}_l^T(\mathbf{I}\\ &\quad +2\mathbf{M}_2)\mathbf{B}_l\mathbf{K}_i\mathbf{x}(t-\tau_{min}-\tau(t))\\ &\quad +\boldsymbol{\omega}^T(t)(\mathbf{M}_2+2\mathbf{M}_2\mathbf{M}_2)\boldsymbol{\omega}(t)\end{aligned}$$

Therefore, for $t_k \le t \le t_{k+1}$, we have

$$\begin{aligned}\dot{V}_3(t) &\le \sum_{l=1}^{L}\sum_{i=1}^{L}\zeta^l\zeta^i(\mathbf{x}^T(t)\mathbf{A}_l^T(\tau_{min}(\mathbf{I}+\mathbf{M}_1+\mathbf{M}_1\mathbf{M}_1)\\ &\quad +\kappa(\mathbf{I}+\mathbf{M}_2+\mathbf{M}_2\mathbf{M}_2))\mathbf{A}_l\mathbf{x}(t)\\ &\quad +\mathbf{x}^T(t-\tau_{min}-\tau(t))\mathbf{K}_i^T\mathbf{B}_l^T(\tau_{min}\mathbf{I}+\kappa\mathbf{I}+2\tau_{min}\mathbf{M}_1\\ &\quad +2\kappa\mathbf{M}_2)\mathbf{B}_l\mathbf{K}_i\mathbf{x}(t-\tau_{min}-\tau(t))\\ &\quad +\boldsymbol{\omega}^T(t)(\tau_{min}\mathbf{M}_1+\tau_{min}\mathbf{M}_1\mathbf{M}_1\\ &\quad +\kappa\mathbf{M}_2+\kappa\mathbf{M}_2\mathbf{M}_2)\boldsymbol{\omega}(t))\end{aligned} \tag{18}$$

Thus, by (14), (16) and (18), we obtain

$$\begin{aligned}\dot{V}(t) &\le \sum_{l=1}^{L}\sum_{i=1}^{L}\zeta^l\zeta^i(\mathbf{x}^T(t)\Pi_{1l}\mathbf{x}(t)\\ &\quad +\mathbf{x}^T(t-\tau_{min}-\tau(t))\Pi_{2il}\mathbf{x}(t-\tau_{min}-\tau(t))\\ &\quad +\boldsymbol{\omega}^T(t)\Pi_3\boldsymbol{\omega}(t))\end{aligned} \tag{19}$$

for $t_k \le t \le t_{k+1}$. With (13a), (13b) and (13c), (19) can be written as

$$\dot{V}(t) \le \mathbf{x}^T(t)\mathbf{R}\mathbf{x}(t)+\rho\boldsymbol{\omega}^T(t)\Pi_3\boldsymbol{\omega}(t) \tag{20}$$

Integrating both sides of (20) yields

$$V(t_f) - V(t_0) \le -\int_{t_0}^{t_f} \mathbf{x}^T(t)\mathbf{R}\mathbf{x}(t)dt + \rho \int_{t_0}^{t_f} \boldsymbol{\omega}^T(t)\boldsymbol{\omega}(t)dt)$$

Thus,

$$\int_{t_0}^{t_f} \mathbf{x}^T(t)\mathbf{R}\mathbf{x}(t)dt \le V(t_0) + +\rho \int_{t_0}^{t_f} \boldsymbol{\omega}^T(t)\boldsymbol{\omega}(t)dt)$$

So, the H_∞ control performance in (3) is achieved. This completes the proof. □

Remark: Consider NCS with $\omega(t) \equiv \mathbf{0}$ If there exists the symmetric positive definite matrices $\mathbf{P}$, $\mathbf{Q}$, $\mathbf{M}_1$ and $\mathbf{M}_2$, and a positive constant ρ such that

$$\Pi_{1l} = \Pi_{1l}^T < 0 \qquad (21a)$$

$$\Pi_{2il} = \Pi_{2ij}^T < 0 \qquad (21b)$$

for $i = 1,2,...,L,\ j = 1,2,...,L$. The overall system would be asymptotical stable. □

Next, in order to solve the control problem more efficiently, the objective can be formulated as the following LMI problem

$$\begin{aligned} &\min_{\mathbf{P},\mathbf{Q},\mathbf{M}_1,\mathbf{M}_2,\sigma} \rho \\ &Subject\ to\ \Pi_{1l} - \mathbf{R} < 0 \\ &\qquad \Pi_{2il} < 0 \\ &\qquad \lambda_{max}(\Pi_{3)} < \rho \\ &\qquad \text{for } i = 1,2,...,L,\ j = 1,2,...,L \end{aligned} \qquad (22)$$

so that the attenuation level ρ in the H_∞ control performance given by (3) is reduced as small as possible. After that, by the Schur complements [12,13], the minimization problem of (22) can be formulated in terms of the following eigenvalue problem (EVP):

$$\begin{aligned} &\min_{\mathbf{P},\mathbf{Q},\mathbf{M}_1,\mathbf{M}_2,\sigma} \rho \\ &Subject\ to\ \Lambda_{1l} < 0 \\ &\qquad \Lambda_{2il} < 0 \\ &\qquad \lambda_{max}(\Pi_{3)} < \rho \\ &\qquad \text{for } i = 1,2,...,L,\ j = 1,2,...,L \end{aligned} \qquad (23)$$

where

$$\Lambda_{1l} = \begin{bmatrix} \begin{matrix}\mathbf{A}_l^T\mathbf{P} + \mathbf{P}\mathbf{A}_l + \mathbf{Q} \\ +\tau_{min}(\mathbf{I}+\mathbf{M}_1) \\ +\kappa(\mathbf{I}+\mathbf{M}_2)\end{matrix} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{P} \\ \mathbf{0} & -\sigma & \mathbf{0} & \mathbf{0} & \mathbf{P} \\ \mathbf{0} & \mathbf{0} & \mathbf{I} & \mathbf{0} & \mathbf{M}_1 \\ \mathbf{0} & \mathbf{0} & \mathbf{0} & -\tau_{min}^{-1} & \mathbf{M}_2 \\ \mathbf{P} & \mathbf{P} & \mathbf{M}_1 & \mathbf{M}_2 & -\kappa^{-1} \end{bmatrix}$$

$$\Lambda_{2il} = \begin{bmatrix} -(\mathbf{I} + 2\tau_{min}\mathbf{I} + 2\kappa\mathbf{I} + \tau_{min}\mathbf{M}_1 + \kappa\mathbf{M}_2)^{-1} & \mathbf{B}_l\mathbf{K}_i \\ \mathbf{K}_i^T\mathbf{B}_l^T & -\mathbf{Q} \end{bmatrix}$$

And the EVP can be solved by the convex optimization techniques.

From **Theorem** and the aforementioned minimization problem, the design procedure for the proposed H_∞ fuzzy controller can be presented as follows:

Design Procedure

Step 1: Based on the T-S fuzzy model (1), specify the controller gain matrices $\mathbf{K}_l\ (l = 1,2,...,L)$

Step 2: Specify $\mathbf{R}$ and a prescribed attenuation level ρ. Then, solve the EVP given by (23) to obtain the symmetric positive definite matrices $\mathbf{P}$, $\mathbf{Q}$, $\mathbf{M}_1$ and $\mathbf{M}_2$, and a positive constant σ.

Step 3: Obtain and the H_∞ fuzzy controller (4) for the networked control system.

4 Illustrative Example

In this section, an inverted pendulum system simulated as the controlled plant to demonstrate the performance of the proposed control strategy. Let be the angular of the pendulum with respect to the vertical line and $x_1(t)$ be the applied the control signal. Define $\mathbf{x}(t) = [x_1(t), \dot{x}_1(t)]^T = [x_1(t), x_2(t)]^T$ the dynamic equations of the inverted pendulum system can be described as follows [14]:

$$\begin{aligned} x_1(t) &= x_2(t) \\ \dot{x}_1(t) &= \frac{g\sin(x_1) - am_p l x_2^2 \sin(x_1)\cos(x_1)}{(4l/3) - am_p l\cos^2(x_1)} \\ &\quad + \frac{a\cos(x_1)u_1}{(4l/3) - am_p l\cos^2(x_1)} + \omega(t) \end{aligned}$$

where $a = m_c + m_p$, $g = 9.8m/s^2$ is the acceleration due to gravity, m_c is the mass of the cart, m_p is the mass of the pole and l is the half length of the pole. In this simulation, we set $m_c = 1kg$,$m_p = 0.1kg$ and $l = 0.5m$ the sampling period be 0.001sec.

Here, the T-S fuzzy model is design as follows:

PlantRule 1:

IF $x_1(t)$ is about 0,

THEN $\dot{\mathbf{x}}(t) = \mathbf{A}_1\mathbf{x}(t) + \mathbf{B}_1 u(t) + \boldsymbol{\omega}(t)$

PlantRule 2:

IF $x_1(t)$ is about $\pm\pi/2$,

THEN $\dot{\mathbf{x}}(t) = \mathbf{A}_2\mathbf{x}(t) + \mathbf{B}_2 u(t) + \boldsymbol{\omega}(t)$

where

$$\mathbf{A}_1 = \begin{bmatrix} 0 & 1 \\ \frac{g}{(4l/3) - am_p l} & 0 \end{bmatrix} \mathbf{B}_1 = \begin{bmatrix} 0 \\ \frac{a}{(4l/3) - am_p l} \end{bmatrix}$$

$$\mathbf{A}_2 = \begin{bmatrix} 0 & 1 \\ \frac{2g}{\pi((4l/3) - am_p l b^2)} & 0 \end{bmatrix} \mathbf{B}_2 = \begin{bmatrix} 0 \\ \frac{ab}{(4l/3) - am_p l b^2} \end{bmatrix}$$

in which $b = \cos(88^\circ)$. Here, we choose the fuzzy

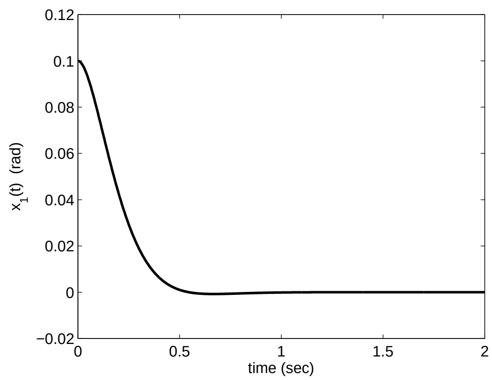

Fig. 3: The trajectory of $x_1(t)$

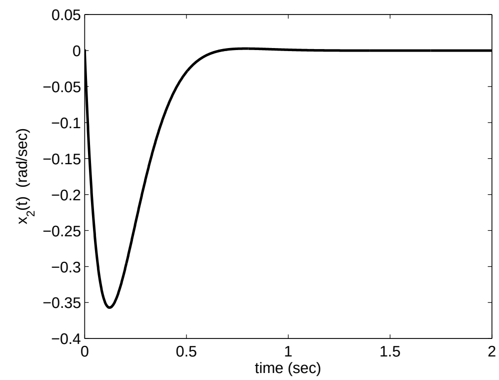

Fig. 4: The trajectory of $x_2(t)$

membership functions as

$$F_{x_1}^1 = (0.5\pi - |x_1|)/(0.5\pi), \text{ and} F_{x_1}^2 = 1 - F_{x_1}^1,$$

and the initial state is chosen as

$$\mathbf{x}(0) = [x_1(0), x_2(0)]^T = [0.1, 0]^T$$

Based on the ***Design Procedure***, the proposed H_∞ fuzzy controller can be designed by the following steps:
Step 1: Based on the T-S fuzzy model (1), specify the controller gain matrices $\mathbf{K}_1 = \mathbf{K}_2 = [1, 5]$
Step 2: Specify $\mathbf{R} = \begin{bmatrix} 25 & 0 \\ 0 & 125 \end{bmatrix}$ and a prescribed attenuation level $\rho = 0.5$. Then, solve the EVP given by (23) to obtain the symmetric positive definite matrices

$$\mathbf{P} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}, \mathbf{Q} = \begin{bmatrix} 10 & 0 \\ 0 & 90 \end{bmatrix} \mathbf{M}_1 = \begin{bmatrix} 0.1 & 0 \\ 0 & 0.1 \end{bmatrix},$$

$$\mathbf{M}_2 = \begin{bmatrix} 0.01 & 0 \\ 0 & 0.01 \end{bmatrix}, \text{ and } \sigma = 0.1.$$

Step 3: Obtain and the H_∞ fuzzy controller (4) for the networked control system.

Simulation results are shown in **Fig.** 3-5. The trajectory of $x_1(t)$ and$x_2(t)$shown in **Fig.** 3 and **Fig.** 4, respectively, and the control signal $u(t)$ is shown in **Fig.** 5. From the simulation results, we can see that the proposed control strategy performs good stability and can inhibit effectively the effect of "network transmission delay" and "data packet dropout".

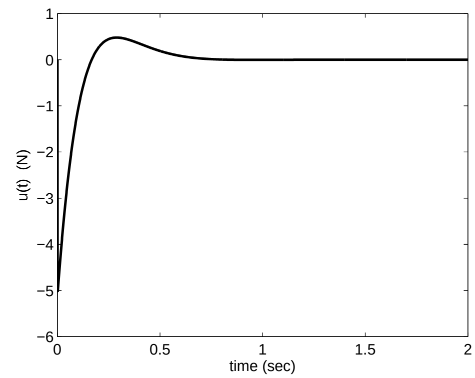

Fig. 5: The trajectory of $u(t)$

5 Conclusions

This paper proposed the H_∞ fuzzy controller design for a class of nonlinear networked control system. First, a T-S fuzzy model would be applied to describe the input-output relation of nonlinear networked control system (NSC). Based on the T-S fuzzy model, the H_∞ fuzzy control is proposed to inhibit the effect of "network transmission delay" and "data packet dropout," and to stabilize the overall system. With chosen the Lyapunov-Krasovskii function, based on Lyapunov stability theorem, we obtain the Lyapunov stability criterion and guarantee the H_∞ control performance of the overall system. The proposed control strategy has the following advantages: (1) The H_∞ control performance and the stability of the overall system can be guaranteed. (2) The effect of "network transmission delay" and "data packet dropout" is inhibited successfully.

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