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Generalized Statistical Convergence in Intuitionistic Fuzzy $2-$ Normed Space

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Abstract: In this paper, we shall introduce the new notion namely, $\mathscr{I}$-λ statistical convergence by using ideal with respect to the intuitionistic fuzzy norm $(\mu,v)_2$. We also study the relation between $\mathscr{I}$-λ statistical convergence and $\mathscr{I}$-statistical convergence.
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1 Introduction

The concepts of fuzzy set and fuzzy set operations were first introduced by Zadeh [28]. Subsequently several authors have discussed various aspects of the theory and applications of fuzzy sets, such as fuzzy topological spaces, similarity relations and fuzzy ordering, fuzzy measures of fuzzy events and fuzzy mathematical programming, population dynamics , chaos control , computer programming , nonlinear dynamical systems , nonlinear operator, etc. Recently, the fuzzy topology proves to be a very useful tool to deal with such situations where the use of classical theories breaks down.

In [10], Park introduced the concept of intuitionistic fuzzy metric space and later on Saadati and Park [18] introduced the concept of intuitionistic fuzzy normed space. Recently Mursaleen and Lohani [12] defined the concept of intuitionistic fuzzy 2-normed space which is generalization of the notion of intuitionistic fuzzy.

The notion of statistical convergence was introduced by Fast [2] and Schoenberg [27] independently. A lot of developments have been made in this areas after the works of Šalát [19], and Fridy [3]. Over the years and under different names statistical convergence has been discussed in the theory of Fourier analysis, ergodic theory and number theory. Recently, Mursaleen and Mohiuddine [12] studied the lacunary statistical convergence with respect to the intuitionistic fuzzy normed space. In [11], Mohiuddine and Lohani introduced the concept of $\lambda-$ statistical convergence with respect to the intuitionistic fuzzy normed space.
Recently, Mursaleen [13] studied the concept of statistical convergence of sequences in random 2-normed space. Quite recently, Savas [23] introduced $\lambda-$ statistical convergence in random 2-normed space.

More investigations in this direction and more applications can be found in ([16], [21], [22] and [24]) where many important references can be found.

In this paper, we shall study $\mathscr{I}-[V,\lambda]$- summable and $\mathscr{I}-\lambda-$ statistical convergence on the intuitionistic fuzzy $2-$ normed space $(\mu,v)_2$. We mainly examine the relation between these two new methods in intuitionistic fuzzy normed space $(\mu,v)_2$.

First, we recall some notations and basic definitions which we will used throughout the paper.

Definition 1.1 [25]. A binary operation $*:[0,1]\times[0,1]\to[0,1]$ is said to be a continuous t-norm if it satisfies the following conditions:
(a) $*$ is associative and commutative,
(b) $*$ is continuous,
(c) $a*1=a$ for all $a\in[0,1]$
(d) $a*b\leq c*d$ whenever $a\leq c$ and $b\leq d$ for each $a,b,c,d\in[0,1]$.

Definition 1.2 [25]. A binary operation $\Diamond:[0,1]\times[0,1]\to[0,1]$ is said to be a continuous t-conorm if it satisfies the following conditions:

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(a) $\diamondsuit$ is associative and commutative,
(b) $\diamondsuit$ is continuous,
(c) $a\diamondsuit 0 = a$ for all $a \in [0,1]$
(d) $a\diamondsuit b \le c\diamondsuit d$ whenever $a \le c$ and $b \le d$ for each $a,b,c,d \in [0,1]$.

Using the nations of continuous t-norm and t-conorm, Saadati and Park [18] have recently introduced the concept of intuitionistic fuzzy norm space as follows:

Definition 1.3. The five-tuple $(X,\mu,v,*,\diamondsuit)$ is said to be an intuitionistic fuzzy norm space (for short, IFNS) if X is a vector space, $*$ is continuous t-norm, $\diamondsuit$ is continuous t-conorm, and μ,v are fuzzy sets on $X\times(0,\infty)$ satisfying the following conditions. For every $x,y \in X$ and $s,t>0$.

(a) $\mu(x,t)+v(x,t) \le 1$,
(b) $\mu(x,t)>0$,
(c) $\mu(x,t)=1$ if and only if $x=0$,
(d) $\mu(\alpha x,t)=\mu(x,\frac{t}{|\alpha|})$ for each $\alpha \neq 0$,
(e) $\mu(x,t)*\mu(y,s) \le \mu(x+y,t+s)$,
(f) $\mu(x,.):(0,\infty)\to[0,1]$ is continuous,
(g) $\lim_{t\to\infty}\mu(x,t)=1$ and $\lim_{t\to 0}\mu(x,t)=0$,
(h) $v(x,t)<1$,
(i) $v(x,t)=1$ if and only if $x=0$,
(j) $v(\alpha x,t)=v(x,\frac{t}{|\alpha|})$ for each $\alpha\neq 0$,
(k) $\mu(x,t)\diamondsuit\mu(y,s) \ge v(x+y,t+s)$,
(l) $v(x,.):(0,\infty)\to[0,1]$ is continuous,
(m) $\lim_{t\to\infty}v(x,t)=0$ and $\lim_{t\to 0}v(x,t)=1$.

In this case (μ,v) is called an intuitionistic fuzzy norm.

In [4],Gähler introduced the following concept of 2-normed space.

Definition 1.4. Let X be a real vector space of dimension d, where $2\le d<\infty$. A 2-norm on X is a function $||.||: X\times X\to\mathbb{R}$ which satisfies,
(a) $||x,y||=0$ if and only if x and y are linearly dependent;
(b) $||x,y||=||y,x||$;
(c) $||\alpha x,y||=|\alpha|||x,y||$;
(d) $||x,y+z|| \le ||x,y||+||x,z||$.
The pair $(X,||.,.||)$ is then called a 2-normed space.
A trivial example of an 2-normed space is $X=\mathbf{R}^2$, equipped with the Euclidean 2-norm $||x_1,x_2||_E =$ the volume of the parallellogram spanned by the vectors x_1,x_2 which may be given expicitly by the formula

$$||x_1,x_2||_E = |det(x_{ij})| = abs(\det(<x_i,x_j>))$$

where $x_i=(x_{i1},x_{i2})\in\mathbf{R}^2$ for each $i=1,2$.

Mursaleen and Lohani [12] used the idea of 2-normed space to define the intuitionistic fuzzy 2-normed space.

Definition 1.5. The five-tuple $(X,\mu,v,*,\diamondsuit)$ is said to be an intuitionistic fuzzy 2-norm space (for short, IF2NS) if X is a vector space, $*$ is continuous t-norm, $\diamondsuit$ is continuous t-conorm, and μ,v are fuzzy sets on $X\times X\times(0,\infty)$ satisfying the following conditions for every $x,y\in X$ and $s,t>0$.
(a) $\mu(x,y;t)+v(x,y;t)\le 1$,
(b) $\mu(x,y;t)>0$,
(c) $\mu(x,y;t)=1$ if and only if x and y are linearly dependent,
(d) $\mu(\alpha x,y;t)=\mu(x,y;\frac{t}{|\alpha|})$ for each $\alpha\neq 0$,
(e) $\mu(x,y;t)*\mu(x,z;s)\le\mu(x,y+z;t+s)$,
(f) $\mu(x,y;.):(0,\infty)\to[0,1]$ is continuous,
(g) $\lim_{t\to\infty}\mu(x,y;t)=1$ and $\lim_{t\to 0}\mu(x,y:t)=0$,
(h) $\mu(x,y;t)=\mu(y,x;t)$
(i) $v(x,y;t)<1$,,
(j) $v(x,y;t)=0$ if and only if x and y are linearly dependent,
(k) $v(\alpha x,y;t)=v(x,y;\frac{t}{|\alpha|})$ for each $\alpha\neq 0$,
(l) $\mu(x,y;t)\diamondsuit\mu(x,z;s)\ge v(x,y+z;t+s)$,
(m) $v(x,y;.):(0,\infty)\to[0,1]$ is continuous,
(n) $\lim_{t\to\infty}v(x,y;t)=0$ and $\lim_{t\to 0}v(x,y;t)=1$.
(o) $v(x,y;t)=v(y,x;t)$
In this case (μ,v) is called an intuitionistic fuzzy 2-norm on X, and we denote it by $(\mu,v)_2$.

Example 1.1. Let $(X,||.||)$ be a 2-normed space, and let $a*b=ab$ and $a\diamondsuit b=\min\{a+b,1\}$ for all $a,b\in[0,1]$. For all $x\in X$ and every $t>0$, consider

$$\mu(x,z;t):=\frac{t}{t+||x,z||} \text{ and } v(x,z;t):=\frac{||x,z||}{t+||x,z||}$$

Then $(X,\mu,v,*,\diamondsuit)$ is an intuitionistic fuzzy 2-normed space.

Definition 1.6. Let $(X,\mu,v,*,\diamondsuit)$ be an intuitionistic fuzzy 2− normed space. A sequence $x=(x_k)$ is said to be convergent to $L\in X$ with respect to $(\mu,v)_2$ if, for every $\varepsilon>0$ and $t>0$, there exists $k_0\in\mathbb{N}$ such that $\mu(x_k-L,z;t)>1-\varepsilon$ and $v(x_k-L,z;t)<\varepsilon$ for all $k\ge k_0$ and for all $z\in X$. In this case we write $(\mu,v)_2-\lim x=L$ or $x_k \overset{(\mu,v)_2}{\to} L$ as $k\to\infty$.

The family $\mathscr{I}\subset 2^Y$ of subsets a nonempty set Y is said to be an ideal in Y if (i) $\emptyset\notin\mathscr{I}$; (ii) $A,B\in\mathscr{I}$ imply $A\cup B\in\mathscr{I}$; (iii) $A\in\mathscr{I}$, $B\subset A$ imply $B\in\mathscr{I}$, while an admissible ideal $\mathscr{I}$ of Y further satisfies $\{x\}\in\mathscr{I}$ for each $x\in Y$. If $\mathscr{I}$ is an ideal in Y then the collection $F(\mathscr{I})=\{M\subset Y: M^c\in\mathscr{I}\}$ forms a filter in Y which is called the filter associated with $\mathscr{I}$.

Definition 1.([9]) *Let $\mathscr{I}\subset 2^{\mathbb{N}}$ be a nontrivial ideal in $\mathbb{N}$. The sequence $\{x_n\}_{n\in\mathbb{N}}$ in X is said to be $\mathscr{I}$-convergent to $x\in X$, if for each $\varepsilon>0$ the set $A(\varepsilon)=\{n\in\mathbb{N}: ||x_n-x||\ge\varepsilon\}$ belongs to $\mathscr{I}$.*

Definition 2.*Let $(X,\mu,v,*,\diamondsuit)$ be an intuitionistic fuzzy 2− normed space. Then, a sequence $x=(x_k)$ is said to be $\mathscr{I}$-statistically convergent to $L\in X$ with respect to $(\mu,v)_2$, if, for every $\varepsilon>0$ and $t>0$, and for non zero $z\in X$ such that*

$$\left\{n\in\mathbb{N}:\frac{1}{n}|\{k\le n:\mu(x_k-L,z;t)\le 1-\varepsilon \text{ or } v(x_k-L,z;t)\ge\varepsilon\}|\ge\delta\right\}\in\mathscr{I}.$$

In this case we write $x_k \overset{(\mu,\nu)}{\to} L\left(S^{(\mu,\nu)_2}(\mathscr{I})\right)$.

2 $\lambda-$Statistical convergence in IF2NS

In this section we study the concept of $\mathscr{I}$-λ- statistical convergence in the intuitionistic fuzzy 2− normed space $(\mu,\nu)_2$. Before proceeding further, we recall the definition of density and related concepts which form the background of the present work.

Definition 2.1. Let K be subset of $\mathbb{N}$, the set of natural numbers. Then the asymptotic density of K denoted by $\delta(K)$, is defined as

$$\delta(K) = \lim_n \frac{1}{n}|\{k \le n : k \in K\}|,$$

where the vertical bars denote the cardinality of the enclosed set.

A sequence $x = (x_k)$ is said to be statistically convergent to the number L if for each $\varepsilon > 0$, the set $K(\varepsilon) = \{k \le n : |x_k - L| > \varepsilon\}$ has asymptotic density zero, i.e.

$$\lim_n \frac{1}{n}|\{k \le n : |x_k - L| \ge \varepsilon\}| = 0.$$

In this case we write $st - \lim x = L$ (see [2], [3]).

Note that every convergent sequence is statistically convergent to the same limit, but converse need not be true.

Let $\lambda = (\lambda_n)$ be a non-decreasing sequence of positive numbers tending to ∞ such that $\lambda_{n+1} \le \lambda_n + 1$, $\lambda_1 = 1$. The collection of such a sequence λ will be denoted by Δ.

The generalized de Valée-Pousin mean is defined by

$$t_n(x) = \frac{1}{\lambda_n}\sum_{k\in I_n} x_k,$$

where $I_n = [n - \lambda_n + 1, n]$.

Let $(X,\mu,\nu,*,\diamondsuit)$ be an intuitionistic fuzzy 2− normed space. A sequence $x = (x_k)$ is said to be $\mathscr{I}$-$[V,\lambda]$-summable to $L \in X$ with respect to $(\mu,\nu)_2$ if for any $\delta > 0$, $t > 0$, and for non zero $z \in X$ such that

$$\{n \in \mathbb{N} : \mu(t_n(x) - L, z; t) \le 1 - \delta \text{ or } \nu(t_n(x) - L, z; t) \ge \delta\} \in \mathscr{I}.$$

Now we define the $\mathscr{I}$-λ- statistical convergence with respect to intuitionistic fuzzy 2− normed space.

Definition 3.*Let* $(X,\mu,\nu,*,\diamondsuit)$ *be an intuitionistic fuzzy* 2− *normed space. A sequence* $x = (x_k)$ *is said to be* $\mathscr{I} - \lambda$ *- statistically convergent or* $\mathscr{I} - S_\lambda$ *convergent to* L *with respect to* $(\mu,\nu)_2$, *, if for every* $\varepsilon > 0$ *and* $\delta > 0$, *and* $t > 0$, *and for non zero* $z \in X$ *such that*

$$\left\{n \in \mathbb{N} : \frac{1}{\lambda_n}|\{k \in I_n : \mu(x_k - L, z; t) \le 1 - \varepsilon \text{ or } \nu(x_k - L, z; t) \ge \varepsilon\}| \ge \delta\right\} \in \mathscr{I}.$$

In this case we write $\mathscr{I} - S_\lambda^{(\mu,\nu)_2} - \lim x = L$ *or* $x_k \to L$ $\left(\mathscr{I} - S_\lambda^{(\mu,\nu)_2}\right)$.

We shall denote by $S^{(\mu,\nu)_2}(\mathscr{I})$, $S_\lambda^{(\mu,\nu)_2}(\mathscr{I})$ and $[V,\lambda]^{(\mu,\nu)_2}(\mathscr{I})$ the collections of all $\mathscr{I}$-statistically convergent, $\mathscr{I} - S_\lambda^{(\mu,\nu)_2}$ convergent and $\mathscr{I}$-$[V,\lambda]^{(\mu,\nu)_2}$-convergent sequences respectively.

Theorem 1.*Let* $(X,\mu,\nu,*,\diamondsuit)$ *be an intuitionistic fuzzy* 2− *normed space. If a sequence* $x = (x_k)$ *in* X *is* $\mathscr{I} - \lambda$ *statistically convergent sequences with respect to* $(\mu,\nu)_2$, *then limit is unique.*

*Proof.*This can be proved by using the techniques similar to those used in Theorem 1 of Savas[23]

Theorem 2.*Let* $(X,\mu,\nu,*,\diamondsuit)$ *be an intuitionistic fuzzy* 2− *normed space. Let* $\lambda = (\lambda_n) \in \Delta$. *Then*

(i) $x_n \to L\,[V,\lambda]^{(\mu,\nu)_2}(\mathscr{I}) \Rightarrow x_k \to L\left(S_\lambda^{(\mu,\nu)_2}(\mathscr{I})\right)$ *and the inclusion* $[V,\lambda]^{(\mu,\nu)_2}(\mathscr{I}) \subset S_\lambda^{(\mu,\nu)_2}(\mathscr{I})$ *is proper for every ideal* $\mathscr{I}$.

(ii) *If* $x \in m(X)$, *the space of all bounded sequences of* X *and* $x_k \to L\left(S_\lambda^{(\mu,\nu)_2}(\mathscr{I})\right)$ *then* $x_k \to L\,[V,\lambda]^{(\mu,\nu)_2}(\mathscr{I})$.

(iii) $S_\lambda^{(\mu,\nu)_2}(\mathscr{I}) \cap m(X) = [V,\lambda]^{(\mu,\nu)_2}(\mathscr{I}) \cap m(X)$.

Proof.(i) Let $\varepsilon > 0$ and $x_k \to L\,[V,\lambda]^{(\mu,\nu)_2}(\mathscr{I})$. We have

$$\sum_{k\in I_n} (\mu(x_k - L, z; t) \text{ or } \nu(x_k - L, z; t))$$
$$\ge \sum_{\substack{k\in I_n \ \&\ \mu(x_k - L, z; t) < 1-\varepsilon \\ \text{or } \nu(x_k - L, z; t) > \varepsilon}} (\mu(x_k - L, z; t) \text{ or } \nu(x_k - L, z; t))$$
$$\ge \varepsilon\,|\{k \in I_r : \mu(x_k - L, z; t) \le 1 - \varepsilon \text{ or } \nu(x_k - L, z; t) \ge \varepsilon\}|.$$

So for a given $\delta > 0$,

$$\frac{1}{\lambda_n}|\{k \in I_n : \mu(x_k - L, z; t) \le 1 - \varepsilon \text{ or } \nu(x_k - L, z; t) \ge \varepsilon\}| \ge \delta$$

$$\Rightarrow \frac{1}{\lambda_n}\sum_{k\in I_n} \mu(x_k - L, z; t) \le (1-\varepsilon)\delta \text{ or } \frac{1}{\lambda_n}\sum_{k\in I_n} \nu(x_k - L, z; t) \ge \varepsilon\delta$$

i.e.

$$\left\{n \in \mathbb{N} : \frac{1}{\lambda_n}|\{k \in I_n : \mu(x_k - L, z; t) \le 1 - \varepsilon \text{ or } \nu(x_k - L, z; t) \ge \varepsilon\}| \ge \delta\right\}$$
$$\subset \left\{n \in \mathbb{N} : \frac{1}{\lambda_n}\left\{\sum_{k\in I_n} \mu(x_k - L, z; t) \le 1 - \varepsilon \text{ or } \sum_{k\in I_n} \nu(x_k - L, z; t) \ge \varepsilon\right\} \ge \varepsilon\delta\right\}.$$

Since $x_k \to L\,[V,\lambda]^{(\mu,\nu)_2}(\mathscr{I})$, so the set on the right-hand side belongs to $\mathscr{I}$ and so it follows that $x_k \to L\left(S_\lambda^{(\mu,\nu)_2}\right)(I)$.

To show that $S_\lambda^{(\mu,\nu)_2}(I) \subsetneq [V,\lambda]^{(\mu,\nu)_2}(\mathscr{I})$, take a fixed $A \in \mathscr{I}$. Let $(\mathbb{R}, |.|)$ denote the space of all real numbers with the usual norm, and let $a * b = ab$ and $ab = min\{a +$

$b,1\}$ for all $a,b\in[0,1]$. For all $x\in\mathbb{R}$ and every $t>0$, consider

$\mu(x,z;t):=\frac{t}{t+||x,z||}$ and $v(x,z;t):=\frac{||x,z||}{t+||x,z||}$.

Then $(\mathbb{R},\mu,v,*,\Diamond)$ is an intuitionistic fuzzy 2−normed space. Now we define a sequence $x=(x_k)$ by

$$x_k=\begin{cases}(k,0) & \text{for } n-\left[\sqrt{\lambda_n}\right]+1\le k\le n,\ n\notin A\\ (k,0) & n-\lambda_n+1\le k\le n,\ n\in A\\ \theta, & \text{otherwise}\end{cases}$$

Then $x\notin m(X)$ and for every $\varepsilon>0$ $(0<\varepsilon<1)$ since

$$\frac{1}{\lambda_n}\left|\{k\in I_n:\mu(x_k-0,z;t)\le 1-\varepsilon \text{ or } v(x_k-0,z;t)\ge\varepsilon\}\right|=\frac{\left[\sqrt{\lambda_n}\right]}{\lambda_n}\to 0$$

as $n\to\infty$ and $n\notin A$, so for every $\delta>0$,

$$\left\{n\in\mathbb{N}:\frac{1}{\lambda_n}\left|\{k\in I_n:\mu(x_k-0,z;t)\le 1-\varepsilon \text{ or } v(x_k-0,z;t)\ge\varepsilon\}\right|\ge\delta\right\}\subset A\cup\{1,2,...,m\}$$

for some $m\in\mathbb{N}$. Since $\mathscr{I}$ is admissible, it follows that $x_k\to\theta\left(S_\lambda^{(\mu,v)}(\mathscr{I})\right)$. Obviously

$$\frac{1}{\lambda_n}\sum_{k\in I_n}(\mu(x_k-\theta,z;t) \text{ or } v(x_k-\theta,z;t))\to\infty$$

i.e. $x_k\nrightarrow\theta[V,\lambda]^{(\mu,v)_2}(\mathscr{I})$. Note that if $A\in\mathscr{I}$ is finite then $x_k\nrightarrow\theta\left(S_\lambda^{(\mu,v)_2}\right)$. This example also shows that $\mathscr{I}-S_\lambda^{(\mu,v)_2}$-convergence is more general than $S_\lambda^{(\mu,v)_2}$-convergence.

(ii) Suppose that $x_k\to L\left(S_\lambda^{(\mu,v)_2}(\mathscr{I})\right)$ and $x\in m(X)$. Let $\mu(x_k-L,z;t)\ge 1-M$ or $v(x_k-L,z;t)\le M$ $\forall$ k. Let $\varepsilon>0$ be given. Now

$$\begin{aligned}&\frac{1}{\lambda_n}\sum_{k\in I_n}(\mu(x_k-L,z;t) \text{ or } v(x_k-L,z;t))\\ &=\frac{1}{\lambda_n}\sum_{\substack{k\in I_n\ \&\ \mu(x_k-L,z;t)\le 1-\varepsilon\\ v(x_k-L,z;t)\ge\varepsilon}}(\mu(x_k-L,z;t) \text{ or } v(x_k-L,z;t))+\\ &+\frac{1}{\lambda_n}\sum_{\substack{k\in I_n\ \&\ \mu(x_k-L,z;t)>1-\varepsilon\\ v(x_k-L,z;t)<\varepsilon}}(\mu(x_k-L,z;t) \text{ or } v(x_k-L,z;t))\\ &\le\frac{M}{\lambda_n}\left|\{k\in I_n:\mu(x_k-L,z;t)\le 1-\varepsilon \text{ or } v(x_k-L,z;t)\ge\varepsilon\}\right|+\varepsilon.\end{aligned}$$

Note that $\left\{n\in\mathbb{N}:\frac{1}{\lambda_n}\left|\{k\in I_n:\mu(x_k-L,z;t)\le 1-\varepsilon \text{ or } v(x_k-L,z;t)\ge\varepsilon\}\right|\ge\frac{\varepsilon}{M}\right\}=A(\varepsilon)\in\mathscr{I}$.If $n\in(A(\varepsilon))^c$ then

$$\frac{1}{\lambda_n}\sum_{k\in I_n}\mu(x_k-L,z;t)>1-2\varepsilon \text{ or } \frac{1}{\lambda_n}\sum_{k\in I_n}v(x_k-L,z;t)<2\varepsilon.$$

Hence

$$\left\{n\in\mathbb{N}:\frac{1}{\lambda_n}\sum_{k\in I_n}\mu(x_k-L,z;t)\le 1-2\varepsilon \text{ or } \frac{1}{\lambda_n}\sum_{k\in I_n}v(x_k-L,z;t)\ge 2\varepsilon\right\}\subset A(\varepsilon)$$

and so belongs to $\mathscr{I}$. This shows that $x_k\to L[V,\lambda]^{(\mu,v)_2}(\mathscr{I})$.

(iii) This readily follows from (i) and (ii).

Theorem 3.*Let $(X,\mu,v,*,\Diamond)$ be an intuitionistic fuzzy 2−normed space. Then $S^{(\mu,v)}(\mathscr{I})\subset S_\lambda^{(\mu,v)}(\mathscr{I})$ if $\liminf_{n\to\infty}\frac{\lambda_n}{n}>0$.*

Proof.(i) For given $\varepsilon>0$,

$$\begin{aligned}&\frac{1}{n}\left|\{k\le n:\mu(x_k-L,z;t)\le 1-\varepsilon \text{ or } v(x_k-L,z;t)\ge\varepsilon\}\right|\\ &\ge\frac{1}{n}\left|\{k\in I_n:\mu(x_k-L,z;t)\le 1-\varepsilon \text{ or } v(x_k-L,z;t)\ge\varepsilon\}\right|\\ &\ge\frac{\lambda_n}{n}\frac{1}{\lambda_n}\left|\{k\in I_n:\mu(x_k-L,z;t)\le 1-\varepsilon \text{ or } v(x_k-L,z;t)\ge\varepsilon\}\right|.\end{aligned}$$

If $\liminf_{n\to\infty}\frac{\lambda_n}{n}=\alpha$ then from definition $\left\{n\in\mathbb{N}:\frac{\lambda_n}{n}<\frac{\alpha}{2}\right\}$ is finite. For $\delta>0$,

$$\begin{aligned}&\left\{n\in\mathbb{N}:\frac{1}{\lambda_n}\left|\{k\in I_n:\mu(x_k-L,z;t)\le 1-\varepsilon \text{ or } v(x_k-L,z;t)\ge\varepsilon\}\right|\ge\delta\right\}\\ &\subset\left\{n\in\mathbb{N}:\frac{1}{n}\left|\{k\in I_n:\mu(x_k-L,z;t)\le 1-\varepsilon \text{ or } v(x_k-L,z;t)\ge\varepsilon\}\right|\ge\frac{\alpha}{2}\delta\right\}\\ &\cup\left\{n\in\mathbb{N}:\frac{\lambda_n}{n}<\frac{\alpha}{2}\right\}.\end{aligned}$$

Since $\mathscr{I}$ is admissible, the set on the right-hand side belongs to $\mathscr{I}$ and this completed the proof.

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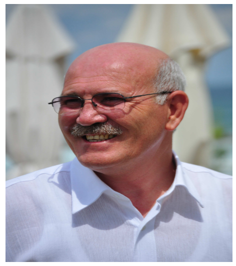

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