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Statistical Analysis related to Exceptional Snow Loads

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Abstract: In the present paper the favorable fit for exceedances related to the accidental snow loads, described in a companion paper, is studied. In particular, the Hill, t-Hill and modified Hill estimators are used and the quality of their distributional fits is assessed. Also t-estimation methods are considered. Assuming that the data is driven by Pareto or exponential distribution, respectively, the exact likelihood ratio tests for homogeneity to validate the results of regression fits are applied. Further, the statistical dependence between the exceedances and the corresponding altitudes is studied.

Keywords: modelling, snow extremes, t-Hill

1 Introduction

An approach to the assessment of accidental snow loads on structures is suggested in the companion paper Sadovský et al. [10], see also [7]. The approach combines engineering and climatology aspects using the data of collection and analysis of snow loads in Slovakia carried out recently Sadovský et al. [8] and Sadovský et al. [9]. The long-term weekly measurements of snow water equivalent (SWE) of snow cover at 660 rain-gauge stations are employed. Out of the rain-gauge stations, 55 meteorological stations were selected at which daily SWE values have been recalculated using other climatological measurements, like depth of snow cover etc. (Sadovský et al. [9]). Preliminary statistical analysis has been made in Stehlík and Sadovský [14].

The SWE records of 52 to 56 winter seasons are well suited for the assessment of the characteristic snow load on the ground, which is defined as 98 % quantile of a suitable extreme value distribution fitted to the yearly snow load maxima. Nevertheless, at some stations an outlying yearly maximum may occur as a result of exceptional heavy snowfalls. It is assumed that the maximum is a member of the same population, however, with a mean return period of say about 1000 years and more. Following Sanpaolesi et al. [11] the largest snow load value is exceptional if the ratio k of the load to the characteristic snow load determined without that value is greater than 1.5. The snow loads identified as exceptional should be treated in accidental design situations as accidental actions (loads), cf. Eurocode EN 1990.

The novelty of the approach for the assessment of the accidental snow loads by Sadovský et al. [10] can be briefly described. First the k values in excess of 1.5 are identified. Then by the expertise of climatologists based on the geomorphology of Slovakia, regions of similar climate conditions for the occurrence of accidental snow loads are determined, see Figure 1. Within a studied region, the values of the empirical distribution function F restricted to the N ordered k values in excess of 1.5 is calculated as

$$F(k_i) = \frac{i}{N_R+1}, \tag{1}$$

where N_R is the sum of winter seasons over all stations in the region and $i \in (N_R - N + 1, \ldots, N_R)$. The obtained empirical upper tail for k ratios is approximated by nonlinear regression analysis using Pareto, exponential and Gumbel distributions. The 0.999 and 0.9999 quantiles of the distributions are of particular interest.

In the present paper tests on the quality of the distributional fits performed in the companion paper (Sadovský et al. [10]) are applied. Particularly the Region 2 and the composite Region 4, within which the mountain basins are considered as one region, are studied. For the

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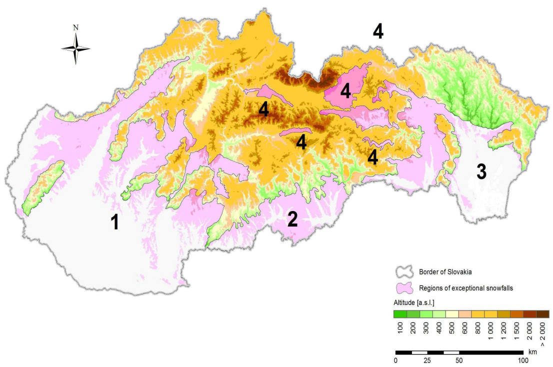

Fig. 1: Regions of exceptional snow loads (Sadovský et al. [7]).

exceptional snow loads and their corresponding k values in Region 1, treated in Sadovský et al. [7], a statistical dependence on the altitude is studied. The idea is to check the anticipated low dependence of k values on the altitude inferred from their definition, which comprises the altitude dependence already in the characteristic values.

2 STATISTICAL ASPECTS OF THE ANALYSIS OF EXCEEDANCES

There are many important issues which should be addressed from the theoretical point of view.

1. Which kind of tails is statistically significant in the data? Authors used Pareto, exponential and Gumbel types. Here we may apply potentially the test based on the ratio between the maximum and mean of the sample of the excesses above some random threshold. Such a test turns out to be useful in the construction of an asymptotical size of the test for the null hypothesis that the distribution comes from the Gumbel domain of attraction, see Neves, Picek and Alves [6].
2. Open question remains, which of the distribution fits better for the quantiles 0.9999 estimation.
3. If a Pareto tail fits well, we may use the statistical inference based on Stehlík et al. [15].
4. Quantile regression.

Related problems:

a) Classical statistics of extremes is largely univariate, but simple modelling is inadequate for many events of interest, which have a temporal aspect, a spatial aspect, or both.

b) Environmental/regulatory changes add urgency: changing probabilities for acceptable design reliability of private and societal structures.

c) Fundamental to all characterizations of extreme value processes is the concept of stability: a model for exceedances over a high threshold should remain valid for exceedances of higher thresholds.

2.1 Testing for Pareto tail Homogeneity

In statistical hypothesis testing, the p-value is the probability of obtaining a test statistic at least as extreme as the one that was actually observed, assuming that the null hypothesis is true. The fact that p-values are based on this assumption is crucial to their correct interpretation. One often rejects a null hypothesis if the p-value is less than 0.05 or 0.01, corresponding to a 5% or 1% chance, respectively, of an outcome at least that extreme, given the null hypothesis.

The following theorem (derived in Stehlík et al.[13]) gives the exact likelihood ratio test (ELRT) statistics of shape parameter homogeneity

$$H0 : a_1 = ... = a_N versus\ non\ H0 \tag{2}$$

for the sample from the Pareto $P(\lambda;a)$ family.

Let $x_1,...,x_N$ be i.i.d. from the Pareto $P(\lambda;a)$ family: $F(x) = 1-(\lambda/x)^a$ where the scale parameter λ is known. Then the likelihood ratio (LR) statistics $-\ln L$ of the hypothesis (2) has the form

$$-\ln N = N\ln\left(\sum_{i=1}^{N} y_i^{\beta}\right) - N\ln N - \sum_{i=1}^{N}\ln y_i^{\beta} \tag{3}$$

with $\beta = 1$ and $y_i = \ln x_i - \ln\lambda$, L being the corresponding likelihood ratio. The likelihood ratio incorporates both the sensitivity and specificity of the test and provides a direct estimate of how much a test result will change the odds of having homogeneity. In statistics, a likelihood ratio test is used to compare the fit of two models one of which is nested within the other. Both models are fitted to the data and their log-likelihood is recorded. In our situation, homogeneity by Pareto model means that we have no statistically significant variation in the tail shape parameter a.

2.2 Testing for exponential tail Homogeneity

Let $x_1,...,x_N$ be i.i.d. from the exponential $Exp(a,b)$ family, with cumulative distribution function (CDF) $F(x) = 1-exp(-a(x-b))$. Then the LR statistics $-\ln L$ of the scale homogeneity hypothesis has the same form as (3) with $y_i = x-b$ and b as known location parameter (for details see Stehlík [12]). Notice, that the difference between exponential and Pareto tail testing is that original Pareto data are first transformed to y_i and then plugged in (3).

For future work:

1)Two parametric families are used by Pareto and exponential in Sadovský et al. [10]. Therefore it will be of interest to modify the test for Pareto and exponential to have a free lower bound.

2)Develop test alternative for another snow load model, e.g. for Gumbel attraction family.

2.3 Pareto tail estimation

Assuming that the data are driven by Pareto, we may employ various estimators to get a good quality of the parameter a estimation. In Stehlík et al. [13] an extensive discussion to the favourable estimation of Pareto tail is given. We will employ here

1)*t-estimator* given by

$$\hat{a} = \frac{1}{x_h - 1}, \tag{4}$$

where x_h stands for the harmonic mean. This is a t-score moment method of estimating introduced by Fabián [3]. Note that the harmonic mean is calculated only for the studied upper tail.

2)*t-Hill estimator* introduced by Fabián and Stehlík [4] and later studied in Stehlík et al. [15]. This is a t-modification of a classical Hill estimator. We have

$$\frac{1}{\hat{a}} = \frac{k/X_{(n-k,n)}}{\sum_{i=1}^{k} \frac{1}{X_{(n-i+1,n)}}} - 1, \tag{5}$$

Here $X(1,n) < \ldots < X(n,n)$ are order statistics. Notice that data is taken over threshold $X(n-k,n)$. Fabián and Stehlík [4]) has proven consistency of the t-Hill estimator, for generalization of t-Hill estimator see Beran et al. [2].

3)*Hill estimator* developed by Hill [5]:

$$\frac{1}{\hat{a}} = \frac{1}{k}\left[\ln\left(\frac{X_{(n,n)}}{X_{(n-k,n)}}\right) + \ldots + \ln\left(\frac{X_{(n-k+1,n)}}{X_{(n-k,n)}}\right)\right]. \tag{6}$$

2.4 Estimation of parameters of exponential distribution

Let $x_1, \ldots, x_N$ be i.i.d. from the exponential $Exp(a,b)$ family, with density $a.exp(a(x-b))$ and support (a,∞). Then the best linear unbiased scale estimator is (see Balakrishnan and Basu, [1]):

$$\hat{a} = \frac{N-1}{N(ar(X) - X_{(1,n)})}. \tag{7}$$

where $ar(X)$ is the arithmetic mean of the data.

Table 1: LR statistics of homogeneity for Pareto on (λ,∞) and exponential family with support on (λ,∞) in Region 2 ($k_i > 1.2$, N = 8).

	Pareto	Exponential
LR statistics	0.0363	0.0398

Table 2: LR statistics of homogeneity for Pareto on (λ,∞) and exponential family with support on (λ,∞) in Region 4.

	Pareto	Exponential
LR statistics ($k_i > 1.5$, N = 11)	0.0891	0.1346
LR statistics ($k_i > 1.3$, N = 17)	0.1621	0.2312

3 REGRESSION FITS USING PARETO AND EXPONENTIAL SCALE ESTIMATORS

Since in the Region 2 only two exceedances k_i above 1.5 have been detected, the threshold has been lowered to 1.2 increasing their number to 8. For the Region 4 two thresholds of 1.5 and 1.3 have been considered resulting in 11 and 17 exceedances. By this choice the influence of threshold level on results of regression fits has been checked (Sadovský et al. [10]).

The LR statistics of homogeneity tests for Pareto and exponential family are shown in Tables 1, 2. For the known parameters λ and b their estimates by non-linear regressions in Sadovský et al. [10] are taken. In all cases shown, very good results are obtained with LR statistics values yielding p-value of 1, which means that it is almost perfect fit (which is in coherence with practical observations).

In the following subsections estimators for Pareto shape parameter (see 2.3) and exponential scale parameter (see 2.4) are calculated. The remaining scale or location parameter is estimated by non-linear regression. The least squares and absolute deviation minimizations by the software Xact (SciLab GmbH) are performed. The results are compared to the non-linear regression of both parameters carried out in Sadovský et al. [10]. For comparison the correlation coefficient r defined as the square root of the determination coefficient r^2

$$r^2 = 1 - \frac{\sum_i (y_i - \hat{y}_i)^2}{\sum_i (y_i - \bar{y})^2} \quad i \in (N_R - N + 1, \ldots, N_R), \tag{8}$$

where $y_i = F(k_i)$, $\hat{y}_i$ is an approximation of y_i by the nonlinear regression and $\bar{y}$ the average of y_i values, is used. Plotted values of empirical cdf and of the theoretical cdf curves allow visual check of approximation.

3.1 Results for the Region 2

The estimators for Pareto shape parameter and exponential scale parameter described in the Subsections 2.3 and 2.4

Table 3: Tail approximation in Region 2 ($k_i > 1.2$, N = 8).

Pareto on (λ,∞)	a	λ	R
a, λ regression	5.197	0.4254	0.9949
t-estimator, λ regression	2.578	0.1356	0.8972
t-Hill estimator, λ regression	6.389	0.5210	0.9860
Hill estimator, λ regression	6.585	0.5355	0.9822
Exponential on (b,∞)	a	b	R
a, b regression	3.787	-0.2291	0.9953
a estimator, b regression	4.697	0.0593	0.9874

Table 4: Estimations of Region 2 quantiles, $k_i > 1.2$, N = 8.

Quantile	0.999	0.9999
Pareto a, λ, on (λ,∞) regression	1.61	2.50
Pareto on (λ,∞) t-Hill estimator, regression	1.54	2.20
Pareto on (λ,∞) Hill estimator, regression	1.53	2.17
Exponential on (b,∞) a, b regression	1.60	2.20
Exponential on (b,∞) a estimator, b regression	1.53	2.02

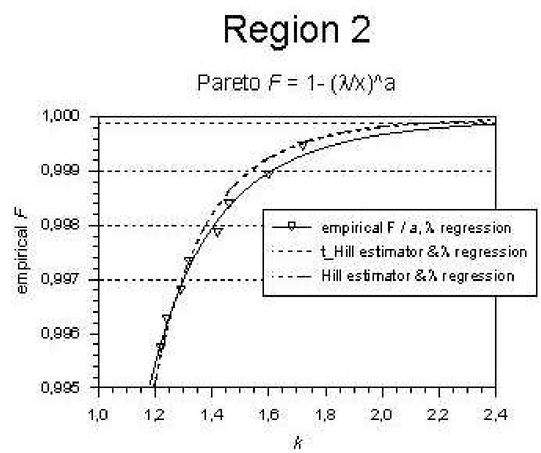

Fig. 2: Tail approximation by Pareto cdf.

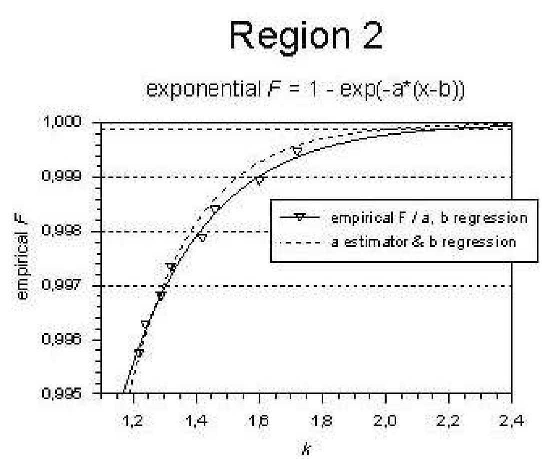

Fig. 3: Tail approximation by exponential cdf.

are shown in Table 3 along with the parameters estimated by non-linear regression and the corresponding correlation coefficient.

Tail approximations are plotted in Figures 2, 3. Table 4 shows 0.999 and 0.9999 quantiles of approximations performed. Due to inferior approximation, the t-estimator results are not included. The Pareto and exponential distributions provide comparable approximation. Slightly better figures of the variance of residua and of the correlation coefficient are obtained for the exponential distribution, see Table 3 and Sadovský et al. [10]. Moreover, the choice of the exponential distribution with parameters estimated by non-linear regression is supported by the visual check of particularly the upper sample quantiles approximation as well as the values of calculated quantiles based on estimators, see Table 4.

3.2 Results for the Region 4

The parameters of tail approximations for $k_i > 1.5$, N = 11 are shown in Table 5. The approximations are plotted in Figures 4, 5 and the corresponding 0.999 and 0.9999 quantiles are in Table 6.

For the alternative threshold of 1.3 yielding N = 17 exceedances, the parameters of tail approximations are shown in Table 7. The corresponding plots and 0.999 and 0.9999 quantiles are in Figures 6, 7 and Table 8.

The reasons for favouring the Pareto distribution with regression fit of parameters in Sadovský et al. [10], which have been the superior tail approximation and its obvious stability by threshold change, are not challenged by the consideration of estimators.

Table 5: Tail approximation in Region 4 ($k_i > 1.5$, N = 11).

Pareto on (λ,∞)	a	λ	R
a, λ regression	4.630	0.5019	0.9791
t-estimator, λ regression	1.331	-	-
t-Hill estimator, λ regression	2.882	0.2488	0.9462
Hill estimator, λ regression	3.211	0.2436	0.9592
Exponential on (b,∞)	a	b	R
a, b regression	2.635	-0.4320	0.9725
a estimator, b regression	3.016	-0.1784	0.9716

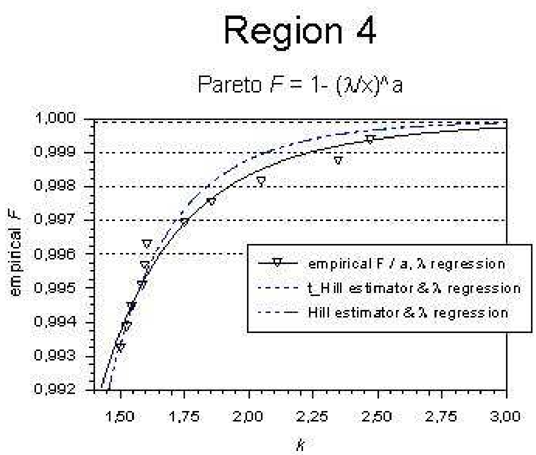

Fig. 4: Tail approximation by Pareto cdf.

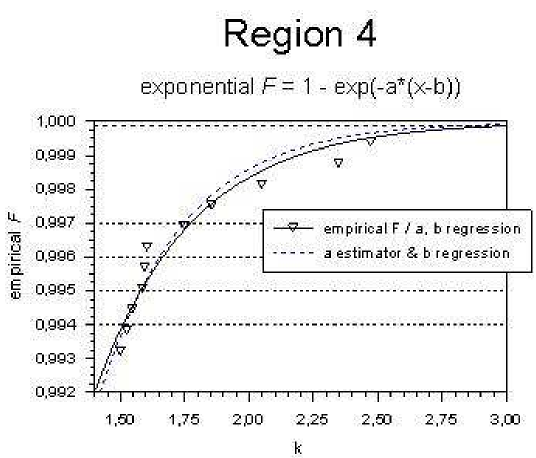

Fig. 5: Tail approximation by exponential cdf.

Table 6: Estimations of Region 4 quantiles, $k_i > 1.5$, N = 11.

Quantile	0.999	0.9999
Pareto a, λ, on (λ,∞) regression	2.23	3.67
Pareto on (λ,∞) t-Hill estimator, regression	2.73	6.08
Pareto on (λ,∞) Hill estimator, regression	2.59	5.31
Exponential on (b,∞) a, b regression	2.19	3.06
Exponential on (b,∞) a estimator, b regression	2.11	2.88

Table 7: Tail approximation in Region 4 ($k_i > 1.3$, N = 17).

Pareto on (λ,∞)	a	λ	R
a, λ regression	4.471	0.4881	0.9800
t-estimator, λ regression	1.619	-	-
t-Hill estimator, λ regression	4.091	0.4413	0.9785
Hill estimator, λ regression	4.292	0.4664	0.9796
Exponential on (b,∞)	a	b	R
a, b regression	3.014	-0.1645	0.9803
a estimator, b regression	2.929	-0.2122	0.9802

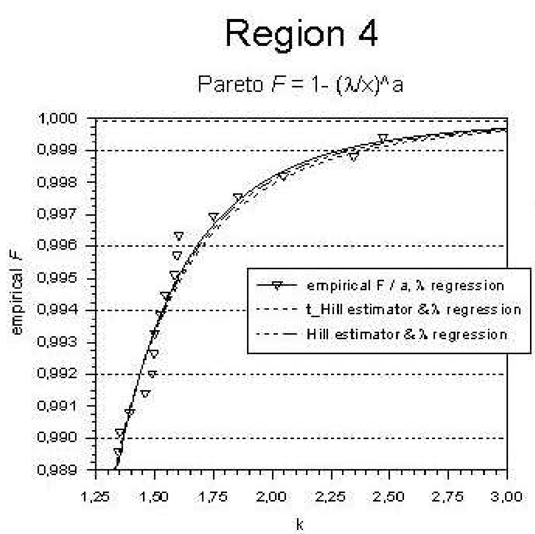

Fig. 6: Tail approximation by Pareto cdf.

Table 8: Estimations of Region 4 quantiles, $k_i > 1.3$, N = 17.

Quantile	0.999	0.9999
Pareto a, λ, on (λ,∞) regression	2.29	3.83
Pareto on (λ,∞) t-Hill estimator, regression	2.39	4.19
Pareto on (λ,∞) Hill estimator, regression	2.33	3.99
Exponential on (b,∞) a, b regression	2.13	2.89
Exponential on (b,∞) a estimator, b regression	2.15	2.93

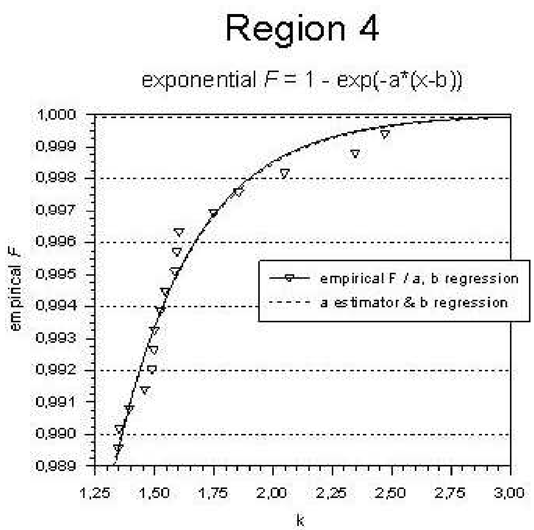

Fig. 7: Tail approximation by exponential cdf.

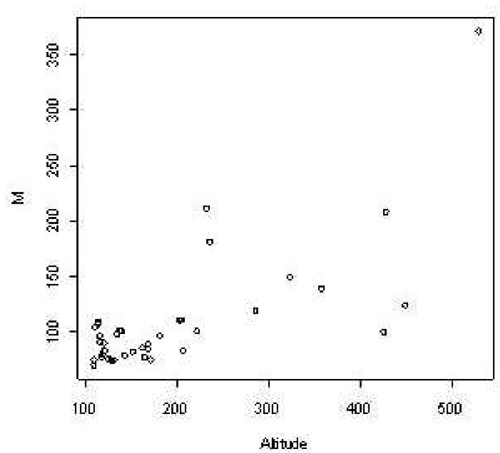

Fig. 8: The dependence of Maximal loads from the altitude.

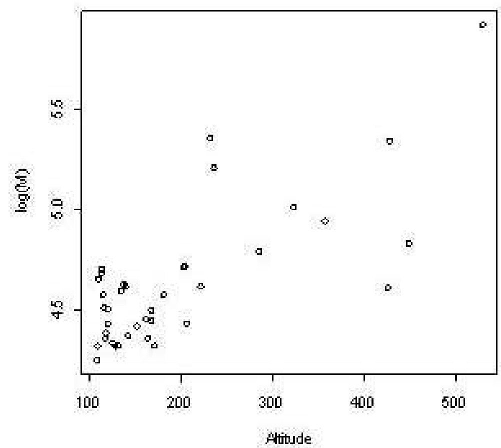

Fig. 9: The dependence of logarithm of the Maximal loads from the altitude.

4 MODELLING THE DEPENDENCE BETWEEN THE ALTITUDE AND THE YEARLY SNOW LOAD MAXIMA.

4.1 Data from Region 1

Figure 8 and Figure 9 show the dependences of maximal loads (M) and logarithm of the Maximal loads from the altitude (A), respectively.

We apply the ordinary least squares (OLS) for the logarithm of M and obtain that the estimated line of regression is

$$log(M)\_est = 4.1824413 + 0.0023254 * A.$$

The standard errors of the coefficients are correspondingly 0.0787975 of the intercept and 0.0003547 for the regression coefficient. The coefficient of determination is R^2 = 0.5374. Therefore

$$M\_est = exp(4.1824413).exp(0.0023254 * A)$$

Further we will consider the error term

$$\varepsilon = log(M) - log(M)\_est.$$

Its chart is given on fig. 10.

The normal QQ plot, together with the Kolmogrov-Smirnov test, show that the errors could be considered as normal (D = 0.1129, p-value = 0.6615). However further we show that there exists a better fit.

We made Generalized Extreme Value distribution (GEV) fit and obtain the following MLE of the parameters

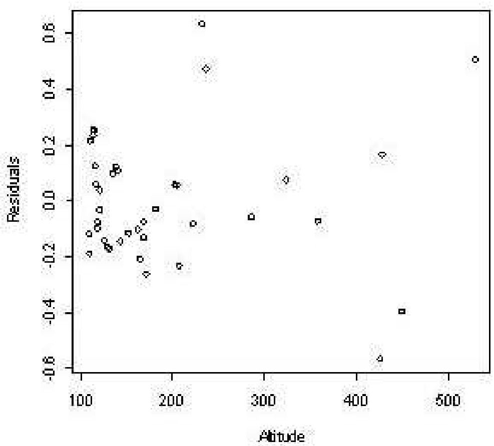

Fig. 10: The error term ε.

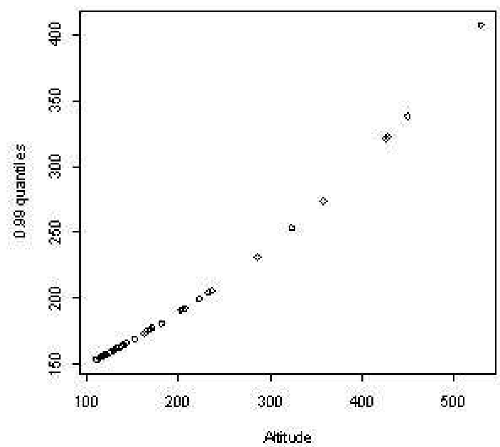

Fig. 11: The estimators for the 0.99 quantiles of the largest snow loads with respect to the altitudes.

μ	σ	ξ
-0.09073322	0.21786729	-0.17540496

with the corresponding standard errors

0.03793636 0.02528167 0.08263649.

The Kolmogorov-Smirnov test shows D = 0.0915 and p-value = 0.87, which is better than in the normal model.

The dependency of M on the altitude is illustrated by estimating its 0.99-quantiles for the given altitudes using this fitting. The results plotted in Figure 11 show that the 0.99-quantiles significantly depend on the altitude. Using the estimated line of the regression and the GEV distribution of the errors we immediately obtain the quantiles of the largest snow loads. For the case of 0.99 they are given on Fig. 11.

4.2 Modelling the dependence between the altitude and k.

The scatter plot from Figure 12 shows that almost there is no dependence between these variables.

We consider the linear model

$$k(A) = k\_est(A) + \varepsilon, \qquad (9)$$

where the coefficients a and b in the regression line

$$k\_est(A) = a + b * A \qquad (10)$$

have been estimated by OLS method. The term ε in (9) is the random error of that estimation. The estimated line of the regression is

$$k\_est(A) = 1.6472866 + 0.0001339 * A$$

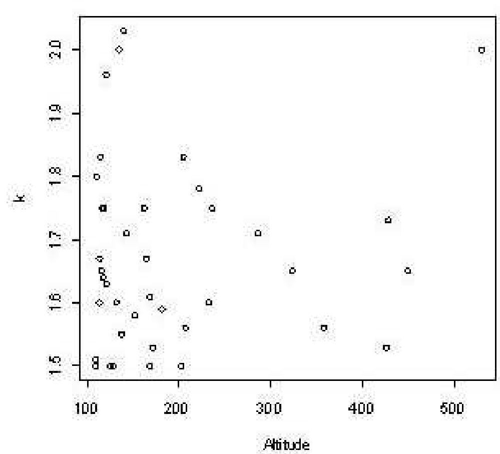

Fig. 12: The scatter plot of the dependence between the altitude of a station and k.

Table 9: Quality of the estimation.

Parameters	Estimate	Std. Error
a	1.6472866	0.0493978
b	0.0001339	0.0002224

The estimators of the parameters are given in Table 10. The coefficient of determination is R^2 = 0.0097.

Further we consider the error term ε. Its dependence on the altitude is given on Figure 13.

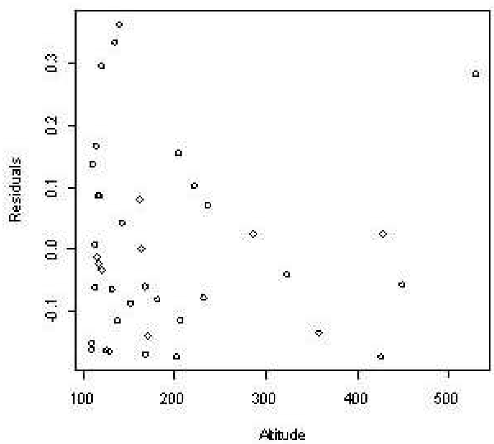

Fig. 13: Scatter plot of the dependence of the residuals on the altitude.

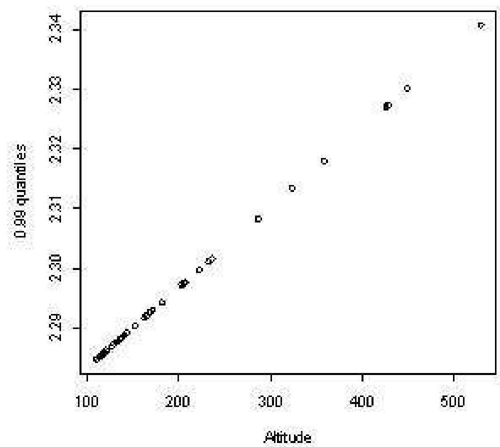

Fig. 14: The estimators for the 0.99 quantiles of k with respect to the altitude.

The Kolmogorov-Smirnov test shows that the error term could be considered as normal. More precisely D = 0.1175 and p-value = 0.6123. It turns out that we could make a better fit. We made stable and GEV fit. The GEV fit seems the best of the considered models. Using the software R we obtain the following MLE of the parameters:

μ	σ	ξ
-0.07531348	0.09941563	0.17254121

with the corresponding standard errors

0.01977435	0.01599755	0.19653951

The Kolmogorov-Smirnov test shows relatively good results D = 0.0795 and p-value = 0.9499. Both the normal and the GEV distributions fit the error term well, but the second one is closer to the empirical distribution due to the value of the Kolmogorov-Smirnov test statistic D. Using the regression line, described in (9) and (10) and the GEV distribution fit of the error term we calculate the estimators of the quantiles of the distribution of k given the altitude. They are such that

$$P(k(A) < x_{0.99}|A = x) = 0.99,$$

i.e.

$$P(1.6472866 + 0.0001339 * A + \varepsilon < x_{0.99}|A = x) = 0.99.$$

In case of 0.99 for different altitudes these quantiles are given on Figure 14. It shows that the dependence of the 0.99-quantiles of k on the altitude is insignificant.

5 CONCLUSIONS

For the method of the assessment of accidental snow loads for the design of structures developed in the companion paper by Sadovský et al. [10] complementary statistical tests and calculations of heavy and light tail parameters estimators have been carried out.

For all situations, homogeneity of the data can be accepted at critical significance level of 0.05. Homogeneity tests show very good results meaning almost perfect fit by the corresponding Pareto and exponential distribution, conditionally that these distributions are fitting well. The reason why the t-estimator (in case of Pareto) differs from the others is its bounded influence function, i.e. its natural robustness.

Parameter estimators used in tails distributional fits yield somewhat worse approximation than by the direct non-linear regression of parameters (Sadovský et al. [10]). However, they can be used as starting values for minimization as well as supporting information for the choice of a heavy or light tail approximation.

We also provided study of dependence of $logM$ and k on the altitude (see section 4). In the case of Region 1, the ratio k of exceptional to characteristic snow loads is Fréchet (almost Gumbel) distributed and the dependence on altitude is not statistically significant.

Throughout the paper several open questions for future work and related problems are outlined. Particularly, the following research tasks are of interest:

1. Development of exact testing procedures (exact mainly because of small samples) for these situations, e.g. considering only exceedances above threshold with inclusion of nonlinear regression issues.
2. Analysis of correlation in the temporal and spatial context and checking whether theory for extremes of stationary processes is adaptable to this situation.

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