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Sensitivity Analysis of Inefficient Supply Chains

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Abstract: The present study is an attempt toward improving the performance of members of supply chain. Improvement of two-stage supply chains is three cases. Inefficients suppliers are improved or inefficient manufactures are improved or both of them are improved. First, New sub-perfect supply chain production possibility set is obtained with efficiency score of α for inefficient suppliers and only all DEA inefficient suppliers are improved. It is proved that as the efficiency score of all points on the main frontier supposed to be 1, the efficiency score on the new frontier is α .Second the procedure is applied for those supply chains which are inefficient in manufacture performance and only all DEA inefficient manufacturers are improved and the last one, it is used to improve the supply chains which are inefficient in supplier and manufacture performances at the same time. In the last two ones it is shown that there are improvements in inefficient manufactures or in both of them at the same time but the improved efficiency is not able to appraise exactly. Overall performance score has an improvement in all cases and the supply chain management can choose the best strategies to maximize overall efficiency score. This paper develops procedures which are referred to each case for performing a sensitivity analysis of the inefficient supply chains in constant returns to scale (CRS). The real case is applied to accept this approach.

Keywords: Data Envelopment Analysis (DEA), Supply Chain Management (SCM), Sensitivity Analysis, Supply Chain, Efficiency

1 Introduction

A supply chain is the combination of equipment, suppliers, manufactures, distributors, retailers and method of controlling inventory, purchasing and distribution, that it tends to improve the way your company finds raw materials it needs to produce a product or service and to deliver it to customers, for example, China Construction Bank [1, 2]

Supply chain effective management has been widely accepted as an important means for supplier or manufacturer or distributor, to obtain the best and high-quality products and services by the least cost and the most profit. The evaluation of the performance and improvement are a great importance and necessity for recognizing in supply chain management (SCM). In supply chain management (SCM), although decreasing the cost and increasing profit is very important, but also partnerships with together is a significant factor for enhancing competitiveness. Among many evaluation methods, data envelopment analysis (DEA) is one of the best ways for assessing the relative efficiency a group of homogenous decision making units (DMUs) that use multiple inputs to produce multiple outputs, originated from the work by Charnes et al [3]. DEA has been applied to evaluate the supply chain performance in several works such as, [4–8] and so on. The traditional DEA models can't be applied directly to the supply chain case because classical DEA treats each DMU, supply chain, as a black box and for example in a supplier-manufacture chain considers only the initial inputs from suppliers and final outputs at the very end of downstream members in the performance evaluation. For the complex nature of supply chain, those intermediate products or linking activities are ignored. Then, several authors have attempted to account these links and consider supply chain as a network DEA by multi-stage [9]. The network DEA model proposed [10] has a multi-stage structure as an extension of the two stage supply chain [11] and DEA model proposed in [12]. In recent years, one of the most important issues in DEA is the sensitivity analysis of efficient DMUs. In 1985, sensitivity analysis of CCR model for a specific efficient DMU with a single output was initiated by Charnes et al. [13]. In 1990 Charnes and Neralic considered additive model and they obtained sufficient conditions for remaining efficient [14]. Then in 1992, Charnes et al. obtained a specific stability region by using L_1 and L_∞ [15]. These researchers have studied the methods which simultaneous proportional change is

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assumed in inputs and outputs for a specific efficient DMU under evaluations. Then Zhu (1996) provides a modified DEA model to compute a stability region which DMU under evaluation remains efficient [16].

In 1998 Seiford and Zhu developed a procedure to determine an input stability region (ISR) and an output stability region (OSR) for efficient DMU [17]. They stated that an efficient DMU will remain efficient after the input increases or output decreases if and only if such changes occur with in the ISR or OSR [17], and this subject are considering in recent years. Jahanshahloo et al. [18] extended the largest stability region for BCC model and Additive model by supporting hyperplanes [19] for DMU under evaluation which all inputs and outputs of DMUs except DMU under evaluation are assumed fixed. In some works sensitivity analysis is based on the super efficiency DEA approach in which the efficient DMU under evaluation is not included in the reference set [20, 21, 23]. Sensitivity analysis of an inefficient DMU is studied less than sensitivity of an efficient DMU.In 1992, Charnes, Haag et al. obtained an improvement for inefficient DMU by using Chebychev norm [15]. The model dealt with improvements in both inputs and outputs that could occur for an inefficient DMU before its statues would change to efficient.

In the recent years sensitivity analysis of inefficient units has been more studied. In 2011 Jahanshahloo et al. supposed that DMU under evaluation is inefficient by the efficiency score of θ_o^* and $\theta_o^* < \alpha < 1$ which α is a fixed constant and defined by the manager .They obtained the new frontier T_v' with efficiency score of α . They proved that as the efficiency score of all points on the main frontier supposed to be 1, the efficiency score on the new frontier is α [20].

But sensitivity analysis of supply chain is still in absence. Improvement in supply chain performance is one of the most important mentioned advantages of progress supply chain.

In 2011, Yang et al. [11] defined two types of supply chain production possibility sets ,which are proved to be equivalent to each other and based upon the production possibility set, a supply chain CRS DEA model is advanced to appraise the overall technical efficiency of supply chain and they obtained the benchmarking units for inefficient supply chains.

Inefficiency of two-stage supply chains is in three cases. Supplier is inefficient or manufacture is inefficient or both of them are inefficient. This paper develops procedures for performing a sensitivity analysis of the inefficient supply chains in three cases. We consider two-stage supply chain which includes supplier and manufacture and the supply chain is under the control of a unique decision maker. By using [11] and the proposed method, new sub-perfect supply chain production possibility set is obtained with efficiency score of α for inefficient suppliers and in this case it has been focused mainly around that only DEA inefficient suppliers have an improvement. It is proved that as the efficiency score of all points on the main frontier supposed to be 1, the efficiency score on the new frontier is α. By different ways such as decreasing inputs, increasing outputs or combination strategies, DEA inefficient suppliers with efficiency scores of θ_{so}^* can obtain efficiency score of α and also the decision maker can choose the best improvement strategies to maximize the overall performance score. Next the model is applied for those supply chains which are DEA inefficient in manufacture performance and only all DEA inefficient manufacturers are improved .The last one , it is used to improve the supply chains which are inefficient in supplier and manufacture performance at the same time.In these cases we show that there are improvements in inefficient manufactures or in both of them at the same time but the improved efficiency is not able to appraise exactly and we have improvement in overall performance score too. What we do is that the inefficient supply chains are appraised respected to a new frontier with efficiency score which is a fixed number (and defined by the manager). The decision maker gives a chance or fortune to them until they can have an improvement and finally the overall efficiency score of supply chain is improved and the overall performance of supply chain is progressed and it is contented the decision maker.

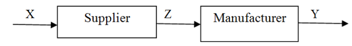

Fig. 1: A supplier-manufacturer chain

This paper proceeds as follows. The next section represents some basic DEA models. Section 3 develops a proposed method for improving the overall performance of inefficient supply chains. Section 4 is a real world application and the proposed method is applied to evaluate the performance of banking chains in a big Chinese commercial bank. Finally conclusions are given in section 5.

2 Background

Suppose there are N two-stage supplier-manufacturer chains as shown in Fig. 1. where S and M represent supplier and manufacturer, respectively. The variable X is the input vector of the supplier (S) and the variable Z is the output vector of the supplier and is also an input of manufacturer (M). The variable Y is the manufacturer's output vector.Any supplier consumes P inputs to generate K intermediate products, and the manufacturer consumes those intermediate products to produce Q outputs. Specially for the j th SC, the inputs and outputs for the supplier are $X_{pj}(p = 1,2,...,P)$ and $Z_{kj}(k = 1,2,...,K)$, and they are $Z_{kj}(k = 1,2,...,K)$ and $Y_{qj}(q = 1,2,...,Q)$ for manufacturer, respectively.

Definition 1(Perfect supply chain CRS production possibility set [11]).

$$T_{SC-P}=\left\{\begin{array}{ll}(x_p,y_q)\,|\,\sum_{j=1}^{N}\lambda_j^S x_{pj}\le x_p, & p=1,2,...,P\\ \sum_{j=1}^{N}\lambda_j^S z_{kj}\ge z_k, & k=1,2,...,K\\ \sum_{j=1}^{N}\lambda_j^M z_{kj}\le z_k, & k=1,2,...,K\\ \sum_{j=1}^{N}\lambda_j^M y_{qj}\ge y_q, & q=1,2,...,Q\\ \lambda_j^S,\lambda_j^M\ge 0, & j=1,2,...,N\end{array}\right\} \quad (1)$$

Definition 2(Sub-perfect supply chain CRS production possibility set [11]).

$$T_{SC-SP}=\left\{\begin{array}{ll}(x_p,y_q)\,|\,\sum_{j=1}^{N}\lambda_j x_{pj}\theta_{Sj}^*\le x_p, & p=1,2,...,P\\ \sum_{j=1}^{N}\lambda_j z_{kj}\ge z_k, & k=1,2,...,K\\ \sum_{j=1}^{N}\lambda_j z_{kj}\le z_k, & k=1,2,...,K\\ \sum_{j=1}^{N}\lambda_j^M y_{qj}/\theta_{Mj}^*\ge y_q, & q=1,2,...,Q\\ \lambda_j^S,\lambda_j^M\ge 0, & j=1,2,...,N\end{array}\right\} \quad (2)$$

Theorem 1.$T_{SC-P}\equiv T_{SC-SP}$

*Proof.*See [11]

Consider DMU_j , ($j=1,...,n$), where each DMU consumes m inputs to produce s outputs. Suppose that the observed input and output vectors of DMU_j are $X_j=(x_{1j},...,x_{mj})$ and $Y_j=(y_{1j},...,y_{sj})$ respectively, and let $X_j\ge 0$ and $X_j\ne 0$ and $Y_j\ge 0$ and $Y_j\ne 0$.

The production possibility set T_c and T_v are defined as follows:

$$T_c=\left\{(X,Y)\,|\,X\ge\sum_{j=1}^{n}\lambda_j X_j, Y\le\sum_{j=1}^{n}\lambda_j Y_j,\lambda_j\ge 0, j=1,...,n\right\}$$

$$T_V=\left\{(X,Y)\,|\,X\ge\sum_{j=1}^{n}\lambda_j X_j, Y\le\sum_{j=1}^{n}\lambda_j Y_j,\sum_{j=1}^{n}\lambda_j=1,\lambda_j\ge 0, j=1,...,n\right\}$$

Let the set of extreme efficient DMUs in T_v be E. By determining the set of E, the set of E' is defined as follows [20]:

$$E'=\left\{(X_j',Y_j')\,|\,(X_j',Y_j')=\left(\frac{1}{\alpha}X_j,Y_j\right), j\in E\right\}$$

And the new production possibility set T_v' :

$$T_v'=\left\{(X',Y')\,|\,X'\ge\frac{1}{\alpha}\sum_{j\in E}\lambda_j X_j, Y'\le\sum_{j\in E}\lambda_j Y_j,\sum_{j\in E}\lambda_j=1,\lambda_j\ge 0, j\in E\right\}$$

To find extreme efficient DMU in BCC model, Anderson and Petersen (AP) model is solved for each DMU [24]:

$$\begin{array}{ll} AP: \min\ \theta_o & \\ S.t.\quad \sum_{j=1, j\ne o}^{n}\lambda_j x_{ij}\le\theta_o x_{io}, & i=1,2,...,m\\ \sum_{j=1, j\ne o}^{n}\lambda_j y_{rj}\ge y_{ro}, & r=1,2,...,s\\ \sum_{j=1, j\ne o}^{n}\lambda_j=1 & \\ \lambda_j\ge 0,\ j=1,...,n, j\ne 0 & \end{array} \quad (3)$$

In the recent years data analysis of inefficient units has been more studied. In 2011 Jahanshahloo et al. supposed that DMU under evaluation is inefficient by the efficiency score of θ_o^* and $\theta_o^*<\alpha<1$ which α is a fixed constant and defined by the manager .They obtained the new frontier T_v' with efficiency score of α . They proved that as the efficiency score of all points on the main frontier supposed to be 1, the efficiency score on the new frontier is α .Then by using different ways such as decreasing inputs, increasing outputs or combination strategies, DMU_o with efficiency score of θ_o^* can obtain efficiency score of α and has an improvement for $\alpha-\theta_o^*$ in efficiency.[20]

Theorem 2.*The efficiency score of each point of E' in T_v is α .*

*Proof.*See [20]

Attention 1. There is one- to- one correspondence between E and E' .

*Proof.*See [20]

Attention 2. There is one-to-one correspondence between T_v and T_v' frontier points.

*Proof.*See [20]

Theorem 3.*The efficiency score of each point on the T_v' frontier is α in T_v.*

*Proof.*See [20]

3 Proposed method

Suppose there are N two-stage supplier-manufacturer chains as shown in Fig. 1. where stage S represents the supplier and the stage M represents a manufacturer, respectively.Any supplier consumes P inputs to generate K intermediate products, and the manufacturer consumes those intermediate products to produce Q outputs. The inputs and outputs for the j th supplier are $X_{pj}(p=1,2,...,P)$ and $Z_{kj}(k=1,2,...,K)$, and they are $Z_{kj}(k=1,2,...,K)$ and $Y_{qj}(q=1,2,...,Q)$ for the j th manufacturer, respectively.

Consider N same supply chains called Decision Making Units (DMUs) in DEA literatures, denoted by $DMU_1,DMU_2,...,DMU_N$ in the context. In most practical situations of supply chain management, the chain operated under the fulfillment of demand from consumers.The performance of a supply chain (SC) is attributed to two main factors: the performances of all SC members, and the co-operation of its members. It means that the performance of each supply chain's member is very important and it is influenced on the overall supply chain efficiency. The influence of supply chain thought on organizational strategy has also significant reflecting. As independent decision makers, each supply chain members maximized its own technical efficiency, thus eliminates that of other members and even of the overall chain.

Suppose that the evaluated supply chain is DEA inefficient with the technical efficiency $\theta_o^* < 1\ (o = 1, 2, .., N)$. The supply chain is under the control of a unique decision maker.The goal is to improve the inefficient supply chain to achieve some desired level of performance which is defined by the supply chain management. This overall inefficiency of supply chain happens when supplier or manufacturer is inefficient or both of them are inefficient at the same time.Therefore these inefficiencies are considered in three cases. The proposed method is employed to improve just inefficient member of supply chain and it is deemed the overall chain and also the other members of SCs.The inefficient supply chains are appraised respected to a new frontier with efficiency score which is a fixed number (and defined by the manager). The decision maker gives a chance or fortune to them until they can have an improvement.In case 1 the exact value of efficiency is obtained and overall performance of supply chain is improved too. In case 2 and 3 it is shown that there are improvements in manufacturer and both of them at the same time and overall performance score of supply chain has an improvment too but the exact score or value is not appraisable.

Case1. Suppose that the evaluated supply chain is DEA inefficient. In the first case, it has been focused mainly around that only DEA inefficient suppliers have an improvement. They are evaluated by Constant Returns to Scale (CRS) assumption. The new sub-perfect frontier with efficiency score α (α is a fixed constant which is defined by the supply chain manager) is obtained and the DEA inefficient suppliers are evaluated by the new frontier. Denote the production possibility set for all supply chains by [11]:

$$T_{SC\_SP} = \left\{ \begin{array}{c} (X,Y) | \sum_{j=1}^{N} \lambda_j \theta_{sj}^* X_j \le X, \\ \sum_{j=1}^{N} \lambda_j Z_j \ge Z, \\ \sum_{j=1}^{N} \lambda_j Z_j \le Z \\ \sum_{j=1}^{N} \lambda_j \varphi_{Mj}^* Y_j \ge Y \\ \lambda_j \ge 0, \quad j = 1, 2, ..., N \end{array} \right\} \quad (4)$$

By the above Production Possibility Set (PPS) θ_d the d^{th} supply chain's DEA efficiency score obtained by the following LP problem:

$$\begin{array}{l} \theta_d = \min \ \theta \\ \quad \sum_{j=1}^{N} \lambda_j X_j^* \le \theta X_d, \\ \quad \sum_{j=1}^{N} \lambda_j Y_j^* \ge Y_d, \\ \quad (X_j^*, Y_j^*) \in T_{SC-SP} \\ \quad \lambda_j \ge 0, \ j = 1, 2, ..., N \end{array} \quad (5)$$

(X^*, Z^*, Y^*) are points that located at the frontier constructed by the sub-perfect supply chain CRS Production Possibility Set. Also E_{SC-SP} is considered as all extreme efficient supply chains. The above model can be rewritten as the following programming:

$$\begin{array}{l} \theta_d = \min \ \theta \\ s.t. \ \sum_{j=1}^{N} \lambda_j X_j^* \le \theta X_d \\ \quad \sum_{j=1}^{N} \lambda_j Y_j^* \ge Y_d \\ \quad \sum_{j=1}^{N} \overline{\lambda}_j X_j \theta_{sj}^* \le X_j^* \\ \quad \sum_{j=1}^{N} \overline{\lambda}_j Z_j \ge Z_j^* \\ \quad \sum_{j=1}^{N} \overline{\lambda}_j Z_j \le Z_j^* \\ \quad \sum_{j=1}^{N} \overline{\lambda}_j Y_j \varphi_{Mj}^* \ge Y_j^* \\ \quad \lambda_j, \overline{\lambda}_j \ge 0, \quad j = 1, ..., N \end{array} \quad (6)$$

We know that the Production Possibility Set of supplier is as follows:

$$T_{CS} = \left\{ (X,Z) | \sum_{j=1}^{N} \lambda_j^S X_j \le X, \ \sum_{j=1}^{N} \lambda_j^S Z_j \ge Z, \ \lambda_j^S \ge 0, \forall j \right\}$$

(AP) model is employed to find all extreme efficient suppliers. Let the set of extreme efficient suppliers in T_{CS} be E_{CS} . Having determined E_{CS} , E'_{CS} is defined as follows:

$$E'_{CS} = \left\{ (X'_j,\ Z'_j) \,|\, (X'_j,\ Z'_j) = \left(\frac{1}{\alpha} X_j,\ Z_j \right), j \in E_{CS} \right\}.$$

The new production possibility set for suppliers T'_{CS} is introduced by:

$$T'_{CS} = \left\{ (X',Z') | \tfrac{1}{\alpha} \sum_{j \in E_{CS}} \lambda_j^S X_j \le X', \ \sum_{j \in E_{CS}} \lambda_j^S Z_j \ge Z', \ \lambda_j^S \ge 0, \forall j \in E_{CS} \right\}$$

Supposed that supplychain$_d$ is inefficient and it is inefficient in supplier$_d$ by the efficiency score of θ_{sd}^* and $\theta_{sd}^* < \alpha < 1$ which α is a fixed constant and defined by the manager. The new frontier T'_{CS} with efficiency score of α is obtained. We know that as the efficiency score of all points on the main frontier supposed to be 1, the efficiency score on the new frontier is α. The CCR model of the d th $(d = 1, 2, ..., N)$ supplier (the supplier in thed th SC) in T'_{CS} is computed by the following model:

$$\begin{array}{l} \theta_{s'} = \min \ \ \theta_s \\ s.t. \ \sum_{j=1}^{N} \lambda_j^S x'_{pj} \le \theta_s x'_{pd}, \ \ p = 1, 2, ..., P \\ \quad \sum_{j=1}^{N} \lambda_j^S z'_{kj} \ge z'_{kd}, \quad k = 1, 2, ..., K \\ \quad \lambda_j^S \ge 0, \qquad j = 1, 2, ..., N \end{array} \quad (7)$$

For improving inefficient supply chains and to find the new supply chain frontier, first of all the extreme points ofT_{SC-SP}should be found. These points are called the setE_{SC-SP} . They are sub set of extreme points of T_{CS}. In the sequel, the set E'_{SC-SP} is found and defines as follows:

$E'_{SC-SP} = \{(X_j^{*'}, Z_j^{*'}, Y_j^{*'}) | \left(X_j^{*'}, Z_j^{*'}, Y_j^{*'} \right) =$

$\left(\theta^*_{s'j}X'_j, Z_j, \varphi^*_{Mj}Y_j\right)\}$. Then T'_{SC-SP} is as follows:

$$T'_{SC-SP} = \left\{ \begin{array}{l} (X',Y')|\ \frac{1}{\alpha}\sum_{j\in E_{SC-SP}} \lambda_j X_j \theta^*_{s'j} \le X', \\ \sum_{j\in E_{SC-SP}} \lambda_j Z_j \ge Z', \\ \sum_{j\in E_{SC-SP}} \lambda_j Z_j \le Z', \\ \sum_{j\in E_{SC-SP}} \lambda_j Y_j \varphi^*_{Mj} \ge Y', \\ \lambda_j \ge 0, \quad j \in E_{SC-SP} \end{array} \right\} \tag{8}$$

Where $\theta^*_{s'j}$ and φ^*_{Mj} are the CCR efficiency score of the supplier and manufacturer in the j^{th} supply chain respectively.

Then by using [11] sub-perfect supply chain CCR production possibility set is obtained and modeled as follows:

$$\begin{array}{l} \theta_{d'} = \min \ \theta \\ s.t. \ \sum_{j\in E'_{SC-SP}} \lambda_j X^{*'}_j \le \theta X'_{d'} \\ \quad \sum_{j\in E'_{SC-SP}} \lambda_j Y^{*'}_j \ge Y'_{d'} \\ \quad \lambda_j \ge 0, \quad j \in E'_{SC-SP} \\ \quad (X^{*'}_j, Y^{*'}_j) \in T'_{SC-SP} \end{array} \tag{9}$$

The above model can be rewritten as the following programming:

$$\begin{array}{l} \theta_{d'} = \min \ \ \theta \\ s.t. \ \sum_{j\in E'_{SC-SP}} \lambda_j X^{*'}_j \le \theta x'_{d'} \\ \quad \sum_{j\in E'_{SC-SP}} \lambda_j y^{*'}_j \ge y'_{d'} \\ \quad \sum_{j\in E'_{SC-SP}} \overline{\lambda}_j X'_j \theta^*_{s'j} \le X^{*'}_{d'}, \\ \quad \sum_{j\in E'_{SC-SP}} \overline{\lambda}_j Z_j \ge Z^{*'}_{d'}, \\ \quad \sum_{j\in E'_{SC-SP}} \overline{\lambda}_j Z_j \le Z^{*'}_{d'}, \\ \quad \sum_{j\in E'_{SC-SP}} \overline{\lambda}_j Y_j \varphi^*_{Mj} \ge Y^{*'}_{d'}, \\ \quad \lambda_j, \overline{\lambda}_j \ge 0, \quad j \in E'_{SC-SP} \end{array} \tag{10}$$

By this procedure, new sub-perfect supply chain production possibility set frontier which is called T'_{SC-SP} is obtained for inefficient supply chains which are inefficient in suppliers and only DEA inefficient suppliers have an improvement. It is proved that as the efficiency score of all points on the main frontier supposed to be 1, the efficiency score on the new frontier is α . By different ways such as decreasing inputs, increasing outputs or combination strategies, DEA inefficient suppliers with efficiency scores of θ^*_{so} can reach to the new frontier T'_{SC-SP} and also the decision maker can choose the best improvement strategies to maximize the overall performance score .Now by following theorems and lemma, It is shown that every point on T'_{SC-SP} frontier has an efficiency score α.

Lemma 1.$M' \in E'_{SC-SP}$ *if and only if* M' *is an extreme efficient unit in* T'_{SC-SP}.

*Proof.*Let M' with coordinate $\left(X^*_{M'}, Z^*_{M'}, Y^*_{M'}\right)$ be an arbitrary extreme efficient point in E'_{SC-SP}. By definition, we have $(X^*_M, Z^*_M, Y^*_M) \in E_{SC-SP}$.

At the first step, we are going to prove M' is efficient in T'_{SC-SP}. By contradiction, let M' not be efficient in T'_{SC-SP}. According to the assumption:

$$\left\{ \begin{array}{l} X^{*'}_j = \frac{1}{\alpha} X^*_j \\ Y^{*'}_j = Y^*_j \end{array} \right.$$

and we have:

$$\begin{array}{l} \min \ \theta_{M'} \\ s.t. \ \sum_{j\in E'_{SC-SP}} \lambda_j X^{*'}_j \le \theta_{M'} X_{M'} = \theta_{M'}\left(\frac{1}{\alpha}X_M\right) \\ \quad \sum_{j\in E'_{SC-SP}} \lambda_j Y^{*'}_j \ge Y_{M'} = Y_M, \\ \quad (X^{*'}_j, Y^{*'}_j) \in T'_{SC-SP} \\ \quad \lambda_j \ge o, \ \ j \in E'_{SC-SP} \end{array}$$

Suppose that the optimal solution of above mentioned problem is $\left(\lambda^*,\ \theta^*_{M'}\right)$. By contradiction suppose $\theta^*_{M'} < 1$. So we get

$$\left\{ \begin{array}{l} \sum_{j\in E'_{SC-SP}} \lambda^*_j X^{*'}_j \le \theta^*_{M'} X_{M'} \\ \sum_{j\in E'_{SC-SP}} \lambda^*_j Y^{*'}_j \ge Y_{M'} = Y_M, \\ (X^{*'}_j, Y^{*'}_j) \in T'_{SC-SP} \\ \lambda^*_j \ge o, \ \ j \in E'_{SC-SP} \end{array} \right\}$$

By multiplying the first constraint to α , we get

$$\left\{ \begin{array}{l} \sum_{j\in E'_{SC-SP}} \lambda^*_j (\alpha X^{*'}_j) \le \theta^*_{M'}(\alpha X_{M'}) \\ \sum_{j\in E'_{SC-SP}} \lambda^*_j Y^{*'}_j \ge Y_{M'} = Y_M, \\ (X^{*'}_j, Y^{*'}_j) \in T'_{SC-SP} \\ \lambda^*_j \ge o, \ \ j \in E'_{SC-SP} \end{array} \right\}$$

According to definition E_{SC-SP} and E'_{SC-SP} :

$$\left\{ \begin{array}{l} \sum_{j\in E_{SC-SP}} \lambda^*_j X^*_j \le \theta^*_{M'} X_M \\ \sum_{j\in E_{SC-SP}} \lambda^*_j Y^*_j \ge Y_M, \\ (X^*_j, Y^*_j) \in T'_{SC-SP} \\ \lambda^*_j \ge o, \ \ j \in E_{SC-SP} \end{array} \right\}$$

The last one has a feasible solution $\left(\lambda^*,\ \theta^*_{M'} < 1\right)$ for the corresponding problem of $M \in E_{SC-SP}$ that is contradiction with $M \in E_{SC-SP}$.

So M' is efficient in T'_{SC-SP} . At the sequel it is going to prove that M' is an extreme unit in T'_{SC-SP} . In contradiction, suppose that M' to be a non-extreme efficient unit in T'_{SC-SP} . Let the set of extreme points in T'_{SC-SP} be:

$$\left\{\left(X^{*'}_1, Z^{*'}_1, Y^{*'}_1\right) \ldots\ldots \left(X^{*'}_t, Z^{*'}_t, Y^{*'}_t\right)\right\}.$$

It has;

$$(X^*_{M'}, Z^*_{M'}, Y^*_{M'}) = \sum_{j=1}^{t} \lambda_j \left(X^{*'}_j, Z^{*'}_j, Y^{*'}_j\right), \ \ \sum_{j=1}^{t} \lambda_j = 1, \ \lambda_j \ge 0, \ \ j = 1, \ldots, t$$

or

$$\left(\frac{1}{\alpha}X_M^*, Z_M^*, Y_M^*\right) = \sum_{j=1}^{t}\lambda_j\left(\frac{1}{\alpha}X_j^*, Z_j^*, Y_j^*\right),$$
$$\sum_{j=1}^{t}\lambda_j = 1,\ \lambda_j \geq 0,\ j = 1,...,t.$$
$$\left(\frac{1}{\alpha}X_M^*, Z_M^*, Y_M^*\right) = (\frac{1}{\alpha}\sum_{j=1}^{t}\lambda_j X_j^*, \sum_{j=1}^{t}\lambda_j Z_j^*,$$
$$\sum_{j=1}^{t}\lambda_j Y_j^*)\ ,\ \sum_{j=1}^{t}\lambda_j = 1,\ \lambda_j \geq 0, \qquad j = 1,...,t$$
$$X_M^* = \sum_{j=1}^{t}\lambda_j X_j^*,\ \ Z_M^* = \sum_{j=1}^{t}\lambda_j Z_j^*,\ \ Y_M^* = \sum_{j=1}^{t}\lambda_j Y_j^*$$
$$\sum_{j=1}^{t}\lambda_j = 1,\ \lambda_j \geq 0,\ j = 1,...,t\ .$$

therefore:

$$(X_M^*, Z_M^*, Y_M^*) = (\sum_{j=1}^{t}\lambda_j X_j^*, \sum_{j=1}^{t}\lambda_j Z_j^*, \sum_{j=1}^{t}\lambda_j Y_j^*)$$
$$= \sum_{j=1}^{t}\lambda_j\left(X_j^*, Z_j^*, Y_j^*\right),$$
$$\sum_{j=1}^{t}\lambda_j = 1,$$
$$\lambda_j \geq 0,\ j = 1,...,t$$

It is represented M with coordinates (X_M^*, Z_M^*, Y_M^*) as a convex combination of extreme efficient points in T_{SC-SP} and this is in contradiction with the assumption. Hence, $M' \in E'_{SC-SP}$ is an extreme efficient unit in T'_{SC-SP}. Conversely if M' with coordinates $\left(X_{M'}^*, Z_{M'}^*, Y_{M'}^*\right)$ be an arbitrary extreme efficient point in T'_{SC-SP}, it is claimed that $\left(\alpha X_{M'}^*, Z_{M'}^*, Y_{M'}^*\right)$ is an extreme efficient point in T_{SC-SP}. By contradiction, let $\left(\alpha X_{M'}^*, Z_{M'}^*, Y_{M'}^*\right)$ not be in E_{SC-SP} . So we have:

$$(\alpha X_{M'}^*, Z_{M'}^*, Y_{M'}^*) = \sum_{j\in E_{SC-SP}}\lambda_j\left(X_j^*, Z_j^*, Y_j^*\right),$$
$$\sum_{j\in E_{SC-SP}}\lambda_j = 1\lambda_j \geq 0,\ j \in E_{SC-SP}.$$
$$(\alpha X_{M'}^*, Z_{M'}^*, Y_{M'}^*) = (\sum_{j\in E_{SC-SP}}\lambda_j X_j^*, \sum_{j\in E_{SC-SP}}\lambda_j Z_j^*, \sum_{j\in E_{SC-SP}}\lambda_j Y_j^*),$$
$$\sum_{j\in E_{SC-SP}}\lambda_j = 1\ ,\ \lambda_j \geq 0,\ j \in E_{SC-SP}.$$

Then:

$$(X_{M'}^*, Z_{M'}^*, Y_{M'}^*) = (\sum_{j\in E_{SC-SP}}\lambda_j(\frac{1}{\alpha}X_j^*), \sum_{j\in E_{SC-SP}}\lambda_j Z_j^*, \sum_{j\in E_{SC-SP}}\lambda_j Y_j^*),$$
$$\sum_{j\in E_{SC-SP}}\lambda_j = 1,\ \lambda_j \geq 0,\ j \in E_{SC-SP}.$$
$$(X_{M'}^*, Z_{M'}^*, Y_{M'}^*) = (\sum_{j\in E'_{SC-SP}}\lambda_j(X_j^{*'}), \sum_{j\in E'_{SC-SP}}\lambda_j Z_j^{*'}, \sum_{j\in E'_{SC-SP}}\lambda_j Y_j^{*'})$$
$$\sum_{j\in E'_{SC-SP}}\lambda_j = 1\ \lambda_j \geq 0,\ j \in E_{SC-SP}.$$

Then $\left(X_{M'}^*, Z_{M'}^*, Y_{M'}^*\right)$ is represented as a convex combination of extreme efficient points in T'_{SC-SP} . It is in contradiction with our assumption. So the prove is completed and $\left(X_{M'}^*, Z_{M'}^*, Y_{M'}^*\right) \in E'_{SC-SP}$.

Theorem 4.*There is a one-to-one correspondence between T_{SC-SP} , T'_{SC-SP} frontier points.*

*Proof.*Let M' with coordinate $\left(X'^*_{M'}, Z'^*_{M'}, Y'^*_{M'}\right)$ be an arbitrary point on the T'_{SC-SP} frontier. Each point on the T'_{SC-SP} frontier is extreme efficient or a non-extreme efficient point.As we know the set of extreme efficient supply chains in T_{SC-SP} is E_{SC-SP} and the set of extreme efficient supply chains in T'_{SC-SP} is E'_{SC-SP} .

1.if M' with coordinate $\left(X_{M'}^*, Z_{M'}^*, Y_{M'}^*\right)$ be an arbitrary extreme efficient point on T'_{SC-SP} By lemma. 1 $M' \in E'_{SC-SP}$ .There is a one- to-one correspondence between E_{SC-SP} and E'_{SC-SP} . Because the set E'_{SC-SP} is generated from E_{SC-SP} . So there is a one- to-one correspondence between extreme points on T_{SC-SP} and T'_{SC-SP} .
2.if M' be an arbitrary non- extreme efficient point on the T'_{SC-SP} frontier, it is shown that there is a one-to-one correspondence between non-extreme efficient points on the T_{SC-SP} and T'_{SC-SP} .There are t extreme efficient points on T'_{SC-SP} so that:

$$(X_{M'}^*, Z_{M'}^*, Y_{M'}^*) = \sum_{j=1}^{t}\mu_j\left(X_j^{*'}, Z_j^{*'}, Y_j^{*'}\right),$$
$$\sum_{j=1}^{t}\mu_j = 1,\ \mu_j \geq 0,\ j = 1,...,t$$

Then:

$$(X_{M'}^*, Z_{M'}^*, Y_{M'}^*) = (\frac{1}{\alpha}\sum_{j=1}^{t}\mu_j X_j^*, \sum_{j=1}^{t}\mu_j Z_j^*, \sum_{j=1}^{t}\mu_j Y_j^*),$$
$$\sum_{j=1}^{t}\mu_j = 1, \quad \mu_j \geq 0, j = 1,...,t.$$
$$(X_M^*, Z_M^*, Y_M^*) = (\sum_{j=1}^{t}\mu_j X_j^*, \sum_{j=1}^{t}\mu_j Z_j^*, \sum_{j=1}^{t}\mu_j Y_j^*),$$
$$\sum_{j=1}^{t}\mu_j = 1, \quad \mu_j \geq 0, j = 1,...,t.$$

We prove that M to be on the T_{SC-SP} frontier. Assume contradiction that M is not on frontier .it is evaluated by the following model:

$$\begin{array}{ll} \min\ \theta_M & \\ s.t.\ \sum_{j\in E_{SC-SP}} \lambda_j X_j^* \le \theta_M X_M & \\ \quad \sum_{j\in E_{SC-SP}} \lambda_j Y_j^* \ge Y_M & \\ \quad (X_j^*, Y_j^*) \in T_{SC-SP} & \\ \quad \lambda_j \ge 0, \quad j \in E_{SC-SP} & \end{array} \tag{11}$$

Assume that (λ^*, θ_M^*) is the optimal solution for the previous problem. Respecting to contradiction assumption, it can be obviously seen that $\theta_M^* \le 1$. Then, the first constraint is multiplied to $\frac{1}{\alpha}$, so:

$$\begin{cases} \sum_{j\in E'_{SC-SP}} \lambda_j^* X_j^{*'} \le \theta_M^* X_{M'} \\ \sum_{j\in E'_{SC-SP}} \lambda_j^* Y_j^{*'} \ge Y_{M'} = Y_M \\ (X_j^{*'}, Y_j^{*'}) \in T_{SC-SP} \\ \lambda_j^* \ge 0, \quad j \in E'_{SC-SP} \end{cases}$$

The above model has a feasible solution $(\lambda^*,\ \theta_M^* \le 1)$ for $M' \in T'_{SC-SP}$ and this is in contradiction with M' that is a non-extreme efficient unit on the T'_{SC-SP} frontier and it is completed our proof.

Theorem 5. *The efficiency score of each point on the T'_{SC-SP} frontier is α in T_{SC-SP} .*

Proof. Let M' with coordinate $(X'^*_{M'}, Z'^*_{M'}, Y'^*_{M'})$ be an arbitrary point on the T'_{SC-SP} frontier. Each point on the T'_{SC-SP} frontier is extreme efficient or a non-extreme efficient point. The following model is employed to evaluate the point M' in T_{SC-SP}:

$$\begin{array}{ll} \min\ \theta_{M'} & \\ s.t.\ \sum_{j\in E_{SC-SP}} \lambda_j X_j^* \le \theta_{M'} (\frac{1}{\alpha} X_M) & \\ \quad \sum_{j\in E_{SC-SP}} \lambda_j Y_j^* \ge Y_{M'} = Y_M & \\ \quad (X_j^*, Y_j^*) \in T_{SC-SP} & \\ \quad \lambda_j \ge 0, \quad j \in E_{SC-SP} & \end{array} \tag{12}$$

The model has a feasible solution $(\theta_{M'} = \alpha, \lambda_M = 1, \lambda_j = 0, (j \in E_{SC-SP}, j \ne M))$

Hence the optimal $\theta_{M'}$, denoted by $\theta_{M'}^*$, is not greater than α . It will be represented $(\theta_{M'}^* \not< \alpha)$. In contradiction, assume that $(\theta_{M'}^* < \alpha)$.Therefore $\theta_{M'}^* = \alpha - \varepsilon$ for some $\varepsilon > 0$. By applying Model (12):

$$\sum_{j\in E_{SC-SP}} \lambda_j X_j^* \le \theta_{M'}^* X_{M'} = \theta_{M'}^* \left(\frac{1}{\alpha} X_M\right) = \left(\frac{\alpha-\varepsilon}{\alpha}\right) X_M$$
$$= \left(1 - \frac{\varepsilon}{\alpha}\right) X_M^*$$
$$\sum_{j\in E_{SC-SP}} \lambda_j Y_j^* \ge Y_{M'} = Y_M$$
$$(X_j^*, Y_j^*) \in T_{SC-SP} \qquad \lambda_j \ge 0,\ \ j \in E_{SC-SP}$$

A feasible solution for this model is:

$$\left(\theta_M = (1 - \frac{\varepsilon}{\alpha}) < 1,\ \lambda_M = 1,\ \lambda_j = 0,\ (j \in Esc-sp, j \ne M)\right)$$

It means that DMU_M is inefficient and this is in contradiction with the assumption. Hence $\theta_{M'}^* = \alpha$ and it is completed the proof.

Case 2. Suppose that the evaluated supply chain is DEA inefficient. In the second case, it has been focused mainly around that only inefficient manufacturers have an improvement. In this case we cannot use the sub-perfect CRS DEA model. Because the sub-perfect CRS DEA model deem the input changes of manufacturers or effective of intermediate units.It means that the sub-perfect CRS DEA model evaluated the changes of the overall input vector and overall output vector of supply chain and it does not consider the changes of intermediate unites. In this model we have one input vector for manufacturer that it is output vector for supplier. But we want to change just the input vector of manufacturer. So we should have two input vectors for manufacturer that one of them is the output vector for supplier and the other is new input vector that is injected to manufacturer and they are called Z_1 and Z_2 respectively. The case is shown in Figure 2.

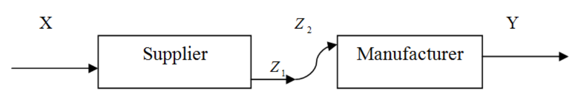

Fig. 2: There are two input vectors for manufacturer.

On the other hand, we know supply chain$_o$ is inefficient and it is inefficient in manufacture$_o$. By (1) the Production Possibility Set (PPS) is obtained and θ_{MC-Sp} is gained by the following LP problem.

$$\begin{array}{ll} \min\ \theta_{MC-SP} & \\ \text{s.t.}\ \sum_{j=1}^{N} \lambda_j^S X_j \le \theta_{MC-SP} X_o & \\ \quad \sum_{j=1}^{N} \lambda_j^S Z_{1j} \ge Z_{1o} & \\ \quad \sum_{j=1}^{N} \lambda_j^M Z_{2j} \le Z_{2o} & \\ \quad \sum_{j=0}^{N} \lambda_j^M Y_j \ge Y_o & \\ \quad \lambda_j^S \ge 0, \quad \lambda_j^M \ge 0, \qquad j = 1,2,\ldots,N & \end{array} \tag{13}$$

(AP)model is employed to find all extreme efficient supply chains [24]. Let the set of extreme efficient supply chains in T_{MC-Sp} be E_{MC-SP} . Having determined

E_{MC-SP} , E'_{MC-SP} is defined as follows:

$$E'_{MC-SP} = \left\{ \left(X'_j,\, Z'_{1j}, Z'_{2j}, Y'_j\right) \mid \left(X'_j,\, Z'_{1j}, Z'_{2j}, Y'_j\right) = \left(X_j, Z_{1j}, \frac{1}{\beta} Z_{2j}, Y_j\right), j \in E_{MC-SP} \right\}$$

We introduce the new production possibility set T'_{MC-SP} :

$$T'_{MC-SP} = \left\{ \begin{array}{c} (X',Y') \mid \sum_{j \in E_{MC-SP}} \lambda_j^S X_j \leq X', \\ \sum_{j \in E_{MC-SP}} \lambda_j^S Z_{1j} \geq Z', \\ \frac{1}{\beta} \sum_{j \in E_{MC-SP}} \lambda_j^M Z_{2j} \leq 1Z', \\ \sum_{j \in E_{MC-SP}} \lambda_j^M Y_j \geq 2Y', \\ \lambda_j^S,\ \lambda_j^M \geq o,\ j \in E_{MC-SP} \end{array} \right\}$$

By the above new Production Possibility Set, θ'_{MC-SP} is obtained by the following LP problem:

$$\begin{array}{ll} \min & \theta'_{MC-SP} \\ \text{s.t.} & \sum_{j=1}^{N} \lambda_j^S X_j \leq \theta'_{MC-SP} X_o \\ & \sum_{j=1}^{N} \lambda_j^S Z_{1j} \geq Z_{1o} \\ & \sum_{j=1}^{N} \lambda_j^M Z_{2j} \leq \frac{1}{\beta} Z_{2o} \\ & \sum_{j=1}^{N} \lambda_j^M Y_j \geq Y_o \\ & \lambda_j^S \geq 0, \quad \lambda_j^M \geq 0, \qquad j = 1, \ldots, N \end{array} \tag{14}$$

Like the case 1, there is the new frontier but the efficiency score on it does not exactly determined. What we sure are that the frontier is close to the main frontier and the decision maker gives chance to those inefficient manufactures to improve themselves. They are DEA inefficient and they cannot obtain efficiency score 1 but they can gain the constant which is closer to 1 and defined by the manager and satisfy him.They have absolutely an improvement but the value of the improvement is not exactly determined. What we do is that the inefficient supply chains which are inefficient in manufacturers performance are appraised respected to a new frontier with efficiency score which is a fixed number (and defined by the manager). SC_o is DEA inefficient and it is inefficient in manufacture performance. The new frontier T'_{MC-SP} is obtained and by choosing different strategies (Input Oriented, Output Oriented, Combination Oriented) , the SC_o can be moved toward the T'_{MC-SP} frontier and can be improved. In different ways the decision maker can give chance to DEA inefficient supply chains to appraise respected to the new frontier and it can has an improvement.

Case 3. Suppose that the evaluated supply chain is DEA inefficient. In this case, it has been focused mainly around that both of them are DEA inefficient at the same time. This means that supplier and manufacture are DEA inefficient and we want to improve them at the same time. They are evaluated with new frontier efficiency which is satisfied the decision maker and because they are appraised to a new frontier with efficiency score of less than one they absolutely have an improvement too. It is not obtained the exact value of improvement but the overall efficiency is improved.When the improvement is compared with case 1 and 2 it is truism to say that an improvement is the best. ω is obtained by solving following model:

$$\begin{array}{ll} \omega = \min\ \theta_{SMC-SP} & \\ \quad \sum_{j=1}^{N} \lambda_j^S X_j \leq \theta_{SMC-SP} X_o, & \\ \quad \sum_{j=1}^{N} \lambda_j^S Z_{1j} \geq Z_{1o}, & \\ \quad \sum_{j=1}^{N} \lambda_j^M Z_{2j} \leq Z_{2o}, & \\ \quad \sum_{j=1}^{N} \lambda_j^M Y_j \geq Y_o, & \\ \quad \lambda_j^S,\ \lambda_j^M \geq o, & j = 1,2,...,N \end{array} \tag{15}$$

(AP) model is employed to find all extreme efficient supply chains [24]. Let the set of extreme efficient supply chain in T_{SMC-Sp} be E_{SMC-SP}. Having determined E_{SMC-SP}, E'_{SMC-SP} is defined as follows:

$$E'_{SMC-SP} = \left\{ \left(X'_j,\, Z'_{1j}, Z'_{2j}, Y'_j\right) \mid \left(X'_j,\, Z'_{1j}, Z'_{2j}, Y'_j\right) = \left(\frac{1}{\alpha} X_j, Z_{1j}, \frac{1}{\beta} Z_{2j}, Y_j\right), j \in E_{SMC-SP} \right\}$$

We introduce the new production possibility set T'_{SMC-SP} :

$$T'_{SMC-SP} = \left\{ \begin{array}{c} (X',Y') \mid \frac{1}{\alpha} \sum_{j \in E_{SMC-SP}} \lambda_j^s X_j \leq X', \\ \sum_{j \in E_{SMC-SP}} \lambda_j^s Z_{1j} \geq Z'_1, \\ \frac{1}{\beta} \sum_{j \in E_{SMC-SP}} \lambda_j^M Z_{2j} \leq Z'_2, \\ \sum_{j \in E_{SMC-SP}} \lambda_j^M Y_j \geq Y', \\ \lambda_j^S,\ \lambda_j^M \geq o,\ j \in E_{SMC-SP} \end{array} \right\}$$

By the above new Production Possibility Set, θ'_{SMC-SP} is obtained by the following LP problem:

$$\begin{array}{ll} \min & \theta'_{SMC-SP} \\ \text{s.t.} & \sum_{j=1}^{N} \lambda_j^S X_j \leq \theta'_{SMC-SP} (\frac{1}{\alpha} X_o) \\ & \sum_{j=1}^{N} \lambda_j^S Z_{1j} \geq Z_{1o} \\ & \sum_{j=1}^{N} \lambda_j^M Z_{2j} \leq \frac{1}{\beta} Z_{2o} \\ & \sum_{j=1}^{N} \lambda_j^M Y_j \geq Y_o \\ & \lambda_j^S \geq 0, \quad \lambda_j^M \geq 0, \qquad j = 1, \ldots, N \end{array} \tag{16}$$

4 Numerical example

In this section, proposed approach is applied to appraise the performance of 17 bank branches of China

Table 1: Data of 17 bank branches (actual supply chains)

No.	Bank	FA	EM	EX	CR	IL	LO	PR
	Branch	(10^8)	(10^3)	(10^8)	(10^8)	(10^8)	(10^8)	(10^8)
A1	Hefei	1.0168	1.221	1.2215	166.9755	8.3098	122.1954	3.7569
A2	Bengbu	0.5915	0.611	0.4758	50.1164	1.7634	19.4829	0.6600
A3	Huainan	0.7237	0.645	0.6061	48.2831	3.4098	34.4120	0.7713
A4	Huaibei	0.5150	0.486	0.3763	35.0704	2.3480	15.2804	0.3203
A5	Maanshan	0.4775	0.526	0.3848	49.9174	5.4613	34.9897	0.8430
A6	Tongling	0.6125	0.407	0.3407	23.1052	1.2413	32.5778	0.4616
A7	Wuhu	0.7911	0.708	0.4407	39.4590	1.1485	30.2331	0.6732
A8	Anqing	1.2363	0.713	0.5547	37.4954	4.0825	20.6013	0.4864
A9	Huangshan	0.4460	0.443	0.3419	20.9846	0.6897	8.6332	0.1288
A10	Fuyang	1.2481	0.638	0.4574	45.0508	1.7237	9.2354	0.3019
A11	Suzhou	0.7050	0.575	0.4036	38.1625	2.2492	12.0171	0.3138
A12	Chuzhou	0.6446	0.432	0.4012	30.1676	2.3354	13.8130	0.3772
A13	Luan	0.7239	0.510	0.3709	26.5391	1.3416	5.0961	0.1453
A14	Xuancheng	0.5538	0.442	0.3555	22.2093	0.9886	13.6085	0.3614
A15	Chizhou	0.3363	0.322	0.2334	16.1235	0.4889	5.9803	0.0928
A16	Chaohu	0.6678	0.423	0.3471	22.1848	1.1767	9.2348	0.2002
A17	Bozhou	0.3418	0.256	0.1594	13.4364	0.4064	2.5326	0.0057

Construction Bank in Anhui province, P.R. China. China Construction Bank (CCB) is one of the largest state-owned commercial banks of China. There are 31 provincial branches of CCB in mainland China, including Anhui provincial branch. An- hui province consists of 17 cities: Hefei, Bengbu, Huainan, Huaibei, Maanshan, Tongling, Wuhu, Anqing, Huangshan, Fuyang, Suzhou, Chuzhou, Luan, Xuancheng, Chizhou, Chaohu and Bozhouas shown in Table1.The data are from Annual Report (2004) of China Construction Bank in Anhui Province [1].

In 2011, This example is used byYang et al. [11].

In the first stage, some inputs such as Fixed Assets (FA), Employee (EM), Expenditure (EX) are consumed to generate outputs such as Credit (CR) and Interbank Loan (IL). In the second stage, the Credit (CR) and Interbank Loan (IL) are used to generate two outputs: Loan (LO) and Profit (PR). In this application, 17 branches of China Construction Bank in Anhui Province are included in the evaluation. Tablereports the CCR efficiency scores of the two stages or subsystems [11].DMUs A1 and A5 are efficient in the first stage (subsystem) and DMUs A1, A6,A7 are efficient in the second stage. The table 2. also reports the overall CCR efficiency. Only DMU A1 is efficient.

Case 1. Suppose that the supply chain is DEA inefficient in first stage.It means that it is inefficient in suppliers.Assume $\alpha = 0.800$. By using AP model, A_1 and $A_5$are extreme efficient in T'_{CS}. The set E'_{CS} is defined as $E'_{CS} = \{A_1, A_5\}$. Then, A_1 is obtained as extreme point in T_{SC-SP}. The first column of table 3 reports the new efficiency for all suppliers. It shows we have improvement for all suppliers. The third column of table 3 reports overall efficiency and it can be compared with the last column of table 2.

Case 2. In this case. asuume $\beta = 0.850$ and DMUs, A1 and A2, are extreme efficient in the second stage and we have an improvement in manufactures. Table 4 show results.

Case 3. In this case asuume $\alpha = 0.8000$, $\beta = 0.85$. Table 5 shows results.

Table 2: Subsystems efficiency & overall efficiency values

No.	Bank Branch	θ^*_{sj}	θ^*_{Mj}	θ^{CCR}_i	θ^*_i
A1	Hefei	1.0000	1.0000	1.0000	1.0000
A2	Bengbu	0.7705	0.7057	0.4510	0.4510
A3	Huainan	0.6318	0.7385	0.5676	0.4320
A4	Huaibei	0.6923	0.4264	0.4059	0.2911
A5	Maanshan	1.0000	0.7729	0.9090	0.7342
A6	Tongling	0.4979	1.0000	0.9558	0.4979
A7	Wuhu	0.6550	1.0000	0.6858	0.5177
A8	Anqing	0.5526	0.5953	0.3713	0.2947
A9	Huangshan	0.4490	0.4759	0.2524	0.1741
A10	Fuyang	0.7205	0.3442	0.2146	0.2146
A11	Suzhou	0.6974	0.3725	0.2976	0.2578
A12	Chuzhou	0.6150	0.5633	0.3442	0.3100
A13	Luan	0.5238	0.2454	0.1373	0.1285
A14	Xuancheng	0.4570	0.7794	0.3827	0.3363
A15	Chizhou	0.5054	0.4648	0.2561	0.1621
A16	Chaohu	0.4689	0.4192	0.2660	0.1963
A17	Bozhou	0.6166	0.2368	0.1588	0.0827

Table 3: Subsystems efficiency & overall efficiency values ($\alpha = 0.8000$)

No.	Bank Branch	$\theta^*_{s'j}$	θ^*_{Mj}	θ'^*_i
A1	Hefei	1.0000	1.0000	1.0000
A2	Bengbu	0.9730	0.7057	0.5600
A3	Huainan	0.7940	0.7385	0.5400
A4	Huaibei	0.8750	0.4264	0.3700
A5	Maanshan	1.0000	0.7729	0.7200
A6	Tongling	0.6270	1.0000	0.6300
A7	Wuhu	0.8280	1.0000	0.6500
A8	Anqing	0.6990	0.5953	0.3700
A9	Huangshan	0.5610	0.4759	0.2200
A10	Fuyang	0.9011	0.3442	0.2700
A11	Suzhou	0.8812	0.3725	0.3200
A12	Chuzhou	0.7713	0.5633	0.3900
A13	Luan	0.6614	0.2454	0.1600
A14	Xuancheng	0.5715	0.7794	0.4200
A15	Chizhou	0.6316	0.4648	0.2000
A16	Chaohu	0.5917	0.4192	0.2500
A17	Bozhou	0.7700	0.2368	0.1000

Conclusion

It is a truism to say that supply chains management needs to innovate to survive. Unless they are prepared to change what they offer (product/service) and the ways in which they create and deliver that offering. Improving quality, speed, and other performance dimensions within a supply Chain is increasingly seen as a shared activity involving the whole chain or network. Some times that innovating process is expensive and it does not satisfy supply chain management's opinion or other enterprises. So if there exists a way which improved the performance of members of supply chain with lower cost or charge, it will

Table 4: Data of 17 bank branches ($\beta = 0.85$)

No.	Bank Branch	θ_i^*
A1	Hefei	1.0000
A2	Bengbu	1.0000
A3	Huainan	1.0000
A4	Huaibei	1.0000
A5	Maanshan	1.0000
A6	Tongling	1.0000
A7	Wuhu	1.0000
A8	Anqing	0.5600
A9	Huangshan	0.9200
A10	Fuyang	1.0000
A11	Suzhou	0.9600
A12	Chuzhou	0.8500
A13	Luan	0.7600
A14	Xuancheng	0.8100
A15	Chizhou	0.9500
A16	Chaohu	0.7200
A17	Bozhou	0.9300

Table 5: Data of 17 bank branches ($\alpha = 0.8000$, $\beta = 0.85$)

No.	Bank Branch	θ_i^*
A1	Hefei	1.0000
A2	Bengbu	1.0000
A3	Huainan	1.0000
A4	Huaibei	1.0000
A5	Maanshan	1.0000
A6	Tongling	1.0000
A7	Wuhu	1.0000
A8	Anqing	0.7000
A9	Huangshan	0.9200
A10	Fuyang	1.0000
A11	Suzhou	1.0000
A12	Chuzhou	0.9300
A13	Luan	0.7600
A14	Xuancheng	0.8100
A15	Chizhou	0.9500
A16	Chaohu	0.7200
A17	Bozhou	0.9300

be better and this way will improve the overall performance of supply chains too. In this paper, a new sub-perfect supply chain production possibility set is obtained with efficiency score of α for inefficient supplier. This model can apply for those supply chains which are inefficient in manufacture performance or supplier and manufacture performance at the same time. The inefficient supply chains are appraised respected to a new frontier with efficiency score which is a fixed number (and defined by the manager).

This model can be applied for other DEA model such as Variant returns to scale and it will be for further researches.

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